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Trivial Solution to the Domain Wall Problem

G. A. Christos

International Centre for Theoretical Physics,
P.O. Box 586, I-34100 Trieste, Italy.
Present address: Department of Theoretical Physics,
Research School of Physical Sciences,
Australian National University,
G.P.O. Box 4, Canberra, A.C.T. 2601.

Abstract

The domain wall problem only arises if one makes the unrealistic pure gauge assumption. The problem is avoided altogether by relaxing this assumption.

Recently much attention has been devoted to solving the domain wall problem (Sikivie 1982). Our contention is that the problem does not really exist.

The domain wall problem arises because it is commonly believed that the anomaly (via instantons) explicitly* breaks U(1) axial invariance down to the centre Z_L (where L is the number of light quark flavours). The subsequent spontaneous breaking† of this centre by the QCD condensate

$$\langle \sum_{i=1}^{L} \bar{q}_i q_i \rangle \neq 0 , \tag{1}$$

then leads to domain walls and consequently to a conflict with cosmology.

As should be well known by now, the anomaly does not explicitly break U(1) axial invariance. The U(1) axial charge generator is derived from the partially conserved (gauge dependent) U(1) axial current $J^L_{\mu 5,\mathrm{sym}}$:

$$Q_5^L \equiv \int \mathrm{d}\boldsymbol{x}\, J^L_{05,\mathrm{sym}}(x) .$$

In the limit of vanishing quark masses there is an exact U(1) axial symmetry ($\dot{Q}_5^L = 0$) irrespective of the anomaly or the presence of any configurations with integer (e.g. instantons), fractional or whatever values of the topological charge operator

$$\frac{g^2}{32\pi^2} \int \mathrm{d}^4 x\, F.\tilde{F}(x) .$$

Incorrect claims usually follow from the anomalous divergence equation for the gauge invariant current $J^L_{\mu 5}$:

$$\partial^\mu J^L_{\mu 5} = 2L \frac{g^2}{32\pi^2} F.\tilde{F} \equiv 2L \partial^\mu K_\mu ,$$

* The breaking is actually spontaneous, signalled by $\langle \det \bar{q}_i q_j \rangle \neq 0$.

† These remarks also apply to $\mathrm{U(1)_{PQ}}$ together with (1) or $\langle \phi_{\mathrm{Higgs}} \rangle \neq 0$.

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and the belief that

$$X(t) \equiv \int \mathrm{d}^3x\, J^L_{05}(x) = \int \mathrm{d}^3x\, \{J^L_{05,\mathrm{sym}}(x) + 2L\, K_0(x)\}$$

is the charge generator for U(1) axial transformations. However, $X(t)$ has non-vanishing commutators with operators constructed from gauge fields alone, known as anomalous commutators (Adler and Boulware 1969), and has nothing whatsoever to do with chirality.

A consistent treatment of the U(1) axial symmetry together with equation (1) has been given (Crewther 1977, 1978, 1979) through an analysis of the chiral $\mathrm{SU}(L) \times \mathrm{SU}(L)$ Ward identities and the anomalous U(1) axial Ward identities. This approach does not involve any centre mismatch; the condensate (1) spontaneously breaks $\mathrm{U}(L)$ axial symmetry together with its axial centre subgroup Z_L. This general analysis makes no restrictions on the values of ν taken by the topological charge operator. In general one requires arbitrary* values of ν (Crewther 1980*a*, 1980*b*) depending on the values of the quark mass parameters. It is interesting to note that for some particular values of the quark mass parameters, integer† values of ν can account for (1), which means that even in this case there is no unbroken centre. Furthermore, the usual notion of a θ vacuum and the usual connection between a θ rotation and a U(1) axial rotation are changed, in a manner that accounts for this.

In short, the domain wall problem is a consequence of a totally unrealistic assumption that instantons give the dominant contribution to all amplitudes. The problem is easily avoided by not specifying *a priori* the infrared boundary conditions as being pure gauge but by allowing the theory to dictate its own infrared behaviour. Proposed solutions (Lazarides and Shafi 1982; Georgi and Wise 1982; Dimopoulos *et al.* 1982; Barr *et al.* 1982; Fujimoto *et al.* 1983; Hayashi and Murayama 1983) to this hypothetical problem involve the embedding of the centre Z_L into some unbroken continuous group. Any restrictions on model building that may subsequently follow are unnecessary.

Note added in proof. Since the gluonic fields are left invariant under U(1) axial transformations it is essential that the U(1) axial charge generator commutes with these fields. That $X(t)$ does not satisfy this requirement, and hence has nothing to do with U(1) axial transformations, is evident from anomalous commutators (Adler and Boulware 1969); for example

$$[\partial_0 A_i(\boldsymbol{x}, t), X(t)] = -\frac{\mathrm{i}\, Lg^2}{2\pi^2} \tilde{F}_{0i}(x) \neq 0\,.$$

However, Q^L_5 does have all of the correct commutators (Adler and Boulware 1969; Adler 1970; Jackiw 1972).

Many authors still believe that $X(t)$ generates U(1) axial transformations because it is thought that as $J^L_{\mu 5,\mathrm{sym}}$ is gauge dependent it cannot generate gauge invariant physical chiralities (i.e. commutators), while $X(t)$ is manifestly gauge invariant and hence physical. Actually the complete opposite is true. It turns out that even though

* A recent attempt (Palmer and Pinsky 1982) to avoid this conclusion has been shown to be incorrect (Crewther 1982).

† This refers to instantons ($\nu = \pm 1$) and multi-instantons, not necessarily restricted to a dilute gas.

$J^L_{\mu 5,\mathrm{sym}}$ is gauge dependent, Q^L_5 still generates gauge invariant commutators (Bardeen 1974; Crewther 1978, 1979, 1980*a*). On the other hand, $X(t)$ is not renormalization group invariant and so its commutators are completely unphysical.

For a detailed derivation of the results regarding the required spectrum of the topological charge operator see the works of Crewther (1980*a*, 1980*b*). The present author is also currently compiling a review of the subject.

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