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States of ^{5}He and ^{5}Li

F. C. Barker[A] *and C. L. Woods*[B]

[A] Department of Theoretical Physics, Research School of Physical Sciences, Australian National University, G.P.O. Box 4, Canberra, A.C.T. 2601.
[B] Department of Nuclear Physics, Research School of Physical Sciences, Australian National University, G.P.O. Box 4, Canberra, A.C.T. 2601.

Abstract

A recent shell model calculation has predicted a low-lying $\frac{1}{2}^+$ level in ^{5}He, in apparent conflict with experimental results. We have confirmed the prediction with an alternative shell model interaction chosen specifically to fit properties of light nuclei. The level is expected to be very broad so that it would not be easily seen in reactions. It should contribute significantly to n–α phase shifts, but did not appear in previous analyses because of the small value of about 3 fm chosen for the channel radius a. A $\frac{1}{2}^+$ level at the expected energy of about 7 MeV is obtained for $a \approx 5{\cdot}1$ fm. Broad $\frac{3}{2}^+$ and $\frac{5}{2}^+$ levels are then found at about 14 MeV in agreement with the shell model calculations. Properties of the low-lying $\frac{3}{2}^-$ and $\frac{1}{2}^-$ levels in ^{5}He and ^{5}Li are discussed, but the measured values now available do not favour a particular value of the channel radius.

1. Introduction

It is generally believed that the only states of ^{5}He and ^{5}Li below the threshold for deuteron breakup at about 17 MeV excitation are the $\frac{3}{2}^-$ ground state and $\frac{1}{2}^-$ first-excited state at about 5 MeV. The energy level diagrams are shown in Fig. 1. Thus it was rather surprising when a recent shell model calculation (van Hees and Glaudemans 1983, 1984), which in general gave good agreement with experiment for the energies and electromagnetic properties of both normal and non-normal parity levels throughout the 1p-shell nuclei, predicted a $\frac{1}{2}^+$ state of ^{5}He (and ^{5}Li) at about $8{\cdot}5$ MeV excitation. An immediate reaction was to suppose that the shell model interaction should be changed to increase the energy of the $\frac{1}{2}^+$ state by 10 MeV or more; earlier shell model calculations had predicted the lowest $\frac{1}{2}^+$ state above the deuteron threshold. We here investigate an alternative possibility, that there is indeed a low-lying $\frac{1}{2}^+$ level that has not previously been identified experimentally.

The predicted $\frac{1}{2}^+$ level is expected to be very broad because a dominant component has the configuration of ^{4}He ground state plus a 2s nucleon. Information about low-lying states of ^{5}He and ^{5}Li comes from only two sources—the elastic scattering of nucleons on ^{4}He, and reactions that populate these nuclei as final (unbound) states. In such reactions, it is very difficult to distinguish between contributions from very broad levels and from alternative reaction modes. Phase shifts extracted from elastic scattering data should, however, contain information about such a broad

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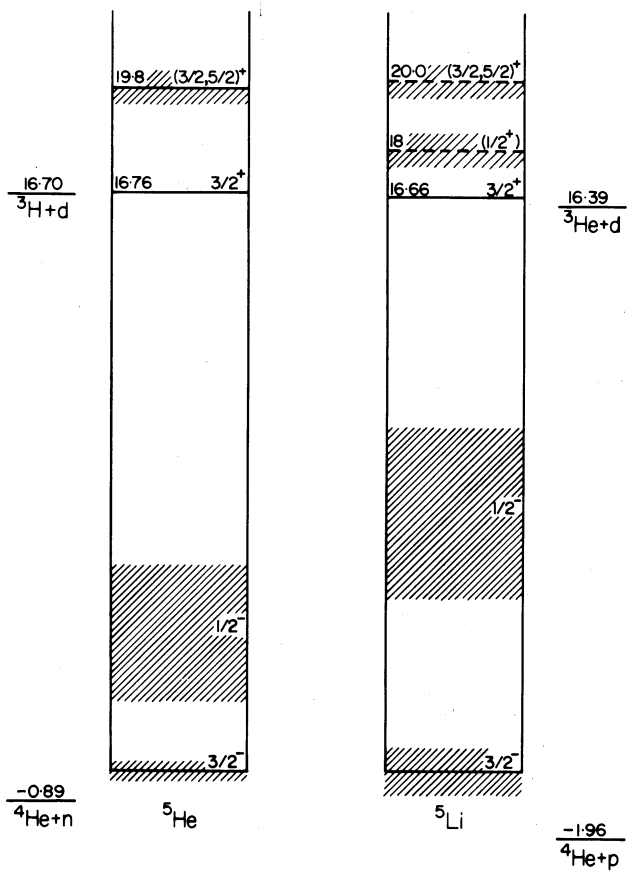

Fig. 1. Energy level diagrams of ^{5}He and ^{5}Li (from Ajzenberg-Selove 1984). The cross-hatching represents the level width for negative-parity states, but serves only to indicate the very broad positive-parity states.

state. Unambiguous values of the n–α and p–α phase shifts for energies up to the deuteron threshold are available (see Bond and Firk 1977; Dodder *et al.* 1977 and references therein). Generally the energy dependence of the phase shifts has been described in terms of R-matrix parameters, one of which is the channel radius a. Ever since Adair (1952) and Dodder and Gammel (1952) chose $a = 2{\cdot}9$ fm in order to fit the $p_{3/2}$ and $p_{1/2}$ phase shifts with one-level approximations, and to account for the Coulomb energy difference between ^{5}Li and ^{5}He, most R-matrix analyses have assumed a value of a near $2{\cdot}9$ fm. This is considerably smaller than the conventional minimum value of the channel radius (Lane and Thomas 1958; Lane 1960), given by $a = 1{\cdot}45(A_1^{1/3}+A_2^{1/3})$ fm $= 3{\cdot}75$ fm. The experimental $s_{1/2}$ phase shifts are well represented by hard-sphere scattering with $a \approx 2{\cdot}5$ fm (Barnard *et al.* 1964; Brown *et al.* 1967; Morgan and Walter 1968), consequently fitting them with $a \approx 2{\cdot}9$ fm would imply a small resonance contribution and so a very high-energy $\frac{1}{2}^+$ level, e.g. at about 28 MeV in ^{5}Li (Dodder *et al.* 1977). The assumption of a larger value of a

would lead to a lower energy $\frac{1}{2}^+$ level, and a simple calculation shows that a $\frac{1}{2}^+$ level at 8·5 MeV would occur for $a \approx 5$ fm. Such a value is not *a priori* unreasonable. Use of $a = 5$ fm would necessitate background contributions to the $p_{3/2}$ and $p_{1/2}$ phase shifts, corresponding to broad higher energy $\frac{3}{2}^-$ and $\frac{1}{2}^-$ states, and would lead to changed parameter values for the low-lying $\frac{3}{2}^-$ and $\frac{1}{2}^-$ states, as well as for the broad d-wave $\frac{3}{2}^+$ and $\frac{5}{2}^+$ states previously found to be well above the deuteron threshold.

In the next section we discuss the reliability of the prediction by van Hees and Glaudemans (1984) of a low-lying $\frac{1}{2}^+$ level in ^{5}He and ^{5}Li, on the basis of the quality of their agreement with the observed properties of these and neighbouring nuclei. We also consider another interaction that is less general than that of van Hees and Glaudemans, but gives a better fit to the properties of these light nuclei; this interaction also predicts a low-lying $\frac{1}{2}^+$ level.

In Section 3, the s-, p- and d-wave nucleon–alpha phase shifts are fitted with R-matrix formulae for a range of values of channel radii, and the value of a is selected that locates the lowest $\frac{1}{2}^+$ level in ^{5}He and ^{5}Li at the value required by the shell model calculations. Properties of the p- and d-wave states for this value of a are compared with the shell model calculations and with experimental results from various reactions.

2. Shell Model Calculations

In their shell model calculation of the properties of both normal and non-normal parity states of the 1p-shell nuclei, van Hees and Glaudemans (1983, 1984), henceforth referred to as Papers I and II, worked within the $(0+1)\hbar\omega$ model space and used a translationally invariant treatment that completely eliminated spurious states. Best values of the 22 relative matrix elements of the interaction and of the harmonic oscillator size parameter $\hbar\omega$ were obtained by fitting the energies of 136 levels of the $A = 4$–16 nuclei, the r.m.s. deviation between theory and experiment being less than 0·7 MeV. Electromagnetic properties of the calculated states in general agree well with experiment, provided an effective mass is used in calculating r.m.s. charge radii and effective (isoscalar) charges are used for E2 and E3 transitions.

Van Hees and Glaudemans assumed the interaction matrix elements and $\hbar\omega$ to be mass independent throughout the range $A = 4$–16. Some evidence that this may not be a good approximation for the smaller A values comes from the values of the r.m.s. charge radii (see I, Fig. 1). As we are particularly interested in the region near $A = 5$, we consider in some detail the predictions that are based on the van Hees and Glaudemans interaction for the lighter nuclei (say $A = 4$–7). Since values of the one-body spectroscopic factors and some of the E1 transition probabilities are not published in I and II, we have obtained them by repeating the van Hees and Glaudemans calculation, using the Oxford–Buenos Aires shell model code (Rae *et al.* 1983).

For comparison, we also use an alternative shell model interaction chosen particularly to fit properties of the light nuclei, without requiring the generality that the van Hees and Glaudemans interaction has throughout the whole 1p shell, since we are mainly interested in the energy predicted for the lowest $\frac{1}{2}^+$ state in ^{5}He.

Several shell model interactions are available for normal-parity states of the light nuclei of the 1p shell. Kumar (1974) has discussed some of these in detail. The

interactions of Cohen and Kurath (1965) were obtained by fitting data from $A = 6$–16 or $A = 8$–16 nuclei, and consequently do not give a very good fit to properties of the $A = 5$–7 nuclei. Kumar (1974) fitted properties of $A = 6$–9 nuclei with the one set of interaction parameters, including energy shifts of the ground-state rotational bands in ^{8}Be and ^{9}Be, while Barker (1966) adjusted a smaller set of interaction parameters separately for each A value to fit $A = 6$–9 properties. Kumar's fit is better than Barker's for $A = 8$ and 9, but not for 6 and 7. In particular, Kumar predicts the second 1^+, $T = 0$ state of ^{6}Li nearly 4 MeV too high (although this could probably be remedied by adjusting parameter values without spoiling other agreement). Also his calculation gives two $\frac{5}{2}^-$ states of ^{7}Li at the correct energies near 7 MeV, but they do not have the observed properties; the lower level should be essentially of α + t structure with a very small ^{6}Li(g.s.) + n spectroscopic factor, and the upper level should have a small α + t component and an appreciable neutron spectroscopic factor (Ajzenberg-Selove 1984). Barker fitted these properties satisfactorily but at the expense of having interaction parameters that depend on A. The parameter values for $A = 6$ and 7 are rather similar, as are those for $A = 8$ and 9. Most of the agreement with experiment for $A = 6$ can be retained if the $A = 7$ interaction parameters (given in equation 11 of Barker 1966) are also used for calculating $A = 6$ properties—the main change from the properties given by Barker for $A = 6$ is that the 0^+, $T = 1$ state is calculated about 0·6 MeV too low (the energies of the four highest states are increased by 1–4 MeV, but this is not significant because these states are unobserved). On the other hand, use of the $A = 6$ parameter values for $A = 7$ leads to poor agreement with the experimental energies and also inverted $\frac{5}{2}^-$ states. Thus, we assume that the interaction between 1p-shell nucleons is given by the $A = 7$ interaction parameters of Barker (1966); these include the $p_{1/2}$–$p_{3/2}$ single-particle energy difference.

For all other matrix elements, we use the Millener and Kurath (1975) interaction, which has been used successfully to explain properties of many non-normal parity levels of $A = 9$–16 nuclei (Millener and Kurath 1975; Teeters and Kurath 1977; Jäger and Kirchbach 1977). Values of these matrix elements in a harmonic oscillator basis and of the single-particle energies appropriate to a no-core shell model calculation were supplied by B. A. Brown (personal communication). In our calculation we retain these values of the 1s and 2s–1d single-particle energies relative to the mean 1p single-particle energy, except that the 1s single-particle energy is increased by 0·54 MeV for the reason given below. As explained in I, such a set of interaction matrix elements and single-particle energies does not have the self-consistency of a translationally invariant interaction, so that spurious and nonspurious states will be mixed. The spurious states are here removed by the method of Gloeckner and Lawson (1974).

We now compare the predictions for the van Hees and Glaudemans interaction and for this alternative interaction with the experimental values for ^{4}He, ^{5}He, ^{6}Li and ^{7}Li separately. From here on, calculated excitation energies are taken relative to the calculated ground-state energy (rather than the experimental ground-state energy, as used in II, Figs 5–7). The quoted values of spectroscopic factors are those calculated directly from the shell model wavefunctions; for nonspurious states in a harmonic oscillator model, the spectroscopic factor defined in terms of relative coordinate wavefunctions is greater by the factor $\{A/(A-1)\}^{\lambda}$, where $\lambda = 1$ for 1p nucleons and $\lambda = 2$ for 2s and 1d nucleons (Anyas-Weiss *et al.* 1974; Millener *et al.* 1983).

(a) States of ^{4}He

The energies of the negative-parity states of ^{4}He predicted by the van Hees and Glaudemans interaction appear from Fig. 5 in II to be in rather good agreement with the experimental values, which were taken from Fiarman and Meyerhof (1973). Some caution should be exercised, however, in accepting these experimental values; those attributed to Werntz and Meyerhof (1968) are values of eigenenergies (from their Fig. 1) rather than the more physical resonance energies (given in their Fig. 2), which are some 2 MeV lower. Also the 1^-, $T = 0$ level, which Werntz and Meyerhof located at 28·5 MeV (resonance energy), has recently been observed at 24·1 MeV (Grüebler *et al.* 1981). These modified experimental energies are shown in Fig. 2*a*, in comparison with the predictions from II and from the alternative interaction. There is reasonable agreement for both interactions. In each case the upper of the two predicted 1^-, $T = 1$ states carries over 80% of the E1 strength to the ^{4}He ground state, in agreement with solution I of Werntz and Meyerhof and with other data (see Fiarman and Meyerhof 1973; Barker 1984).

(b) States of ^{5}He

The calculated spectra of normal and non-normal parity states of ^{5}He from II and for the alternative interaction are compared with experiment (Ajzenberg-Selove 1984) in Fig. 2*b*. The splitting of the low-lying $\frac{1}{2}^- - \frac{3}{2}^-$ doublet is calculated to be 3·1 MeV in II and 2·50 MeV for the alternative interaction, in comparison with the accepted experimental value of 4 ± 1 MeV; see, however, Section 3 below. Both the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states have neutron spectroscopic factors of unity. The small width ($\approx 0{\cdot}1$ MeV) of the $\frac{3}{2}^+$ level observed at 16·76 MeV implies a very small spectroscopic factor $\mathscr{S}$ for the neutron channel ($\mathscr{S} \lesssim 0{\cdot}02$); the lowest calculated $\frac{3}{2}^+$ state in II is at 15·1 MeV and has $\mathscr{S} = 0{\cdot}29$ while the next $\frac{3}{2}^+$ state at 17·9 MeV has $\mathscr{S} = 0{\cdot}11$, and therefore neither can readily be identified with the experimental level. For the alternative interaction, the lowest $\frac{3}{2}^+$ state at 14·51 MeV has $\mathscr{S} = 0{\cdot}58$, and the next $\frac{3}{2}^+$ state at 16·76 MeV has $\mathscr{S} = 0{\cdot}005$; because of this small $\mathscr{S}$ value, the second $\frac{3}{2}^+$ state has been identified with the observed level, and the 1s single-particle energy was adjusted to make the energies agree.

The alternative interaction predicts the lowest $\frac{1}{2}^+$ state at 6·35 MeV, compared with the value 7·4 MeV from II. Such a state would be very broad, since it has a ^{4}He + n s-wave spectroscopic factor of 0·6, the maximum possible for a nonspurious state. Similar broad $\frac{3}{2}^+$ and $\frac{5}{2}^+$ states with d-wave spectroscopic factors of about 0·6 are predicted by both interactions at 13–15 MeV.

(c) States of ^{6}Li

The calculated and experimental (Ajzenberg-Selove 1984) level spectra of ^{6}Li are shown in Fig. 2*c*. Although the level sequence of normal-parity states calculated in II is correct, the observed spacings are not well reproduced. The first- and second-excited states are each about 1·3 MeV too low, while the third-excited state is 1·3 MeV too high, these discrepancies being rather similar to those found for the (8–16)POT interaction of Cohen and Kurath (1965). As mentioned in II, the second 1^+, $T = 0$ state is predicted about 1·5 MeV too high. There is better agreement with experiment for the alternative interaction.

Non-normal parity states predicted in II are at 9 MeV and above for $T = 0$ and

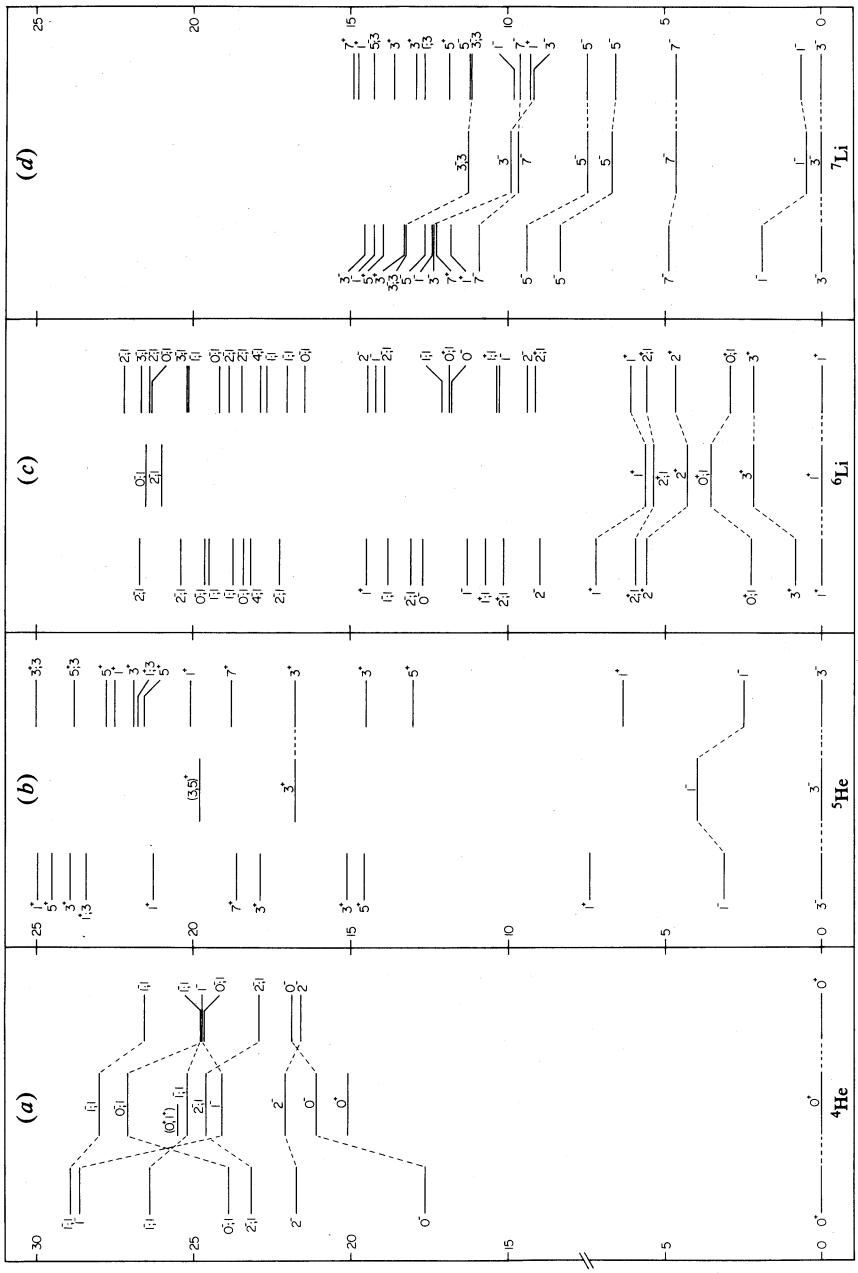

Fig. 2. Spectra of ^{4}He, ^{5}He, ^{6}Li and ^{7}Li. Each state is labelled by J^π; T (or $2J^\pi$; $2T$ for odd-A nuclei). The value of T is not given for $T = T_z$. Experimental data in the central columns are taken from Fiarman and Meyerhof (1973), modified as explained in Section 2, and Ajzenberg-Selove (1984). The calculated values from II and for the alternative interaction of Section 2 are shown on the left- and right-hand sides respectively. The values from II are relative to the calculated ground-state energy (in contrast with those given in Figs 5–7 of II). A dashed line connects a calculated state with the experimental level with which it is identified. For clarity, only negative-parity $T = 1$ states of ^{6}Li are shown above 15 MeV. No states are shown in ^{5}He, ^{6}Li and ^{7}Li above 25, 23 and 15 MeV respectively.

13 MeV and above for $T = 1$. The photoneutron cross section of ^{6}Li (Berman *et al.* 1965; Dytlewski *et al.* 1984) shows broad peaks at 12 and 16 MeV, indicating E1 absorption to $T = 1$ negative-parity levels, or groups of levels, of ^{6}Li at these energies; the decrease in the photoneutron cross section above 16 MeV could well be due to the opening of the t+^{3}He channel at 15·8 MeV, so that the total photoabsorption cross section would peak at about 20 MeV (Shin *et al.* 1975) rather than 16 MeV. In II, the lowest predicted states with large E1 strengths are the ($T = 1$) 2^- state at 13·1 MeV, 1^- at 18·7 MeV and 2^- at 21·7 MeV, and thus it would seem that these are 1 MeV or more too high in energy. The calculated E1 strength of the 13·1 MeV state is somewhat smaller than that required to fit the observed 12 MeV peak; however, the main contribution to the strength comes from a $1p_{3/2} \rightarrow 1d_{5/2}$ excitation and involves small components of the 2^- wavefunction. For the alternative interaction, the lowest negative-parity $T = 1$ state is a 1^- state at 12·1 MeV, but this has a rather small E1 strength to the ground state, and the low-energy photoabsorption cross section of ^{6}Li should be dominated by the contribution from the 2^-, $T = 1$ state at 13·91 MeV. Other $T = 1$ states with large E1 strengths are calculated to lie at 18·9, 20·1 and 22·2 MeV. Thus, the calculated energy of the 13·9 MeV state is about 2 MeV too high, and also its E1 strength is less than that of the corresponding state in II, so that in both these respects the alternative interaction gives a greater discrepancy than II.

(d) States of ^{7}Li

The calculated and experimental (Ajzenberg-Selove 1984) level spectra of ^{7}Li are shown in Fig. 2*d*. The spectrum of normal-parity states calculated in II shows discrepancies with experiment similar to those for the Cohen and Kurath (8–16)POT interaction. In particular the separation of the lowest two states is too large by a factor of 4. The $\frac{5}{2}^-$ states are close together, as required, but the neutron spectroscopic factor of the lower state is four times that of the upper state, contrary to observation. The alternative interaction gives a good fit to the measured properties of the normal-parity levels, as in Barker (1966).

The lowest non-normal parity state of ^{7}Li is calculated in II to be a $\frac{1}{2}^+$ state at 11·8 MeV, followed by $\frac{7}{2}^+$, $\frac{3}{2}^+$ and $\frac{5}{2}^+$ states spread over the next 2 MeV. These energies agree well with those calculated by Aswad *et al.* (1973). The lowest states for the alternative interaction are somewhat lower, with a $\frac{1}{2}^+$ state at 9·28 MeV, $\frac{5}{2}^+$ at 11·86 MeV and $\frac{3}{2}^+$ at 12·92 MeV. The only experimental non-normal parity level given in II (Fig. 7) is a suggested $\frac{3}{2}^+$ level at 9·5 MeV, taken from Ajzenberg-Selove (1979). The published evidence for this level came from an *R*-matrix analysis of ^{6}Li(n, n) and ^{6}Li(n, α) data by Holt *et al.* (1975), in which it was assumed that the ^{7}Li level at about 6·6 MeV was $\frac{1}{2}^+$, as was once thought to be a possibility (Ajzenberg-Selove and Lauritsen 1959), although it is now established to be $\frac{5}{2}^-$ (Ajzenberg-Selove 1984). Two similar *R*-matrix analyses have been published recently, each requiring a $\frac{1}{2}^+$ level below a $\frac{3}{2}^+$ level, but at somewhat different energies. Chiu and Firk (1981) found a $\frac{1}{2}^+$ level at 6·25 MeV, making it bound with respect to the ^{6}Li(g.s.)+n channel, and a $\frac{3}{2}^+$ level at 9·38 MeV, while Knox and Lane (1983) required an unbound $\frac{1}{2}^+$ level at 8·81 MeV and a $\frac{3}{2}^+$ level at 9·97 MeV. The data fitted in these two analyses were not identical, and also different assumptions and approximations were made. Knox and Lane have mentioned some of these differences. It seems, however, to be the choice of channel radius a_n for the ^{6}Li+n channel that is critical in determining whether the $\frac{1}{2}^+$ level is bound or

unbound. In the one-level approximation, which was used for both $\frac{1}{2}^+$ and $\frac{3}{2}^+$ states by Knox and Lane and (approximately) by Chiu and Firk, the s-wave scattering length in the ^{6}Li+n channel with channel spin s ($=\frac{1}{2}, \frac{3}{2}$) is given by (see e.g. Barker 1977, equations 10 and 11)

$$a_s \equiv x_s + i\, y_s = a_n\left(1 - \frac{\gamma_{1n}^2}{E_1 - \sum_c \{S_c(0) - B_c + i\, P_c(0)\}\gamma_{1c}^2}\right). \quad (1)$$

Thus the $\frac{1}{2}^+$ level will be unbound if $a_n > x_{1/2}$ or bound if $a_n < x_{1/2}$. Experimental values of $x_{1/2}$ are 3·98 fm (Barker 1980) and 3·88 fm (Knox and Lane 1983). The different choices $a_n = 3{\cdot}9$ fm by Chiu and Firk and $a_n = 4{\cdot}2$ fm by Knox and Lane therefore account for their different energies for the $\frac{1}{2}^+$ level; since $x_{3/2} = 0{\cdot}72$ fm (Barker 1980) or 1·15 fm (Knox and Lane 1983), the $\frac{3}{2}^+$ level is unbound in both analyses. This sensitivity to the channel radius means that a precise value of the $\frac{1}{2}^+$ energy is unlikely to come from analyses of this type. As Knox and Lane pointed out, recent ^{7}Li(γ, t)^{4}He measurements (Skopik *et al.* 1979; Junghans *et al.* 1979) show a broad peak at about 8 MeV, suggesting E1 absorption to $\frac{1}{2}^+$, $\frac{3}{2}^+$ or $\frac{5}{2}^+$ states of ^{7}Li at about 8 MeV excitation energy. Legendre polynomial fits to the angular distributions give an a_2 coefficient that is substantially negative, being about $-0{\cdot}6$ (Skopik *et al.* 1979) or $-0{\cdot}2$ (Junghans *et al.* 1979), which indicates an appreciable $\frac{5}{2}^+$ contribution (Junghans *et al.* 1979). This is consistent with all the calculations, which give large B(E1↑) values for the lowest $\frac{1}{2}^+$ and $\frac{5}{2}^+$ states. Since the calculations predict the $\frac{1}{2}^+$ state about 2 MeV below the $\frac{5}{2}^+$ state, it therefore seems that the $\frac{5}{2}^+$ state should be at about 8–10 MeV and the $\frac{1}{2}^+$ state should be close to the ^{6}Li+n threshold at 7·25 MeV. Then the energies of the non-normal parity states calculated in II would be too high by about 4 MeV and those for the alternative interaction too high by about 2 MeV. This latter discrepancy at least can be attributed to the effects of deformation, since these positive-parity states of ^{7}Li are about 70% 1s-hole states in low-lying states of ^{8}Be, and Kumar (1974) has shown that deformation causes the energies of these ^{8}Be states to be depressed by about 3 MeV relative to what one might expect from fitting properties of ^{5}He, ^{6}Li and ^{7}Li. For the alternative interaction, with the energies of the $\frac{1}{2}^+$ and $\frac{5}{2}^+$ states of ^{7}Li reduced by 2 MeV, these states would contribute to the ^{7}Li photoabsorption integrated cross section about 3·6 MeV mb; the corresponding contribution from II, with a 4 MeV reduction in the calculated energies, is about 1·1 MeV mb. These values may be compared with experimental values for the 8 MeV peak in the phototriton cross section of 6·2 MeV mb given by Skopik *et al.* (1979), which would seem to be an overestimate since it takes no account of any background contribution, and about 3 MeV mb from Junghans *et al.* (1979).

In summary, there are discrepancies between the calculations of II and experiment as regards the energies and other properties of both normal and non-normal parity levels in the $A = 4$–7 nuclei. An alternative interaction that we have considered gives better agreement in most respects; the one case where II is better concerns the energy and E1 strength of the low-lying $T = 1$ negative-parity states of ^{6}Li. Our main interest is, however, the low-lying $\frac{1}{2}^+$ state in ^{5}He, and both interactions predict this state at an excitation energy of about 7 MeV, with a large spectroscopic factor for the ^{4}He+n channel so that the level would be very broad. Similar broad $\frac{3}{2}^+$ and $\frac{5}{2}^+$ levels are predicted by both interactions at about 13–15 MeV. Some earlier shell model calculations that obtained such levels at much higher energies (Fraser and Spicer 1966; Wagner and Werntz 1971) specifically chose their parameter values,

in particular the 2s–1d single-particle energies, to ensure this, on the basis that there was no experimental evidence for the existence of these states at lower energies.

In the next section, we see to what extent the nucleon–alpha phase shifts and other experimental data are consistent with the existence of broad, low-lying $\frac{1}{2}^+$, $\frac{3}{2}^+$ and $\frac{5}{2}^+$ states in ^{5}He and ^{5}Li.

3. *R*-matrix Fits to Scattering and Reaction Data

In the energy region below the threshold of the deuteron channel, the nucleon–alpha phase shifts are real and may be represented in the one-channel approximation of *R*-matrix theory (Lane and Thomas 1958). The nuclear phase shift for the system with total angular momentum J, parity π and channel relative orbital angular momentum l may be written

$$\delta_{lJ} = \beta_{lJ} - \phi_l, \tag{2}$$

where the resonant phase shift is defined in terms of the single element of the R matrix (or R function) by

$$\beta_{lJ} = \arctan\left(\frac{P_l}{R_{lJ}^{-1} - (S_l - B_{lJ})}\right). \tag{3}$$

Here P_l, S_l and $-\phi_l$ are the penetration factor, shift factor and hard-sphere phase shift, which are all functions of energy and depend on the channel radius a (assumed to be the same for all J^π), and B_{lJ} is the constant boundary condition parameter. The element R_{lJ} is written as a sum over levels:

$$R_{lJ} = \sum_\lambda \gamma_{\lambda lJ}^2 / (E_{\lambda lJ} - E), \tag{4}$$

where $E_{\lambda lJ}$ is the eigenenergy and $\gamma_{\lambda lJ}^2$ the reduced width of the level λ, and E is the c.m. energy of the system measured from the nucleon–alpha threshold.

We assume a two-level approximation ($\lambda = 1, 2$), with the upper level lying well above the energy range being fitted and so providing a background contribution. We are interested only in the properties of the lower level, and so define its resonance energy by

$$E_r(J^\pi) = E_{1lJ} - \gamma_{1lJ}^2 [S_l\{E_r(J^\pi)\} - B_{lJ}]. \tag{5}$$

By choosing

$$B_{lJ} = S_l\{E_r(J^\pi)\}, \tag{6}$$

we then have

$$E_r(J^\pi) = E_{1lJ}, \tag{7}$$

$$\beta_{lJ}\{E_r(J^\pi)\} = \tfrac{1}{2}\pi. \tag{8}$$

Then $E_r(J^\pi)$ is directly comparable with the shell model level energy for the J^π level. A spectroscopic factor for comparison with the shell model value may be obtained

from

$$\gamma_{1lJ}^2 = \mathscr{S}(J^\pi)\tfrac{1}{2}a\left(u_{lJ}^2(a)\Big/\int_0^a u_{lJ}^2(r)\,\mathrm{d}r\right)\frac{\hbar^2}{Ma^2}; \tag{9}$$

the radial wavefunction $u_{lJ}(r)$ is calculated for a Woods–Saxon potential with radius $R = 1{\cdot}25(A-1)^{1/3}$ fm $= 1{\cdot}984$ fm and diffuseness $a_0 = 0{\cdot}65$ fm, the depth being adjusted to fit the resonance energy $E_r(J^\pi)$. Reasonable changes in the values of R and a_0 have little effect on the derived values of $\mathscr{S}(J^\pi)$.

In a reaction that produces ^{5}He and ^{5}Li as a final unbound state, the contribution to the cross section from J^π states has the energy dependence (Barker 1967, equation 11)

$$\sigma(J^\pi, E) \propto \frac{\sin^2\beta_{lJ}}{P_l}\left|\sum_\lambda G_\lambda^{\frac{1}{2}}\gamma_{\lambda lJ}/(E_{\lambda lJ}-E)\Big/\sum_\lambda \gamma_{\lambda lJ}^2/(E_{\lambda lJ}-E)\right|^2. \tag{10}$$

In the one-level approximation, if the energy dependence of the feeding factor G_λ is neglected, this reduces to

$$\sigma(J^\pi) \propto \sigma^{(a)}(J^\pi) = (\sin^2\beta_{lJ})/P_l. \tag{11}$$

In the two-level approximation used here, the same form for $\sigma(J^\pi)$ is obtained if $G_\lambda^{\frac{1}{2}}$ is proportional to $\gamma_{\lambda lJ}$, an approximation that may be reasonable for a reaction such as ^{4}He(d, n)^{5}Li, assumed to proceed as a direct transition. For a reaction like ^{6}Li(p, d)^{5}Li, it is reasonable to assume that the upper level will not be fed, because it belongs to higher configurations, giving $G_2 = 0$ and

$$\sigma(J^\pi) \propto \sigma^{(b)}(J^\pi) = \frac{\sin^2\beta_{lJ}}{P_l}\left(1+\frac{\gamma_{2lJ}^2(E_{1lJ}-E)}{\gamma_{1lJ}^2(E_{2lJ}-E)}\right)^{-2}; \tag{12}$$

this necessarily peaks at a higher energy than does $\sigma^{(a)}(J^\pi)$. We denote the peak energy of $\sigma(J^\pi)$, given by either (11) or (12), as $E_{\max}(J^\pi)$ and the FWHM as $\Gamma_{\frac{1}{2}}(J^\pi)$.

Our procedure is to fit experimental values of the $s_{1/2}$, $p_{1/2}$, $p_{3/2}$, $d_{3/2}$ and $d_{5/2}$ nucleon–alpha phase shifts, for a range of values of the channel radius a, by adjusting the level parameters $E_{\lambda lJ}$ and $\gamma_{\lambda lJ}^2$. With the choice (6) of the boundary condition parameter, the values of $E_r(J^\pi)$ and of the spectroscopic factor $\mathscr{S}(J^\pi)$ derived from (9) are compared with shell model values. The level parameters are also used to calculate values of $\sigma(J^\pi)$ from (11) and (12), from which are derived values of $E_{\max}(J^\pi)$ and $\Gamma_{\frac{1}{2}}(J^\pi)$ for comparison with experimental values.

As representative of the experimental phase shifts below the threshold of the deuteron channel, we use the n–α phase shifts and errors given in Table 5 of Bond and Firk (1977) for $E_n = 0{\cdot}2$–$20{\cdot}0$ MeV, and the p–α phase shifts and errors in Table XI of Dodder *et al.* (1977) for $E_p = 0{\cdot}94$–$17{\cdot}84$ MeV. These authors compare their values with those of other groups; although at some energies and for some lJ values the discrepancies exceed the assigned errors, the agreement is sufficiently good for the present purpose.

We take $E_{2lJ} = 100$ MeV for all lJ values; this locates the second level well above the energy range being fitted so that it provides a background contribution.

Changing the value of $E_{2I,J}$ does not significantly change the quality of fit or the parameter values obtained for the level 1. Since it is only these latter parameter values that interest us, we restrict the energy range fitted so that the resonant phase does not increase by more than about 180°; this affects only the $p_{1/2}$ and $p_{3/2}$ fitting, for which the ranges become $E_n \leqslant 10$ and 5 MeV and $E_p \leqslant 13{\cdot}65$ and $7{\cdot}5$ MeV respectively. The resultant parameter values do not depend sensitively on the choice of these limits. Values of a from 3 to 6 fm are considered, the lower limit being about the value assumed in most earlier analyses and the upper limit being about as large as might be considered acceptable. Since the phase shifts of Bond and Firk and of Dodder *et al.* were actually obtained from R-matrix fits to the original cross section and polarization data with $a = 3{\cdot}3$ and $2{\cdot}9$ fm respectively, the quality of our fits is better for the smaller values of a, but even for $a = 6$ fm the fits are reasonable. We note that the arguments given by Bond and Firk and by Dodder *et al.* for their

Table 1. Values of resonance energies and spectroscopic factors as functions of channel radius

^{5}He and ^{5}Li values are denoted by n and p respectively. Values for $J^\pi = \frac{5}{2}^+$ are not essentially different from those for $\frac{3}{2}^+$

a (fm)	$E_r(\frac{3}{2}^-)$ (MeV)		$\mathscr{S}(\frac{3}{2}^-)$		$E_x(\frac{1}{2}^-)$ (MeV)		$\mathscr{S}(\frac{1}{2}^-)$	
	n	p	n	p	n	p	n	p
3	0·985	2·087	1·76	1·79	5·55	6·14	2·03	1·98
4	0·944	2·002	1·12	1·12	3·66	3·80	1·25	1·24
5	0·913	1·906	0·99	0·99	2·72	2·71	1·08	1·07
6	0·881	1·816	0·97	0·95	2·12	2·03	1·01	1·00

a (fm)	$E_x(\frac{1}{2}^+)$ (MeV)		$\mathscr{S}(\frac{1}{2}^+)$		$E_x(\frac{3}{2}^+)$ (MeV)		$\mathscr{S}(\frac{3}{2}^+)$	
	n	p	n	p	n	p	n	p
3	24·3	24·8	0·43	0·48	40·9	25·7	1·75	0·71
4	15·0	15·0	0·84	0·95	22·9	19·9	1·28	0·80
5	7·6	7·3	1·10	1·08	14·3	14·1	1·02	0·93
6	3·9	3·6	1·07	1·06	10·0	9·8	0·97	0·99

Table 2. Values of peak energies and FWHM in reaction cross sections as functions of channel radius

Cases a and b refer to reaction cross sections calculated from equations (11) and (12) respectively

Case	a (fm)	$E_{max}(\frac{3}{2}^-)$ (MeV)		$\Gamma_{\frac{1}{2}}(\frac{3}{2}^-)$ (MeV)		$E_{x,max}(\frac{1}{2}^-)$ (MeV)		$\Gamma_{\frac{1}{2}}(\frac{1}{2}^-)$ (MeV)	
		n	p	n	p	n	p	n	p
a	3	0·871	2·11	0·75	1·92	2·98	3·16	8·6	10·2
	4	0·862	1·84	0·71	1·39	2·65	2·76	5·7	6·5
	5	0·847	1·79	0·67	1·23	2·15	2·15	4·1	4·7
	6	0·827	1·72	0·62	1·13	1·73	1·64	3·2	4·0
b	3	0·879	2·15	0·78	2·15	4·00	4·22		
	4	0·874	1·87	0·76	1·56	3·22	3·46	9·3	10·8
	5	0·871	1·86	0·74	1·48	2·76	2·89	6·2	7·1
	6	0·868	1·86	0·72	1·40	2·42	2·46	4·7	5·3
Exp.[A]		0·89±0·05	1·96±0·05	0·60±0·02	≈1·5	4±1	5–10	4±1	5±2

[A] Adopted values, given in Ajzenberg-Selove (1984).

particular choices of a are not very convincing. For the s- and d-wave cases and for the smaller values of a, the level 1 is outside the energy range being fitted and there is not a clear distinction between the contribution from level 1 and the background, so that the value of $\mathscr{S}(J^\pi)$ in particular is uncertain in such cases.

The resultant values of the resonance energies and spectroscopic factors are given in Table 1 for selected values of a. For convenience, the energies of the excited states with $J^\pi = \frac{1}{2}^-, \frac{1}{2}^+, \frac{3}{2}^+$ and $\frac{5}{2}^+$ are expressed as excitation energies $E_x(J^\pi) = E_r(J^\pi) - E_r(\frac{3}{2}^-)$. The values of $E_x(J^\pi)$ and $\mathscr{S}(J^\pi)$ are essentially the same for ^{5}He and ^{5}Li, as expected, at least for the larger values of a. Also $\mathscr{S}(J^\pi) \approx 1$ for all J^π for $a \gtrsim 5$ fm; these values are in reasonable agreement with the shell model expectations from Section 2. The values of $E_x(J^\pi)$ for the $\frac{1}{2}^+$, $\frac{3}{2}^+$ and $\frac{5}{2}^+$ cases decrease rapidly as a increases. In particular $E_x(\frac{1}{2}^+) = 7$ MeV, as expected from Section 2, for $a \approx 5{\cdot}1$ fm for both ^{5}He and ^{5}Li. Then $E_x(\frac{3}{2}^+)$ and $E_x(\frac{5}{2}^+)$ are both about 14 MeV, in good agreement with the expectations of Section 2, and $E_x(\frac{1}{2}^-)$, the excitation energy of the first excited state, is 2·6 MeV. This agrees well with the values of 3·1 MeV for the van Hees and Glaudemans interaction and 2·5 MeV for the alternative interaction of Section 2.

From values of $\sigma(J^\pi)$ calculated from (11) and (12), the values of $E_{max}(J^\pi)$ and $\Gamma_{\frac{1}{2}}(J^\pi)$ for the positive-parity cases are found to decrease rapidly as a increases; however, even for a as large as 5·1 fm, the value of $\Gamma_{\frac{1}{2}}$ in each case is greater than 10 MeV, so that it is unlikely that the contributions from these J^π values to any reaction cross section could be distinguished from other contributions or from background due to other modes of decay. For the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ cases, the values of $E_{max}(J^\pi)$ and $\Gamma_{\frac{1}{2}}(J^\pi)$ are given in Table 2 as functions of a; again the energy of the $\frac{1}{2}^-$ state is given as an excitation energy $E_{x,\,max}(\frac{1}{2}^-) = E_{max}(\frac{1}{2}^-) - E_{max}(\frac{3}{2}^-)$. The adopted experimental values of these quantities, taken from Ajzenberg-Selove (1984), are also given; these values are almost entirely based on measurements made before 1966. The differences between the values for cases a and b may be taken as an indication of the uncertainty in the calculated values, although for some reactions (e.g. direct reactions) and in some circumstances (e.g. high bombarding energies, forward angles), it may be reasonable to assume that either case a or b is applicable. The formula (11) of case a has been used previously in many papers with reference to the ground state of ^{5}He or ^{5}Li, but always with $a \approx 3$ fm (in particular see Hamburger and Cameron 1960; Ohlsen and Young 1964; Franke *et al.* 1985). Comparison of the calculated and experimental values in Table 2 does not enable one to select a best value of the channel radius; for some properties the calculated values are insensitive to the value of a, other properties favour a large value of a and still others a small value, while there appears to be a clear discrepancy for the energy of the $\frac{1}{2}^-$ excited state in ^{5}Li. The large width of the $\frac{1}{2}^-$ state, however, makes measurements of its properties very difficult. If we accept the value $a = 5{\cdot}1$ fm, there appears to be a discrepancy for the ground-state energy in ^{5}Li. There is, however, a considerable spread in the experimental values from which the adopted value (Ajzenberg-Selove 1984; Wapstra *et al.* 1985) was obtained: 1·65±0·20 MeV (Sher *et al.* 1951), 1·99±0·15 MeV (Almqvist *et al.* 1953), 1·95±0·20 MeV (Blair *et al.* 1954), 1·90±0·10 MeV (Titterton 1955), 1·76±0·15 MeV (Frost and Hanna 1958) and 2·03±0·05 MeV (Rybka and Katz 1958). A new measurement of the ^{5}Li ground-state energy, and of other properties of the low-lying $\frac{3}{2}^-$ and $\frac{1}{2}^-$ levels in both ^{5}Li and ^{5}He, is planned here at the A.N.U.

4. Summary

The prediction in a recent shell model calculation (van Hees and Glaudemans 1983, 1984) of a broad low-lying $\frac{1}{2}^+$ state in ^{5}He and ^{5}Li is supported by calculations with an alternative shell model interaction chosen to give better agreement with measured properties of the $A = 4$–7 nuclei.

The absence of such a level from the currently accepted energy level spectra of ^{5}He and ^{5}Li is shown to be due to the small value of about 3 fm assumed for the channel radius in previous analyses of the nucleon–alpha phase shifts. A $\frac{1}{2}^+$ level at the expected excitation energy of about 7 MeV is obtained for a channel radius of about 5·1 fm, which also leads to broad $\frac{3}{2}^+$ and $\frac{5}{2}^+$ levels at about 14 MeV in agreement with the shell model calculations. We suggest that phase shift analyses of the scattering and polarization data, similar to those performed by Bond and Firk (1977) and Dodder *et al.* (1977), should be repeated with such a larger value of the channel radius.

The properties of the low-lying $\frac{3}{2}^-$ and $\frac{1}{2}^-$ states of ^{5}He and ^{5}Li measured in reactions are at present not sufficiently unambiguous to permit a clear preference for a particular value of the channel radius.

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