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The Relationship between r.f. Current Drive and the Measured Loop Voltage in Tokamak r.f. Heating Experiments

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Abstract

The properties of tokamak discharges are known to undergo noticeable variations during r.f. heating. In particular, a significant drop in the loop voltage has been observed in a number of experiments, which can be attributed to a decrease in the plasma resistance or inductance or to 'r.f. current generation'. The problem is analysed from the circuit theory point of view. The ambiguities encountered in interpreting the loop voltage measurements in 'small' tokamak experiments are discussed.

1. Introduction

There is continued interest in the use of r.f. waves to drive the toroidal current in tokamaks (Thonemann *et al.* 1952; Ohkawa 1970; Wort 1971; Klima 1975; Fisch 1978; Ehst *et al.* 1982). The use of r.f. current drive may make it possible to operate a tokamak reactor in a steady state mode, a very desirable feature from the engineering point of view. Alternatively, r.f. current drive may be utilized to supplement the magnetically coupled current so that longer burn times could be achieved using ohmic heating circuits of modest volt–second ratings. There is also a potential for using r.f. current drive to control the current distribution so that equilibria with optimum properties can be obtained.

In most r.f. current drive experiments, the r.f. power is applied during the steady phase of an ohmic heating discharge (see e.g. Parlange and Van Houtte 1982; Nakamura *et al.* 1982; Stevens *et al.* 1982). The noticeable drop in the loop voltage during the r.f. heating is usually attributed to r.f. current generation. However, the decrease in the loop voltage can be caused by a decrease in the plasma resistance or inductance. The interpretation of the loop voltage measurements is particularly ambiguous in small experiments where many parameters change simultaneously and a true steady state is never established.

A brief theoretical background of r.f. current drive is given in Section 2 and the equivalent circuits are developed in Section 3. The application of the results to small tokamak experiments is discussed in Section 4. The generalized Ohm's law for a plasma with r.f. current drive is derived from the kinetic equation in the Appendix.

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2. Theoretical Background

The role of the r.f. current drive is best explained by considering the generalized Ohm's law:

$$\eta \boldsymbol{J} = \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} - (1/ne)\boldsymbol{J} \times \boldsymbol{B} + (1/ne)\nabla P_e . \tag{1}$$

For a stationary toroidal equilibrium we have

$$\boldsymbol{v} = 0 , \tag{2}$$

$$\nabla n \times (\boldsymbol{J} \times \boldsymbol{B}) = 0 , \tag{3}$$

$$\nabla n \times \nabla P_e = 0 . \tag{4}$$

Hence, one obtains

$$\oint \eta \boldsymbol{J} . \mathrm{d}\boldsymbol{l} = \oint \boldsymbol{E} . \mathrm{d}\boldsymbol{l} , \tag{5}$$

where the integration is carried out along any closed curve. For a toroidal filament (Fig. 1), one obtains [using Faraday's law and equation (5)]

$$I_j R_j = -\mathrm{d}\phi_j/\mathrm{d}t , \tag{6}$$

where I_j is the toroidal current in the jth filament, R_j is its resistance and ϕ_j is the total poloidal flux linked to it. It follows that the poloidal flux produced by the ohmic heating primary coil must increase linearly with time in order that a steady current is maintained. This cannot continue indefinitely, and the discharge has to be eventually terminated because of the physical limitations of the ohmic heating circuit.

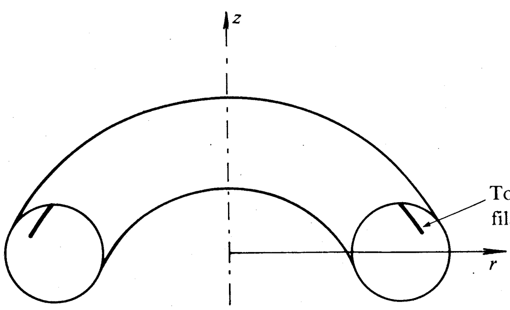

Fig. 1. Schematic diagram of an axisymmetric toroidal plasma showing a toroidal filament.

It is known that r.f. travelling wave fields exert a unidirectional force on the plasma (Ohkawa 1970; Klima 1973). Under certain circumstances, this unidirectional force can be utilized to sustain a unidirectional steady current in the plasma. In this situation, the generalized Ohm's law has the form (see the Appendix)

$$\eta \boldsymbol{J} = \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} - (1/ne)\boldsymbol{J} \times \boldsymbol{B} + (1/ne)\nabla P_e + \boldsymbol{F}_{\mathrm{rf}} , \tag{7}$$

where $\boldsymbol{F}_{\mathrm{rf}}$ is the average (over the electron distribution function) unidirectional force exerted by the r.f. field on the electron fluid per unit charge. It follows that the loop equation for a toroidal filament has the form

$$I_j R_j = -(\mathrm{d}\phi_j/\mathrm{d}t) + \mathscr{E}_{\mathrm{rf}j}, \tag{8}$$

where $\mathscr{E}_{\mathrm{rf}j}$ is the effective e.m.f. induced by the r.f. wave in the toroidal filament j:

$$\mathscr{E}_{\mathrm{rf}j} = \oint_j \boldsymbol{F}_{\mathrm{rf}} \cdot \mathrm{d}\boldsymbol{l}. \tag{9}$$

We note that the e.m.f. has, in general, different values for different toroidal filaments (different values of j), i.e. $\mathscr{E}_{\mathrm{rf}}$ is not necessarily uniform over a poloidal cross section. For a steady state discharge maintained by an r.f. wave, the condition

$$\mathrm{d}\phi_j/\mathrm{d}t = 0 \tag{10}$$

and hence

$$\mathscr{E}_{\mathrm{rf}j} = I_j R_j \tag{11}$$

must be satisfied for all values of j. It should be noted that the current distribution in a discharge maintained by r.f. waves is not necessarily the same as the distribution of the plasma conductivity; it can be altered by varying the distribution of the r.f. induced e.m.f. $\mathscr{E}_{\mathrm{rf}}$. It is, in principle, possible to design the r.f. launching structure so that the distribution of the e.m.f. corresponds to an optimum current profile.

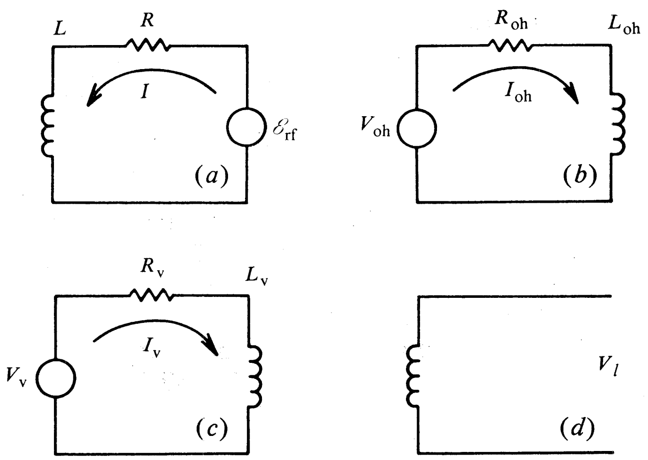

Fig. 2. Equivalent circuit of a tokamak plasma: (*a*) lumped element circuit; (*b*) ohmic heating circuit; (*c*) vertical field circuit; (*d*) open-circuited loop.

3. Equivalent Circuit

In the following discussion the tokamak plasma is considered to be a lumped element (R–L) circuit which is magnetically coupled to the various field circuits. The toroidal plasma current is strongly coupled to the ohmic heating circuit; it is also coupled to the vertical field circuit. There is a very weak coupling to the toroidal field circuit (and to the plasma poloidal current as well) because of the anisotropy of the plasma resistivity; this weak coupling will be neglected in the following analysis.

The equivalent circuit is shown in Fig. 2. The plasma is represented by a lumped element circuit (*a*) which consists of a resistance R and an inductance L. The r.f. current drive is included as a lumped e.m.f. $\mathscr{E}_{\mathrm{rf}}$. This is equivalent to the common practice of introducing a 'toroidal current without Joule losses I_{rf}' (Parlange and Van Houtte 1982), provided that $\mathscr{E}_{\mathrm{rf}} = RI_{\mathrm{rf}}$. The ohmic heating circuit (*b*) has an inductance L_{oh}, a resistance R_{oh} and is coupled to the plasma by a mutual inductance M_{oh}, to the vertical field circuit by a mutual inductance M_{voh} and to the measuring loop by mutual inductance $M_{l\mathrm{oh}}$. The vertical field circuit (*c*) has a resistance R_{v} and inductance L_{v} and is also coupled magnetically to the other circuits. The voltage sources in the ohmic heating circuit and the vertical field circuit (V_{oh} and V_{v}) can represent capacitor banks, passive or active crow-bar or feedback controlled sources. The loop voltage V_l is the voltage induced in an open-circuited toroidal loop (*d*) situated as close as possible to the plasma. The circuit equations are

$$V_{\mathrm{oh}} = R_{\mathrm{oh}} I_{\mathrm{oh}} + L_{\mathrm{oh}} \frac{\mathrm{d}I_{\mathrm{oh}}}{\mathrm{d}t} + \frac{\mathrm{d}}{\mathrm{d}t}(M_{\mathrm{oh}} I) + M_{\mathrm{voh}} \frac{\mathrm{d}I_{\mathrm{v}}}{\mathrm{d}t}, \tag{12}$$

$$V_{\mathrm{v}} = R_{\mathrm{v}} I_{\mathrm{v}} + L_{\mathrm{v}} \frac{\mathrm{d}I_{\mathrm{v}}}{\mathrm{d}t} + \frac{\mathrm{d}}{\mathrm{d}t}(M_{\mathrm{v}} I) + M_{\mathrm{voh}} \frac{\mathrm{d}I_{\mathrm{oh}}}{\mathrm{d}t}, \tag{13}$$

$$\mathscr{E}_{\mathrm{rf}} = RI + \frac{\mathrm{d}}{\mathrm{d}t}(LI) + \frac{\mathrm{d}}{\mathrm{d}t}(M_{\mathrm{oh}} I_{\mathrm{oh}}) + \frac{\mathrm{d}}{\mathrm{d}t}(M_{\mathrm{v}} I_{\mathrm{v}}), \tag{14}$$

$$V_l = -M_{l\mathrm{oh}} \frac{\mathrm{d}I_{\mathrm{oh}}}{\mathrm{d}t} - M_{l\mathrm{v}} \frac{\mathrm{d}I_{\mathrm{v}}}{\mathrm{d}t} - \frac{\mathrm{d}}{\mathrm{d}t}(M_l I), \tag{15}$$

where the subscripts oh, v and l refer to the ohmic heating circuits, the vertical field circuit and the open-circuited loop respectively. The minus signs in equation (15) are chosen arbitrarily so that the loop voltage is positive for positive plasma current.

The circuit parameters R_{oh}, L_{oh}, M_{voh}, R_{v}, L_{v}, $M_{l\mathrm{oh}}$ and $M_{l\mathrm{v}}$ are constant and can, in principle, be measured accurately. The ohmic heating coil is usually designed such that the stray field in the discharge region is negligible. The mutual inductance M_{oh} is therefore very nearly constant.

The lumped parameters R, L, $\mathscr{E}_{\mathrm{rf}}$, M_{oh}, M_{v} and M_l are, of course, related to the local properties of the discharge. Because of the toroidal symmetry, it is sufficient to consider the plasma as an aggregate of toroidal filaments. The toroidal current in the jth filament is I_j. The jth filament has a resistance R_j, a self-inductance M_{jj} and is magnetically coupled to the ohmic heating circuit, the vertical field circuit, the measurement loop and to the other filaments by $M_{\mathrm{oh}j}$, $M_{\mathrm{v}j}$, M_{lj} and M_{jk}. The circuit equations are

$$V_{\mathrm{oh}} = R_{\mathrm{oh}} I_{\mathrm{oh}} + L_{\mathrm{oh}} \frac{\mathrm{d}I_{\mathrm{oh}}}{\mathrm{d}t} + M_{\mathrm{voh}} \frac{\mathrm{d}I_{\mathrm{v}}}{\mathrm{d}t} + \sum_j M_{\mathrm{oh}j} \frac{\mathrm{d}I_j}{\mathrm{d}t}, \tag{16}$$

$$V_{\mathrm{v}} = R_{\mathrm{v}} I_{\mathrm{v}} + L_{\mathrm{v}} \frac{\mathrm{d}I_{\mathrm{v}}}{\mathrm{d}t} + M_{\mathrm{voh}} \frac{\mathrm{d}I_{\mathrm{oh}}}{\mathrm{d}t} + \sum_j M_{\mathrm{v}j} \frac{\mathrm{d}I_j}{\mathrm{d}t}, \tag{17}$$

$$\mathscr{E}_{\mathrm{rf}j} = R_j I_j + \sum_k M_{jk}\frac{\mathrm{d}I_k}{\mathrm{d}t} + M_{\mathrm{oh}j}\frac{\mathrm{d}I_{\mathrm{oh}}}{\mathrm{d}t} + M_{\mathrm{v}j}\frac{\mathrm{d}I_{\mathrm{v}}}{\mathrm{d}t}, \tag{18}$$

$$V_l = -M_{l\mathrm{oh}}\frac{\mathrm{d}I_{\mathrm{oh}}}{\mathrm{d}t} - M_{l\mathrm{v}}\frac{\mathrm{d}I_{\mathrm{v}}}{\mathrm{d}t} - \sum_j M_{lj}\frac{\mathrm{d}I_j}{\mathrm{d}t}. \tag{19}$$

The inductances M_{jk}, $M_{\mathrm{oh}j}$, $M_{\mathrm{v}j}$ and M_{lj} are not time-dependent since the toroidal filaments are stationary in this model; this does not rule out plasma motion because the filaments are not assigned to certain fluid elements.

Equations (12)–(15) are required to be equivalent to equations (16)–(19). The toroidal current I is obviously equal to the sum of the toroidal currents in the filaments:

$$I = \sum_j I_j. \tag{20}$$

It follows that

$$M_{\mathrm{oh}} = \frac{1}{I}\sum_j I_j M_{\mathrm{oh}j}, \qquad M_{\mathrm{v}} = \frac{1}{I}\sum_j I_j M_{\mathrm{v}j}, \tag{21, 22}$$

$$M_l = \frac{1}{I}\sum_j I_j M_{lj}, \qquad L = \frac{1}{I^2}\sum_{j,k} I_j I_k L_{jk}, \tag{23, 24}$$

$$\mathscr{E}_{\mathrm{rf}} = \frac{1}{I}\sum_j I_j \mathscr{E}_{\mathrm{rf}j}, \qquad R = \frac{1}{I^2}\sum_j I_j^2 R_j. \tag{25, 26}$$

We note that $M_{\mathrm{oh}j}$ is approximately constant for all j; it follows that $M_{\mathrm{oh}} \approx M_{\mathrm{oh}j}$ is not time-dependent. On the contrary $M_{\mathrm{v}j}$, M_{lj} and L_{jk} are not the same for all j; consequently M_{v}, M_l and L depend on the current distribution and are therefore (in general) time-dependent. They may vary because of a change of the current profile or because of a shift of the plasma as a whole. The definition of $\mathscr{E}_{\mathrm{rf}}$ (equation 14) implies that the energy supplied by the equivalent e.m.f. $\mathscr{E}_{\mathrm{rf}}$ to the lumped plasma circuit is equal to the sum of the energy supplied by the elemental sources $\mathscr{E}_{\mathrm{rf}j}$ to the toroidal plasma filaments. Similarly, the energy dissipated in the lumped resistance R is equal to the sum of the energy dissipated in the filament resistances R_j. We also note that the resistance R is equal to the parallel sum of the resistors R_j in the special case of an ohmic heating discharge during the steady period in which $\mathrm{d}I_j/\mathrm{d}t = 0$. In this case we have

$$I_j = \alpha/R_j, \tag{27}$$

where

$$\alpha = (\mathrm{d}/\mathrm{d}t)\phi_{\mathrm{oh}} = M_{\mathrm{oh}}\,\mathrm{d}I_{\mathrm{oh}}/\mathrm{d}t \tag{28}$$

is a constant. It follows that

$$R = 1\Big/\sum \frac{1}{R_j}. \tag{29}$$

Equation (28) is not in general applicable during r.f. current drive.

4. Loop Voltage Measurements

The loop voltage measurement is a routine diagnostic in tokamaks. It is relatively simple and inexpensive, yet it provides valuable information about the discharge. The loop voltage is a global property of the configuration and depends on a large number of variables. The interpretation of the loop voltage measurements in r.f. heating and current drive experiments usually entails some assumptions. In the following discussion these assumptions are identified and the conditions for their validity are examined.

The relationship between the measured loop voltage and the discharge parameters is obtained using equations (14) and (15):

$$V_l = RI - \mathscr{E}_{\mathrm{rf}} + (\mathrm{d}/\mathrm{d}t)\{(L-M_l)I\}$$
$$+(\mathrm{d}/\mathrm{d}t)\{(M_{\mathrm{oh}}-M_{l\mathrm{oh}})I_{\mathrm{oh}}\}+(\mathrm{d}/\mathrm{d}t)\{(M_{\mathrm{v}}-M_{l\mathrm{v}})I_{\mathrm{v}}\}\,. \tag{30}$$

This relationship can be simplified for two special cases:

Special case 1: A pure ohmic heating discharge during the steady phase. For this special case $\mathscr{E}_{\mathrm{rf}} = 0$ and all other quantities except I_{oh} are constant. It follows that

$$V_l = RI + (M_{\mathrm{oh}} - M_{l\mathrm{oh}})\,\mathrm{d}I_{\mathrm{oh}}/\mathrm{d}t\,. \tag{31}$$

Using equation (14) one obtains

$$M_{\mathrm{oh}}\,\mathrm{d}I_{\mathrm{oh}}/\mathrm{d}t = RI \tag{32}$$

and hence

$$V_l = (1+\varepsilon_1)IR\,, \tag{33}$$

where

$$\varepsilon_1 = (M_{\mathrm{oh}} - M_{l\mathrm{oh}})/M_{\mathrm{oh}} \tag{34}$$

is the ratio of the ohmic heating stray (error) flux in the area between the plasma and the measuring loop to the ohmic heating magnetic flux linking the plasma. For the majority of experiments, the error field is negligible and the loop is situated very close to the plasma so that $|\varepsilon_1| \ll 1$ and equation (33) can be approximated to

$$V_l \approx RI\,. \tag{35}$$

Special case 2: The steady phase of a discharge with r.f. current drive. It can be shown using equations (30) and (34) that

$$V_l' = (1+\varepsilon_1)(R'I' - \mathscr{E}_{\mathrm{rf}}) \approx R'I' - \mathscr{E}_{\mathrm{rf}}\,, \tag{36}$$

where V_l', I' and R' are the loop voltage, the plasma toroidal current and the plasma lumped resistance during the r.f. current drive. In most situations, the r.f. power is applied to a normal ohmic heating tokamak plasma. The change in the loop voltage $\Delta V_l = V_l - V_l'$ is given by

$$\Delta V_l = \mathscr{E}_{\mathrm{rf}} - (I'R' - IR)\,. \tag{37}$$

If the toroidal current does not vary (due to a feedback control of the ohmic heating circuit, for example) and assuming that the plasma resistance does not vary as well, one obtains

$$\mathscr{E}_{\mathrm{rf}} = \Delta V_l\,, \qquad I_{\mathrm{rf}} = \mathscr{E}_{\mathrm{rf}}/R = (\Delta V_l/V_l)I\,. \tag{38, 39}$$

The assumption that $R' = R$ is, however, unjustifiable. The plasma resistance during r.f. current drive can be different from the resistance of an ohmic heating plasma for three reasons: (*a*) the electron temperature (and possibly the effective Z) can vary, (*b*) the current distribution can vary and (*c*) the current may be carried by a different class of electrons. The change in the loop voltage is, therefore, attributed to the combined effect of the r.f. induced e.m.f. and the change in the plasma resistance. The contributions of these two effects are indistinguishable.

For small tokamak experiments, a quasi-steady state may not be established during the discharge (or during the application of the r.f. current drive) and the inductive terms in equation (30) have to be retained. It follows that the decrease in the loop voltage, $\Delta V_l = V_l - V_l'$, is given by

$$\Delta V_l = \mathscr{E}_{\mathrm{rf}} - (RI - R'I') + g + g_{\mathrm{oh}} + g_{\mathrm{v}}\,, \tag{40}$$

where the correction terms g, g_{oh} and g_{v} represent the effect of the plasma self-inductance and the mutual inductance with ohmic heating circuit and with the vertical field circuit respectively. These corrections are given by

$$g = (\mathrm{d}/\mathrm{d}t)\{(L - M_l)I\} - (\mathrm{d}/\mathrm{d}t)\{(L' - M_l')I'\}\,, \tag{41}$$

$$g_{\mathrm{oh}} = (\mathrm{d}/\mathrm{d}t)\{(M_{\mathrm{oh}} - M_{l\mathrm{oh}})I_{\mathrm{oh}}\} - (\mathrm{d}/\mathrm{d}t)\{(M_{\mathrm{oh}}' - M_{l\mathrm{oh}})I_{\mathrm{oh}}'\}\,, \tag{42}$$

$$g_{\mathrm{v}} = (\mathrm{d}/\mathrm{d}t)\{(M_{\mathrm{v}} - M_{l\mathrm{v}})I_{\mathrm{v}}\} - (\mathrm{d}/\mathrm{d}t)\{(M_{\mathrm{v}}' - M_{l\mathrm{v}})I_{\mathrm{v}}'\}\,. \tag{43}$$

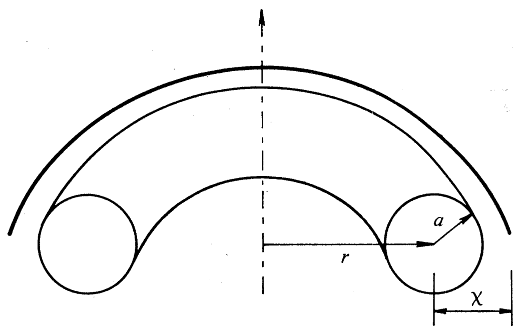

Fig. 3. Schematic diagram showing the position of the measuring loops.

It is very important to note that the inductive terms in equations (41)–(43) are proportional to $L - M_l$, $M_{\mathrm{oh}} - M_{l\mathrm{oh}}$ and $M_{\mathrm{v}} - M_{l\mathrm{v}}$, whereas the induced e.m.f.s in the plasma loop are proportional to L, M_{oh} and M_{v} respectively (see equation 14). It follows that the inductive terms g, g_{oh} and g_{v} in equation (40) can sometimes be neglected for some experiments [i.e. $\Delta V_l \approx \mathscr{E}_{\mathrm{rf}} - (IR - I'R')$] even though the corresponding terms play a significant role in equation (14).

We now estimate the order of magnitude of the correction terms g, g_{oh} and g_{v} for a tokamak plasma with major radius r and minor radius a. The distance between the plasma centre and the measurement loop is χ (see Fig. 3). The following approximate formulae are obtained for a large aspect ratio tokamak:

$$L-M \sim \mu_0 r\{\ln(\chi/a)+\tfrac{1}{2}l_i\}, \tag{44}$$

$$(M_{oh}-M_{loh})I_{oh} \sim 2\pi r(\chi-\tfrac{1}{2}a)B_e, \tag{45}$$

$$(M_v-M_{lv})I_v \sim 2\pi r(\chi-\tfrac{1}{2}a)B_v, \tag{46}$$

where l_i is the internal inductance per unit length (Furth 1981), B_e is the ohmic heating error field and B_v is the vertical field. The orders of magnitude of the correction terms are estimated using equations (44)–(46):

$$O\{g\} \sim \mu_0 r\{\ln(\chi/a)+\tfrac{1}{2}l_i\}\frac{\Delta I}{\Delta t}+\tfrac{1}{2}\mu_0 rI\frac{\Delta l_i}{\Delta t}-\mu_0 I\frac{r}{a}\left(\frac{\Delta a}{\Delta t}\pm\frac{\Delta r}{\Delta t}\right), \tag{47}$$

$$O\{g_{oh}\} \sim 2\pi r(\chi-\tfrac{1}{2}a)\frac{1}{I_{oh}}\frac{\Delta I_{oh}}{\Delta t}B_e+2\pi rB_e\left(\pm\frac{\Delta r}{\Delta t}-\frac{\Delta a}{2\Delta t}\right), \tag{48}$$

$$O\{g_v\} \sim 2\pi r(\chi-\tfrac{1}{2}a)\frac{\Delta B_v}{\Delta t}+2\pi rB_v\left(\pm\frac{\Delta r}{\Delta t}-\frac{\Delta a}{2\Delta t}\right). \tag{49}$$

To illustrate the use of the above analysis we apply it to the results obtained from the LHR current drive experiments in the Petula tokamak (Parlange and Van Houtte 1982). The Petula tokamak has a major radius $r = 0\cdot72$ m, a minor radius $a = 0\cdot14$ m, a toroidal field $2\cdot7$ T and a toroidal current $I = 10$ kA. It follows that the vertical field is $B_v \approx 0\cdot02$ T. The ohmic heating circuit has a feedback control that maintains the toroidal current very nearly constant during the experiment. It was observed that the loop voltage dropped by about $1\cdot8$ V when 180 kW of r.f. power was applied. Many other effects were observed. The most important observation (for the purpose of this application) was an outward shift of about 2 cm that took place in about 50 ms (i.e. $\Delta r/\Delta t = 0\cdot02/0\cdot05 = 0\cdot4\ \mathrm{m\,s^{-1}}$). The inductive corrections associated with this motion are estimated by using equations (47)–(49), assuming that $\Delta I = 0$, $\Delta l_i = 0$, $\Delta a = 0$, $B_e = 0$ and $\Delta B_v = 0$. Thus we have

$$O\{g) \sim \mu_0 I(r/a)(\Delta r/\Delta t) \sim 0\cdot025\ \mathrm{V},$$

$$O\{g_v\} \sim 2\pi rB_v\Delta r/\Delta t \sim 0\cdot036\ \mathrm{V},$$

i.e. these corrections are much smaller than the observed decrease in the loop voltage ($\sim 1\cdot8$ V) for the Petula experiments.

5. Conclusions

The relationship between r.f. current drive and the measured loop voltage in tokamak r.f. heating experiments has been considered. The tokamak plasma was treated as a lumped element circuit which is magnetically coupled to the ohmic heating and the vertical field circuit. It was shown that the decrease in the loop voltage observed during r.f. current drive in tokamak experiments operating in a quasi-steady mode could be attributed to an r.f. induced e.m.f. (or current) and/or to a decrease in the plasma resistance. The contributions of these two effects are indistinguishable. Inductive effects also contribute to the loop voltage for small experiments where a quasi-steady state is not established. Expressions for these inductive terms were obtained as functions of the circuit parameters, as well as simple formulae that can be used to estimate the order of magnitude of these terms.

Acknowledgments

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Appendix. Kinetic Derivation of the Generalized Ohm's Law appropriate for a Plasma with r.f. Current Drive

The Boltzmann equation for a charged particle species j is

$$\partial f_j/\partial t + \boldsymbol{v}\,.\nabla f_j + (e_j/m_j)(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})\,.\nabla_{\mathrm{v}} f_j = \sum_k C_{kj}(f_j, f_k) + D_j\,. \tag{A1}$$

The fields $\boldsymbol{E}$ and $\boldsymbol{B}$ are the macroscopic smoothed fields. The fluctuating microscopic fields arising from the corpuscular nature of the plasma are introduced as a sum of collision terms C_{kj}. The inelastic term D_j accounts for the atomic interactions (ionization, recombination, charge exchange etc.) that may take place.

We now consider a plasma equilibrium with an ignorable space coordinate z. This situation may represent an axisymmetric toroidal equilibrium, an axisymmetric mirror or an infinitely long pinch. The ignorable coordinate may be either a linear or an angular coordinate and the conjugate momentum is accordingly either linear or angular momentum. To this equilibrium a travelling wave field of a certain angular frequency ω and wave vector k is applied using an external source. The fields may be written as sums of slowly varying fields $\boldsymbol{E}_0$ and $\boldsymbol{B}_0$ and r.f. fields whose amplitudes E_1 and B_1 vary on a slow time scale:

$$\boldsymbol{E} = \boldsymbol{E}_0(x,y,t) + \boldsymbol{E}_1(x,y,t)\exp\{\mathrm{i}(\omega t - kz)\}, \tag{A2}$$

$$\boldsymbol{B} = \boldsymbol{B}_0(x,y,t) + \boldsymbol{B}_1(x,y,t)\exp\{\mathrm{i}(\omega t - kz)\}. \tag{A3}$$

In general a number of harmonic and 'daughter' waves are excited but the effect of these waves will not be considered here. The distribution function can be similarly expanded into the sum of a slowly varying part f_0 and a rapidly varying r.f. part f_1:

$$f_j = f_{0j} + f_{1j}\exp\{\mathrm{i}(\omega t - kz)\}. \tag{A4}$$

Substituting equations (A2)–(A4) into the Boltzmann equation (A1) we obtain

$$\mathrm{i}(\omega - kv_z)f_{1j} + v_x\frac{\partial f_{1j}}{\partial x} + v_y\frac{\partial f_{1j}}{\partial y} + \frac{e_j}{n_j}(\boldsymbol{E}_0 + \boldsymbol{v}\times\boldsymbol{B}_0).\nabla_{\mathrm{v}} f_{1j}$$

$$\approx \sum_k C_{1kj} + D_{1j} - \frac{e_j}{m_j}(\boldsymbol{E}_1 + \boldsymbol{v}\times\boldsymbol{B}_1).\nabla_{\mathrm{v}} f_{0j}, \tag{A5}$$

$$\frac{\partial f_{0j}}{\partial t} + v_x\frac{\partial f_{0j}}{\partial x} + v_y\frac{\partial f_{0j}}{\partial y} + \frac{e_j}{m_j}(\boldsymbol{E}_0 + \boldsymbol{v}\times\boldsymbol{B}_0).\nabla_{\mathrm{v}} f_{0j}$$

$$= \sum_k C_{0kj} + D_{0j} + \frac{e_j}{2m_j}(\boldsymbol{E}_1 + \boldsymbol{v}\times\boldsymbol{B}_1).\nabla_{\mathrm{v}} f^*_{1j}, \tag{A6}$$

where the asterisk denotes the complex conjugate. Equations (A5) and (A6), together with Maxwell's equations and the appropriate boundary conditions, form a complete set. However, solutions can be obtained only for simple special cases (see e.g. Vedendov 1967; Galeev and Sagdeev 1979). No attempt will be made here to obtain a general solution; nevertheless, some interesting general results can be obtained without having to solve the equations.

Taking moments of equation (A6) we obtain the continuity equation

$$\partial n_j/\partial t + \boldsymbol{V}_j.\nabla n_j = -n_j\nabla.\boldsymbol{V}_j + S_j, \tag{A7}$$

the momentum equation

$$m_j n_j \partial \boldsymbol{V}_j/\partial t + m_j n_j \boldsymbol{V}_j.\nabla\boldsymbol{V}_j = -\nabla.{}^2P_j + e_j n_j(\boldsymbol{E}_0 + \boldsymbol{V}_j\times\boldsymbol{B}_0)$$

$$+\boldsymbol{R}_j + \boldsymbol{R}^{\mathrm{s}}_j + e_j n_j \boldsymbol{F}_j, \tag{A8}$$

and the energy equation

$$\tfrac{3}{2}(\partial/\partial t)n_j T_j + \tfrac{3}{2}\nabla.(n_j T_j \boldsymbol{V}_j) + ({}^2P_j.\nabla)\boldsymbol{V}_j + \nabla.\boldsymbol{q}_j = Q_j + Q^{\mathrm{s}}_j + W_j. \tag{A9}$$

The macroscopic parameters are defined as follows:

$$n_j = \int \mathrm{d}^3v\, f_{0j}, \tag{A10}$$

$$V_j = \int d^3v\, \boldsymbol{v} f_{0j}, \tag{A11}$$

$$^2P_j = \int d^3v\, m_j(\boldsymbol{v}-\boldsymbol{V}_j)(\boldsymbol{v}-\boldsymbol{V}_j) f_{0j}, \tag{A12}$$

$$T_j = \int d^3v\, \tfrac{1}{3} m_j(\boldsymbol{v}-\boldsymbol{V}_j).(\boldsymbol{v}-\boldsymbol{V}_j) f_{0j}, \tag{A13}$$

$$\boldsymbol{q}_j = \int d^3v\, \tfrac{1}{2} m_j(\boldsymbol{v}-\boldsymbol{V}_j).(\boldsymbol{v}-\boldsymbol{V}_j)(\boldsymbol{v}-\boldsymbol{V}_j) f_{0j}. \tag{A14}$$

The transport coefficients are defined as follows:

$$S_j = \int d^3v\, D_j \tag{A15}$$

is the rate of creation of particles j through ionization, recombination, charge exchange etc.;

$$\boldsymbol{R}_j = \sum_k \int d^3v\, m_j \boldsymbol{v} C_{0kj} \tag{A16}$$

is the rate of momentum transfer to particles j due to collision with other charged particle species;

$$\boldsymbol{R}_j^{\mathrm{s}} = \int d^3v\, m_j \boldsymbol{v} D_{0j} \tag{A17}$$

is the rate of momentum transfer to particles j due to atomic processes;

$$Q_j = \sum_k \int d^3v\, \tfrac{1}{2} m_j(\boldsymbol{v}-\boldsymbol{V}_j).(\boldsymbol{v}-\boldsymbol{V}_j) C_{0kj} \tag{A18}$$

is the rate of increase of the internal energy of particle species j due to collisions with charged particles;

$$Q_j^{\mathrm{s}} = \int d^3v\, \tfrac{1}{2} m_j(\boldsymbol{v}-\boldsymbol{V}_j).(\boldsymbol{v}-\boldsymbol{V}_j) D_{0j} \tag{A19}$$

is the rate of increase of the internal energy of particle species j due to the atomic processes. The ponderomotive coefficients are defined as follows:

$$e_j n_j \boldsymbol{F}_j = \int d^3v\, m_j \boldsymbol{v}(-e_j/2m_j)(\boldsymbol{E}_1 + \boldsymbol{v} \times \boldsymbol{B}_1).\nabla_{\mathrm{v}} f_{1j}^* \tag{A20}$$

is the rate of transfer of momentum from the r.f. wave to the particle species j;

$$W_j = \int d^3v\, \tfrac{1}{2} m_j(\boldsymbol{v}-\boldsymbol{V}_j).(\boldsymbol{v}-\boldsymbol{V}_j)(-e_j/2m_j)(\boldsymbol{E}_1 + \boldsymbol{v} \times \boldsymbol{B}_1).\nabla_{\mathrm{v}} f_{1j}^* \tag{A21}$$

is the rate of increase of the internal energy of the particle species j due to their interaction with the r.f. wave. We note that the rate of transfer of energy from the r.f. wave to the particle species j is

$$\begin{aligned} P_j &= \int d^3v\, \tfrac{1}{2} m_j \boldsymbol{v}.\boldsymbol{v}(-e_j/2m_j)(\boldsymbol{E}_1 + \boldsymbol{v} \times \boldsymbol{B}_1).\nabla_{\mathrm{v}} f_{1j}^* \\ &= W_j + e_j n_j \boldsymbol{V}_j.\boldsymbol{F}_j. \end{aligned} \tag{A22}$$

In deriving the above equations, we have used the conditions

$$\int \mathrm{d}^3 v\, C_{kj} = 0, \tag{A23}$$

$$\int \mathrm{d}^3 v\, (-e_j/2m_j)(\boldsymbol{E}_1 + \boldsymbol{v} \times \boldsymbol{B}_1) . \nabla_v f_{1j}^* = 0, \tag{A24}$$

which follow from the fact that neither Coulomb collisions nor r.f. waves can create or annihilate particles.

According to Klima (1973) the total momentum transfer from the r.f. wave to the plasma is related to the r.f. wave energy dissipation through the general formula

$$\sum_j \int \mathrm{d}^3 r\, P_j = \frac{\omega}{k} \sum_j \int \mathrm{d}^3 r\, \boldsymbol{F}_j . \hat{z}, \tag{A25}$$

where the integration is carried out over the volume occupied by the plasma (this volume must be bounded by a vacuum region). It is noted that only the component of the ponderomotive force parallel to the wave phase velocity contributes to equation (A25). It is therefore convenient to decompose the ponderomotive force into a parallel component $\boldsymbol{F}_{\parallel j}$ and a perpendicular component $\boldsymbol{F}_{\perp j}$. The total perpendicular force is obviously zero since the wave imparts net momentum only in the parallel direction:

$$\sum_j \int \mathrm{d}^3 r\, \boldsymbol{F}_{\perp j} = 0. \tag{A26}$$

Nevertheless $\boldsymbol{F}_\perp$ can be nonzero and can contribute to the plasma confinement.

We now derive the one-fluid equations for a plasma with r.f. current drive. For simplicity we assume that there is only one ion species (j = i for ions and e for electrons). We also assume that the off-diagonal terms of the kinetic pressure tensors can be neglected and that the plasma flow velocity $\boldsymbol{V}$ is small. Using equation (A8) we obtain

$$n m_\mathrm{i}\, \partial \boldsymbol{V}/\partial t = -\nabla P + \boldsymbol{J} \times \boldsymbol{B} + \boldsymbol{R}^\mathrm{s} + en(\boldsymbol{F}_\mathrm{i} - \boldsymbol{F}_\mathrm{e}), \tag{A27}$$

$$\boldsymbol{E} + \{\boldsymbol{F}_\mathrm{e} + (m_\mathrm{e}/m_\mathrm{i})\boldsymbol{F}_\mathrm{i}\} + \boldsymbol{V} \times \boldsymbol{B} - (1/ne)\boldsymbol{J} \times \boldsymbol{B} + (1/ne)\nabla P_\mathrm{e} = \eta \boldsymbol{J}, \tag{A28}$$

where

$$\boldsymbol{V} = (m_\mathrm{e} + m_\mathrm{i})^{-1}(m_\mathrm{e} \boldsymbol{V}_\mathrm{e} + m_\mathrm{i} \boldsymbol{V}_\mathrm{i}), \tag{A29}$$

$$P = P_\mathrm{e} + P_\mathrm{i}, \tag{A30}$$

$$\boldsymbol{R}^\mathrm{s} = \boldsymbol{R}_\mathrm{i}^\mathrm{s} + \boldsymbol{R}_\mathrm{e}^\mathrm{s}, \tag{A31}$$

$$\eta = m_\mathrm{e}(\nu_\mathrm{ei} + \nu_\mathrm{en})/ne^2, \tag{A32}$$

with ν_en the electron–neutral collision frequency, and we have assumed that

$$\boldsymbol{R}_\mathrm{ei} = -\boldsymbol{R}_\mathrm{ie} = m_\mathrm{e}\, \nu_\mathrm{ei}(\boldsymbol{V}_\mathrm{i} - \boldsymbol{V}_\mathrm{e}), \tag{A33}$$

$$\boldsymbol{R}_\mathrm{e}^\mathrm{s} = -m_\mathrm{e}\, \nu_\mathrm{en}\, \boldsymbol{V}_\mathrm{e} \approx m_\mathrm{e}\, \nu_\mathrm{en}(\boldsymbol{V}_\mathrm{i} - \boldsymbol{V}_\mathrm{e}). \tag{A34}$$

We now define

$$\boldsymbol{F}_{\mathrm{rf}} = \boldsymbol{F}_{\mathrm{e}} + (m_{\mathrm{e}}/m_{\mathrm{i}})\boldsymbol{F}_{\mathrm{i}} \tag{A35}$$

and hence equation (A28) can be written in the form

$$\eta \boldsymbol{J} = \boldsymbol{E} + \boldsymbol{V} \times \boldsymbol{B} - (1/ne)\boldsymbol{J} \times \boldsymbol{B} + (1/ne)\nabla P_{\mathrm{e}} + \boldsymbol{F}_{\mathrm{rf}}\,. \tag{A36}$$

Before concluding, it is important to note that the resistivity η in equations (A32) and (A36) can be vastly different from the classical resistivity since the distribution function f can be severely distorted by the r.f. field.

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