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Derivation of Muon Intensities in Sea-water Depths up to 1400 M.W.E. from a Recent Primary Cosmic Ray Spectrum

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Abstract

The muon intensities in sea-water depths up to 1400 M.W.E. have been derived from a recent primary cosmic ray spectrum. The scaling hypothesis of Feynman has been used in the calculation of meson spectra in the atmosphere. The range–energy relation for muons in sea water, used in the present work, accounts for the muon energy loss in sea water due to collisions, pair production, bremsstrahlung and nuclear interactions. The calculated muon range spectrum in sea water is well in accord with the experimental data obtained by Higashi *et al.* (1966), Davitaev *et al.* (1969), and Rogers and Tristam (1981, 1983).

1. Introduction

The primary cosmic ray spectrum and muon spectrum on Earth help in making an estimate of the high energy interaction parameters above accelerator energies. Usually the primary cosmic rays from space collide with nuclei in the atmosphere at average depths ranging from about $70\,\mathrm{g\,cm^{-2}}$ for protons to about $15\,\mathrm{g\,cm^{-2}}$ for iron nuclei. The atmosphere has a vertical thickness of $1033\,\mathrm{g\,cm^{-2}}$ to sea level, and a large number of generations of interactions can occur with predominantly pions, kaons and nucleons produced in each interaction. Neutral pions decay to γ rays and produce electromagnetic cascades which are not expected to be important sources of high energy muons. Mesons, similar to charged pions and kaons, may decay to produce muons, but they also interact with the atmosphere to produce more hadrons. Cosmic muons arriving at sea level are produced by the decay of pions and kaons generated in the atmosphere in reactions by primary particles with air nuclei. It is of phenomenological interest to derive the muon range spectrum in sea water and in the Earth's crust from the precisely measured primary cosmic ray spectrum by using accelerator results with the scaling hypothesis of Feynman (1969), and the usual meson atmospheric diffusion equations, followed by appropriate range–energy relations. The muon spectrum plays a vital role because it is related to the spectrum of parent mesons such as charged pions and kaons and, through the nuclear interaction characteristics, to the primary spectrum.

The calculation of the range spectrum from the energy spectrum of muons is of two-fold importance, namely to establish the validity of the usual range–energy relation by comparison with the experimental data, and to establish, through different model calculations, whether the calculated muon flux adequately correlates with the primary spectrum.

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In the present work we used our recently estimated primary nucleon spectrum (Bhattacharyya 1983) as the hadron source, and Fermilab accelerator data (Johnson *et al.* 1978) for the $\mathrm{pp} \rightarrow \pi^{\pm} X$ inclusive reaction data with Feynman (1969) scaling, to estimate the production spectrum of pions in the atmosphere. It is evident, from recent measurements on multiparticle production, that Feynman scaling is weakly violated in the central region. However, in the forward region, namely $x_{\mathrm{f}} > 0{\cdot}1$, Feynman scaling is valid within the experimental errors of 10%, as shown by a comparison with the cross sections surveyed by Rushbrook (1982) at ISR at about 2 TeV and the CERN SPS $\mathrm{p\bar{p}}$ collider at 155 TeV energy. Recently Muraki *et al.* (1983) have pointed out that if Feynman scaling is violated then the charged hadron production at the SPS $\mathrm{p\bar{p}}$ collider energy is only 10% less than that of the ISR energies.

In the present paper we apply the meson atmospheric diffusion equation of Ramana Murthy and Subramanian (1972) to calculate the energy spectrum of muons at sea level and then compare the results with the measured data of Holmes *et al.* (1961), Allkofer *et al.* (1971), Nandi and Sinha (1972), Ayre *et al.* (1975) and Green *et al.* (1978). By using the range–energy relation for sea water, the muon spectrum at certain sea depths is estimated and the result compared with those experimentally determined by Higashi *et al.* (1966), Davitaev *et al.* (1969), and Rogers and Tristam (1981, 1983). The composition of sea water is taken to be uniform in density, so that the average atomic weight and atomic number can be obtained accurately. However, a precise determination of the detector depth in sea water is difficult.

2. Nuclear Physics and Kinematics

The primary cosmic rays provide a beam of strongly interacting particles composed of a mixture of different elemental species ranging from protons to iron nuclei. Near the top of the atmosphere most of the primary nuclei divide into their constituents, namely protons and neutrons. The total primary nucleon flux at energy E ($\mathrm{GeV\,n^{-1}}$) has been found to obey the form

$$N(E)\,\mathrm{d}E = AE^{-\gamma}\,\mathrm{d}E\,(\mathrm{cm^2\,s\,sr\,GeV\,n^{-1}})^{-1}. \tag{1}$$

The power law exponent of the differential spectrum of incident nucleons is taken to be constant for the energy range considered, and the production spectrum of pions can be written as

$$\pi(E)\,\mathrm{d}E = (Z_{\mathrm{p}\pi^+} + Z_{\mathrm{p}\pi^-})N(E)\,\mathrm{d}E, \tag{2}$$

where the fractional energy moments for the $\mathrm{pp} \rightarrow \pi^{\pm} X$ inclusive reactions can be estimated from

$$Z_{\mathrm{p}\pi} = \int_0^1 x^{\gamma-2} f_{\mathrm{p}\pi}(x)\,\mathrm{d}x. \tag{3}$$

In very high energy collisions Feynman scaling predicts an energy-independent structure function which can explain the production cross section of a given type of particle, for example meson production by protons striking other protons. The structure function can be expressed as

$$f(x,p_{\mathrm{T}}) = E\,\mathrm{d}^2\sigma/\mathrm{d}p_{\mathrm{L}}\,\mathrm{d}p_{\mathrm{T}} = E\,\mathrm{d}^3\sigma/\mathrm{d}^3p, \tag{4}$$

where p_L and p_T are the centre-of-mass longitudinal and transverse momenta of the secondary particle whose momentum is p, E is the energy of the particle produced, σ is the cross section, and the usual Feynman scaling variable is $x \approx 2p_L s^{-\frac{1}{2}}$. The scaling function is estimated from the Lorentz invariant cross section $E\mathrm{d}^3\sigma/\mathrm{d}^3p$ by by the relation

$$f_{p\pi}(x) = \frac{\pi}{\sigma_{\text{in}}}\int_0^1 E(\mathrm{d}^3\sigma/\mathrm{d}^3p)\,\mathrm{d}p_T^2, \tag{5}$$

where σ_{in} is the total cross section.

3. Meson Atmospheric Diffusion Equation and Muon Flux Calculation

The conventional one-dimensional diffusion equation for the propagation of charged pions in the atmosphere can be written as

$$\delta n_\pi(E,t)/\delta t = -n_\pi(E,t)(1/\lambda_\pi + B_\pi/Et) + \{\pi(E)/\lambda_n\}\exp(-t/\Lambda), \tag{6}$$

where $n_\pi(E,t)$ is the number of charged pions having energies between E and $E+\mathrm{d}E$ at a depth t (g cm^{-2}) in the atmosphere, λ_π is the interaction mean free path of pions, $B_\pi = 118$ GeV is the critical energy for pion decay, which depends on the mass and lifetime of pions and on the density distribution of the atmosphere, while Λ and λ_n are the absorption and interaction mean free paths of nucleons in the atmosphere, taken to be 110 and 80 g cm^{-2} respectively and assumed to be independent of energy. Equation (6) accounts for the loss of pions by interaction and decay, along with the gain of pions by their production in nucleon–nucleon collisions. Pion production by pions has been neglected. The solution of equation (6) is

$$n_\pi(E,t) = \{\pi(E)/\lambda_n\}\exp(-t/\lambda_\pi)\,t^{-B_\pi/E}\int_0^t \exp\{t(1/\lambda_\pi - 1/\Lambda)\}\,t^{B_\pi/E}\,\mathrm{d}t. \tag{7}$$

Assuming $\lambda_\pi = 110\ \mathrm{g\,cm^{-2}} = \Lambda$ we get

$$n_\pi(E,t) = \{\pi(E)/\lambda_n\}\exp(-t/\Lambda)\,t/(1 + B_\pi/E). \tag{8}$$

The muon energy spectrum takes the form

$$\begin{aligned}\mu(E)\,\mathrm{d}E &= \int_0^{1033} n_\pi(E,t)(B_\pi r/Et)r^{\gamma-1}\,\mathrm{d}t \\ &= N(E)(Z_{p\pi^+} + Z_{p\pi^-})(\Lambda/\lambda_n)r^{\gamma-1}\{B_\pi r/(B_\pi r + E)\}h(E,\mathrm{d}E/\mathrm{d}x)\,y(\mathrm{K}/\pi, E), \end{aligned} \tag{9}$$

where $r = 0{\cdot}76$ is the energy degradation factor in $\pi \to \mu$ decay.

The sea-water constituents are the elements H, O, Na, Cl, Mg and S. In general the energy loss rate of muons in sea water due to ionization, pair production, bremsstrahlung and nuclear interactions is given by

$$-\mathrm{d}E/\mathrm{d}x\,|_{\text{total}} = a + bE + c\ln(E_m'/m_\mu c^2)\ \mathrm{MeV\,g^{-1}\,cm^2}, \tag{10}$$

where E_m' is the maximum energy transfer to electrons in sea water, a and c include the muon energy loss due to collisions, while b includes the pair creation, bremsstrahlung and nuclear interactions of muons. The simplified form of the usual range–energy relation follows:

$$E = a'\{\exp(bx) - 1\}/b, \tag{11}$$

where

$$a' = a + c\ln(E'_{\mathrm{m}}/m_\mu c^2),$$

$$b = b_{\mathrm{pair}} + b_{\mathrm{bremss}} + d_{\mathrm{nucl\ int}},$$

$$E'_{\mathrm{m}} = E^2/e(E+eA),$$

with $e = 2{\cdot}718$, $A = 11{\cdot}3$ GeV and $m_\mu c^2 = 0{\cdot}105$ GeV.

Table 1. Parameters for fitting the invariant cross section $E\,\mathrm{d}^3\sigma/\mathrm{d}^3p$ after Johnson *et al.* (1978)

Particle produced	A_{h}	$\bar{n}_{\mathrm{h}}$	m_{h}^2
π^+	0·72	2·86	0·66
π^-	0·55	3·52	0·74

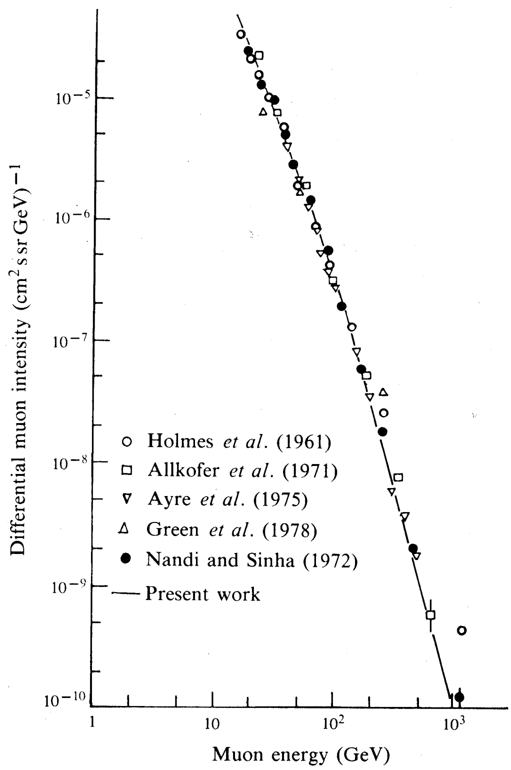

Fig. 1. Calculated muon energy spectrum at sea level, compared with experimental results from other workers.

4. Results and Discussion

The recent Japanese–American Cooperative Emulsion Experiments (Gregory *et al.* 1981) yielded the primary proton and helium spectra in the energy range 2–300 TeV, which followed a power law fit with an integral spectral index of 1·7. The absence of a break in the primary cosmic ray spectrum was also noticed in their measurements. From a closer study of the chemical composition data given by Simon *et al.* (1980)

and Gregory *et al.* (1981), the all-particle primary spectrum with energy E (GeV n^{-1}) has been estimated (Bhattacharyya 1983) to have the form

$$N(E)\,\mathrm{d}E = (2\cdot237\times10^4)E^{-2\cdot7}\,\mathrm{d}E\,(\mathrm{cm}^2\,\mathrm{s\,sr\,GeV\,n}^{-1})^{-1}\,. \qquad (12)$$

The fractional energy moments for the pp $\rightarrow \pi^{\pm}X$ inclusive reactions have been estimated from the approximate scaling distribution in the 100–400 GeV p–p collision Fermilab data of Johnson *et al.* (1978), giving

$$\sigma_{\mathrm{in}}^{-1}\,E\,\mathrm{d}^3\sigma/\mathrm{d}^3p \approx A_{\mathrm{h}}(1-x)^{\bar{n}_{\mathrm{h}}}/(1+p_{\mathrm{T}}^2/m_{\mathrm{h}}^2)^4\,, \qquad (13)$$

where the values of the parameters A_{h}, $\bar{n}_{\mathrm{h}}$ and m_{h}^2 are presented in Table 1 for positively and negatively charged pion production. The estimated fractional energy moments for charged pion production, calculated from the data of Johnson *et al.*, are

$$Z_{\mathrm{p}\pi^+} = 0\cdot039565, \qquad Z_{\mathrm{p}\pi^-} = 0\cdot0264$$

(with spectral index $\gamma = 2\cdot7$).

Table 2. Parameters for the estimation of energy loss rate in sea water

Parameter	Units	Value	Parameter	Units	Value
Atomic number Z		7·43	b_{pair}	$\mathrm{g^{-1}\,cm^2}$ [B]	$1\cdot10\times10^{-6}$
Mass number A		14·79	b_{bremss}	$\mathrm{g^{-1}\,cm^2}$ [B]	$1\cdot30\times10^{-6}$
Density	$\mathrm{g\,cm^{-3}}$	1·025	$d_{\mathrm{nucl\,int}}$	$\mathrm{g^{-1}\,cm^2}$ [B]	$(0\cdot44\text{–}0\cdot55)\times10^{-6}$ [C]
a	$\mathrm{MeV\,g^{-1}\,cm^2}$	1·75 [A]	c	$\mathrm{MeV\,g^{-1}\,cm^2}$	0·085

[A] Hayakawa (1969).
[B] The units of E are MeV.
[C] From Roychoudhury *et al.* (1983) for energy values ranging from 10–200 GeV. The shadowing effect has been accounted for by using the carbon target results of Goodman *et al.* (1981), namely $A_{\mathrm{eff}}/A = 0\cdot67$.

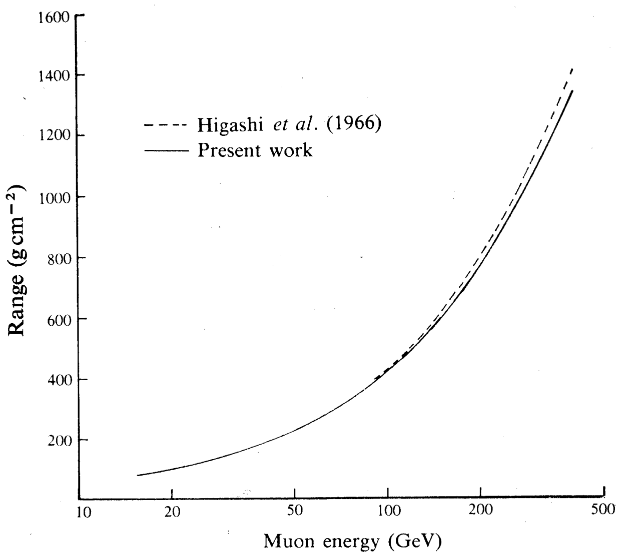

Fig. 2. Calculated range–energy curve for muons in sea water compared with that found by Higashi *et al.* (1966).

The survival probability $h(E, \mathrm{d}E/\mathrm{d}x)$ (see equation 9) of muons produced at a mean depth of 100 g cm^{-2} and reaching sea level has been computed. The function $y(\mathrm{K}/\pi, E)$ gives the contribution of muons from the K $\rightarrow \mu_2$ decay mode and our adopted value of K/π is 0·2. By using the expression (9), the sea-level muon spectrum has been estimated and is shown in Fig. 1 along with the magnetic spectrograph results of other workers.

The fair agreement of our calculated results with the experimental data in Fig. 1 encouraged us to correlate the muon energy spectrum with the sea-water depth–intensity results by using a modified range–energy relation for sea water. Table 2 gives the parameters of the range–energy relation for sea water adopted in this analysis. Fig. 2 shows our calculation for the range–energy relation of muons in sea water along with that calculated by Higashi *et al.* (1966). In a previous survey Pal and Bhattacharyya (1984) found that the sea-water range–energy results of Higashi *et al.* (1966) lie appreciably above those of Miyake *et al.* (1964) and ours for the Kolar Gold Field high density rock. In the present case we have chosen sea water for its uniform density, although accurate deep-sea depth determination is an extremely difficult problem for oceanographers.

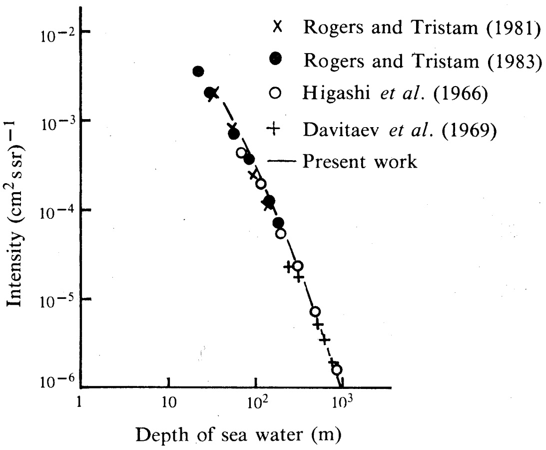

Fig. 3. Calculated muon intensity as a function of sea-water depth, compared with the experimental results from other workers.

In the present study the low range muon intensity in sea water has been calculated from the derived muon energy spectrum and is shown in Fig. 3 along with the experimental data of other workers. It is evident from Fig. 3 that the calculated muon intensity is in accord with the experimental data, which leads to the conclusion that the sea-water depth–intensity data can be well correlated with the latest precise primary cosmic ray spectrum.

5. Conclusions

The sea-water depth–intensity spectrum, calculated from the recent primary spectrum by using Fermilab accelerator data in the framework of Feynman scaling and from our range–energy relation for muons in sea water, is in agreement with the experimental data of Higashi *et al.* (1966), Davitaev *et al.* (1969) and Rogers and Tristam (1981, 1983). This agreement supports our adopted muon energy loss rate due to nuclear interactions of muons in sea water, as expected from the quantum chromodynamical based ξ-scaling model of Georgi and Politzer (1976).

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