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Calculation of the $^{13}N(p, \gamma)^{14}O$ Cross Section at Low Energies

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Abstract

R-matrix formulae are used to calculate the $^{13}N(p, \gamma)^{14}O$ cross section in the energy region of astrophysical interest. Values of the *R*-matrix parameters are determined by fitting experimental data in ^{14}C and ^{14}N as well as ^{14}O, and by making use of shell model calculations where necessary. With the presently available data, there is considerable uncertainty in the predicted cross section, but this could be reduced appreciably by a measurement of the lifetime of the first excited state of ^{14}C. With the preferred channel radius of about 5 fm, the predicted cross section is lower than those found in two previous calculations, which are discussed in some detail.

1. Introduction

Knowledge of the $^{13}N(p, \gamma)^{14}O$ cross section at low energies is of interest in calculations of energy generation in stars during the hot CNO cycle (Mathews and Dietrich 1984). The cross section has not yet been measured directly, but it has been calculated recently by two different methods (Mathews and Dietrich 1984; Langanke *et al.* 1985). There appears to be sufficient uncertainty in the accuracy of these calculations to justify another calculation using a different method.

At low energies, the $^{13}N(p, \gamma)^{14}O$ cross section is dominated by E1 transitions to the ^{14}O ground state from the $J^{\pi} = 1^{-}$ state at $5 \cdot 173$ MeV, which is produced by s-wave protons at $0 \cdot 547$ MeV channel energy. The situation is similar to that for the $^{12}C(p, \gamma)^{13}N$ reaction, where the low-energy cross section is dominated by ground-state E1 transitions from the $\frac{1}{2}^{+}$ level of ^{13}N at $2 \cdot 365$ MeV, produced by s-wave protons at $0 \cdot 421$ MeV. This cross section and other properties of the low-lying levels of ^{13}C and ^{13}N have been well fitted by *R*-matrix formulae, and the resultant parameter values are in reasonable agreement with shell model predictions (Barker and Ferdous 1980, henceforth referred to as BF). We here use formulae and methods similar to those used by BF in order to fit properties of the $T = 1$, 1^{-} and 0^{+} levels of the $A = 14$ isobars, and to calculate the $^{13}N(p, \gamma)^{14}O$ cross section.

The *R*-matrix formulae involve parameters whose values are obtained by fitting level energies and widths, thermal neutron scattering and capture data, and E1 transition probabilities. The E1 matrix elements include external contributions calculated using wavefunctions with the correct asymptotic forms. Use is also made of spectroscopic

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factors measured in single-nucleon transfer reactions, and of the results of shell model calculations. The levels of prime interest here are the 0^+ ground states and 1^- first excited states of ^{14}C and ^{14}O and the 1^- analogue state in ^{14}N. Because of a lack of experimental information about these states (in particular only a lower limit is known for the radiative width of the 1^- state of ^{14}C), it is also useful to consider properties of the 0^+ second excited states of ^{14}C and ^{14}O and their analogue in ^{14}N (which we denote by 0^{+*}).

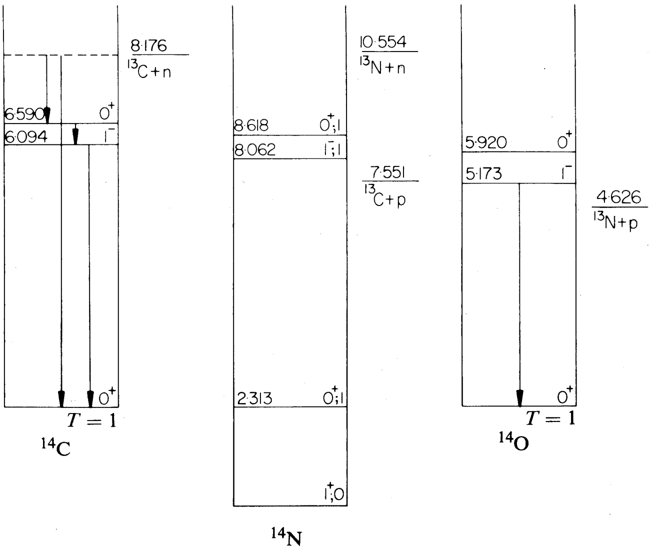

Fig. 1. Relevant levels of ^{14}C, ^{14}N and ^{14}O and the E1 transitions between them.

Table 1. Values of quantities related to 1^- and 0^+ levels of ^{14}C, ^{14}N and ^{14}O

Relevant J^π value	Iso-bar	Quantity	Adopted value	Best fit $a = 5$ fm
1^-	^{14}C	E_b (MeV)	$-2\cdot082\pm0\cdot002$	$-2\cdot082^A$
	^{14}N	E_r (MeV)	$0\cdot511\pm0\cdot001$	$0\cdot511^A$
	^{14}O	E_r (MeV)	$0\cdot547\pm0\cdot010$	$0\cdot547^A$
	^{14}N	Γ^o (keV)	33 ± 2	$33\cdot8$
	^{14}O	Γ^o (keV)	$38\cdot1\pm1\cdot8$	$35\cdot9$
	^{14}C	a_s (fm)	$5\cdot47\pm0\cdot09$	$5\cdot47$
$1^- \to 0^+$	^{14}C	Γ_γ^o (eV)	$>0\cdot066$	$1\cdot21^B$
	^{14}C	$\sigma_{n\gamma}$(thermal) (mb)	$1\cdot15\pm0\cdot04$	$1\cdot15^A$
0^{+*}	^{14}N	Γ^o (keV)	$5\cdot6\pm1\cdot9$	$5\cdot23$
	^{14}O	Γ^o (keV)	$\leqslant50$	$11\cdot8$
$0^{+*} \leftrightarrow 1^-$	^{14}C	$\Gamma_\gamma^o(0^{+*}\to1^-)$ (meV)	$0\cdot15\pm0\cdot02$	$0\cdot150$
	^{14}C	$\sigma_{n\gamma}$(thermal) (mb)	$0\cdot12\pm0\cdot01$	$0\cdot120$

^{A} Exact fit.
^{B} Solution A.

The fits to the data and the calculation of the $^{13}N(p, \gamma)^{14}O$ cross section are given in Section 4. Comparison with and comments on the earlier calculations are made in Section 5.

2. Experimental Data

Experimental values are taken from Ajzenberg-Selove (1981), unless another reference is given. The relevant levels of ^{14}C, ^{14}N and ^{14}O, and the E1 transitions between them, are shown in Fig. 1. The E1 transitions in ^{14}N are not considered, since those between $T = 1$ states are isospin forbidden. Experimental values of the relevant quantities are collected in Table 1. Here E_b and E_r are the energies of bound and resonance levels measured from the appropriate nucleon threshold. The Γ^o and Γ^o_γ are total and radiative widths in the c.m. system, the superscript o denoting observed width (see BF). The value of $\Gamma^o(^{14}N; 1^-)$ given by Ajzenberg-Selove (1981) is 30 ± 1 keV, which comes solely from the value $\Gamma^o_{lab} = 32\cdot5\pm1$ keV measured by Seagrave (1952). Other values of Γ^o_{lab} are $\sim$40 keV (Fowler *et al.* 1948; Fowler and Lauritsen 1949), 36 keV (given by Kashy *et al.* 1961 from analysis of the data of Milne 1954) and 37 keV (Vogl 1963). An error of $\pm0\cdot6$ keV may be assigned to Vogl's value corresponding to the error of ±10 keV that he ascribes to the reduced width $\gamma^2 = 570$ keV. The value of $\Gamma^o(^{14}N; 1^-)$ adopted in Table 1 is an average of these values, with a reasonably large error to take account of their spread. The value of $\Gamma^o(^{14}O; 1^-)$ in Table 1 is from a recent measurement by Chupp *et al.* (1985). Ajzenberg-Selove (1981) gives $\Gamma^o(^{14}N; 0^{+*}) = 7\pm1$ keV, which is the average of the values $\Gamma^o_{lab} = 6\pm2$ keV from $^{13}C(p, \gamma)^{14}N$ (Seagrave 1952) and $\Gamma^o_{lab} = 9\pm2$ keV from $^{13}C(p, p)^{13}C$ (Latorre and Armstrong 1966). Kashy *et al.* (1961) also fitted (p, p) data with $\Gamma^o_{lab} = 10$ keV. Whereas the peak due to this 0^+ level is very pronounced in the (p, γ) reaction, the level shows as a small perturbation on a large background in the (p, p) reaction so that its width there is poorly determined.† We here take the value of $\Gamma^o(^{14}N; 0^{+*})$ from the (p, γ) measurement alone. Values of a_s, the free coherent scattering length of thermal neutrons on ^{13}C for the 1^- channel, and $\sigma_{n\gamma}$(thermal), the thermal neutron capture cross section to the ground state and to the 0^+, $6\cdot590$ MeV state of ^{14}C, are taken from Mughabghab *et al.* (1982). Table 1 also gives our best fit values for $a = 5$ fm.

3. The *R*-matrix Formulae

Formulae and notation are similar to those used previously in BF and Barker (1984). We are dealing with energy regions where there is at most one open channel for each J^π value, and the one-channel approximation is used in extracting parameter values from fitting experimental data, except for the observed widths in ^{14}N where the contribution of the closed ^{13}N(g.s.)+n channel is also taken into account. In each isobar, we assume a one-level approximation [equation (1) of BF] for each of the 0^+ levels, since the properties of each level are required only at the energy of the level itself, but a two-level approximation [equation (2) of BF] for $J^\pi = 1^-$. The second 1^- level is introduced to simulate the contributions of higher 1^- levels, e.g.

† In the fits of Latorre and Armstrong (1966), the reduced width did not change from its starting value of $0\cdot19$ MeV fm, and the difference between their value of Γ^o_{lab} and that of Seagrave is due entirely to use of an inaccurate value of the penetration factor by Woodbury *et al.* (1953).

the known or suspected 1^- levels of ^{14}C at 9·80, 11·40 and 12·96 MeV and the giant dipole resonance, and its presence is significant because the properties of the 1^- states of the systems are required not only at the energy of the lowest 1^- level in ^{14}C, ^{14}N and ^{14}O, but also at the ^{13}C+n threshold in ^{14}C and over a wide range of energies in ^{14}O. As in BF, for each assumed value of the channel radius a, we use $B_n = S_n(E_{1n})$ and $B_p = S_p(E_{1p})$ so that E_{1n} and E_{1p} are just the observed energies of the levels as given in Fig. 1 or Table 1. The formulae for $a_s(^{14}\mathrm{C})$ and $\Gamma^o(^{14}\mathrm{O})$ are equations (8) and (12) of BF. The corresponding formula for $\Gamma^o(^{14}\mathrm{N}; 1^-)$ is somewhat more complicated because of the two-channel approximation; it can be written

$$\Gamma^o(^{14}\mathrm{N};\, 1^-) = \delta_+ - \delta_-\,, \tag{1}$$

where δ_+ and δ_- are those solutions of the quadratic equations

$$A\delta_\pm^2 + (B\pm C)\delta_\pm \pm D = 0 \tag{2}$$

that vanish as $D \to 0$, where

$$A = 1+(\gamma_{1p}^2+\gamma_{2p}^2)S_p'+(\gamma_{1n}^2+\gamma_{2n}^2)S_n'+(\gamma_{1p}\gamma_{2n}-\gamma_{1n}\gamma_{2p})^2 S_p' S_n'\,, \tag{3a}$$

$$B = -(1+\gamma_{1p}^2 S_p' + \gamma_{1n}^2 S_n')(E_{2p}-E_r)\,, \tag{3b}$$

$$C = -\{\gamma_{1p}^2+\gamma_{2p}^2+(\gamma_{1p}\gamma_{2n}-\gamma_{1n}\gamma_{2p})^2 S_n'\}P_p\,, \tag{3c}$$

$$D = \gamma_{1p}^2 P_p(E_{2p}-E_r)\,. \tag{3d}$$

Here we have written $P_p(E_r) = P_p$, $\mathrm{d}S_p(E_r)/\mathrm{d}E = S_p'$ and $\mathrm{d}S_n(E_r)/\mathrm{d}E = S_n'$. To a good approximation, this gives

$$\Gamma^o(^{14}\mathrm{N};\, 1^-) = -2D/B = \frac{2\gamma_{1p}^2 P_p}{1+\gamma_{1p}^2 S_p' + \gamma_{1n}^2 S_n'}\,. \tag{4}$$

This formula (4) is also used for $\Gamma^o(^{14}\mathrm{N}; 0^{+*})$.

Formulae for the E1 radiative widths and the thermal neutron capture cross sections in ^{14}C and for the ^{13}N(p, γ)^{14}O cross section are taken from equations (26) and (24) of BF:

$$\Gamma_\gamma^o(\mathrm{i}\to\mathrm{f}) = f_{\mathrm{if}}(a^2/N_\mathrm{i} N_\mathrm{f})\,|\mathscr{M}_{\mathrm{if}}+2\Theta_\mathrm{i}\Theta_\mathrm{f} J_{\mathrm{if}}|^2\,, \tag{5}$$

$$\sigma_\gamma(\mathrm{i}\to\mathrm{f}) = \frac{2\pi M}{\hbar^2 k_\mathrm{i}^3}\,\frac{2J_\mathrm{i}+1}{2}\,f_{\mathrm{if}}\,\frac{a^3 u_\mathrm{i}^2(a)}{2N_\mathrm{f}\Theta_\mathrm{i}^2}\,|\mathscr{M}_{\mathrm{if}}+2\Theta_\mathrm{i}\Theta_\mathrm{f} J_{\mathrm{if}}|^2\,. \tag{6}$$

The additional factor of $\frac{1}{2}$ in equation (6) comes from $(2I_t+1)^{-1}$, where $I_t = \frac{1}{2}$ is the target spin. In the present cases, f_{if} is given by

$$f_{\mathrm{if}} = \tfrac{4}{3}(\tfrac{6}{14})^2 e^2(E_\gamma/\hbar c)^3(l_\mathrm{i}\,100\,|\,l_\mathrm{f}0)^2\, U^2(1j_\mathrm{f}\,J_\mathrm{i}\,I_t;\,j_\mathrm{i}\,J_\mathrm{f})\, U^2(1\,l_\mathrm{f}\,j_\mathrm{i}\,\tfrac{1}{2};\,l_\mathrm{i}\,j_\mathrm{f})\,. \tag{7}$$

For the $1^-\to 0^+$ transitions of interest, we use $l_\mathrm{i} = 0$, $j_\mathrm{i} = \frac{1}{2}$, $l_\mathrm{f} = 1$ and $j_\mathrm{f} = \frac{1}{2}$, giving

$$f_{if} = \tfrac{4}{147}\, e^2 (E_\gamma/\hbar c)^3 \,. \qquad (8)$$

The radial integrals J_{if} are evaluated analytically for the ^{14}C cases and numerically for ^{14}O.

4. Fits to Data

(a) Properties Involving 1^- States Alone

Because there are not sufficient data to determine all the R-matrix parameters for the 1^- states, we make some assumptions that are reasonable or should not affect the results significantly. We assume that

$$E_{2p} - E_{1p} = E_{2n} - E_{1n} = 10\ \mathrm{MeV}\,,$$

and that

$$\gamma_{2n}^2(^{14}C) = 2\gamma_{2p}^2(^{14}N) = 2\gamma_{2n}^2(^{14}N) = \gamma_{2p}^2(^{14}O)\,,$$

as is true for analogue states of pure isospin $T = 1$, and we denote this by γ_2^2. Also we put [cf. BF, equations (25) and (32)], where $N = p, n$,

$$\gamma_{1N} = (\hbar^2/Ma^2)\Theta_{1N}\,, \qquad \Theta_{1N} = \mathscr{S}_N^{\frac{1}{2}}\,\Theta_{sp,N}\,, \qquad (9a, b)$$

$$\Theta_{sp,N} = u_N(a)\left(\tfrac{1}{2}a \Big/ \int_0^a u_N^2(r)\,dr\right)^{\frac{1}{2}}, \qquad (9c)$$

and assume that $\mathscr{S}_n(^{14}C) = 2\mathscr{S}_p(^{14}N) = 2\mathscr{S}_n(^{14}N) = \mathscr{S}_p(^{14}O)$, which we denote by $\mathscr{S}(1^-)$. The internal transition moment $\mathscr{M}_{if}$ is assumed to have the same value for each of the three ground-state E1 transitions shown in Fig. 1, and to have the same value for each of the two transitions involving the excited 0^+ state.

Table 2. Best fits to properties of 1^- levels of ^{14}C, ^{14}N and ^{14}O

a (fm)	$\Gamma^o(^{14}N)$ (keV)	$\Gamma^o(^{14}O)$ (keV)	X	$\mathscr{S}(1^-)$	γ_2^2 (MeV)
3·0	33·8	35·4	0·845	1·502	6·76
3·5	34·1	34·8	1·284	0·686	1·47
4·0	34·0	35·0	1·119	0·560	1·34
4·5	33·9	35·4	0·853	0·547	1·49
5·0	33·8	35·9	0·586	0·563	1·59
5·5	33·6	36·4	0·375	0·589	1·64
6·0	33·5	36·8	0·215	0·616	1·67
6·5	33·3	37·2	0·104	0·641	1·68
7·0	33·2	37·6	0·036	0·665	1·68

The two parameters $\mathscr{S}(1^-)$ and γ_2^2 are adjusted to give a best fit to the experimental values of the three quantities $\Gamma^o(^{14}N; 1^-)$, $\Gamma^o(^{14}O; 1^-)$ and $a_s(^{14}C)$ that involve only the 1^- states, for each choice of the channel radius a. Because of the rapid energy dependence of $P_p(E)$, the appreciable uncertainty in the value of $E_r(^{14}\mathrm{O}; 1^-)$ is equivalent to an additional uncertainty in the value of $\Gamma^o(^{14}\mathrm{O}; 1^-)$ equal to $(\partial \ln P_p/\partial E)_{E_r} \Gamma^o \delta E_r$, which is approximately 2·8 keV. In the fitting, we therefore take the experimental value of $\Gamma^o(^{14}\mathrm{O}; 1^-)$ as $38 \cdot 1 \pm 3 \cdot 3$ keV. The uncertainty in $E_r(^{14}\mathrm{N}; 1^-)$ does not change appreciably the uncertainty in $\Gamma^o(^{14}\mathrm{N}; 1^-)$. The parameter $\mathscr{S}(1^-)$ is determined essentially by fitting the values of $\Gamma^o(^{14}\mathrm{N}; 1^-)$ and $\Gamma^o(^{14}\mathrm{O}; 1^-)$, since these are more or less independent of γ_2^2; the value of γ_2^2 is then determined by an almost exact fit to the value of $a_s(^{14}\mathrm{C})$. The best fit values of $\Gamma^o(^{14}\mathrm{N}; 1^-)$ and $\Gamma^o(^{14}\mathrm{O}; 1^-)$ and the corresponding parameter values are given in Table 2, for a range of values of a. Values of a as small as 3 fm are included, because a value near this was found in a previous fit (Mughabghab *et al.* 1982) to some of the $A = 14$ properties that we fit; we note that the conventional minimum value of the channel radius (Lane and Thomas 1958; Lane 1960) is $a = 1 \cdot 45(A_1^{1/3} + A_2^{1/3})$ fm $=$ 4·86 fm. The quality of fit, measured by X [see BF, equation (28)], is best for the larger a values but is acceptable for all the values of a considered. Except for $a \approx 3$ fm, the best fit values of $\mathscr{S}(1^-)$ are somewhat smaller than the shell model values of 0·76 (Millener and Kurath 1975), 0·84 (Lie 1972), 0·85 (Jäger *et al.* 1971), 0·78 (Hsieh and Horie 1970) and 0·83 (Sebe 1963), and the experimental values from one-nucleon stripping reactions on $^{13}\mathrm{C}$ of 0·75 (Peterson *et al.* 1984), 0·78 and 0·87 (Datta *et al.* 1978) and 0·74 (Bobbitt *et al.* 1973), but much larger than an experimental value 0·07 also obtained from stripping (Peterson and Hamill 1981).

(b) Properties Involving 1^- and 0^+ States

The expressions (5) and (6) for $\Gamma_\gamma^o(\mathrm{i}\to\mathrm{f})$ and $\sigma_\gamma(\mathrm{i}\to\mathrm{f})$, applied to the $1^- \to 0^+$ ground-state transitions in $^{14}\mathrm{C}$ and $^{14}\mathrm{O}$, contain two as yet undetermined parameters, namely $\mathscr{M}_{\mathrm{if}} \equiv \mathscr{M}(1^- \to 0^+)$ and $\Theta_\mathrm{f} \equiv \Theta(0^+)$ [or equivalently $\mathscr{S}^{\frac{1}{2}}(0^+)$]. Fitting the experimental value of $\sigma_{\mathrm{n}\gamma}$(thermal) to the $^{14}\mathrm{C}$ ground state gives one restriction on these. Absolute values of $\mathscr{S}(0^+)$ obtained from one-nucleon stripping reactions vary considerably, including 1·84 (Peterson and Hamill 1981), 2·09 and 2·61 (Datta *et al.* 1978), 0·97 (Bobbitt *et al.* 1973), 2·48 (Mutchler *et al.* 1971), 2·05 (Schiffer *et al.* 1967) and 1·64 (Holbrow *et al.* 1966). A shell model value is $\mathscr{S}(0^+) = 1 \cdot 73$ (Cohen and Kurath 1967). A value of $\mathscr{M}(1^- \to 0^+)$ may be obtained from the shell model calculation of Kozub *et al.* (1981) for the $A = 14$ system, which assumed a $1\hbar\omega$ basis for the negative-parity states and a $0\hbar\omega + 2\hbar\omega$ basis for the positive-parity states, and included both the cases of no mixing and mixing of the $0\hbar\omega$ and $2\hbar\omega$ configurations. The value of B(E1) for the $1^- \to 0^+$ transition in $^{14}\mathrm{C}$ does not depend sensitively on whether or not mixing is included. Their value for no mixing, calculated using harmonic oscillator single-particle wavefunctions, leads to

$$|\mathscr{M}(1^- \to 0^+)| = \{\tfrac{196}{3}\pi\, B_\downarrow(\mathrm{E1})/e^2 a^2\}^{\frac{1}{2}} = 0 \cdot 739\, b/a,$$

where b is the harmonic oscillator length parameter. Kozub *et al.* pointed out that the major components of the 1^- wavefunction [in an SU(3) basis] do not contribute to the E1 matrix element. Also the calculated B(E1) is very sensitive to the 1s-hole components of the 1^- wavefunction, which is similar to the situation for $A = 13$ E1 transitions found by Teeters and Kurath (1977); a calculation similar to that of Kozub *et al.* showed that although the 1^- state has only about 0·4% admixture of 1s-hole components, the value of B(E1) is reduced by a factor of three if the 1s shell is kept closed (C. L. Woods, personal communication). The earlier weak-coupling calculation of Lie (1972) gave B(E1) = 0, due to the 1^- and 0^+ states in his model differing by only a neutron p–h excitation, and the use of a c.m. selection rule; the latter, however, does not strictly apply because of small spurious admixtures in the 1^- state.

Table 3. Values of quantities related to $1^- \to 0^+$ ground state transitions in ^{14}C and ^{14}O

a (fm)	$\mathscr{M}(1^- \to 0^+)$ A	$\mathscr{M}(1^- \to 0^+)$ B	$\lvert\mathscr{M}(1^- \to 0^+)\rvert$ Shell model	$\Gamma_\gamma^0(1^- \to 0^+)$ (eV) ^{14}C A	^{14}C B	^{14}O A	^{14}O B
3·0	−0·578	0·174	0·419	1·64	5·57	1·38	3·58
3·5	−0·229	0·316	0·359	1·27	5·74	1·32	3·89
4·0	−0·015	0·395	0·314	1·45	6·27	1·48	4·26
4·5	0·071	0·399	0·279	1·42	6·20	1·42	4·20
5·0	0·096	0·371	0·251	1·21	5·69	1·20	3·85
5·5	0·093	0·329	0·228	0·90	4·92	0·92	3·32
6·0	0·078	0·286	0·209	0·60	4·13	0·64	2·78
6·5	0·058	0·243	0·193	0·36	3·34	0·41	2·25
7·0	0·039	0·204	0·179	0·18	2·64	0·24	1·77

We use the shell model value of $\mathscr{S}(0^+)$, which is reasonably consistent with the stripping values, and adjust the value of $\mathscr{M}(1^- \to 0^+)$ to fit the value of $\sigma_{n\gamma}$(thermal) given in Table 1. There are two solutions, which are given in columns A and B of Table 3 [for $\Theta_{1N}(1^-)\,\Theta_{1N}(0^+) > 0$]; these values are not sensitive to the assumed value of $\mathscr{S}(0^+)$. Except for $a \lesssim 4$ fm, there is appreciable cancellation between the internal and external contributions to the E1 matrix element for each solution, with the external contribution dominating in solution A (as happened for the thermal neutron capture cross sections in BF), and the internal contribution dominating in solution B. Table 3 also gives the shell model value of $\lvert\mathscr{M}(1^- \to 0^+)\rvert$ for $b = 1{\cdot}699$ fm, as used by Kozub *et al.* (1981). For $a \gtrsim 4$ fm, the shell model value of $\mathscr{M}$, assumed positive, falls between the solutions A and B. For the $\frac{1}{2}^+ \to \frac{1}{2}^-$ E1 transition in ^{13}C and ^{13}N, the value $\lvert\mathscr{M}(\frac{1}{2}^+ \to \frac{1}{2}^-)\rvert = 0{\cdot}704\ \text{fm}/a$ similarly calculated from the B(E1) value given by Teeters and Kurath (1977) is for $a = 5$ fm about twice the value of 0·066 required in BF to fit $A = 13$ experimental data, and so we prefer the values of solution A in Table 3. The predicted values of $\Gamma_\gamma^0(1^- \to 0^+)$ in ^{14}C are also given in Table 3 for each solution; they are all much larger than the experimental lower limit, and so do not lead to any further restriction.

Using these parameter values, we may calculate the ^{13}N(p, γ)^{14}O cross section from equation (6). Since the low-energy cross section is dominated by contributions from the 1^-, 5·173 MeV level, and the total width of this level has already been fitted, a more or less equivalent calculation is of the radiative width of the level from equation (5). The calculated values of $\Gamma_\gamma^0(1^- \to 0^+)$ in ^{14}O are given in Table 3. It is seen that they are close to the values of Γ_γ^0 for the mirror transition in ^{14}C, particularly for solution A, and that they have a similar variation with a. Because of the different transition energies in the two nuclei, the ratio of the calculated E1 strengths in ^{14}O and ^{14}C is approximately 1·7 for solution A (1·1 for solution B), which is somewhat smaller than the ratios of about 2·5 for the mirror E1 strengths in ^{13}N and ^{13}C (see BF, Table 2 and note added in proof); the smaller ratio for $A = 14$ as compared with $A = 13$ may be attributed to the larger binding energies of the $A = 14$ ground states.

(c) Selection of Optimum Channel Radius from Properties Involving 1^- and 0^{+*} States

Although there is some preference for the larger values of a from the fits in Table 2, the acceptable range of a values is still large and leads to a wide range of $\Gamma_\gamma^o(^{14}\mathrm{O}; 1^- \to 0^+)$ values (even if solution B is excluded). We seek to restrict the range of a values by fitting data involving the excited 0^{+*} levels, in addition to the 1^- levels. Two new parameters are involved, $\mathscr{S}(0^{+*})$ and $\mathscr{M}(1^- \leftrightarrow 0^{+*})$. Values of $\mathscr{S}(0^{+*})$ obtained from one-nucleon stripping reactions include $0 \cdot 042$ (Peterson and Hamill 1981), $0 \cdot 10$ (Bobbitt *et al.* 1973) and $<0 \cdot 24$ (Fortune *et al.* 1971), as well as $0 \cdot 14$ (Peterson *et al.* 1984) from a bad fit to the data. The shell model calculation of Kozub *et al.* (1981) gave $|\mathscr{M}(1^- \leftrightarrow 0^{+*})| = 0 \cdot 170 b/a$ in the no-mixing case; however, this value is sensitive to changes in the $2s_{\frac{1}{2}}$ single-particle energy and is very sensitive to $0\hbar\omega$–$2\hbar\omega$ mixing. The calculation of Lie (1972) gave $\mathscr{M}(1^- \leftrightarrow 0^{+*}) = 0$. We therefore obtain values of $\mathscr{S}(0^{+*})$ and $\mathscr{M}(1^- \leftrightarrow 0^{+*})$ by fitting the three pieces of data in Table 1 involving the 0^{+*} states. The parameters are essentially determined by almost exact fits to the values of $\Gamma_\gamma^o(0^{+*} \to 1^-)$ and $\sigma_{\mathrm{n}\gamma}$(thermal), and the quality of fit is then dominated by the fit to $\Gamma^o(^{14}\mathrm{N}; 0^{+*})$. The best fit values of $\Gamma^o(^{14}\mathrm{N}; 0^{+*})$ and the parameters are given in Table 4 [for $\Theta_{1\mathrm{N}}(1^-)\, \Theta_{1\mathrm{N}}(0^{+*}) > 0$]. A good fit is obtained over the whole range of a values. The predicted value of $\Gamma^o(^{14}\mathrm{O}; 0^{+*})$ is less than 15 keV for all a values, and so is consistent with the experimental upper limit given in Table 1.

Table 4. Best fits to properties of 0^{+*} levels of ^{14}C and ^{14}N

a (fm)	$\Gamma^o(^{14}\mathrm{N})$ (keV)	X	$\mathscr{S}(0^{+*})$	$\mathscr{M}(1^- \leftrightarrow 0^{+*})$	$\mathscr{R}_{\mathscr{S}}$	$\mathscr{R}_{\mathscr{M}}$ A	B	C
3·0	4·90	0·140	0·0365	−0·233	0·145	0·404	−1·344	−0·082
3·5	5·24	0·038	0·0398	−0·106	0·152	0·464	−0·336	−0·076
4·0	5·24	0·037	0·0420	−0·049	0·156	3·182	−0·124	−0·072
4·5	5·22	0·042	0·0448	−0·019	0·161	−0·263	−0·047	−0·067
5·0	5·23	0·039	0·0481	−0·001	0·167	−0·010	−0·003	−0·061
5·5	5·30	0·027	0·0518	0·010	0·173	0·106	0·030	−0·055
6·0	5·43	0·009	0·0589	0·016	0·180	0·211	0·057	−0·048
6·5	5·60	0·000	0·0605	0·020	0·187	0·351	0·084	−0·041
7·0	5·89	0·025	0·0658	0·023	0·195	0·585	0·113	−0·033

It is convenient to consider the values of the ratios $\mathscr{R}_{\mathscr{S}} \equiv \mathscr{S}^{\frac{1}{2}}(0^{+*})/\mathscr{S}^{\frac{1}{2}}(0^+)$ and $\mathscr{R}_{\mathscr{M}} \equiv \mathscr{M}(1^- \leftrightarrow 0^{+*})/\mathscr{M}(1^- \to 0^+)$; values of these given in Table 4 use $\mathscr{S}(0^+) = 1 \cdot 73$ and the two solutions A and B in Table 3. In the simplest approximation, the 0^+ ground state of ^{14}C and ^{14}O belongs to the $1s^4 1p^{10}$ configuration, and is a $0\hbar\omega$ state such as is given by the wavefunctions of Cohen and Kurath (1965), while the 0^{+*} state is a $2\hbar\omega$ state belonging to the configurations $1s^4 1p^8 (2s1d)^2$, $1s^4 1p^9 (2p1f)$ and $1s^3 1p^{10} (2s1d)$. In this approximation, denoted in this paragraph by a superscript zero, the calculation of Kozub *et al.* (1981) gives $|\mathscr{R}^0_{\mathscr{M}}| = 0 \cdot 230$; one also expects $\mathscr{R}^0_{\mathscr{S}} \approx 0$, since only the $1s^4 1p^9 2p_{\frac{1}{2}}$ component of the 0^{+*} state contributes to $\mathscr{S}(0^{+*})$. The mixing of the $0\hbar\omega$ and $2\hbar\omega$ states has been considered in the shell model calculations of Lie (1972) and of Kozub *et al.* (1981), and also from an empirical point of view by Fortune and Stephans (1982). In the Fortune and Stephans model, the unmixed 0^+ and 0^{+*} states are assumed to mix, without any higher 0^+ states being involved; with a simple description of the unmixed 0^{+*} state, they fitted the measured ^{12}C(t, p)^{14}C cross sections with a mixing coefficient $\epsilon = -0\cdot35\pm0\cdot02$, corresponding to $\mathscr{R}_{\mathscr{S}} = -\epsilon(1-\epsilon^2)^{-\frac{1}{2}} = 0\cdot37$. Their model also gives the formula $\mathscr{R}_{\mathscr{M}} = (\mathscr{R}_{\mathscr{S}}+\mathscr{R}^0_{\mathscr{M}})/(1-\mathscr{R}_{\mathscr{S}}\mathscr{R}^0_{\mathscr{M}})$. If we take the value of $\mathscr{R}^0_{\mathscr{M}}$ from Kozub *et al.* [with a negative sign in order to obtain the destructive interference that they found in B(E1; $0^{+*}\to1^-$)] and the values of $\mathscr{R}_{\mathscr{S}}$ from Table 4, this formula leads to the values of $\mathscr{R}_{\mathscr{M}}$ given in the column labelled C in Table 4. The values C agree with the solution A for $a \approx 4\cdot8$ fm (and with solution B for $a \approx 4\cdot3$ fm), the corresponding value of $\mathscr{R}_{\mathscr{S}}$ being about $0\cdot16$, which is much less than the value found by Fortune and Stephans. The calculation of Lie (1972) suggests $|\mathscr{R}_{\mathscr{S}}| \lesssim 0\cdot2$. In the calculation of Kozub *et al.* (1981), however, the mixing does not follow the Fortune and Stephans model and the main $2\hbar\omega$ admixtures to the lowest 0^+ state come from highly excited states, also the good agreement of the unmixed 0^+ energies with experiment is destroyed by the mixing. One should not take the Kozub *et al.* value of $\mathscr{R}^0_{\mathscr{M}}$ too seriously; however, it seems likely that the tendency for cancellation between $\mathscr{R}_{\mathscr{S}}$ and $\mathscr{R}^0_{\mathscr{M}}$ would lead to a small value of $|\mathscr{R}_{\mathscr{M}}|$, implying a value of a near 5 fm (at least for solution A). Such a value is consistent with all the data fitted here, and is also in agreement with the value found in BF from fitting $A = 13$ data.

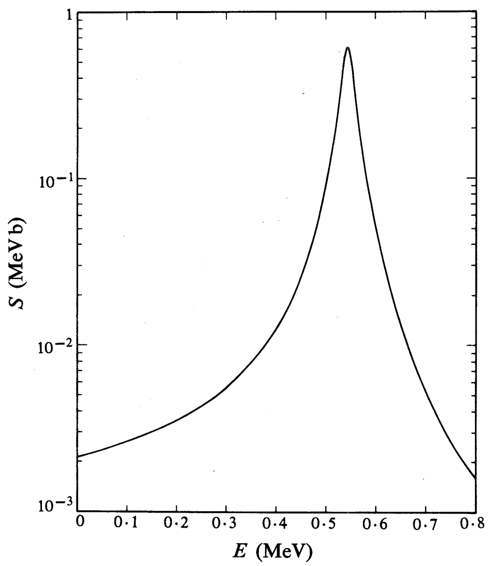

Fig. 2. The S factor for the ^{13}N(p, γ)^{14}O reaction calculated for $a = 5$ fm and for solution A of Table 3.

(d) Calculation of the ^{13}N(p,γ)^{14}O *Cross Section*

For $a = 5$ fm, the predicted lifetime of the 1^- first excited state of ^{14}C is $0\cdot54$ fs for solution A ($0\cdot12$ fs for solution B). Such a lifetime should be measurable in inelastic electron scattering on ^{14}C (see Plum *et al.* 1984) or by resonance fluorescence (see Moreh *et al.* 1981). The radiative width for the $1^- \to 0^+$ transition in ^{14}O is $1\cdot20$ eV ($3\cdot85$ eV for solution B). The corresponding ^{13}N(p, γ)^{14}O cross section, calculated from equation (6), is shown in Fig. 2 as the S factor, defined by

$$S(E) = \sigma E \exp(2\pi\eta),$$

where E is the c.m. proton energy and η is the Sommerfeld parameter. A least-squares fit to $S(E)$ for $E \leqslant 0\cdot3$ MeV with a quadratic function of E (Langanke *et al.* 1985) gives $S(0) = 2\cdot26$ keV b, $S'(0) = -0\cdot94\times10^{-3}$ b and $S''(0) = 0\cdot80\times10^{-4}$ keV^{-1} b. For other values of a, the peak magnitude of S is closely proportional to the value of $\Gamma^{o}_{\gamma}(^{14}\mathrm{O}; 1^{-}\rightarrow 0^{+})$ as given in Table 3, while the shape of S is approximately independent of a.

5. Discussion

From the previous section, the favoured value of the channel radius is $a \approx 5$ fm. This is appreciably different from the value found by Mughabghab *et al.* (1982), who fitted the values of a_{s} and $\sigma_{n\gamma}$(thermal) for the ^{14}C ground-state transition, as given in Table 1, and obtained $a = 3\cdot07\pm0\cdot05$ fm. They used a formula for the cross section that was derived from the Lane and Lynn (1960) theory of direct capture. Their formula is essentially the same as equation (6) with $\mathscr{M}_{\mathrm{if}} \equiv \mathscr{M}(1^{-}\rightarrow 0^{+}) = 0$, which corresponds to their assumption that compound nuclear processes are negligibly small. Also they took $\mathscr{S}(0^{+}) = 2\cdot09$ from Datta *et al.* (1978) which, as seen above, is only one measurement among several that have a considerable spread. The small error that they assign to the value of a makes no allowance for uncertainties in the values of $\mathscr{M}(1^{-}\rightarrow 0^{+})$ and $\mathscr{S}(0^{+})$. The value of a is particularly sensitive to the choice of $\mathscr{M}(1^{-}\rightarrow 0^{+})$ because of the large amount of cancellation that must take place in order to obtain a cross section that is only a few per cent of the hard-sphere value. Mughabghab *et al.* also neglected an alternative solution giving $a \approx 3\cdot7$ fm. To fit the value of $\sigma_{n\gamma}$(thermal) for the transition to the 0^{+*} excited state, Mughabghab *et al.* assumed $\mathscr{M}(1^{-}\leftrightarrow 0^{+*}) = 0$ and then required $\mathscr{S}(0^{+*}) = 0\cdot060\pm0\cdot004$; these are in reasonable agreement with our values in Table 4.

In the earlier calculations of the ^{13}N(p, γ)^{14}O cross section, Mathews and Dietrich (1984) deduced a value for $\Gamma^{o}_{\gamma}(^{14}\mathrm{O}; 1^{-}\rightarrow 0^{+})$ of $2\cdot44$ eV, and Langanke *et al.* (1985) gave a value of $1\cdot50$ eV, which are both appreciably larger† than our preferred value of $1\cdot20$ eV (for $a = 5\cdot0$ fm and solution A). Mathews and Dietrich used the experimental value of the ^{12}C(p, γ)^{13}N cross section for normalization purposes, and Langanke *et al.* used it as a test of their model (although neither model fits the data as well as does BF). The peak cross section for this reaction σ_{peak} is proportional to the radiative width $\Gamma^{o}_{\gamma}(^{13}\mathrm{N}; \frac{1}{2}^{+}\rightarrow \frac{1}{2}^{-}) = \Gamma^{o}_{\gamma}$ say, and as Mathews and Dietrich state, there has been some confusion over the value of this quantity. Their discussion, however, does not help to clarify the situation. Of the thin-target high-resolution measurements that they claim all give $\Gamma^{o}_{\gamma} = 0\cdot62$–$0\cdot67$ eV, Fowler and Lauritsen (1949) gave Γ^{o}_{γ}(lab) $= 0\cdot63$ eV corresponding to $\Gamma^{o}_{\gamma} = 0\cdot58$ eV; Hunt and Jones (1953) did not make an absolute measurement and so gave no value of Γ^{o}_{γ}; Hebbard and Vogl (1960) and Vogl (1963) normalized their measurements to $\sigma_{\mathrm{peak}} = 127$ μb as measured by Seagrave (1951, 1952), so that the value of $130\cdot0$ μb that Vogl gave for the cross section at a single energy should not be regarded as an independent absolute

† Actually the cross section given by Langanke *et al.* (1985), with a resonance S factor of $0\cdot88$ MeV b at $E_{\mathrm{r}} = 0\cdot547$ MeV and a total width of $\Gamma^{o} = 40\cdot1$ keV, corresponds to $\Gamma^{o}_{\gamma} = 1\cdot96$ eV [from $\Gamma^{o}_{\gamma} = 5\cdot55\times10^{-5}\,\Gamma^{o} S(E_{\mathrm{r}})$ MeV^{-1} b^{-1}]. Also Chupp *et al.* (1985) referred to unpublished calculations by B. A. Brown and by D. J. Millener giving $\Gamma^{o}_{\gamma} = 5$ eV.

measurement, and moreover the error of $\pm 3\%$ given by Vogl is stated to be a relative error only (this was previously pointed out in BF); the measurement of Rolfs and Azuma (1974) is the only one that satisfies the claim of Mathews and Dietrich, with $\sigma_{\text{peak}} = 125 \pm 15\ \mu b$ corresponding to $\Gamma_\gamma^\circ = 0 \cdot 63 \pm 0 \cdot 08$ eV (for $\Gamma_p^0 = 33 \cdot 7$ keV). Mathews and Dietrich entirely ignored the measurement of Riess *et al.* (1968) giving $\Gamma_\gamma^\circ = 0 \cdot 45 \pm 0 \cdot 05$ eV in obtaining their adopted value $\Gamma_\gamma^\circ = 0 \cdot 65 \pm 0 \cdot 02$ eV, which is obviously dominated by the Vogl value with its assumed 3% error. Langanke *et al.* (1985) also dismissed the Riess *et al.* measurement by saying that it is given without supporting details, but accepted the value $\Gamma_\gamma^\circ = 0 \cdot 64 \pm 0 \cdot 07$ eV given by Fox *et al.* (1975) [note this is an unpublished preprint which merely says that this value is a weighted average of four references (including Riess *et al.*) but does not say what values were averaged or what weights were given to them]. In BF, the experimental value of Γ_γ° was taken as $0 \cdot 50 \pm 0 \cdot 04$ eV (not $0 \cdot 45$ eV as implied by Mathews and Dietrich). Mathews and Dietrich give an approximate formula connecting the values of $\Gamma_\gamma^\circ(^{14}O; 1^- \to 0^+)$ and $\Gamma_\gamma^\circ(^{13}N; \frac{1}{2}^+ \to \frac{1}{2}^-)$ in their model; if $\Gamma_\gamma^\circ(^{13}N; \frac{1}{2}^+ \to \frac{1}{2}^-)$ is assumed to be $0 \cdot 50$ eV, this formula gives $\Gamma_\gamma^\circ(^{14}O; 1^- \to 0^+) = 1 \cdot 72$ eV, which is still appreciably larger than our preferred value.

Mathews and Dietrich (1984) used a direct–semidirect radiative capture model with a hydrodynamic core-polarization correction for quenching the E1 transitions. They initially assumed single-particle configurations for the initial and final states, fitted their calculated cross sections with an analytical form depending on certain parameters, and then modified the values of some of these parameters (Γ_p^0, Γ_γ° and S_{nr}^0) by multiplying by spectroscopic factors. Their model fails to give the correct value of $\Gamma_\gamma^\circ(^{13}C; \frac{1}{2}^+ \to \frac{1}{2}^-)$, and they attributed this to the greater importance of the core-polarization contributions in this transition as compared with the corresponding ^{13}N transition. Their treatment of the core-polarization contribution is open to question, since it is not clear why this contribution to Γ_γ°, which comes mainly from the internal region, should be multiplied by the same factor $\mathscr{S}(2s_{\frac{1}{2}})\,\mathscr{S}(1p_{\frac{1}{2}})$ as the valence nucleon contribution in the external region, especially since this factor for $^{13}N(p,\gamma)^{14}O$ is more than twice that for $^{12}C(p,\gamma)^{13}N$. It also seems strange that Mathews and Dietrich obtained $\mathscr{S}(1^-) \equiv \mathscr{S}(2s_{\frac{1}{2}}) = 0 \cdot 75$ from fitting $\Gamma^\circ(^{14}N; 1^-) = 30$ keV, whereas from Table 2 (for $a = 5 \cdot 0$ fm), a smaller value $\mathscr{S}(1^-) = 0 \cdot 563$ corresponds to a larger width $\Gamma^\circ(^{14}N; 1^-) = 33 \cdot 8$ keV. This discrepancy is apparently due to the assumption made by Mathews and Dietrich that their value of 80 keV for the single-particle width of the ^{14}N 1^- state, obtained from a Woods–Saxon well calculation, should be divided by two in order to give the width for a state with 50% proton and 50% neutron configuration. This is not what one expects from equations (4) and (9). For $a = 5 \cdot 0$ fm for example, $\mathscr{S}_p = 1$, $\mathscr{S}_n = 0$ leads to $\Gamma^\circ(^{14}N; 1^-) = 77$ keV, in approximate agreement with the value of Mathews and Dietrich, but $\mathscr{S}_p = 0 \cdot 5$, $\mathscr{S}_n = 0 \cdot 5$ gives $\Gamma^\circ(^{14}N; 1^-) = 48$ keV. The reason for this is that $\Theta_{\text{sp,n}}^2$ is smaller than $\Theta_{\text{sp,p}}^2$ and S_n' is smaller than S_p'. Also the assumption by Mathews and Dietrich in their equation (13) that the proton width is proportional to $\mathscr{S}(2s_{\frac{1}{2}})$ is not consistent with our equation (4).

Langanke *et al.* (1985) calculated the $^{13}N(p,\gamma)^{14}O$ cross section in a microscopically founded potential model. The ^{14}O ground state (as well as the $p + {}^{13}N$ scattering states) was described as a totally antisymmetrized ^{13}N-cluster plus proton wavefunction, with the internal structure of the ^{13}N cluster fixed and assumed to be the lowest shell model state with highest spatial symmetry. Langanke *et al.* found that the ^{14}O ground state has a 96·5% probability of being in the (harmonic-oscillator) shell model ground state, which must then also have the highest spatial symmetry. In this approximation, the spectroscopic factor $\mathscr{S}(0^+)$ has the value $\frac{10}{9}$ (compared with the intermediate coupling value of 1·73 given by Cohen and Kurath 1967), which is close to the value that Langanke *et al.* gave for the normalization kernel $\mu_{01} = 1{\cdot}1966$. Similarly for the ^{12}C(p, γ)^{13}N case, which Langanke *et al.* used as a test of the validity of their model, $\mathscr{S}(\frac{1}{2}^-) = \frac{1}{3}$ for states of highest spatial symmetry (compared with 0·613 in intermediate coupling), which is close to their value $\mu_{01} = 0{\cdot}3611$. Because of this small value of μ_{01} in their approximation, Langanke *et al.* were able to get agreement with the ^{12}C(p, γ)^{13}N peak cross section in an essentially one-channel approximation, whereas BF required appreciable cancellation between contributions involving the ^{12}C first-excited state as parent and those involving the ^{12}C ground state. It is unlikely that the model of Langanke *et al.* would give agreement with the observed E1 transition strength between the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ states of the mirror nucleus ^{13}C (cf. Marrs *et al.* 1975), although they used the energies of these states to obtain their potential parameters.

Langanke *et al.* (1985) found a slight increase in the S factor as the energy approaches zero, a feature which does not occur in our calculation or that of Mathews and Dietrich (1984). A similar low-energy upturn occurs in the S factor for ^{7}Be(p, γ)^{8}B, where it is due to the small binding energy of the final state of 136 keV (Tombrello 1965; Williams and Koonin 1981; Barker 1983); such a behaviour is not expected for ^{13}N(p, γ)^{14}O where the final state is bound by 4·6 MeV.

Table 5. Comparison of results for the 1^- level of ^{14}O and the ^{13}N(p, γ)^{14}O cross section

Ref.	E_r (MeV)	Γ^o (keV)	Γ^o_γ (eV)	S_{peak} (MeV b)	$S(0)$ (keV b)
		Calculated values			
A	$0{\cdot}547^D$	34·7	$2{\cdot}44^G$	$1{\cdot}27^G$	2·2
B	$0{\cdot}547^D$	40·1	$1{\cdot}50^H$	$0{\cdot}88^I$	2·6
C	$0{\cdot}547^D$	35·9	1·20	0·615	2·16
		Experimental values			
	$0{\cdot}547\pm0{\cdot}010^E$	$38{\cdot}1\pm1{\cdot}8^F$			

^{A} Mathews and Dietrich (1984).
^{B} Langanke *et al.* (1985).
^{C} Present calculation for $a = 5$ fm and solution A.
^{D} Exact fit.
^{E} Ajzenberg-Selove (1981).
^{F} Chupp *et al.* (1985).
^{G} Corresponds to $\Gamma^o_\gamma(^{13}\mathrm{N}; \frac{1}{2}^+ \to \frac{1}{2}^-) = 0{\cdot}65$ eV; for $\Gamma^o_\gamma(^{13}\mathrm{N}; \frac{1}{2}^+ \to \frac{1}{2}^-) = 0{\cdot}50$ eV, $\Gamma^o_\gamma = 1{\cdot}72$ eV and $S_{peak} = 0{\cdot}89$ MeV b.
^{H} Should be 1·96 eV (see footnote, p. 666).
^{I} Value of $S(E_r)$.

In fairness it should be said that the present calculation also has shortcomings. Most quantities are dependent on the channel radius, and while this may be an advantage when one is fitting data, it is a handicap in making predictions. If an accurate value of the lifetime of the first excited state of ^{14}C was available, then the present method should give a fairly reliable estimate of $\Gamma_\gamma^o(^{14}O; 1^- \to 0^+)$ and of the $^{13}N(p,\gamma)^{14}O$ cross section. Since, however, the lifetime is not known, we have had to make use of less appropriate data and of calculated values, so that there are considerable uncertainties in our predictions. Nevertheless, we favour a value of $\Gamma_\gamma^o(^{14}N; 1^- \to 0^+)$ somewhat smaller than those obtained in previous calculations, and consequently a smaller $^{13}N(p,\gamma)^{14}O$ cross section. Table 5 compares results of the three different calculations.

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