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Reflection of Magnetoionic Waves from a Steep Density Gradient. II* Incident Ordinary Mode

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Abstract

We consider the case of an ordinary mode wave incident on a density discontinuity in an anisotropic plasma. The relative magnitudes of the two transmitted and two reflected modes produced by the incident o mode are calculated, and the dominant secondary mode is determined for different incident wave parameters and plasma parameters. An application of this work to the interpretation of the polarization of certain solar radio bursts is considered.

1. Introduction

In a previous paper (Hayes 1985; henceforth denoted by H—see present issue p. 687) we developed a method for calculating the relative magnitudes of the four secondary modes produced when a magnetoionic mode strikes a density discontinuity within a plasma. The plasma regions on either side of the density discontinuity were assumed to be homogeneous and in each plasma region it was possible to have two modes propagating towards the density boundary and two modes propagating away from the density boundary. Only five of these possible eight modes were considered; one incident mode propagating towards the density boundary and two reflected and two transmitted modes propagating away from the density boundary. The term 'mode' in H referred specifically to one of these five modes and this notation is also used in the present paper.

The method in H involved solving the dispersion relations for all five modes, and then solving the boundary conditions on the electric and magnetic fields of the modes. The quantity chosen to represent the energy of each mode was the component of the Poynting flux normal to the density boundary. The relative energy in each secondary mode was then taken to be the ratio of the normal Poynting flux of that mode to the normal Poynting flux of the incident mode.

The calculations to determine the relative normal Poynting flux for each mode were performed for many different values of the incident mode parameters (i.e. wave frequency and angle of propagation), for different plasma parameters, and for different external magnetic fields. The relative energy in each of the secondary modes changed as the wave parameters and physical conditions varied, and it was found that in general it is possible to determine (*a*) which parameters have the largest effect on the

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relative strength of each secondary mode and (*b*) what the effect of changing each parameter is. The results obtained in H were for an incident extraordinary mode.

In the present paper the results of similar calculations for the case of an incident ordinary mode are presented. An important application of these results is in the investigation of the polarization of Type III solar radio bursts (Wild and McCready 1950). At the fundamental frequency, these bursts are observed with an average degree of circular polarization of 35% in the o mode sense; the harmonic has an average polarization of 11%, also in the o mode sense (Dulk and Suzuki 1980). Theoretical models for the generation of Type III bursts predict bursts polarized in the o mode sense but they have not been able to explain the observed low degree of polarization of the bursts. However, the polarization may also be influenced by propagation effects such as repeated reflection of the radio waves off the walls of an under-dense duct in the solar corona. Such ducts have already been proposed to explain the anomolous source heights found for Type III radio bursts (Duncan 1979). Hence we are interested in the conditions where an initial o mode wave generates strong reflected x and o modes at a density discontinuity. This application is discussed briefly in Section 4.

The method of determining the relative energy in each of the secondary modes produced by a mode incident on a steep density gradient is summarized in Section 2. Our conclusions on the effect of varying different wave and plasma parameters are presented in Section 3.

2. Method of Analysis

In order to determine the energy (relative to the incident mode) of each of the four secondary modes produced when an incident o mode strikes a density discontinuity, we follow the procedure outlined in H, Sections 2 and 3. In brief, values of the plasma and wave parameters $\omega/\omega_{\mathrm{p1}}$, $\omega_{\mathrm{p2}}/\omega_{\mathrm{p1}}$, $\Omega_{\mathrm{e}}/\omega_{\mathrm{p1}}$, l_1 and θ_{in} are chosen (these parmeters are defined in H, Section 2) and the Poynting flux normal to the density boundary S_z is calculated for each mode. The relative energy in the secondary modes is taken to be $S_{z\alpha}/S_{z\mathrm{in}}$ where the subscript α is 'or', 'xr', 'ot' or 'xt' for the reflected o mode, reflected x mode, transmitted o mode and transmitted x mode respectively. The values of the plasma and wave parameters are chosen from within the ranges

$$1 < \omega/\omega_{\mathrm{p1}} < 20, \qquad 1 < \omega_{\mathrm{p2}}/\omega_{\mathrm{p1}} < 10, \qquad 0 < \Omega_{\mathrm{e}}/\omega_{\mathrm{p1}} < 1$$

$$\text{or } 1 < \Omega_{\mathrm{e}}/\omega_{\mathrm{p1}} < 10, \qquad 0 < l_1 < 1, \qquad 0^\circ < \theta_{\mathrm{in}} < 90^\circ.$$

The method for determining the variations in $S_{z\alpha}/S_{z\mathrm{in}}$ as each of these five parameters changes is described in H, Section 3. The results of the calculations are plotted as pairs of graphs of $S_{z\alpha}/S_{z\mathrm{in}}$ against frequency. The upper graph shows the normal Poynting fluxes of the two reflected modes and the lower graph shows the normal Poynting fluxes of the two transmitted modes. On both graphs the dashed curves correspond to o modes and the solid curves correspond to x modes. In many cases the x-mode graphs have two sections covering two different frequency regions. The lower frequency section corresponds to the low frequency branch of the x mode, i.e. the z mode. In the next two sections, the term 'x mode' is used to refer only to the higher frequency branch of the extraordinary mode; we refer to the lower frequency branch as the z mode. For any given incident mode, one of the transmitted modes is an o mode and the other transmitted mode is either an x mode or a z mode.

3. Results

Figs 1–5 show the changes that occur in S_{za}/S_{zin} as ω/ω_{p1}, ω_{p2}/ω_{p1}, Ω_e/ω_{p1}, l_1 and θ_{in} are varied for the case $\Omega_e/\omega_{p1} < 1$. This is the parameter region which is relevant to the study of solar Type III radio bursts. Detailed descriptions of the variations between graphs are given in the figure captions.

Summary of Results—Small Magnetic Field

(*a*) The transmitted o mode is the dominant mode at frequencies above the cut-off frequency for that mode. This cut-off frequency increases as ω_{p2}/ω_{p1} increases and as θ_{in} increases, but the variation of the other parameters has little effect on the transmitted o mode.

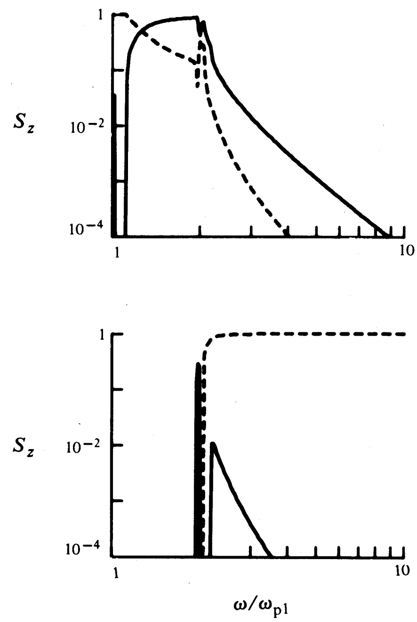

Fig. 1. Graphs of S_z against frequency showing the normal Poynting fluxes of the two reflected modes (upper) and of the two transmitted modes (lower). Dashed curves correspond to o modes and solid curves to x modes. Graphs are for $\omega_{p2}/\omega_{p1} = 2$, $\Omega_e/\omega_{p1} = 0{\cdot}2$, $l_1 = 0{\cdot}707$ and $\theta_{in} = 20°$. The following observations are noted: (i) At high frequencies and moderate angles of incidence the dominant mode is the transmitted o mode, while for large angles of incidence the initial o mode is almost completely reflected in the o mode. (ii) At frequencies where the transmitted o mode propagates it is much stronger than any of the other modes. However, this mode changes over a small frequency range from the dominant mode to a very weak mode. (iii) The transmitted x mode is never very strong; it usually contains less than $0{\cdot}01$ of the initial mode's normal Poynting flux, and it only propagates over a small range of frequencies. The maximum in the normal Poynting flux of this mode occurs at frequencies just above the cut-off frequency for the x mode. The transmitted z mode also propagates over a narrow range of frequencies but the mode is usually stronger than the transmitted x mode. However, it is never the dominant mode. (iv) Significant reflection of the incident energy only occurs below the cut-off frequency of the transmitted o mode. At these lower frequencies either the reflected o mode or the reflected x or z mode may be stronger. A strong reflected x mode only occurs for small angles of incidence, whereas a strong reflected z mode can occur at all angles of incidence.

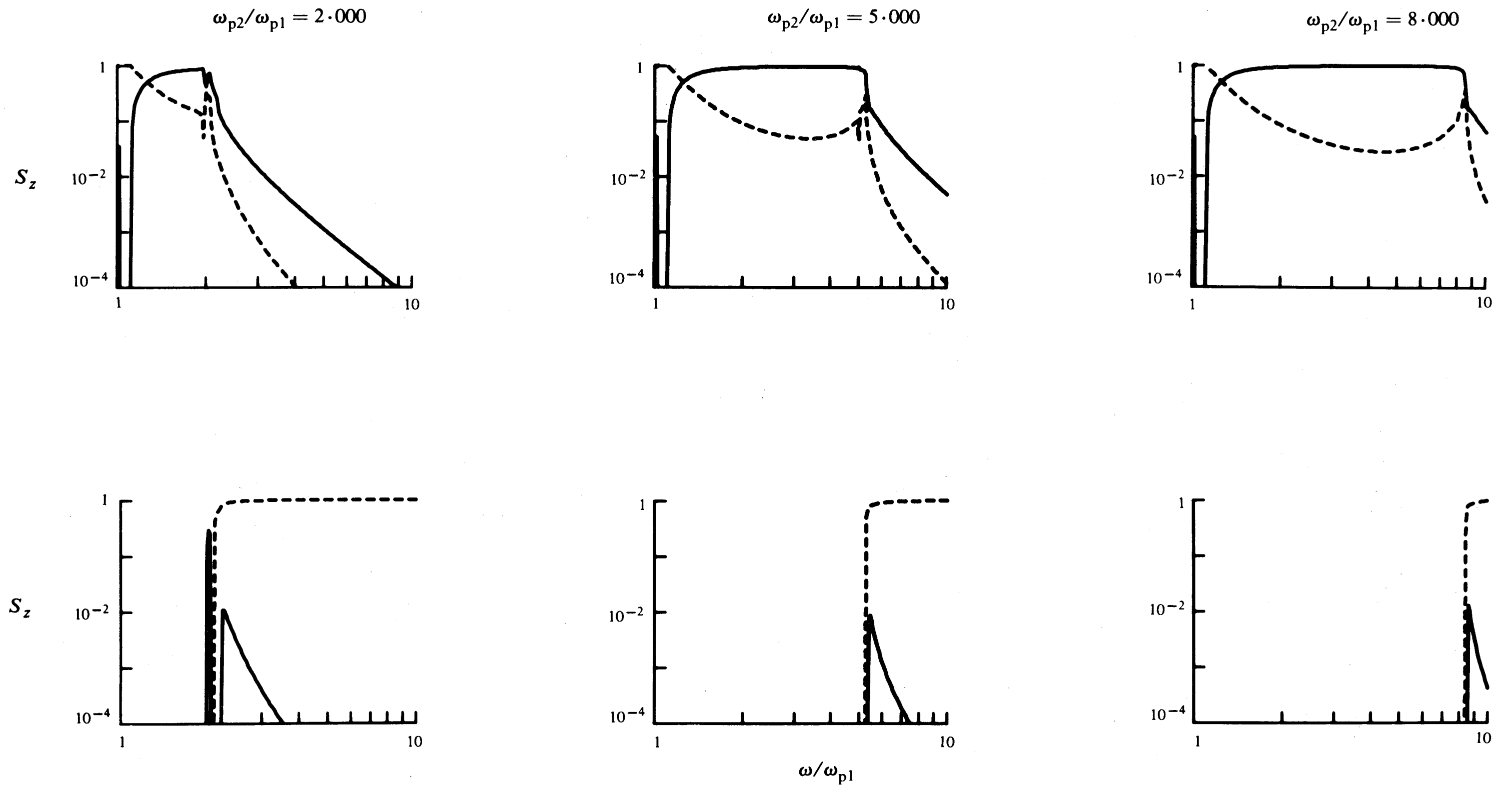

Fig. 2. Graphs of S_z against frequency for ω_{p2}/ω_{p1} changing, $\Omega_e/\omega_{p1} = 0\cdot2$, $l_1 = 0\cdot707$ and $\theta_{in} = 20°$. As ω_{p2}/ω_{p1} increases: (i) The cut-off frequency of the transmitted o mode increases and hence this mode is the dominant mode at higher frequencies only. (ii) The frequency range over which the transmitted x and z modes propagate becomes narrower and for some angles of incidence both these modes may become negligible, i.e. $S_{zxt}/S_{zin} \ll 10^{-4}$. There is no consistent trend in the relative strengths of either the transmitted x mode or the transmitted z mode. (iii) All the reflected modes become stronger and remain strong modes over a wider range of frequencies (corresponding to a transfer of energy into these modes rather than into the transmitted o mode). The reflected x mode becomes stronger with respect to the reflected o mode except for frequencies near the reflected o mode cut-off and the transmitted o mode cut-off. (iv) If the reflected z mode propagates it may be the dominant mode over a small range of frequencies just above ω_{p1}.

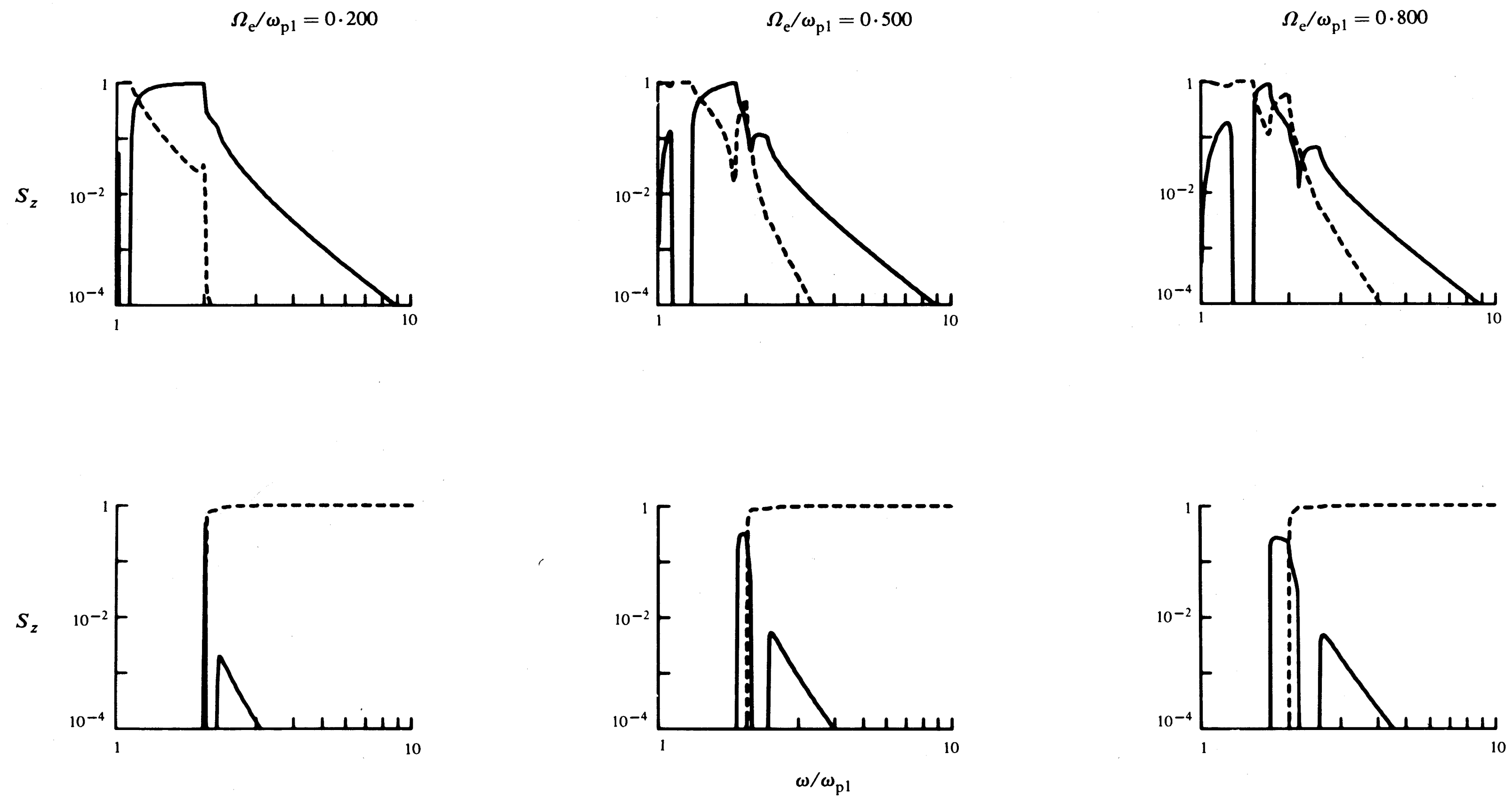

Fig. 3. Graphs of S_z against frequency for Ω_e/ω_{p1} changing, $\omega_{p2}/\omega_{p1} = 2$, $l_1 = 1$ and $\theta_{in} = 20°$. As Ω_e/ω_{p1} increases: (i) There is no apparent change in the transmitted o mode. (ii) The strength of the transmitted x mode may increase slightly but there is no significant trend in the relative energy in the transmitted z mode. The transmitted z mode propagates over a wider frequency range. (iii) The average strength of the reflected o mode increases slightly, although at some frequencies it may become weaker due to the transfer of energy to the reflected z mode instead of the reflected o mode. (iv) The reflected z mode propagates over a wider frequency range whereas the reflected x mode propagates at higher frequencies and over a narrower frequency range. There is no consistent trend in the strength of the reflected z mode but the reflected x mode generally becomes weaker.

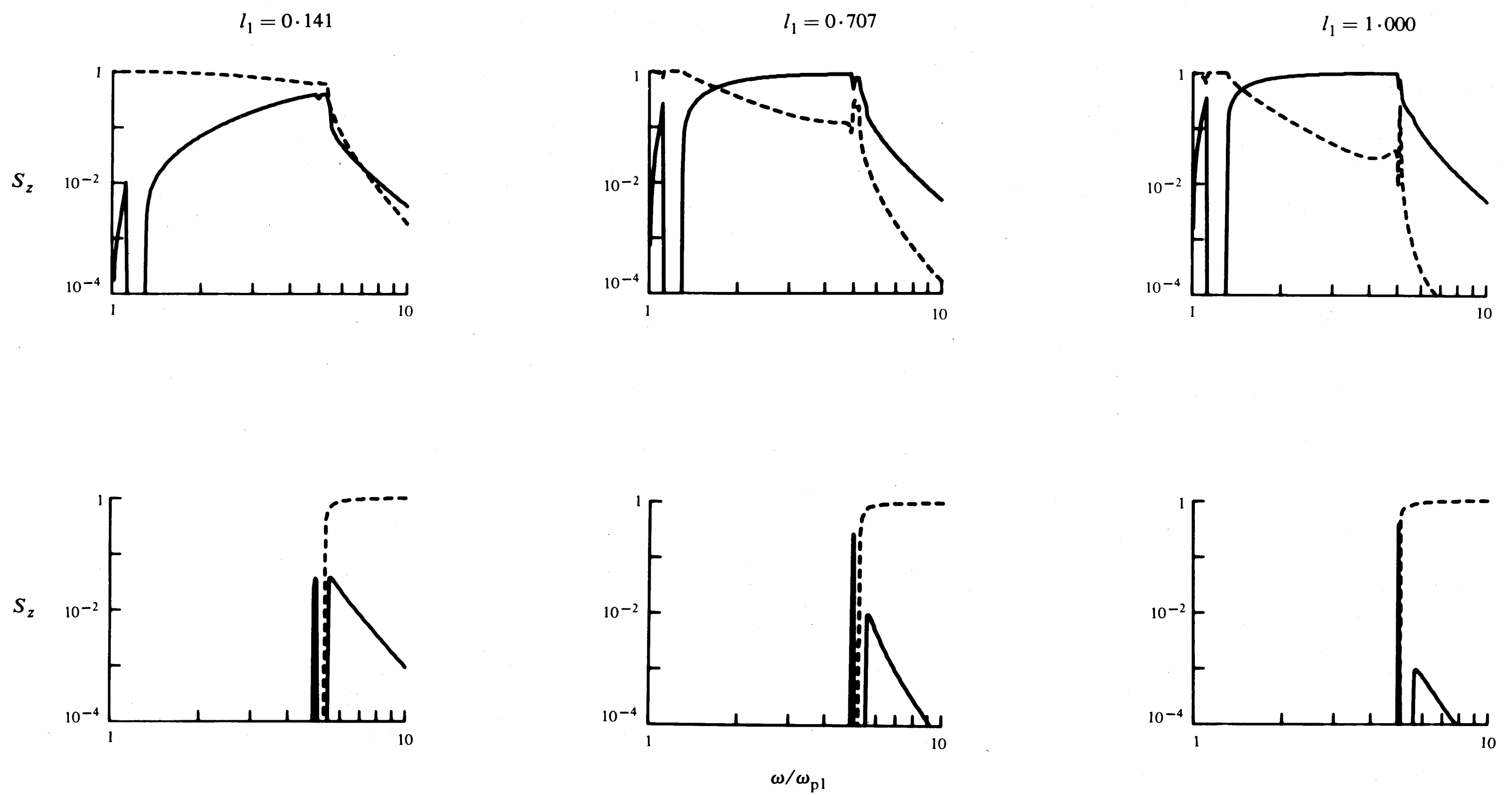

Fig. 4. Graphs of S_z against frequency for l_1 changing, $\omega_{p2}/\omega_{p1} = 5$, $\Omega_e/\omega_{p1} = 0\cdot5$ and $\theta_{in} = 20°$. As l_1 increases: (i) There is little change in the transmitted o mode. (ii) The transmitted x mode becomes weaker and it propagates over a narrower frequency range. However, the transmitted z mode shows no consistent trends. (iii) The reflected o mode becomes weaker at all frequencies where it propagates. (iv) The reflected z mode becomes stronger. The reflected x mode also becomes stronger when the angle of incidence is small, although it may become weaker for large angles of incidence.

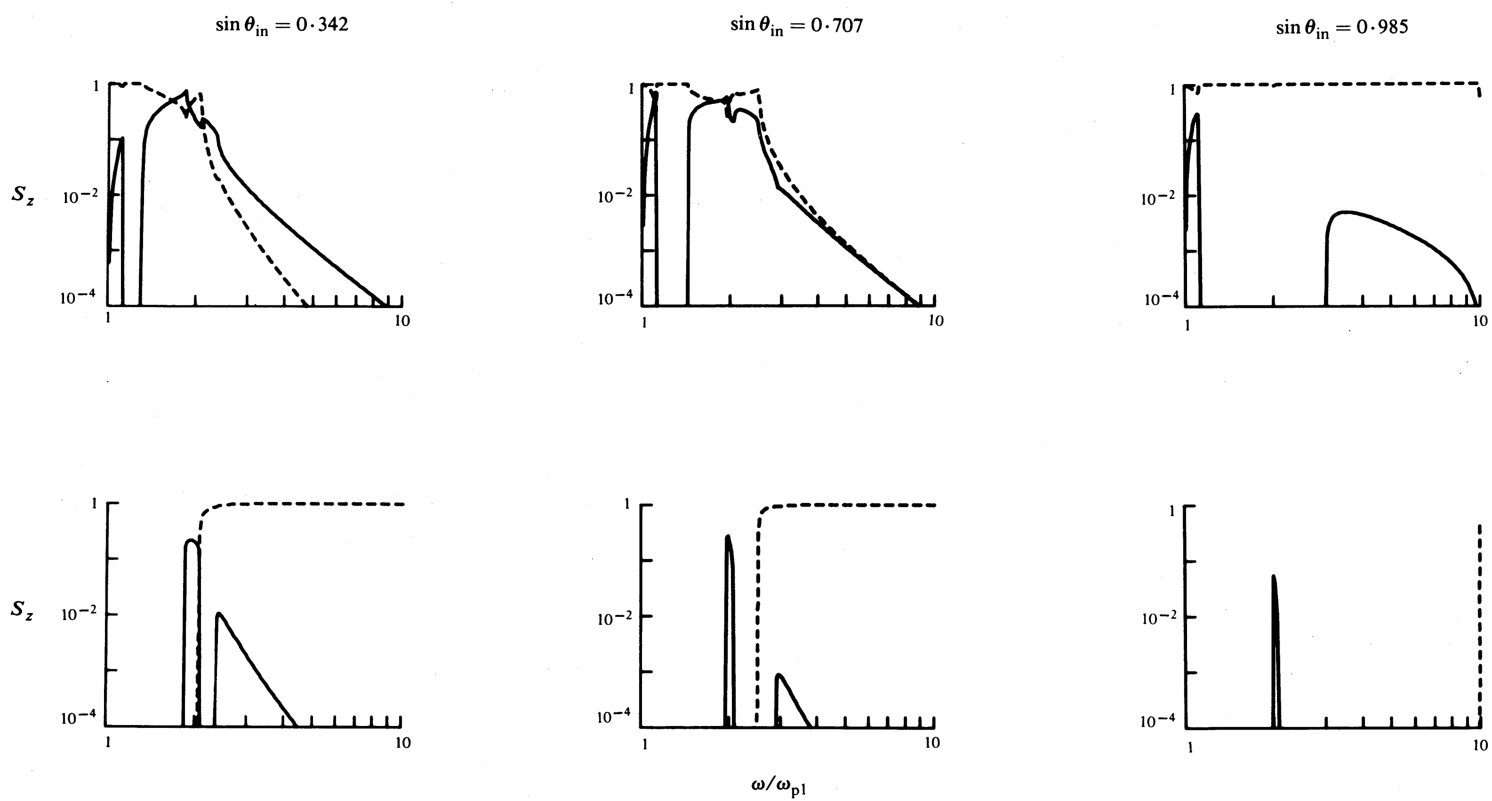

Fig. 5. Graphs of S_z against frequency for θ_{in} changing, $\omega_{p2}/\omega_{p1} = 2$, $\Omega_e/\omega_{p1} = 0 \cdot 5$ and $l_1 = 0 \cdot 707$. As θ_{in} increases: (i) The transmitted o mode is the dominant mode at higher frequencies only. (ii) The frequency ranges in which the high and low frequency branches of the transmitted x mode propagate become narrower. The strength of the transmitted z mode does not change noticeably whereas the transmitted x mode becomes weaker. (iii) The frequency range over which the reflected o mode is a strong mode becomes larger and this mode becomes much stronger. (iv) There is no consistent trend in the strength of the reflected z mode but the reflected x mode becomes weaker and only propagates at higher frequencies.

(*b*) The transmitted x mode (which is never a very strong mode) becomes stronger as l_1 decreases or as θ_{in} decreases. The strength of the transmitted z mode is fairly insensitive to the variation of all parameters except the wave frequency. However, the frequency range over which the transmitted x (or z) mode propagates and has values of $S_{zxt}/S_{zin} > 10^{-4}$ increases when ω_{p2}/ω_{p1} decreases, when l_1 decreases or when θ_{in} decreases.

(*c*) Significant reflection of the incident o mode only occurs at frequencies below the cut-off frequency for the transmitted o mode. Either the reflected o mode or the reflected x (or z) mode may be the dominant mode. The strength of the x mode with respect to the o mode increases as ω_{p2}/ω_{p1} increases, Ω_e/ω_{p1} decreases, or as θ_{in} decreases.

(*d*) The dominant mode is typically the transmitted o mode at higher frequencies and the reflected o mode at lower frequencies. Under specific conditions the dominant mode may also be the reflected x mode or the reflected z mode. The reflected x mode is more likely to be the dominant mode for small or moderate angles of incidence. The variation of the other parameters has little effect on the strength of the reflected x mode relative to the reflected o mode.

Summary of Results—Large Magnetic Field or Small Initial Plasma Density

The behaviour of the transmitted and reflected o modes is very similar for all values of Ω_e/ω_{p1} (i.e. >1 or <1). However, the x modes are generally weaker when $\Omega_e/\omega_{p1} > 1$ than when $\Omega_e/\omega_{p1} < 1$, and the z modes are much stronger. Furthermore, general trends in the strength of both the x and z modes are more pronounced when $\Omega_e/\omega_{p1} > 1$. Conclusions (*a*) and (*b*) above are also applicable to the results for $\Omega_e/\omega_{p1} > 1$. The results for $\Omega_e/\omega_{p1} > 1$ which are different from those for $\Omega_e/\omega_{p1} < 1$ may be summarized by:

(*a′*) The transmitted x mode is still a weak mode; however, the transmitted z mode is generally a strong mode and may be the dominant secondary mode over a small range of frequencies. Both these modes become stronger as ω_{p2}/ω_{p1} increases, Ω_e/ω_{p1} decreases, l_1 increases or as θ_{in} decreases.

(*b′*) In general the low frequency reflected z mode is stronger than the higher frequency reflected x mode. The z mode may also be stronger than the reflected o mode. The reflected x (and z) modes become stronger as ω_{p2}/ω_{p1} increases, Ω_e/ω_{p1} decreases, or as l_1 increases. As θ_{in} increases the reflected x mode becomes weaker whereas the reflected z mode becomes stronger.

4. Discussion

The important features of the secondary modes produced when an incident o mode strikes a steep density gradient are:

(i) The dominant secondary mode is usually the transmitted o mode or, if this is an evanescent mode, the reflected o mode.

(ii) Either the reflected z mode or the transmitted z mode may be the dominant secondary mode, but only over a narrow range of frequencies. The z modes are stronger for larger magnetic fields.

(iii) The transmitted x mode is rarely a strong mode, but the reflected x mode can be the dominant reflected mode (at some frequencies) for weaker external magnetic fields and for incident waves striking the density boundary at angles less than $\sim 45°$.

If we replace 'x mode' by 'o mode' and vice versa, there are many similarities between these results and the behaviour of the secondary modes when the incident wave is in the x mode. For example, in H we found that the dominant secondary modes are generally either the transmitted or reflected x (or z) modes (i.e. the same type of mode as the incident mode). The transmitted o mode is never a strong mode, and the reflected o mode is stronger for weaker external magnetic fields and smaller angles of incidence.

The differences between the secondary modes produced by an incident o mode and those produced by an incident x mode are largely due to the behaviour of the low frequency branch of the extraordinary mode (the z mode). The trends in the strength of this mode relative to the incident mode are often different to the trends in the relative strength of the higher frequency x mode. There are also large differences in the values of the wave frequency, plasma frequencies, gyrofrequency and angle of incidence at which a different mode becomes the dominant secondary mode. For example, the transmitted o mode is the dominant secondary mode produced by an incident o mode at frequencies above a cut-off frequency, say ω_{co}, whereas the transmitted x mode is the dominant mode produced by an incident x mode at frequencies above a different cut-off frequency, say $\omega_{cx} > \omega_{co}$.

Hence, if we consider reflected and transmitted modes with the same polarization as the incident mode (or with opposite polarization to the incident mode), the trends in the energies of these modes relative to the incident mode energy as the plasma and wave parameters vary are almost independent of the polarization (o or x) of the incident mode. However, for specific values of the plasma and wave parameters, the relative energies of these secondary modes can be quite different for an incident x and an incident o mode.

Finally we comment on the possibility that reflection of o modes (producing both o and x modes) is responsible for the observed low polarization of Type III solar radio bursts. In this paper we have found that the reflected x mode produced by an incident o mode is stronger for larger density changes, for weaker external magnetic fields, and for smaller angles of incidence. The ratio Ω_e/ω_{p1} in the solar corona is generally small and this favours the generation of a strong reflected x mode. Observations of thin, over-dense structures in the solar corona (Pick *et al.* 1979; Trottet *et al.* 1982) indicate that density changes (possibly at the edges of plasma ducts) may be as large as a factor of 10, which also favours x mode generation. However, our results show that the largest variation in the strength of the reflected x mode is due to changes in the angle of incidence and for large angles of incidence the reflected x mode is very weak and may be evanescent. Hence, the generation of a strong reflected x mode requires small angles of incidence. Ducts in the solar corona are probably aligned with the solar magnetic field, which in our model corresponds to the magnetic field lying in the plane of the density discontinuity. Therefore a small angle of incidence corresponds to a large angle between the wavevector of the incident o mode and the magnetic field. An o mode propagating outwards through the solar corona in the absence of small-scale density inhomogeneities is refracted in the direction of the radial solar density gradient. It can be argued (Wentzel 1984) that the radial density gradient is not necessarily parallel to the magnetic field, so the angle between the magnetic field and an incident o mode may be quite large.

It is possible, in principle, for reflection of an initially purely o mode wave off the walls of a duct to lead to a mixture of modes compatible with the observed polarization of Type III solar radio bursts. However, whether or not this mechanism provides a plausible explanation for the observed polarization depends on the details of the propagation of the escaping radiation. To discuss this requires a detailed model for the duct and for local gradients in the plasma. In view of the absence of other plausible explanations for the observed polarization, it is desirable that a detailed model which could account for the inferred depolarization of Type III bursts be formulated and explored.

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