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Comment on the Brans–Dicke–Kasner Solution

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Abstract

We point out that the Brans–Dicke–Kasner solution recently given by Johri and Goswami (1983) is nothing but the general Bianchi type-I vacuum solution first given by Ruban and Finkelstein (1972).

1. Introduction

In a recent paper by Johri and Goswami (1983) the Brans–Dicke–Bianchi type-I vacuum field equations have been solved. The resulting solution turns out to be the generalization of the Kasner (1921) solution of the general theory of relativity. However, we show that this solution is not new; rather, it can be transformed into the Kasner-type solution first given by Ruban and Finkelstein (1972). The Ruban–Finkelstein (1972, 1975) papers were not mentioned by Johri and Goswami. Even their own paper on perfect fluid Brans–Dicke–Bianchi type-I solutions (Johri and Goswami 1981) given two years earlier is not mentioned. These perfect fluid solutions were also first given by Ruban and Finkelstein (1975) in a somewhat different form.

The Brans–Dicke–Bianchi type-I vacuum field equations have also been considered by Shri Ram (1983*a*, 1983*b*) and Shri Ram and Singh (1983). However, it has been shown by us (Lorenz-Petzold 1984*a*, 1984*b*, 1984*c*) that the corresponding solutions are either wrong or not new. The whole class of the Brans–Dicke–Bianchi types I–IX vacuum solutions has been obtained by us recently (Lorenz-Petzold 1984*d*). In the present paper we discuss the results given by Johri and Goswami (1983) and present the explicit transformation which proves our statement that the Johri–Goswami solution is nothing but the general Ruban–Finkelstein solution.

2. Field Equations and Solutions

The Bianchi type-I metric is given by

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + R_i^2(t)\{\mathrm{d}x^i\}^2, \qquad i = 1,2,3. \tag{1}$$

The corresponding Brans–Dicke vacuum field equations to be solved are

$$\dot{H}_i + 3HH_i + (\ln\phi)^{\cdot}(\ln R_i)^{\cdot} = 0, \tag{2a}$$

$$H_1 H_2 + H_1 H_3 + H_2 H_3 + 3H(\ln\phi)^{\cdot} - \tfrac{1}{2}\omega(\ln\phi)^{\cdot 2} = 0, \tag{2b}$$

$$(R^3\dot{\phi})^{\cdot} = 0, \tag{2c}$$

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where $H_i = \dot{R}_i/R_i$ are the Hubble parameters, $3H = \sum H_i$, $R^3 = R_1 R_2 R_3$ and $(\ )^{\cdot} = \mathrm{d}/\mathrm{d}t$. By setting $g = R^3\phi$ we obtain the decoupled field equations

$$\ddot{g} = 0, \tag{3a}$$

$$\dot{H}_i + H_i(\ln g)^{\cdot} = (\ln R)^{\cdot\cdot} + (\ln g)^{\cdot}(\ln R)^{\cdot}, \tag{3b}$$

$$R^3\dot{\phi} = c, \tag{3c}$$

from which we obtain the general expressions

$$R_i = r_i R \exp\left(c_i \int g^{-1}\,\mathrm{d}t\right), \tag{4a}$$

$$\phi = \phi_0 \exp\left(c \int g^{-1}\,\mathrm{d}t\right), \tag{4b}$$

$$\sum c_i = 0, \qquad \sum c_i^2 = \tfrac{2}{3}a(a+c) - (\omega + \tfrac{4}{3})c^2, \tag{4c}$$

where

$$g = at + b \tag{4d}$$

is the solution of equation (3a) and a, b, c, r_i and ϕ_0 are constant.

We obtain two different kinds of solutions: one for $a = 0$,

$$R_i = r_i \exp(c_i - \tfrac{1}{3}b)t/b, \qquad \phi = \phi_0 \exp(ct/b); \tag{5a}$$

and one for $a \neq 0$,

$$R_i = r_i(at+b)^{(a-c+3c_i)/3a}, \qquad \phi = \phi_0(at+b)^{c/a}. \tag{5b}$$

By setting

$$p_i = (c_i - \tfrac{1}{3}c)/b, \qquad c = -b; \tag{6a}$$

$$p_i = \tfrac{1}{3} + c_i C/c, \qquad c/a = C/(1+C), \tag{6b}$$

respectively, where C denotes the Ruban–Finkelstein (1972) constant, we obtain the corresponding forms first given by Ruban and Finkelstein (1972):

$$\sum p_i = 1, \qquad \sum p_i^2 = -(1+\omega); \tag{7a}$$

$$\sum p_i = 1, \qquad \sum p_i^2 = 1 - C(\omega C - 2). \tag{7b}$$

The connection with the Johri–Goswami (1983) work is given by the simple relations

$$R_1 = A, \qquad R_2 = B, \qquad R_3 = C, \tag{8a}$$

$$p_1 = c/a, \qquad p_i = (a-c+3c_i)/3a, \tag{8b}$$

if $a \neq 0$. The special solution with $a = 0$ has been overlooked by Johri and Goswami (1983). Thus, we have shown that the Brans–Dicke–Kasner solution obtained by these authors is the general ($a \neq 0$) vacuum solution first given by Ruban and Finkelstein (1972).

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