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Electron Capture Decay Energies of ^{153}Gd and ^{175}Hf

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Abstract

The electron capture Q values of ^{153}Gd and ^{175}Hf are analysed by using properties of their β decays to different levels belonging to the same rotational family in the daughter atoms. The results are $Q = 500^{+100}_{-50}$ keV for ^{153}Gd and $Q = 700^{+100}_{-50}$ keV for ^{175}Hf. They agree with data from nuclear reactions but reject smaller values based on K capture probabilities.

1. Introduction

Masses of isotopes close to the line of β stability are generally quite well understood. They constitute the 'backbone' of mass adjustments (Wapstra *et al.* 1985), which attempt to construct a set of mutually consistent nuclear mass data throughout the table of isotopes. These in turn are most valuable for understanding the detailed behaviour of the mass surface as well as extrapolating it towards the 'exotic' nuclei with their unusual compositions of protons and neutrons.

To this end it is important to infer ground state mass differences between pairs of isotopes as precisely as possible. Among the various techniques employed, the most powerful ones include electron capture (EC) decay, or β decay in general, and nuclear reaction studies, supplemented by direct mass spectroscopy for stable species. For the vast majority of nuclides these complementary results agree, often to better than ± 1 keV. In fact, a significant improvement of this precision is not envisaged for the near future, since at the level of 10–100 eV sensitivity, both the possibly finite rest mass of the neutrino and the difficult-to-calculate atomic excitations will start to play a role.

The two long-lived radioactive nuclides ^{153}Gd and ^{175}Hf are exceptional. For both species the K electron capture probability P_K to excited nuclear states of the respective daughter atom has been measured. Thus the respective decay energies Q have been inferred, but were found to deviate by more than 10 standard deviations from the corresponding quantities derived from nuclear reactions [for references see below and the discussions in Wapstra *et al.* (1977, p. 16; 1985, p. 216)].

It is the objective of the present paper to assess the decay energies of the two nuclides with a third, independent method and thus clarify the reported ambiguity. The method to be employed relies on the well established deformed character of the two isotopes and will be explained with ^{153}Gd.

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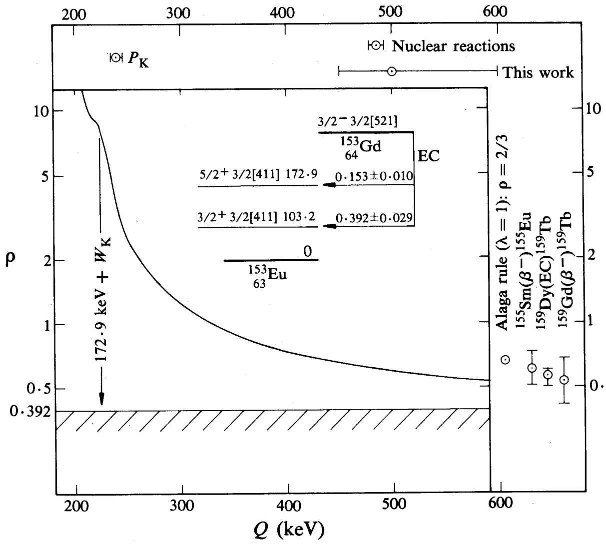

Fig. 1. Ratio of reduced transition probabilities ρ plotted (semi-logarithmic) as a function of the ^{153}Gd(EC)^{153}Eu decay energy Q. The nuclear levels involved are at excitation energies of 103 and 173 keV. Below ~221 keV no K capture is energetically possible to the 173 keV level, resulting in a discontinuity of the slope $\rho(Q)$. Experimental values for ρ from other isotopes as well as the prediction of the Alaga rule (multipolarity $\lambda = 1$) are given near the right-hand ordinate. The extracted Q value as well as published data on Q are inserted next to the top abscissa. A simplified decay scheme showing only the feeding to the levels involved is indicated.

2. Decay of Rotational Nuclei via Electron Capture

A simplified decay scheme of ^{153}Gd is shown as an inset in Fig. 1. The branching ratio $b(E_x)$ for electron capture to a nuclear level at excitation energy E_x is proportional to the phase space factor f and the reduced transition probability $B(\lambda)$, with λ the multipole order of the β transition. Alaga *et al.* (1955) have shown that for deformed nuclei $B(\lambda)$ can be further factorized, namely $B(\lambda) = \mu(\lambda)\langle CG\rangle^2$. Here $\langle CG\rangle$ is a calculated geometrical factor, given as a Clebsch–Gordan coefficient, which is a function solely of λ and the total nuclear angular momentum I, as well as its projection K, onto the nuclear symmetry axis. In contrast, the matrix element $\mu(\lambda)$ is more complicated since it retains the dependence on the specific features of the intrinsic nuclear structure and might involve the operators $\boldsymbol{\sigma}.\boldsymbol{r}$, $\beta\gamma_5$ (for $\lambda = 0$), $\boldsymbol{r}$, $\boldsymbol{r}\times\boldsymbol{\sigma}$, $\boldsymbol{\alpha}$ (for $\lambda = 1$), and B_{ij} (for $\lambda = 2$) when we consider the specific case of first forbidden non-unique β decay. [For these operators we have used the standard nomenclature; for more details the reader is referred to the recent monograph by Behrens and Bühring (1982).]

When one compares the transition probabilities to different members (at energies E_x and E'_x respectively) of the *same* rotational family, the factor μ involving the intrinsic wavefunctions is the same. Hence it drops from their ratio and we find

$$b(E'_x)/b(E_x) = \rho f(Q-E'_x)/f(Q-E_x),$$

with ρ being the *ratio* of reduced transition probabilities, $\rho = \langle CG' \rangle^2 / \langle CG \rangle^2$. This simple form for ρ is expected to be valid if only one multipole order contributes to the β decay.

The f function for electron capture of a nucleus ^{A}Z corresponds to the integrated Fermi function of (continuous) β decay and is given by

$$f(Q - E'_{\mathrm{x}}, Z) = \frac{1}{m^2 c^4} \sum_i \tfrac{1}{2}\pi g_i^2 (Q - E_{\mathrm{x}} - W_i)^2 B_i,$$

where the summation is over all allowed transitions, $mc^2 = 511$ keV is the electron rest mass and g_i is the amplitude of the radial electron wavefunction, with the symbolic index i denoting the triplet of accessible atomic quantum numbers *nls*. The sum is restricted to $j \equiv l + s = \frac{1}{2}$ orbitals, when we operate within the allowed approximation, which was found to be valid (in the ξ approximation) for the majority of first forbidden non-unique β transitions (Schopper 1966; Vatai 1973). Thus no electron capture to orbitals such as $p_{3/2}$, $d_{5/2}$ etc. is possible. The g_i values have been compiled by Bambynek *et al.* (1977), from where also the numerical values for B_i, the electron exchange and overlap corrections are taken. The term $Q - E_{\mathrm{x}} - W_i$ is the neutrino energy, assuming a vanishing neutrino mass, where W_i is the binding energy of the *nls* electron of the daughter atom (Sevier 1979).

The functional dependence of $\rho(Q)$ for ^{153}Gd is displayed in Fig. 1. For the calculations, branching ratios of $(15 \cdot 3 \pm 1 \cdot 0)\%$ for the $\frac{3}{2} \rightarrow \frac{5}{2}$ and $(39 \cdot 2 \pm 2 \cdot 9)\%$ for the $\frac{3}{2} \rightarrow \frac{3}{2}$ β transitions have been used (Lee 1982). It is apparent that a knowledge of ρ allows one to infer the decay energy Q.

We now proceed to demonstrate how one can indeed reliably estimate ρ and thus extract $Q(^{153}\mathrm{Gd})$. The ground state of this nucleus is described by the Nilsson quantum numbers $I^{\pi}\Omega[N n_z \Lambda] = \frac{3}{2}^{-}\frac{3}{2}[521]$. The β decay leads to the $I^{\pi} = \frac{3}{2}^{+}$ and $\frac{5}{2}^{+}$ members of the rotational family $I^{\pi}\frac{3}{2}[411]$. If both β transitions were of multipolarity 1, the Alaga intensity rule would predict the ratio of reduced transition probabilities

$$\rho = \langle \tfrac{3}{2}\,\tfrac{3}{2}\,1\,0 | \tfrac{5}{2}\,\tfrac{3}{2} \rangle^2 / \langle \tfrac{3}{2}\,\tfrac{3}{2}\,1\,0 | \tfrac{3}{2}\,\tfrac{3}{2} \rangle^2 = \tfrac{2}{3}\,.$$

However for the $\frac{3}{2} \rightarrow \frac{3}{2}$ transition, multipolarity $\lambda = 0$ may also contribute. Therefore ρ can be more realistically estimated from data of the same type of β transitions in neighbouring nuclei involving the same Nilsson quantum numbers. These have, in fact, been observed in the (continuous) β decays of ^{155}Sm and ^{159}Gd as well as the electron capture decay of ^{159}Dy. The Q values for these nuclei are well known (Wapstra *et al.* 1985), and hence we may apply the above formulae in order to *extract* ρ. The phase space factor for continuous β decay has been calculated in the allowed approximation as parametrized by Wilkinson and Macefield (1974). The data for ρ thus obtained are in mutual statistical agreement. Their average, $\rho = 0 \cdot 56 \pm 0 \cdot 05$, is somewhat smaller than the Alaga prediction $\rho = 0 \cdot 67$, indicating that $\sim 20\%$ of the $\frac{3}{2} \rightarrow \frac{3}{2}$ transitions are of multipolarity $\lambda = 0$. The individual results are plotted in Fig. 1; their average implies $Q = 500^{+100}_{-50}$ keV. The precision of this value is only moderate (because $Q \gg W_{\mathrm{K}}$). However, as can be seen from Fig. 1, the precision with which the Q value could be extracted improves rapidly as Q decreases.

In spite of the $\approx 10\%$ uncertainty, the deduced Q value thus *clearly* contradicts the lower of the previous results which was based on K capture probabilities,

$Q = 240 \pm 5$ keV, while being in good accord with that based on nuclear reactions, $Q = 477 \pm 5$ keV. The latter value is an average of several experiments and was used as an input value by Wapstra *et al.* (1985). The former value, $Q = 240 \pm 5$ keV, is the average of five measurements of P_K (Blok *et al.* 1962; Cretzu *et al.* 1964; Boyer *et al.* 1967; Sergienko *et al.* 1980; Singh *et al.* 1985), after the four older entries had been corrected for changes which occurred over the last 20 years in the value adopted for the K shell fluorescence yield ω_K. Another similar but much less sensitive correction (of a few per cent) has been applied to the conversion coefficients of the transitions involved, which we have taken from measured values (Lederer and Shirley 1978) or, if not available, from the calculations of Hager and Seltzer (1968). The original as well as the corrected data for P_K are given in Table 1. After all values have thus been adjusted to the same $\omega_K = 0\cdot932$ (Krause 1979) the consistency amongst the ensemble of P_K data is not significantly improved but yields a satisfactory chi-squared per degree-of-freedom of 1·4. However, why these five experiments commonly fail in ascertaining Q is still unclear at present. It is worth mentioning in this context that only one of the groups (Cretzu *et al.* 1964) made an early attempt to observe coincidences between K and L X rays on one hand, and the 173 keV photon on the other hand, which is potentially the most precise method for determining P_K. The average results from the three different techniques for Q are shown in Fig. 1.

Table 1. The K capture probabilities P_K of the transition ^{153}Gd(EC)^{153}Eu to its 172·9 keV excited nuclear level

P_K^0 is the original value as given in the reference; P_K is the respective quantity after adjustment to $\omega_K = 0\cdot932$ has been made (see text for more details); $\frac{1}{4}\chi^2 = 1\cdot36$; $Q(P_K) = 240 \pm 5$ keV

P_K^0	Reference	Adjusted $P_K \pm \Delta P_K$
0·43	Singh *et al.* (1985)	$0\cdot43 \pm 0\cdot04^A$
0·34	Sergienko *et al.* (1980)	$0\cdot32 \pm 0\cdot04$
0·31	Boyer *et al.* (1967)	$0\cdot28 \pm 0\cdot07$
0·38	Cretzu *et al.* (1964)	$0\cdot37 \pm 0\cdot03$
0·42	Blok *et al.* (1962)	$0\cdot36 \pm 0\cdot03$
		Av. $P_K = 0\cdot364 \pm 0\cdot016$

[A] No adjustment made.

In complete analogy we now analyse the decay of ^{175}Hf. Again, first forbidden non-unique electron capture links the $\frac{5}{2}^-\frac{5}{2}[512]$ ground state of ^{175}Hf to the two members $\frac{7}{2}^+\frac{5}{2}[402]$ and $\frac{5}{2}^+\frac{5}{2}[402]$ of a rotational family in ^{175}Lu. The decay scheme, also simplified and only showing the levels involved (Minor 1976; Lederer and Shirley 1978), and the function $\rho(Q)$ are displayed in Fig. 2. Experimentally, we find $\rho = 0\cdot39 \pm 0\cdot03$ from an analysis of the β decay of ^{171}Er, in agreement with the Alaga rule prediction $\rho = \frac{2}{5}$, and indicating a small $\lambda = 0$ contribution to the $\frac{5}{2} \rightarrow \frac{5}{2}$ transition. The result for the energy liberated in the ^{175}Hf(EC)^{175}Lu decay is $Q = 700^{+100}_{-50}$ keV. This, again, favours the findings of the nuclear reaction studies, $Q = 686 \pm 3$ keV (Wapstra *et al.* 1985, p. 280), while contradicting the electron capture data, $Q = 611 \pm 8$ keV. Here, however, the rejection is not as decisive as in the previous case, since our result leaves still about 5% likelihood for $Q(^{175}$Hf) being smaller than 620 keV.

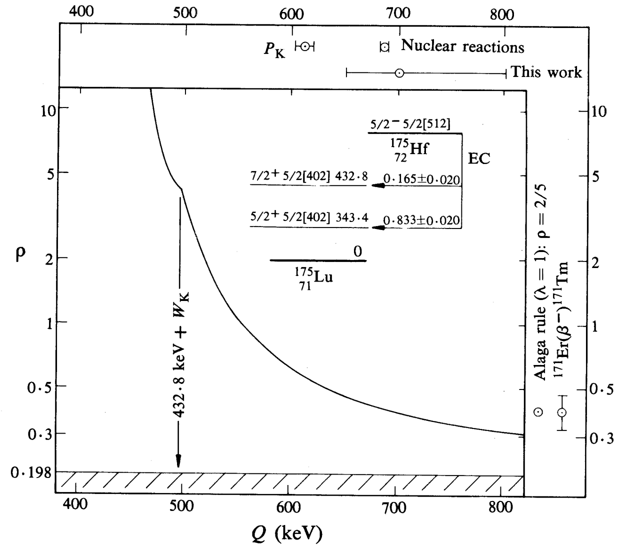

Fig. 2. Analogous plot to Fig. 1 for the decay ^{175}Hf(EC)^{175}Lu. The discontinuity of $\rho(Q)$ is shifted to 500 keV, while the nuclear levels involved are at 343 and 433 keV, as given in the simplified decay scheme.

3. Conclusions

In summary, the electron conversion decay energies of ^{153}Gd and ^{175}Hf have been examined by a method which employs their deformed nature. As such, our analysis is independent of earlier approaches, namely studies of nuclear reactions as well as K capture probabilities. With each of these two methods a number of consistent measurements have been reported in the past. Their two average results, however, cannot be reconciled, since they differ by several standard deviations. In spite of the comparably low precision of our finding, which is in fact inferior to each of the two mentioned 'standard' procedures, it nevertheless allows us to reject the data inferred from the capture studies with about 95% confidence for ^{175}Hf and better than 99% confidence for ^{153}Gd. Therefore, we conclude by calling for new coincidence experiments with the aim of investigating the electron capture decays to the 173 keV level of ^{153}Eu and to the 433 keV level of ^{175}Lu. These experiments have the potential of determining the decay energies to better than a few keV.

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