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The Excitation of Ion-acoustic Waves by a Magnetic Pump in Cylindrical Geometry*

N. F. Cramer[A] *and I. J. Donnelly*[B]

[A] School of Physics, University of Sydney, Sydney, N.S.W. 2006.
[B] Australian Atomic Energy Commission Research Laboratories, Private Mail Bag, Sutherland, N.S.W. 2232.

Abstract

The modulational, parametric and purely growing mode instabilities of a magnetic pump with a finite radial wavenumber in cylindrical geometry are investigated. The modulational instability is compared with the instability of a parallel propagating pump wave, and the growth rates are found to be similar. The growth rate of a pair of ion-acoustic waves is shown to be zero for a collisionless plasma, in agreement with less general results found previously. The growth rate in the collisional case is found to be nonzero. The purely growing mode growth rate is calculated for excitation of ion-acoustic plus torsional Alfvén waves.

1. Introduction

In the field of plasma heating, ion-acoustic waves are expected to play a large role because of their low phase speed, leading to efficient coupling of their energy to the ions of the plasma. Some experimental evidence of the role of ion-acoustic waves parametrically excited by a magnetic pump has been obtained by Demirkhanov *et al.* (1977) who attributed the observed ion heating to either ion Landau damping (collisionless case) or ion viscous damping (collisional case) of ion-acoustic waves.

The role of ion-acoustic waves (or more generally, in the magnetized plasma, slow magnetosonic waves) in parametric instabilities of magnetic pumps has been investigated theoretically in a number of papers. Lashmore-Davies and Ong (1975) found that, using a slab geometry, a pair of ion-acoustic waves is not parametrically excited by a magnetic pump. Chhabra *et al.* (1981) subsequently derived the same result with an approximate treatment of cylindrical geometry, retaining the first few terms in a long radial wavelength expansion of the pump fields. These results were all for the case of low β. In fact Elfimov and Nekrasov (1973) had already shown this result in cylindrical geometry; they calculated the growth rate to be proportional to β, so that it would therefore vanish in the limit considered in the above two papers. We show here a simple proof of this result for arbitrary radial wavelength pump and excited waves, and also show that the result is restricted to a collisionless plasma; for a collisional plasma a nonzero growth rate is obtained.

The decay instability of the magnetic pump to an ion-acoustic plus an Alfvén wave has been considered by Elfimov and Nekrasov (1973) (cylindrical geometry)

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and Cramer (1977) (slab geometry), who found a growth rate proportional to $\beta^{-\frac{1}{2}}$. Clearly, the analysis must be faulty in the limit $\beta \to 0$; in fact, as we show here, the decay instability is replaced by a purely growing mode instability with finite growth rate.

A further role of the ion-acoustic mode in instabilities of a magnetic pump arises in the modulational instability, in which forced sideband Alfvén waves plus a density perturbation are excited. Such an instability may be important in limiting the amplitude of Alfvén waves in the solar wind. Previous work (Lashmore-Davies 1976; Goldstein 1978; Derby 1978) has only considered a pump wave propagating parallel to the ambient magnetic field. Here we use a formalism set up to investigate general parametric excitation of magnetohydrodynamic (MHD) waves by an axisymmetric pump with a finite radial wavelength in cylindrical geometry, in order to consider the modulational instability of the pump wave travelling perpendicularly to the magnetic field, and to show the connection of this instability to the abovementioned parametric instabilities.

2. Basic Equations

Rather than use a full kinetic theory treatment, we model the low-β plasma with the MHD equations describing an isothermal collisionless plasma. (We later consider a collisional plasma also.) The wave frequencies are assumed to be much less than the ion-cyclotron frequency. The basic equations are

$$\rho(\partial \boldsymbol{v}/\partial t + \boldsymbol{v}\,.\nabla\boldsymbol{v}) = \mu_0^{-1}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} - C_S^2 \nabla\rho\,, \tag{1a}$$

$$\partial\boldsymbol{B}/\partial t = \nabla \times (\boldsymbol{v}\times\boldsymbol{B}), \qquad \partial\rho/\partial t + \nabla\,.(\rho\boldsymbol{v}) = 0\,, \tag{1b, c}$$

and the pump fields are (Cramer and Sy 1979)

$$\boldsymbol{B}^{(0)} = \boldsymbol{B}_0 + \varepsilon\bar{\boldsymbol{b}} = B_0\{1 + \varepsilon\,\mathrm{J}_0(k_0 r)\cos\omega_0 t\}\hat{\boldsymbol{z}}\,, \tag{2a}$$

$$\boldsymbol{v}^{(0)} = \varepsilon\bar{\boldsymbol{v}} = \varepsilon(C_S^2 + V_A^2)^{\frac{1}{2}}\mathrm{J}_1(k_0 r)\sin\omega_0 t\,\hat{\boldsymbol{r}}\,, \tag{2b}$$

$$\rho^{(0)} = \rho_0 + \varepsilon\bar{\rho} = \rho_0\{1 + \varepsilon\,\mathrm{J}_0(k_0 r)\cos\omega_0 t\}\,, \tag{2c}$$

$$\boldsymbol{j}^{(0)} = \varepsilon\bar{\boldsymbol{j}} = (\varepsilon B_0 k_0/\mu_0)\mathrm{J}_1(k_0 r)\cos\omega_0 t\,\hat{\boldsymbol{\theta}}\,, \tag{2d}$$

where the cylindrical coordinates (r, θ, z) are used and ε is a small parameter. Here V_A is the Alfvén speed and C_S is the sound speed, and we define $\beta = C_S^2/V_A^2$. The frequency and wavenumber of the pump are related by

$$\omega_0 = k_0(C_S^2 + V_A^2)^{\frac{1}{2}}\,.$$

The excited wave equations are then (Cramer and Sy 1979)

$$\rho_0\,\partial\boldsymbol{v}/\partial t - \mu_0^{-1}(\nabla\times\boldsymbol{b})\times\boldsymbol{B}_0 + C_S^2\nabla\rho = \varepsilon\boldsymbol{F}\,, \tag{3}$$

$$\partial\boldsymbol{b}/\partial t - \nabla\times(\boldsymbol{v}\times\boldsymbol{B}_0) = \varepsilon\nabla\times\boldsymbol{G}\,, \tag{4}$$

$$\partial\rho/\partial t + \nabla\,.(\rho_0\boldsymbol{v}) = \varepsilon H\,, \tag{5}$$

where

$$\begin{aligned} \boldsymbol{F} = &-\bar{\rho}(\partial\boldsymbol{v}/\partial t) - \rho(\partial\bar{\boldsymbol{v}}/\partial t) - \rho_0\,\boldsymbol{v}\,.\nabla\bar{\boldsymbol{v}} - \rho_0\,\bar{\boldsymbol{v}}\,.\nabla\boldsymbol{v} \\ &+ \mu_0^{-1}(\nabla\times\boldsymbol{b})\times\bar{\boldsymbol{b}} + \mu_0^{-1}(\nabla\times\bar{\boldsymbol{b}})\times\boldsymbol{b}\,, \end{aligned}$$

$$\boldsymbol{G} = \bar{\boldsymbol{v}}\times\boldsymbol{b} + \boldsymbol{v}\times\bar{\boldsymbol{b}}\,, \qquad H = -\nabla\,.(\bar{\rho}\boldsymbol{v} + \rho\bar{\boldsymbol{v}})\,.$$

In the low-β case, the slow magnetosonic wave becomes the ion-acoustic wave with v_z the dominant velocity component. Thus the equations required to describe the excited ion-acoustic wave are equation (5) and the z component of equation (3), which may be combined to yield

$$\left(\frac{\partial^2}{\partial t^2} - C_S^2 \frac{\partial^2}{\partial z^2}\right) v_z = \varepsilon\left(\frac{1}{\rho_0}\frac{\partial}{\partial t}(\hat{z}.\boldsymbol{F}) - \frac{C_S^2}{\rho_0}\frac{\partial H}{\partial z}\right). \tag{6}$$

A natural variable for describing the shear Alfvén wave is j_z (cf. Woods 1962; Cramer and Sy 1979), so that the equation describing the excited Alfvén wave is gained by taking the curl of equation (4) and eliminating $\boldsymbol{v}$ by means of (3):

$$\left(\frac{\partial^2}{\partial t^2} - V_A^2 \frac{\partial^2}{\partial z^2}\right) \mu_0 j_z = \varepsilon\left(\frac{\partial}{\partial t}\hat{z}.\{\nabla \times (\nabla \times \boldsymbol{G})\} + \frac{B_0}{\rho_0}\frac{\partial}{\partial z}\{\hat{z}.(\nabla \times \boldsymbol{F})\}\right). \tag{7}$$

Since the pump wave providing the coupling has no spatial variation in the z direction, all the excited wave fields may be assumed to have the same wavenumber $k_z = k$. Then the z, θ and time variation of v_z is written as $\exp(\mathrm{i}kz + \mathrm{i}m\theta - \mathrm{i}\omega_1 t)$ and that of j_z as $\exp(\mathrm{i}kz + \mathrm{i}m\theta - \mathrm{i}\omega_2 t)$.

3. Excitation Processes

Excitation of Ion-acoustic plus Alfvén Waves

Since we are considering only the interaction between the Alfvén and ion-acoustic waves, and not the self-interactions of these waves, we need retain in the right-hand sides of equations (5) and (6) only those terms in Alfvén wave variables, and in equation (7) only those terms in ion-acoustic wave variables. We find then that

$$\boldsymbol{G} \sim 0, \quad \hat{z}.\boldsymbol{F} \sim -j_\theta b_r, \quad \hat{z}.(\nabla \times \boldsymbol{F}) \sim \mathrm{i}mr^{-1}\rho\,\partial\bar{v}_r/\partial t, \quad H \sim -(\partial\bar{p}/\partial r)v_r,$$

neglecting the self-interaction coupling terms. We are assuming that the compressional, or fast, Alfvén wave is not excited in the interaction, so that the natural variables (Woods 1962) belonging to the fast wave, namely b_z and $\nabla.\boldsymbol{v} - \partial v_z/\partial z$, remain very small. This assumption is justified by noting that the excited wave frequencies are taken to be lower than the cutoff frequency for the fast mode, so that excited fast wave oscillations will be far from resonant oscillations for the system. (However, in a cylindrical plasma surrounded by a narrow vacuum annulus the fast wave can exist at frequencies below the body wave cutoff as a surface wave; we do not consider this case here.)

By using the linear equations to express v_r and b_r on the RHS of equation (6) in terms of j_z, and ρ on the RHS of equation (7) in terms of v_z, the coupled wave equations become

$$\left(\frac{\partial^2}{\partial t^2} + V_A^2 k^2\right)\mu_0 j_z = -\frac{\varepsilon m B_0 k^2}{\omega_1 r} v_z \frac{\partial \bar{v}_r}{\partial t}, \tag{8}$$

$$\left(\frac{\partial^2}{\partial t^2} + C_S^2 k^2\right) v_z = -\frac{\varepsilon m \mu_0 j_z}{\rho_0 k_{cn}^2 r}\left(\omega_1 \bar{j}_\theta - \frac{\omega_2 C_S^2}{B_0}\frac{\partial \bar{p}}{\partial r}\right). \tag{9}$$

If the nonlinear interaction causes an appreciable shift in the acoustic wave frequency then care must be taken when using the normal mode relations for the acoustic wave to express ρ in terms of v_z in the RHS of equation (7). We have found *a posteriori* that, for the purely growing and modulational instabilities, both terms on the LHS of the z component of equation (3) are of the same magnitude as the interaction term on the RHS, whereas the RHS of equation (5) is of order $\beta^{\frac{1}{2}}$ smaller than the LHS terms. Therefore, the normal mode relation $\omega_1 \rho = \rho_0 k v_z$ remains valid whereas the relation $\rho_0 \omega_1 v_z = C_S^2 k\rho$ does not.

The second term in the RHS parentheses of equation (9) is of order $\beta^{\frac{1}{2}}$ compared with the first and so may be neglected. Also, on the RHS of equation (9) we have assumed that the radial variation of j_z may be expressed by a sum of radial modes, each of which has the Bessel function dependence $\mathrm{J}_m(k_{cn} r)$. Thus, we assume the following expansions for the non-dimensionalized current density and velocity:

$$j = \mu_0 j_z / B_0 k = \sum_n a_n(t)\, \mathrm{J}_m(k_{cn} r)\,,$$

$$v = v_z / V_A = \sum_{n'} b_{n'}(t)\, \mathrm{J}_m(k_{cn'} r)\,.$$

The analysis proceeds by considering an interaction between single radial modes (as in Cramer and Sy 1979) so that in equation (9) only the nth radial mode of j_z appears on the RHS. The values of k_{cn} and $k_{cn'}$ are determined by the boundary conditions: in the case of the metal-wall bounded plasma, E_θ and E_z vanish at the wall. We note that E_θ and E_z may be identically zero for some of the low-frequency waves considered here; however, in any realistic plasma, effects such as finite frequency, resistivity and viscosity cause these electric fields to be nonzero.

Now for the nth radial mode of the Alfvén wave E_θ is proportional to $m\,\mathrm{J}_m(k_{cn} r)/k_{cn} r$, so that for $m \neq 0$ the k_{cn} are determined by the zeros of $\mathrm{J}_m(k_{cn} R)$, where R is the wall radius. Also, if v_z goes to zero in the viscous boundary layer at the wall, the set of $k_{cn'}$ for the ion-acoustic wave is the same as the set of k_{cn}.

With the explicit forms of the pump fields, equations (8) and (9) take the form

$$\left(\frac{\partial^2}{\partial t^2} + V_A^2 k^2\right) j = -\varepsilon m \frac{\omega_0}{\omega_1} k V_A^2 \cos\omega_0 t \frac{\mathrm{J}_1(k_0 r)}{r} v\,, \tag{10}$$

$$\left(\frac{\partial^2}{\partial t^2} + C_S^2 k^2\right) v = -\varepsilon m \omega_1 \frac{k_0 k V_A}{k_{cn}^2} \cos\omega_0 t \frac{\mathrm{J}_1(k_0 r)}{r} j\,. \tag{11}$$

We may note firstly that there is no wave coupling if $m = 0$, i.e. the excited waves must be non-axisymmetric. Individual radial modes are now treated by multiplying equations (10) and (11) by $r\,\mathrm{J}_m(k_{cl} r)$ and integrating over r:

$$\left(\frac{\partial^2}{\partial t^2} + V_A^2 k^2\right) a_l = -\varepsilon m(\omega_0/\omega_1) k_{cl} k V_A^2 \cos\omega_0 t \sum_n C_{ln} b_n\,, \tag{12}$$

$$\left(\frac{\partial^2}{\partial t^2} + C_S^2 k^2\right) b_l = -\varepsilon m \omega_1 k_{cl} k_0 k V_A \cos\omega_0 t \sum_n (C_{ln}/k_{cn}^2) a_n\,, \tag{13}$$

where

$$C_{ln} = \int_0^R \mathrm{J}_1(k_0 r)\, \mathrm{J}_m(k_{cl} r)\, \mathrm{J}_m(k_{cn} r)\, \mathrm{d}r \Big/ k_{cl} \int_0^R r\, \mathrm{J}_m^2(k_{cl} r)\, \mathrm{d}r\,.$$

Equations (12) and (13) constitute a set of coupled ordinary differential equations for the a_l and b_l, with the sums over n resulting from the mixing of radial modes due to the radial variation of the pump. We shall simplify matters by considering only the coupling of the first, longest wavelength, radial modes, i.e. we retain only the first term in the sum over n, so that $n = l = 1$. Higher order (shorter wavelength) radial modes are commonly heavily damped, and so would be relatively harder to excite. We plot in Fig. 1 the factor C_{11} against k_0, for $m = 1$ and a fixed value of k_{c1} of $0 \cdot 383\ \mathrm{cm}^{-1}$, corresponding to a plasma radius of 10 cm. Note that the maximum value of C_{11} occurs at $k_0 \sim k_{c1}$, corresponding to a radial resonance of the pump and excited waves. The region $k_0 < k_{c1}$ is of most interest here, because this corresponds to the pump wave frequency being less than the fast wave cutoff frequency, thus ensuring that no fast waves are excited.

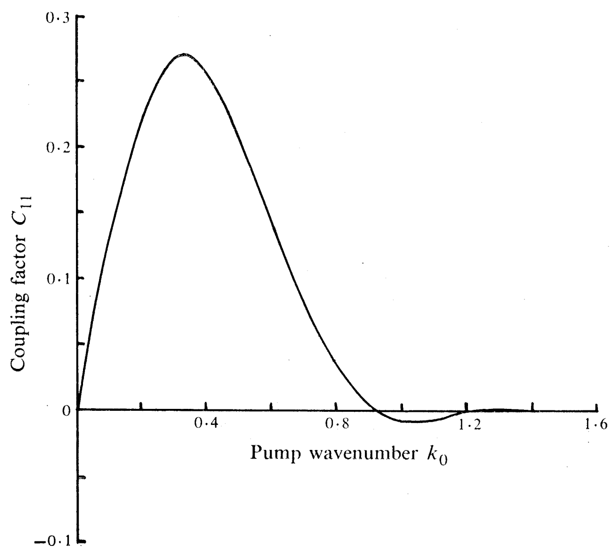

Fig. 1. Coupling factor C_{11} as a function of pump wavenumber k_0 (in cm^{-1}). The excited radial wavenumber k_{c1} is $0 \cdot 383\ \mathrm{cm}^{-1}$ and the azimuthal wavenumber m is 1.

Modulational Instability. Lashmore-Davies (1976) has shown that a finite amplitude Alfvén wave propagating along the magnetic field, as opposed to the propagation across the field considered here, can have a modulational instability, with the Alfvén wave breaking up into upper and lower sideband waves and a longitudinal density perturbation. His analysis was, however, for linear polarization of the large amplitude Alfvén wave, which is not an exact solution of the nonlinear MHD equations. Later work (Derby 1978; Goldstein 1978) extended the analysis to the circularly polarized finite amplitude Alfvén wave, which is an exact solution of the MHD equations. Such a pump leads to a simplified linear dispersion relation for the instability. The role of the instabilities of these waves in limiting the amplitude of Alfvén waves in the solar wind was investigated by Lashmore-Davies and Stenflo (1979, 1981), who also considered the circularly polarized pump wave which is not necessarily a natural mode of the plasma, i.e. a frequency and wavenumber imposed externally as in a laboratory plasma experiment.

In the present paper we consider the complementary case of the pump wave propagating perpendicularly to the magnetic field. Thus, we suppose that the pump wave beats with a density perturbation, varying as $\exp(\mathrm{i}kz + \mathrm{i}m\theta - \mathrm{i}\omega t)$, to produce forced waves at upper ($\omega_+ = \omega + \omega_0$) and lower ($\omega_- = \omega - \omega_0$) sideband frequencies, so that the variables $a_1(\omega_+)$, $a_1(\omega_-)$ and $b_1(\omega)$ describe the amplitudes of the three excited waves, assuming the first radial mode only for all three waves.

We proceed by using an analysis similar to that of Lashmore-Davies (1976). The amplitudes of the forced sideband Alfvén waves are, from equation (12),

$$a_1(\omega_+) = Lb_1/(\omega_+^2 - V_A^2 k^2), \qquad a_1(\omega_-) = Lb_1/(\omega_-^2 - V_A^2 k^2), \tag{14a, b}$$

where $L = \frac{1}{2}\varepsilon m(\omega_0/\omega)k_{c1} k V_A^2 C_{11}$. The equation for the density perturbation is obtained from equation (13), with contributions on the RHS from both $a_1(\omega_+)$ and $a_1(\omega_-)$:

$$(\omega^2 - C_S^2 k^2)b_1 = M\{a_1(\omega_+) + a_1(\omega_-)\}, \tag{15}$$

where $M = \frac{1}{2}\varepsilon m\omega(k_0 k/k_{c1})V_A C_{11}$.

Substituting the expression for the forced waves from equation (14) into (15) gives the dispersion relation for the density perturbations

$$\omega^2 - C_S^2 k^2 = LM\{(\omega_+^2 - V_A^2 k^2)^{-1} + (\omega_-^2 - V_A^2 k^2)^{-1}\}. \tag{16}$$

We now solve this relation observing that we are dealing with a low-β plasma. The density perturbations have $\omega \ll \omega_0$, since they are close to being ion-acoustic modes of the plasma with ω close to $C_S k$. The dispersion relation reduces to

$$\omega^2 = C_S^2 k^2 - \tfrac{1}{2}\omega_0^2\{k^2/(k^2 - k_0^2)\}m^2 C_{11}^2 \varepsilon^2. \tag{17}$$

This result is similar to that by Lashmore-Davies (1976) for the parallel propagating pump wave, except for the factor $m^2 C_{11}^2$. Thus, for $|k| > k_0$, there is instability if the amplitude of the pump wave exceeds the threshold given by

$$\varepsilon = 2^{\frac{1}{2}} C_S (k^2 - k_0^2)^{\frac{1}{2}}/m\omega_0 |C_{11}|, \tag{18}$$

provided that $m \neq 0$.

Parametric Instability. The dispersion relation (16) is obviously not valid in the vicinity of the singularities at $\omega_\pm^2 = V_A^2 k^2 = \omega_A^2$. The connection of this case to that of parametric resonance was demonstrated by Lashmore-Davies (1976). When $\omega_- \sim -\omega_A$ the forced sideband $a_1(\omega_-)$ is in resonance with a natural mode of the plasma, i.e. a shear Alfvén wave, and the density perturbation resonates with the ion-acoustic mode. The resonance condition $\omega_0 = \omega + \omega_A$ is approximately satisfied and the parametric decay instability results, as has been discussed by Elfimov and Nekrasov (1973) for cylindrical geometry and Cramer (1977) for slab geometry. The forced wave $a_1(\omega_+)$ is non-resonant, provided that we do not have $\omega \ll \omega_A$, and may be neglected. In both these papers the parametric growth rate was found to be proportional to $\beta^{-\frac{1}{2}}$, so that the analysis is obviously inapplicable in the limit of low β. In this case, with $\omega_+ \sim -\omega_- \sim \omega_A \sim \omega_0$, the wave $a_1(\omega_+)$ is no longer non-resonant and the full dispersion relation (16) must be solved. The analysis of such a case has been considered in detail by Nishikawa (1968). We define $\delta = \omega_0 - \omega_A$ and make the approximation

$$\omega_\pm^2 - \omega_A^2 \approx \pm 2\omega_A(\omega \pm \delta). \tag{19}$$

This leads to the following expression for the dispersion relation (16):

$$\omega^2 - C_S^2 k^2 = (K/2\omega_A)\{(\omega+\delta)^{-1} - (\omega-\delta)^{-1}\}, \tag{20}$$

where $K = LM$. Equation (20) has a purely growing mode solution (cf. Nishikawa 1968) in which the ion-acoustic wave response occurs at zero real frequency, and the Alfvén wave response occurs close to the pump frequency. The wave fields grow at the maximum rate

$$y = (K/2\omega_A)^{1/3} \approx \tfrac{1}{2}\omega_0(|m|\varepsilon|C_{11}|)^{2/3}. \tag{21}$$

This expression differs from the decay instability growth rate (Cramer 1977) $y_D \approx \frac{1}{2}\omega_0 |m| \varepsilon\beta^{-\frac{1}{2}}$ mainly in the factor $\beta^{-\frac{1}{2}}$, and would be the correct growth rate for instabilities occurring in low-β experiments such as those reported by Demirkhanov *et al.* (1977).

Excitation of Ion-acoustic Waves

In this decay instability, we need to consider only self-interaction terms on the RHS of equation (6). Otherwise, we retain the r dependence of the pump and excited wave fields. Thus, we have

$$F_z \approx -\bar{p}(\partial v_z/\partial t) - \rho_0 \bar{v}_r(\partial v_z/\partial r),$$

$$H \approx -\bar{p}(\partial v_z/\partial z) - \rho\nabla . \bar{v} - (\partial\rho/\partial r)\bar{v}_r.$$

By using the relation $kv_z = \omega_1 \rho/\rho_0$ on the RHS of (6) and the explicit forms of the pump fields, equation (6) becomes

$$\left(\frac{\partial^2}{\partial t^2} + C_S^2 k^2\right)v_z = \varepsilon\frac{\omega_1}{\omega_2}\left(\{\omega_2(\omega_2-\omega_1)\cos\omega_0 t + \mathrm{i}\,\omega_0\omega_1 \sin\omega_0 t\}\mathrm{J}_0(k_0 r)v_z \right.$$
$$\left. + \mathrm{i}\,\omega_0(\omega_1+\omega_2)\sin\omega_0 t\,\mathrm{J}_1(k_0 r)\frac{1}{k_0}\frac{\partial v_z}{\partial r}\right). \tag{22}$$

On Fourier transforming equation (22) in time (cf. Cramer and Sy 1979) and assuming exact parametric resonance, i.e. $\omega_1 = -\omega_2 = \frac{1}{2}\omega_0$, the RHS vanishes. Thus, the ion-acoustic wave pair is not excited by a magnetic pump for a general radial dependence of both pump and excited wave field: this result generalizes that of Chhabra *et al.* (1981) who considered only radially uniform excited wave fields and the small-r approximation to the pump fields.

The above result is valid for a collisionless plasma in which the effective ratio of specific heats γ is 1. However, in a collisional plasma, an extra coupling term will occur on the RHS of equation (22) which arises from the pressure gradient term of equation (1a). If we assume adiabaticity, the pressure perturbation in the pump wave is (cf. Cramer 1977)

$$P^{(0)} = P_0\{1 + \varepsilon\gamma\,\mathrm{J}_0(k_0 r)\cos\omega_0 t\},$$

where P_0 is the ambient pressure. The pressure perturbation in the excited wave is then

$$P = C_S^2 \rho\{1 + \varepsilon(\gamma-1)\mathrm{J}_0(k_0 r)\cos\omega_0 t\}.$$

Substitution into equations (1) then leads to in place of equation (22), assuming exact parametric resonance and neglecting all the non-contributing terms,

$$(\partial^2/\partial t^2 + C_S^2 k^2)v_z = \varepsilon C_S^2 k^2(\gamma - 1)\cos\omega_0 t\, \mathrm{J}_0(k_0 r)v_z\,. \tag{23}$$

The resultant growth rate of ion-acoustic waves in the collisional plasma is, well above threshold,

$$y = \varepsilon\tfrac{1}{8}\omega_0(\gamma - 1)|\,C_{11}\,|\,.$$

We note finally that the extra term proportional to $\gamma - 1$ in equation (3) does not contribute to the purely growing or modulational instabilities.

4. Conclusions

We have used a formalism developed by Elfimov and Nekrasov (1973) and Cramer and Sy (1979) for the calculation of the growth rates of parametric instabilities of a magnetic pump with nonzero radial wavelength in cylindrical geometry to compute the excitation of ion-acoustic waves by such a pump. Our results may be summarized as follows:

(1) The modulational instability of the pump wave propagating perpendicularly to the ambient magnetic field has a growth rate very similar to that of the parallel propagating pump considered by previous authors.

(2) In the low-β limit a purely growing mode instability occurs which modifies the divergent growth rate of the parametric decay into the Alfvén and ion-acoustic waves found by other authors.

(3) The result of zero growth rate for a pair of excited ion-acoustic waves found by previous authors has been generalized to the case of a more realistic radial variation of the pump fields and excited wave fields, and found to be not true in the case of a collisional plasma.

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