Aust. J. Phys., 1985, **38**, 233–7

A Modified Solar Collector Flow Factor

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Abstract

The mean fluid temperature in a solar collector is often approximated by the average of the inlet and outlet temperatures. When this is done a simpler expression for the collector flow factor can be derived, the use of which permits the formal proof that collector efficiency will decrease with any maldistribution of fluid flow in the risers of the collector.

1. Introduction

The thermal efficiency η of a flat-plate collector is calculated from the inlet and outlet temperatures T_i and T_o and the total mass flow rate $\dot{m}$ as

$$\eta = \dot{m}C_p(T_o - T_i)/GA\,, \tag{1}$$

where C_p is the specific heat of the working fluid (J kg^{-1} °C^{-1}), G is the incident solar radiation (W m^{-2}) and A is the intercepting area of the collector (m^2). The efficiency is also expressed in terms of experimentally derived collector factors $\tau\alpha$, F', F''_f and U_t as (Duffie and Beckman 1980)

$$\eta = F'\{\tau\alpha - U_t(T_f - T_a)/G\} \tag{2}$$

$$= F''_f F'\{\tau\alpha - U_t(T_i - T_a)/G\}\,, \tag{3}$$

where F' is the collector efficiency factor, $\tau\alpha$ the collector transmission–absorptance product, U_t the collector heat loss coefficient (W m^{-2} °C^{-1}), T_f the mean fluid temperature (°C), T_a the ambient air temperature (°C) and F''_f the collector flow factor.

Equation (1) defines the efficiency of any heat collecting device while (2) and (3), which arise from heat transfer analysis of flat-plate collectors, give the efficiency in terms of specific collector factors. These last two expressions show how the efficiency will vary with changes, for example, in the collector factors $\tau\alpha$ and U_t. Equation (2) uses the mean fluid temperature as the main temperature parameter, while (3) uses the more readily measured fluid inlet temperature. All three expressions for the efficiency are entirely equivalent.

0004-9506/85/020233$02.00

The collector flow factor is defined by

$$F_{\mathrm{f}}'' = \mu\{1-\exp(-1/\mu)\}, \tag{4}$$

where

$$\mu = \dot{m}C_p/F'U_{\mathrm{t}}A, \tag{5}$$

and, by using equations (1)–(3), the mean fluid temperature can be written as

$$T_{\mathrm{f}} = T_{\mathrm{i}} + (T_{\mathrm{o}} - T_{\mathrm{i}})\mu(1-F_{\mathrm{f}}'')/F_{\mathrm{f}}''. \tag{6}$$

We note that if we write $\xi = \mu(1-F_{\mathrm{f}}'')/F_{\mathrm{f}}'' = 0\cdot5$, then $T_{\mathrm{f}} = \frac{1}{2}(T_{\mathrm{o}}+T_{\mathrm{i}}) = T_{\mathrm{m}}$, i.e. the mean of the inlet and outlet temperatures. Further, as shown in Fig. 1, the parameter ξ tends to $0\cdot5$ as μ increases and, in the normal range of operation of solar collectors, μ lies between 10 and 15. Thus T_{m} is a good approximation to T_{f}, lying within 5% over this range.

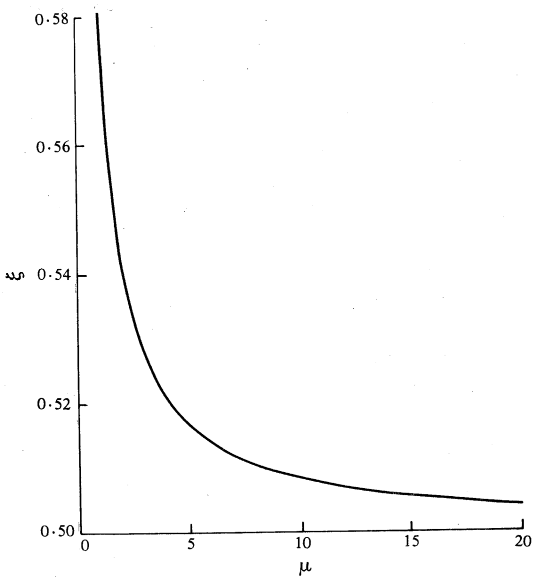

Fig. 1. The parameter $\xi = \mu(1-F_{\mathrm{f}}'')/F_{\mathrm{f}}''$ as a function of the flow parameter μ. For this illustration the collector efficiency factor F' was chosen as $0\cdot93$ and the heat loss coefficient was taken as $9\cdot0\ \mathrm{W\,m^{-2}\,{}^{\circ}C^{-1}}$.

The experimental determination of the collector characteristics F' and U_{t} often involves the approximation $T_{\mathrm{f}} = T_{\mathrm{m}}$ and, for $T_{\mathrm{o}} - T_{\mathrm{i}}$ in the range 5–10°C, this is recommended by the Australian Standard for collector testing.* The usefulness of the approximation lies in its applicability to the determination of F' and U_{t} for a given $\tau\alpha$ from equation (2) when all the quantities on the RHS of equation (1) are measured. When it is necessary to calculate T_{f} from equation (6), the unknown quantities F' and U_{t} must be found by a self-consistent iterative procedure.

* Australian Standard 2535 (1982). Glazed flat-plate solar collector with water as the heat-transfer fluid—method for testing thermal performance (Standards Assn of Australia).

2. Modified Flow Factor

Substitution of the expression $T_f = \frac{1}{2}(T_o + T_i)$ into equation (2) yields

$$\eta = (F'U_t/G)\{\tau\alpha G/U_t - \tfrac{1}{2}(T_o + T_i) + T_a\}$$

$$= (F'U_t/G)\{\tau\alpha G/U_t - \tfrac{1}{2}(T_o - T_i) - T_i + T_a\}, \quad (7)$$

and substitution of equation (5) into (1) yields

$$\eta = (F'U_t/G)\{\mu(T_o - T_i)\}. \quad (8)$$

Use of this expression in (7) gives

$$T_o - T_i = (\mu + \tfrac{1}{2})^{-1}\{\tau\alpha G/U_t - (T_i - T_a)\},$$

which inserted into (8) gives

$$\eta = (\mu F'U_t/G)\{\tau\alpha G/U_t - (T_i - T_a)\}/(\mu + \tfrac{1}{2})$$

$$= \{1 + (2\mu)^{-1}\}^{-1} F'\{\tau\alpha - U_t(T_i - T_a)/G\}$$

$$= F''_m F'\{\tau\alpha - U_t(T_i - T_a)/G\}, \quad (9)$$

where F''_m is the modified flow factor:

$$F''_m = \{1 + (2\mu)^{-1}\}^{-1}. \quad (10)$$

The modified flow factor appears to be different from F''_f defined by equation (4) but, in fact, in the operating range of solar collectors with $\mu > 10$, the two expressions are almost identical as a series expansion of the exponential function shows.

Thus in many circumstances it makes very little difference whether F''_f or F''_m is taken as the collector flow factor but, as explained above, when F''_m is used the collector factor F' and the heat loss coefficient U_t are much more readily found from experiment.

3. Flow Distribution and Efficiency

We consider a single tube-fin element of a flat-plate collector of area ΔA and assume that the collector coefficients F' and U_t apply locally over the collector surface. The efficiency of the kth riser ($k = 1 \rightarrow N$) is given by

$$\eta_k = F''_m F'\{\tau\alpha - U_t(T_i - T_a)/G\}.$$

The flow rate is usually non-uniform in the different risers (McPhedran *et al.* 1983) and the mean collector efficiency may be written

$$\eta_m = I\left(\sum_{k=1}^{N} F''_m(\dot{m}_k)\right)/N. \quad (11)$$

Here we have

$$I = F'\{\tau\alpha - U_t(T_i - T_a)/G\},$$

$$\dot{m}_k = \dot{m}_t/N + \Delta_k = \dot{m}_o + \Delta_k,$$

where $\dot{m}_k$ is the flow rate in the kth riser, Δ_k is the deviation from the mean and

$$\sum_{k=1}^{N} \Delta_k = 0.$$

Substitution for μ in equation (10) gives

$$\begin{aligned} F''_m &= [1 + \{2\dot{m}_k C_p/(F' U_t \Delta A)\}^{-1}]^{-1} \\ &= \{1 + (\beta \dot{m}_k)^{-1}\}^{-1}, \qquad \beta = 2C_p/(F' U_t \Delta A). \end{aligned}$$

Hence, we obtain the expression for the mean collector efficiency

$$\eta_m = (I/N) \sum_{k=1}^{N} \{1 + (\beta \dot{m}_k)^{-1}\}^{-1}.$$

With some manipulation this becomes

$$\eta_m = I[1 - (1 + \beta \dot{m}_o)^{-1} N^{-1} \sum_{k=1}^{N} \{1 + \beta \Delta_k/(1 + \beta \dot{m}_o)\}^{-1}].$$

When the flow is uniform, $\Delta_k = 0$ for all k and we define $\eta_m = \eta_u$; then

$$\eta_u - \eta_m = I[1 - (1 + \beta \dot{m}_o)^{-1} - 1 + (1 + \beta \dot{m}_o)^{-1} N^{-1} \sum_{k=1}^{N} \{1 + \beta \Delta_k/(1 + \beta \dot{m}_o)\}^{-1}].$$

Again, with some manipulation this becomes

$$\eta_u - \eta_m = (I/N) \frac{1}{1 + \beta \dot{m}_o} \sum_{k=1}^{N} \{-\beta \Delta_k/(1 + \beta \dot{m}_k)\}. \tag{12}$$

In this summation the negative Δ_k will contribute more than the positive values because for negative Δ_k the denominator will be smaller. This means that the expression (12) is always positive. A formal proof is given in the Appendix. This shows that any deviation from identical flow rates in the various risers of a flat-plate collector will decrease the overall efficiency of the collector.

4. Conclusions

In many cases of potential interest the mean fluid temperature in a flat-plate collector may be taken as the arithmetic mean of the collector inlet and outlet temperatures. This allows ready determination of the important collector characteristics F' and U_t. In the same circumstances a modified collector flow factor may be defined, the use of which facilitates a proof that deviations from equal flow rates in each of the risers of the collector always decrease the efficiency. Further, the relative effects of different (known) flow configurations on the efficiency can be determined using equation (11).

Acknowledgment

M. J. O'Keefe gratefully acknowledges financial support of a Monash University Graduate Scholarship.

References

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Appendix

We consider the summation in equation (12):

$$\sum_{k=1}^{N} \{-\beta\Delta_k(1+\beta\dot{m}_k)^{-1}\}.$$

If the Δ_k are divided into negative and positive components as

$$\Delta_j^- < 0 \qquad \text{for all } j = 1\text{–}N^-,$$

$$\Delta_l^+ > 0 \qquad \text{for all } l = 1\text{–}N^+,$$

then

$$\sum_{k=1}^{N} \Delta_k = \sum_{j=1}^{N^-} \Delta_j^- + \sum_{l=1}^{N^+} \Delta_l^+ = -C+C = 0,$$

where C is a positive constant. There exists a number K^- and a number K^+ satisfying

$$\dot{m}_o+\Delta_j^- < K^- < \dot{m}_o < K^+ < \dot{m}_o+\Delta_l^+,$$

for all j and l, so that

$$\sum_{k=1}^{N} \{-\beta\Delta_k(1+\beta\dot{m}_k)^{-1}\} = \sum_{j=1}^{N^-} [-\beta\Delta_j^-\{1+\beta(\dot{m}_o+\Delta_j^-)^{-1}\}]$$

$$+ \sum_{l=1}^{N^+} [-\beta\Delta_l^+\{1+\beta(\dot{m}_o+\Delta_l^+)^{-1}\}]$$

$$> -\beta(1+\beta K^-)^{-1} \sum_{j=1}^{N-} \Delta_l^- + (-\beta)(1+\beta K^+)^{-1} \sum_{l=1}^{N^+} \Delta_l^+$$

$$= \beta C\{(1+\beta K^-)^{-1} - (1+\beta K^+)^{-1}\}$$

$$> 0 \quad \text{since} \quad K^- < K^+.$$

Manuscript received 19 December 1984, accepted 8 February 1985