Aust. J. Phys., 1985, **38**, 715–32

X-ray Propagation in Strongly Magnetized Plasmas

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Abstract

A new treatment of the calculation of the dielectric tensor of a magnetized plasma in an extremely strong field, such as occurs in magnetized neutron stars, is presented. An accurate numerical scheme for the evaluation of the hermitian and antihermitian parts of the tensor is used to calculate the refractive index and absorption cross section for magnetic fields and plasma temperatures typical of pulsating X-ray sources and γ-ray burst sources.

1. Introduction

The quantum effects produced by a strong magnetic field ($\geqslant 10^{12}$ G) on the propagation of waves close to the cyclotron frequency in a plasma have been studied by several authors (Canuto and Ventura 1977; Kirk 1980; Pavlov *et al.* 1980) with a view to applications in various astrophysical situations involving magnetized neutron stars. In these papers, a nonrelativistic approach to the problem is adopted, since the energy corresponding to the magnetic field strengths of interest is typically one order of magnitude smaller than the electron rest mass energy, i.e.

$$\hbar\Omega/mc^2 \sim 10^{-1},$$

where Ω $(=eB/mc)$ is the electron gyro-frequency.

On the other hand, there have recently been extensive investigations of relativistic effects upon the propagation of waves at or about the gyro-frequency in plasmas with a more modest magnetic field. Thus, Wu and Lee (1979) considered relativistic effects on the theory of the electron cyclotron maser when proposing a mechanism for the terrestrial kilometric radiation, and Fidone *et al.* (1982) and Airoldi and Orefice (1982) have been concerned with cyclotron absorption in laboratory plasmas.

The main catalyst of these investigations has been the realization that the classical, nonrelativistic resonance condition for gyro-magnetic absorption

$$\omega - s\Omega - k_{\|}\, v_{\|} = 0 \tag{1}$$

0004-9506/85/050715$02.00

is different in character from the (classical) relativistic condition

$$\omega - s\Omega/\gamma - k_{\parallel}\, v_{\parallel} = 0. \tag{2}$$

[In (1) and (2), ω is the wave frequency, s is the harmonic number, $k_{\parallel}$ and $v_{\parallel}$ are the projections along the magnetic field of the wave vector $\boldsymbol{k}$ and the velocity $\boldsymbol{v}$ respectively, and $\gamma = 1/(1-v^2/c^2)^{\frac{1}{2}}$. In fact, equation (1) possesses a single root $v_{\parallel}$ for given ω, s, Ω and $k_{\parallel}$, whereas equation (2) can have either one root, two roots or no root at all. Consequently, the use of equation (1) can introduce errors into a calculation even if the frequencies and temperatures involved are nonrelativistic in the usual sense.

This same situation occurs also when the nonrelativistic and relativistic resonance conditions are written for a quantum plasma. Herold *et al.* (1981) investigated the relativistic case using parameters appropriate for pulsating X-ray sources. They found significant deviations from the nonrelativistic treatments mentioned above, mainly in the neighbourhood of perpendicular propagation. In particular, they noticed that the response function of the plasma is singular at a critical frequency unless the finite lifetime of the first excited Landau level is explicitly included in the calculation. This is a remarkable property of the quantum plasma. In more conventional plasmas, singularities in the response function are removed by the introduction of a finite temperature. Herold *et al.* were able to include finite (though nonrelativistic) temperatures in their calculations, but still obtained the singular behaviour.

The aim of the present paper is to re-evaluate the response function for the strongly magnetized plasma. A slightly different approach to that of Herold *et al.* is adopted by using the method of Svetozarova and Tsytovich (1962). This approach, using statistically averaged propagators, has recently been clarified and extended in a series of papers (Melrose and Parle 1983*a*, 1983*b*; Melrose 1983). In the present evaluation, the nonrelativistic approximations employed by Herold *et al.* are lifted, allowing more accurate computation of the refractive index in the case of the application to pulsating X-ray sources, as well as permitting the calculation of new results of interest in relation to the theory of γ-ray bursts. In addition, the behaviour of the response function close to the singularity is examined in detail. The classical limit of the antihermitian part of the response function is then obtained, and compared with the results of Fidone *et al.* (1982). This calculation is particularly instructive, as it demonstrates how the singularities in the quantum expression merge into the continuous, finite classical result.

2. Plasma Susceptibility

The linear reponse of a plasma may be characterized by the susceptibility tensor χ_{ij}, related to the dielectric tensor $\mathscr{E}_{ij}$ by

$$\mathscr{E}_{ij} = \delta_{ij} + \chi_{ij}.$$

In the case of a strong magnetic field, it is also necessary to include the linear response of the vacuum:

$$\chi_{\text{vac}} = -2\delta\{\mathbf{1} - n^2(\mathbf{1} - \hat{\boldsymbol{k}}\hat{\boldsymbol{k}})\} + 4n^2\delta(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{B}})(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{B}}) + 7\delta\,\hat{\boldsymbol{B}}\hat{\boldsymbol{B}}, \tag{3}$$

where n is the refractive index,

$$\delta := \frac{\alpha}{45\pi}\left(\frac{B}{B_{\rm c}}\right)^2 ,$$

$B_{\rm c}$ is the critical magnetic field strength ($=4 \cdot 414 \times 10^{13}$ G), $\hat{\boldsymbol{k}}$ and $\hat{\boldsymbol{B}}$ are unit vectors along the direction of wave propagation and the magnetic field respectively, and α is the fine-structure constant (Gnedin *et al.* 1978; Melrose and Stoneham 1976; Mészáros and Ventura 1979). Equation (3) is valid only when $B/B_{\rm c} \ll 1$ and $\hbar\omega/mc^2 \ll 1$.

The susceptibility of an electron plasma, summed over electron spin, is given by (Pavlov *et al.* 1980)

$$\chi_{ij} = -\frac{\omega_{\rm p}^2}{\omega^2}\int_{-\infty}^{+\infty} {\rm d}p \sum_{n,n'=0}^{\infty}\left(\frac{Q_{ij}^{+}\{f(\epsilon)-f(\epsilon')\}}{\epsilon-\epsilon'+\omega+{\rm i}0} + \frac{Q_{ij}^{-} f(\epsilon)}{\epsilon+\epsilon'+\omega+{\rm i}0} + \frac{Q_{ij}^{-} f(\epsilon')}{\epsilon+\epsilon'-\omega-{\rm i}0}\right). \tag{4}$$

Here, $\epsilon := (1+p^2+2nB)^{\frac{1}{2}}$, $\epsilon' := (1+p'^2+2n'B)^{\frac{1}{2}}$, $p' = p+k_{\|}$ and $f(\epsilon)$ is the electron distribution function, normalized such that

$$\sum_{n=0}^{\infty}\int {\rm d}p\, g_n f(\epsilon) = 1, \qquad g_n = 2-\delta_{n,0}\,.$$

The quantities ϵ and ϵ' are energies expressed in units of the electron rest mass mc^2. Similarly, the frequencies ω and $\omega_{\rm p}$ (the plasma frequency) and the momenta p, p' and $\hbar k_{\|}$ are in units of $mc^2/\hbar$ and mc respectively, and the magnetic field B is in units of $B_{\rm c}$. The parameter Q_{ij} is given by

$$Q_{xx}^{\pm} = \tfrac{1}{2}\left(1 \mp \frac{1+pp'}{\epsilon\epsilon'}\right)(F_{n'-1,n}^2 + F_{n',n-1}^2) \pm \frac{2B}{\epsilon\epsilon'}(nn')^{\frac{1}{2}} F_{n'-1,n}\, F_{n',n-1}\,, \tag{5a}$$

$$Q_{yy}^{\pm} = Q_{xx}^{\pm} \mp \frac{4B}{\epsilon\epsilon'}(nn')^{\frac{1}{2}} F_{n'-1,n}\, F_{n',n-1}\,, \tag{5b}$$

$$Q_{zz}^{\pm} = \tfrac{1}{2}\left(1 \mp \frac{1-pp'}{\epsilon\epsilon'}\right)(F_{n'-1,n-1}^2 + F_{n',n}^2) \mp \frac{2B}{\epsilon\epsilon'}(nn')^{\frac{1}{2}} F_{n'-1,n-1}\, F_{n',n}\,, \tag{5c}$$

$$Q_{xy}^{\pm} = -Q_{yx}^{\pm} = -\tfrac{1}{2}{\rm i}\left(1 \mp \frac{1+pp'}{\epsilon\epsilon'}\right)(F_{n'-1,n}^2 - F_{n',n-1}^2)\,, \tag{5d}$$

$$Q_{yz}^{\pm} = -Q_{zy}^{\pm} = \mp{\rm sign}(k_x)\frac{\rm i}{\epsilon\epsilon'}(\tfrac{1}{2}B)^{\frac{1}{2}}\{n^{\frac{1}{2}}p'(F_{n'-1,n-1}\,F_{n'-1,n} - F_{n',n}\,F_{n',n-1}) + n'^{\frac{1}{2}}p(F_{n',n}\,F_{n'-1,n} - F_{n'-1,n-1}\,F_{n',n-1})\}\,, \tag{5e}$$

$$Q_{xz}^{\pm} = Q_{zx}^{\pm} = \mp\frac{{\rm sign}(k_x)}{\epsilon\epsilon'}(\tfrac{1}{2}B)^{\frac{1}{2}}\{n^{\frac{1}{2}}p'(F_{n'-1,n-1}\,F_{n'-1,n} + F_{n',n}\,F_{n',n-1}) + n'^{\frac{1}{2}}p(F_{n',n}\,F_{n'-1,n} + F_{n'-1,n-1}\,F_{n',n-1})\}\,, \tag{5f}$$

where $F_{n',n}$ is related to the associated Laguerre polynomials $L_n^s(x)$ (Abramowitz and Stegun 1965) by

$$F_{n',n} = (u^{n-n'}\,\mathrm{e}^{-u}\,n'!/n!)^{\frac{1}{2}}\mathrm{L}_{n'}^{n-n'}(u)$$
$$= (-1)^{n+n'}F_{n,n'}$$

for $n \geqslant n'$. Here, $u := k_x^2/2B = k_\perp^2/2B$, since the wave vector $\boldsymbol{k}$ is assumed to lie in the z–x plane, with the magnetic field along the z-axis. In order to facilitate discussion of the symmetries of Q_{ij}, the dependence on the sign of k_x has been given explicitly in equations (5e) and (5f).

The expression (4) may be simplified somewhat by noting the property of $Q_{ij}^{\pm}$ under the exchange of n, p and n', p':

$$Q_{ij}^{\pm}(n, n', p, -\boldsymbol{k}) = Q_{ij}^{\pm *}(n', n, p-k_{\|}, \boldsymbol{k}),$$

so that

$$\chi_{ij} =: \chi_{ij}^{(1)}(\omega, \boldsymbol{k}) + \chi_{ij}^{(1)*}(-\omega^*, -\boldsymbol{k})$$

with

$$\chi_{ij}^{(1)} := -\frac{\omega_{\mathrm{p}}^2}{\omega^2}\int_{-\infty}^{\infty}\mathrm{d}p\sum_{n,n'=0}^{\infty}\left(\frac{Q_{ij}^{+}\,f(\epsilon)}{\epsilon-\epsilon'+\omega+\mathrm{i}0}+\frac{Q_{ij}^{-}\,f(\epsilon)}{\epsilon+\epsilon'+\omega+\mathrm{i}0}\right). \tag{6}$$

Since only frequencies around the gyro-frequency $\omega \approx B$ will be considered, and we shall assume $B \ll 1$, the second term in equation (6) is nowhere resonant and is insensitive to the effects of a finite electron temperature. Consequently, we may replace $f(\epsilon)$ by $\delta(p)\delta_{n,0}$ in this term, obtaining

$$\int_{-\infty}^{\infty}\mathrm{d}p\sum_{n,n'=0}^{\infty}\left(\frac{Q_{ij}^{-}\,f(\epsilon)}{\epsilon+\epsilon'+\omega+\mathrm{i}0}+\frac{Q_{ij}^{-}\,f(\epsilon)}{\epsilon+\epsilon'-\omega-\mathrm{i}0}\right) \approx \delta_{ij}+\mathrm{O}(B).$$

The problem of evaluating χ_{ij} is eased by observing that the potential applications of the calculation are in problems in which $\omega_{\mathrm{p}}^2/\omega^2 \ll 1$. The refractive index differs only slightly from unity in this case, and we may restrict our attention to real values of ω and $\boldsymbol{k}$ with $\omega \approx |\boldsymbol{k}|$. It follows that the antihermitian part of χ_{ij} is obtained by taking the semi-residue of the integrands in equation (4) evaluated at the roots of the resonant denominator

$$D_1 := \epsilon-\epsilon'+\omega,$$

and the hermitian part is given by the principal values of the integrals.

(a) Resonance Condition

Before proceeding with the calculation, it is instructive to consider the resonant denominator in more detail (Melrose *et al.* 1982). The classical resonance condition

$$\omega - sB/\gamma - k_{\|}\,v_{\|} = 0$$

may be represented as a semi-ellipse in $v_\perp - v_{||}$ space, with its centre at

$$v_{||} = \omega k_{||}/(s^2 B^2 + k_{||}^2), \qquad v_\perp = 0,$$

with eccentricity

$$e^2 = k_{||}^2/(s^2 B^2 + k_{||}^2)$$

and semi-major axis parallel to the $v_\perp$ axis:

$$V^2 = (s^2 B^2 - \omega^2 + k_{||}^2)/(s^2 B^2 + k_{||}^2).$$

The evaluation of the plasma response, given the distribution function of particles in velocity space, reduces to an integral along the physically relevant sections of this ellipse.

In the quantum case one may construct a similar diagram by defining the quantities $v_{||}$ and $v_\perp$ as follows:

$$v_{||} := p/\epsilon, \qquad v_\perp := (2nB)^{\frac{1}{2}}/\epsilon.$$

Although these quantities have in general no physical meaning, they coincide with the classical velocity in the classical limit. The resonance condition takes the form

$$\omega - (sB - \tfrac{1}{2}\omega^2 + \tfrac{1}{2}k_{||}^2)/\gamma - k_{||}\, v_{||} = 0 \tag{7}$$

and is once again represented by a semi-ellipse in $v_\perp - v_{||}$ space, in this case centred at

$$v_{||} = \omega k_{||}/\{(sB - \tfrac{1}{2}\omega^2 + \tfrac{1}{2}k_{||}^2)^2 + k_{||}^2\}, \qquad v_\perp = 0.$$

This ellipse is called the resonant ellipse.

However, the particles themselves are not distributed uniformly in $v_\perp - v_{||}$ space, but are confined to lie on curves determined by the condition

$$v_{||}^2 + v_\perp^2(1 + 1/2nB) = 1, \tag{8}$$

i.e. on ellipses centred at $v_\perp = v_{||} = 0$ and contained within the unit circle. These ellipses are called 'n-ellipses'. The evaluation of the plasma response, given the distribution function of particles in p and n, reduces to a summation over the points of intersection of the resonant ellipse (equation 7) with the n-ellipses (equation 8). These intersections can be found by defining three additional functions similar in form to D_1:

$$D_2 := \epsilon - \epsilon' - \omega, \qquad D_3 := \epsilon + \epsilon' + \omega, \qquad D_4 := \epsilon + \epsilon' - \omega.$$

One then obtains a quadratic function D of p with roots p_1 and p_2:

$$D := D_1 D_2 D_3 D_4 = -4q^2(p - p_1)(p - p_2),$$

with

$$q^2 := \omega^2 - k_{||}^2\,,$$

$$p_{1,2} = -\tfrac{1}{2}k_{||}\{1+(\epsilon_n^2-\epsilon_{n'}^2)/q^2\}\pm\Delta\,, \tag{9}$$

where

$$\epsilon_n := (1+2nB)^{\frac{1}{2}}\,,$$

$$\Delta := \frac{\omega}{2q^2}\{(\epsilon_n+\epsilon_{n'}+q)(\epsilon_n+\epsilon_{n'}-q)(\epsilon_n-\epsilon_{n'}+q)(\epsilon_n-\epsilon_{n'}-q)\}^{\frac{1}{2}}\,. \tag{10}$$

If $q^2 > (\epsilon_n+\epsilon_{n'})^2$ or $q^2 < (\epsilon_n-\epsilon_{n'})^2$, then p_1 and p_2 are real (for real ω) and thus represent the momenta of resonant particles. Both roots belong to the product $D_1\,D_2$ provided that $q^2 < (\epsilon_n-\epsilon_{n'})^2$ (Melrose *et al.* 1982), but occur singly, i.e. one root for D_1 and one for D_2 if $q^2 < 0$ (or $n = n'$). As the only denominator of interest in equation (4) is D_1, and since we shall obtain wave solutions in which $k_{||} < \omega$, it suffices to consider the range

$$0 < q^2 < (\epsilon_n-\epsilon_{n'})^2\,,$$

in which case

$$D_1(p_1) = D_1(p_2) = 0$$

for $(n'-n)\omega > 0$.

Clearly, a point of special interest occurs where q is such that $\Delta = 0$ and it is here that Herold *et al.* (1981) encountered the singularity of the response function (for $n' = 1$ and $n = 0$). However, each pair of Landau levels n and n', such that $|n-n'| = 1$, defines a value of $q_n \approx B$ at which $\Delta = 0$. Expanding ϵ_n and $\epsilon_{n'}$ in powers of nB and $n'B$ gives

$$q_n = B\{1-\tfrac{1}{2}(n+n')B\}\,. \tag{11}$$

Thus, for waves propagating at an angle θ to the magnetic field, and assuming $|k| = \omega$, the points defined by equation (11) occur at the frequencies

$$\omega_n = \{1-\tfrac{1}{2}(n+n')B\}\,B/\sin\theta\,.$$

At frequencies $\omega > \omega_0$ there are no real roots of D_1 for any n and n' such that $|n-n'| = 1$. The momenta of particles resonant at ω_0 is, according to equation (9),

$$p_1 = p_2 = \epsilon_n \cos\theta/|\sin\theta|$$

for $n' > n$. At low temperatures only particles with $p \ll 1$ are of importance. In this case the singularity appears at angles of propagation close to $\frac{1}{2}\pi$ where $\sin\theta \approx 1$ and, consequently, $\omega_0 \approx B$. In terms of the pictorial representation of the resonance condition as a resonant ellipse in $v_\perp - v_{||}$ space, a singularity appears when one of the n-ellipses touches the resonant ellipse. A singularity of this kind does not arise in the classical treatment in which particles are assumed to be evenly distributed in $v_\perp - v_{||}$ space.

Using the properties described above, the denominator in equation (4) may be rewritten as

$$\begin{aligned} 1/D_1 &= D_2\, D_3\, D_4/D \\ &= \frac{D_2\, D_3\, D_4}{4q^2(p_1-p_2)}\left(\frac{1}{p-p_2}-\frac{1}{p-p_1}\right). \end{aligned} \tag{12}$$

It remains to determine the position of the contour of integration with respect to the poles in equation (12). Two cases must be considered. Firstly, if $\mathrm{Real}\{q^2-(\epsilon_n-\epsilon_{n'})^2\} > 0$ then no resonant particles exist for that particular $n,\, n'$ transition; the integration contour remains the real p-axis in this case. Secondly, if $\mathrm{Real}\{q^2-(\epsilon_n-\epsilon_{n'})^2\} < 0$, resonant particles do exist, and the location of the contour is found by considering ω to have a small positive imaginary part. In complex p-space this corresponds to placing the p_1 pole below the contour and the p_2 pole above it, when Δ is defined such that

$$\mathrm{Real}(2q^2\Delta/\omega) > 0\,, \qquad p_1-p_2 = 2\Delta\,.$$

This prescription can be written

$$\frac{1}{D_1+\mathrm{i}0} = \frac{D_2\, D_3\, D_4}{4q^2(p_1-p_2)}\left(\frac{1}{p-p_2-\mathrm{i}0}-\frac{1}{p-p_1+\mathrm{i}0}\right) \tag{13}$$

for $\mathrm{Real}(q^2) < (\epsilon_n-\epsilon_{n'})^2$.

(b) Hermitian Response

Equation (13) may be used to rewrite the first term in equation (6), resulting in expressions for the elements of the resonant part of $\chi_{ij}^{(1)}$ which are of the form

$$\int_{-\infty}^{+\infty} \mathrm{d}p \sum_{n,n'=0}^{\infty} \frac{g_{ij}}{p_1-p_2}\left(\frac{1}{p-p_1+\mathrm{i}0}-\frac{1}{p-p_2-\mathrm{i}0}\right). \tag{14}$$

The hermitian part of the response tensor is then found from the principal part of these integrals. An approximate expression may be obtained by replacing g_{ij} as follows:

$$g_{ij}(n, n', p, k) \approx \mathrm{e}^{-p^2/2T}(a_0+pa_1+p^2a_2)\,, \tag{15}$$

where a_0, a_1 and a_2 are the coefficients of the first three terms in the Taylor series of $\mathrm{e}^{p^2/2T}g_{ij}$. The result can easily be expressed in terms of the plasma dispersion function (Fried and Conte 1961). This approximation is accurate provided the temperatures considered are nonrelativistic, i.e.

$$T \ll 1\,. \tag{16}$$

It has the advantage of preserving the exact form of the resonant denominator; only the numerator is subjected to the nonrelativistic approximation. However, the momenta of particles resonant at the critical frequency ω_0 become large for small angles of propagation, so that the expansion (15) is inadequate there. Another widely used approximation scheme (Wu and Lee 1979; Herold *et al.* 1981; Fidone *et al.* 1982) consists of expanding the resonant denominator in equation (6) to second order in p as well as expanding the numerator. The resonant ellipses are thus approximated by circles and the range of validity of the results is once again reduced by the requirement

$$k_{\|}^{2} \ll \omega^{2} \tag{17}$$

(Hewitt *et al.* 1982). This approximation is often termed 'semi-relativistic'. Since both approximations fail at small θ, we adopt a numerical evaluation of the integrals in equation (14). In the examples presented in the following section, the summations over n and n' are performed explicitly up to limits determined on the one hand by the electron distribution amongst the Landau levels and, on the other, by the exclusion of all terms which are not resonant close to the fundamental frequency B. In addition, the small gyro-radius approximation is employed which, in our notation, uses the smallness of the quantity $nk_{\perp}^{2}/2B$ to justify retaining only the leading term of the Laguerre polynomials contained in Q_{ij}^{+} of equations (5). At frequencies close to the gyro-frequency one has

$$k_{\perp} \ll B, \tag{18}$$

so that the small gyro-radius approximation applies provided that equation (16) is fulfilled.

(c) Antihermitian Response

In contrast to the hermitian part, χ_{ij}^{H}, it is relatively simple to obtain an exact expression for the antihermitian part, χ_{ij}^{A}, of the plasma susceptibility. To this end, the resonant denominator in equation (4) is replaced according to equation (13), and the semi-residue taken at each pole. The resulting expressions are simplified by noting that for many distributions of interest (including thermal equilibrium) one has

$$f(\epsilon') = \mathrm{e}^{-\omega/T} f(\epsilon)\, h_{n'} \qquad (h_{n'} = 0 \text{ or } 1) \tag{19}$$

and that for positive values of ω only those terms in which $n' > n$ contribute to the double summation. Thus, we have

$$\chi_{ij}^{\mathrm{A}} = \mathrm{i}\pi \frac{\omega_{\mathrm{p}}^{2}}{\omega^{2}} \sum_{n'>n} \int_{-\infty}^{+\infty} \mathrm{d}p\, f_n(\epsilon)\, Q_{ij}^{+} \frac{2\omega\epsilon\epsilon'}{q^{2}(p_1-p_2)} \{\delta(p-p_1)+\delta(p-p_2)\}$$
$$\times (1-h_{n'}\,\mathrm{e}^{-\omega/T}), \tag{20}$$

where $\epsilon' = \epsilon+\omega$, and $f_n(\epsilon)$ is here understood to represent a distribution at temperature T such that

$$f_n(\epsilon) = A h_n \mathrm{e}^{-\epsilon/T}, \tag{21}$$

with the constant A determined by

$$A^{-1} = \int_{-\infty}^{+\infty} \mathrm{d}p \sum_{n=0}^{\infty} g_n\, h_n \mathrm{e}^{-\epsilon/T}. \tag{22}$$

(d) Behaviour at Threshold

Several properties of the singularity encountered by Herold *et al.* (1981) can be deduced from expressions (14) and (20). At a singular point we have $p_1 = p_2$ and $\Delta = 0$. For higher frequencies, Δ is purely imaginary. The lack of real values of p_1 and p_2 implies that no resonant particles exist so that, denoting the frequency of the singularity by ω_0, one has

$$\chi_{ij}^{\mathrm{A}} = 0 \quad \text{for} \quad \omega > \omega_0 \,. \tag{23}$$

On the other hand, as $\omega \to \omega_0$ from below, equation (20) shows that χ_{ij}^{A} diverges:

$$\chi_{ij}^{\mathrm{A}} \sim (\omega_0 - \omega)^{-\frac{1}{2}} \quad \text{as} \quad \omega \to (\omega_0)_- \,, \tag{24}$$

provided that $|\boldsymbol{k}| \approx \omega$. Thus, the frequency ω_0 is a threshold for the onset of absorption.

Turning to the hermitian part of the response (14), with the replacement indicated by equation (15), leads one to expressions of the form

$$\chi_{ij}^{\mathrm{H}} \sim b_0 + \sum_{l=1}^{3} b_l \,\mathrm{Real}\left[\left\{p_1^{l-1}\, W\!\left(\frac{-p_1}{(2T)^{\frac{1}{2}}}\right) + p_2^{l-1}\, W\!\left(\frac{p_2}{(2T)^{\frac{1}{2}}}\right)\right\}\Big/ (p_1 - p_2)\right], \tag{25}$$

where $W(z)$ is the plasma dispersion function (Fried and Conte 1961) and the b_l are well-behaved at the point $p_1 = p_2$. Defining

$$\delta := p_1 - p_2$$

one has δ purely imaginary for $\omega > \omega_0$ and purely real for $\omega < \omega_0$. As $\delta \to 0$ equation (25) yields

$$\chi_{ij}^{\mathrm{H}} \sim b_0 + \sum_{l=1}^{3} b_l \,\mathrm{Real}\left[\frac{p_2^{l-1}}{\delta}\left\{W\!\left(\frac{-p_2}{(2T)^{\frac{1}{2}}}\right) + W\!\left(\frac{p_2}{(2T)^{\frac{1}{2}}}\right)\right\}\right.$$
$$\left. + \left((l-1)p_2^{l-2} - \frac{p_2^l}{T}\right) W\!\left(\frac{-p_2}{(2T)^{\frac{1}{2}}}\right) + \frac{2p_2^{l-1}}{(2T)^{\frac{1}{2}}}\right]. \tag{26}$$

Application of the symmetry property

$$W(z) = -W^*(-z^*)$$

to equation (26) enables the second term to be written as

$$\sum_{l=1}^{3} b_l \,\mathrm{Real}\left[\frac{p_2^{l-1}}{\delta}\left\{W\!\left(\frac{p_2}{(2T)^{\frac{1}{2}}}\right) - W^*\!\left(\frac{p_2^*}{(2T)^{\frac{1}{2}}}\right)\right\}\right].$$

For $\omega < \omega_0$, this term vanishes, and χ_{ij}^{H} tends to a finite limit as $\delta \to 0$. However, for $\omega > \omega_0$, one has

$$\chi_{ij}^{\mathrm{H}} \sim \delta^{-1}$$
$$\sim (\omega - \omega_0)^{-\frac{1}{2}}, \quad \text{as} \quad \omega \to (\omega)_+ \,. \tag{27}$$

In summary, χ_{ij}^{A} is zero above the frequency ω_0 and χ_{ij}^{H} is finite below it; χ_{ij}^{A} diverges below ω_0 and χ_{ij}^{H} above it. However, the divergences are sufficiently weak that a bounded quantity can be constructed by averaging over a finite bandwidth in frequency. Thus the quantities

$$\langle\chi_{ij}^{A}\rangle_{\omega} := \frac{1}{2\delta\omega}\int_{\omega-\delta\omega}^{\omega+\delta\omega} \mathrm{d}\omega'\, \chi_{ij}^{A}(\omega'), \tag{28}$$

$$\langle\chi_{ij}^{H}\rangle_{\omega} := \frac{1}{2\delta\omega}\int_{\omega-\delta\omega}^{\omega+\delta\omega} \mathrm{d}\omega'\, \chi_{ij}^{H}(\omega') \tag{29}$$

are bounded for all ω provided $\delta\omega \neq 0$.

(e) Classical Limit

The antihermitian part of the plasma susceptibility may be obtained from equation (20) by taking the limit $n, n' \to \infty$ with $s := n'-n$ finite and B, ω and $|k| \to 0$. The momenta of the resonant particles p_1 and p_2 take on the form

$$p_{1,2} = sBk_{\|}/q^2 \pm \omega(s^2B^2/q^2 - \epsilon_n^2)^{\frac{1}{2}}/q \tag{30}$$

and the summation over n, which is converted in this limit into an integral, extends up to a maximum value given by the condition that $p_{1,2}$ remain real:

$$n \leqslant (s^2B^2/q^2 - 1)/2B. \tag{31}$$

The singularity obtained when $p_1 = p_2$ is lost in this conversion of the n-summation into an integral. To see this, one first defines

$$\mu := \{1 - 2nB/(s^2B^2/q^2 - 1)\}^{\frac{1}{2}}. \tag{32}$$

Then, in a procedure exactly analogous to that used in treating equation (20), one expresses the singular quantity

$$S := \sum_n (p_1 - p_2)^{-1} \tag{33}$$

as an integral:

$$S \to \int_0^{n_{\max}} \mathrm{d}n/(p_1 - p_2) \tag{34}$$

$$= \int_0^1 \mathrm{d}\mu\; q(s^2B^2/q^2 - 1)^{\frac{1}{2}}/2\omega B, \tag{35}$$

which is non-singular, and trivial to evaluate:

$$S \to q(s^2B^2/q^2 - 1)^{\frac{1}{2}}/2\omega B. \tag{36}$$

Straightforward calculations of this kind lead from equation (20) to the following expression for the antihermitian part of the response tensor in the classical limit:

$$\chi^{\mathrm{A}} = \frac{\omega_{\mathrm{p}}^2}{\omega} \frac{\mathrm{i}\pi}{4T\mathrm{K}_2(1/T)\,|k_{\|}|} \sum_s \mathrm{e}^{-sB\omega/q^2 T} \int_{-\xi}^{+\xi} \mathrm{d}u\, \mathrm{e}^{-u} \mathbf{\Pi}, \tag{37}$$

where K_2 is a modified Bessel (McDonald) function,

$$\xi := a|k_{\|}|/qT, \tag{38}$$

with

$$a := (s^2 B^2/q^2 - 1)^{\frac{1}{2}} \tag{39}$$

and

$$\mathbf{\Pi} = \begin{bmatrix} \dfrac{2s^2B^2}{k_\perp^2}\mathrm{J}_s^2 & -\mathrm{i}\dfrac{2sB\epsilon_\perp}{k_\perp}\mathrm{J}_s\mathrm{J}_s' & \mathrm{sign}(k_x)\dfrac{2sBp}{k_\perp}\mathrm{J}_s^2 \\ \mathrm{i}\dfrac{2sB\epsilon_\perp}{k_\perp}\mathrm{J}_s\mathrm{J}_s' & 2\epsilon_\perp^2\mathrm{J}_s'^2 & \mathrm{i}\,\mathrm{sign}(k_x)2\epsilon_\perp p\mathrm{J}_s\mathrm{J}_s' \\ \mathrm{sign}(k_x)\dfrac{2sBp}{k_\perp}\mathrm{J}_s^2 & -\mathrm{i}\,\mathrm{sign}(k_x)2\epsilon_\perp p\mathrm{J}_s\mathrm{J}_s' & p^2\mathrm{J}_s^2 \end{bmatrix}. \tag{40}$$

In equation (40) the notation

$$\epsilon_\perp := a(1-u^2/\xi^2)^{\frac{1}{2}}, \qquad p := \frac{\omega T}{k_{\|}}(u + sBk_{\|}^2/\omega T q^2)$$

is used, the argument of the Bessel functions J_s is $k_\perp \epsilon_\perp/B$ and J_s' represents $\mathrm{d}/\mathrm{d}z(\mathrm{J}_s(z))$. In the small gyro-radius approximation, the Bessel functions are expanded in a power series of their argument, which enables the remaining integration to be performed. The results obtained are in agreement with those presented by Fidone *et al.* (1982), when allowance is made for the fact that Fidone *et al.* retained terms of first order in the small parameter $k_\perp^2\, T/B^2$ in some elements of χ_{ij}^{A}, whilst omitting them in others.

3. Wave Properties

In astrophysical situations the density of the plasma of interest is usually so small that the refractive index differs very little from unity. This is the case for the accretion columns of X-ray pulsars, for example, where

$$\delta n := |n^2 - 1| \sim \omega_{\mathrm{p}}^2/\omega^2 \leqslant 10^{-6},$$

n being the refractive index. It is then a simple matter to solve the wave equation to lowest order in the parameter δn, and one finds, in agreement with formulae given by (amongst others) Pavlov *et al.* (1980):

$$n = 1 + n_{\mathrm{I}} \pm (n_{\mathrm{L}}^2 + n_{\mathrm{C}}^2)^{\frac{1}{2}}, \tag{41}$$

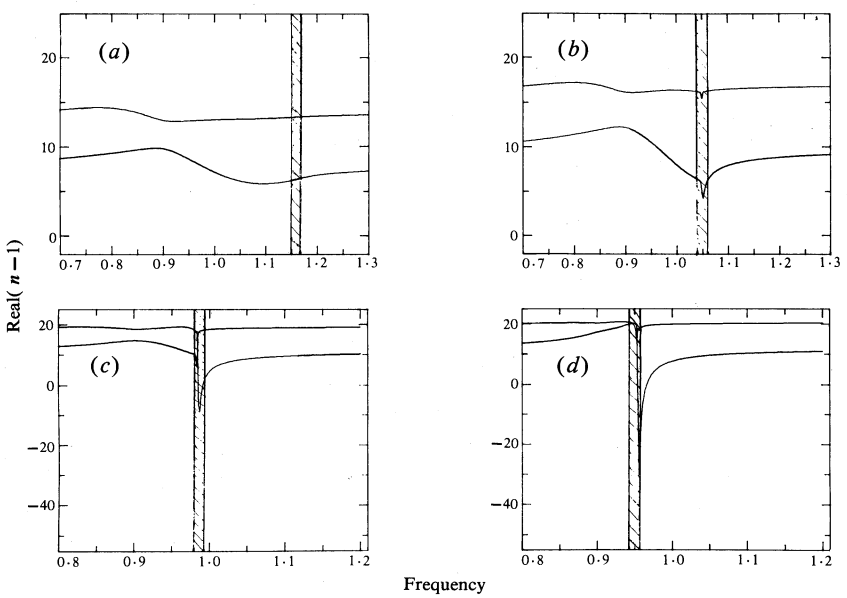

Fig. 1. Real part of the refractive index n of a plasma. Plotted is the quantity $(n-1)\omega_B^2/\omega_p^2$ where ω_B is the cyclotron frequency and ω_p the plasma frequency. The parameters are such that $\hbar\omega_B = 55$ keV, $\omega_B^2/\omega_p^2 = 10^7$ and the temperature is 20 keV. The angles of propagation are (*a*) 55°, (*b*) 65°, (*c*) 75° and (*d*) 85°. These curves result from a numerical integration of equation (14). Around the critical frequency, given by $\omega_0 = (\epsilon_1 - 1)/\sin\theta$, accuracy is lost, and this region is indicated by the shaded zone.

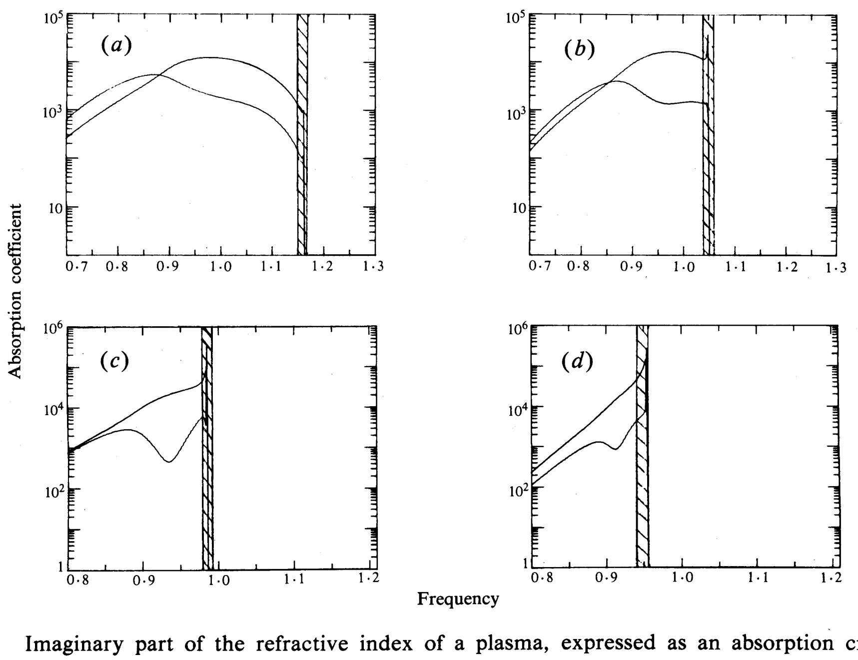

Fig. 2. Imaginary part of the refractive index of a plasma, expressed as an absorption cross section in units of the Thomson cross section. The parameters and shaded zone are explained in Fig. 1.

with

$$n_{\mathrm{I}} = \tfrac{1}{4}(\chi_{yy} + \chi_{xx}\cos^2\theta - \chi_{xz}\sin 2\theta + \chi_{zz}\sin^2\theta),$$

$$n_{\mathrm{L}} = \tfrac{1}{4}(-\chi_{yy} + \chi_{xx}\cos^2\theta - \chi_{xz}\sin 2\theta + \chi_{zz}\sin^2\theta),$$

$$n_{\mathrm{C}} = \tfrac{1}{2}\,\mathrm{i}(\chi_{xy}\cos\theta + \chi_{yz}\sin\theta).$$

In this notation the polarization vectors of the normal modes in cartesian coordinates with $\boldsymbol{k}$ along z and $\boldsymbol{x}$ in the plane defined by $\boldsymbol{k}$ and $\boldsymbol{B}$ are given by

$$\boldsymbol{e}_{1,2} = \alpha\{\, n_{\mathrm{L}} \pm (n_{\mathrm{L}}^2 + n_{\mathrm{C}}^2)^{\frac{1}{2}},\ -\mathrm{i}\, n_{\mathrm{C}},\ 0\}\,, \tag{42}$$

where

$$\alpha = \{\,|\,n_{\mathrm{L}} + (n_{\mathrm{L}}^2 + n_{\mathrm{C}}^2)^{\frac{1}{2}}\,|^2 + |\,n_{\mathrm{C}}|^2\}^{\frac{1}{2}}.$$

(a) X-ray Pulsar

As a first application of our calculations, we may compare the refractive index for parameters corresponding to an X-ray pulsar with the results of Herold *et al.* (1981). Fig. 1 shows the real part of the refractive index for four different angles of propagation, and for an electron temperature of 20 keV. All electrons are assumed to be in the Landau ground state. Significant deviations from the results of Herold *et al.* (cf. their Fig. 6) occur only for angles of propagation well away from $\theta = 90°$. This is to be expected, since the semi-relativistic approximation employed by Herold *et al.* (which they term 'p_z-quadratic') applies only when equation (17) is satisfied, i.e. at angles such that $\cos\theta \ll 1$.

At angles close to 90°, our results agree, except that the damping introduced by Herold *et al.* leads to a refractive index which is continuous across the singularity, whereas our calculation produces a discontinuity at this point, as discussed in Section 2*d*. Fig. 2 displays the imaginary part of the refractive index. The discontinuity at the singularity is the dominant feature of this function. The exclusion of any damping of the first excited Landau level results in a kinematic restriction upon those electrons which are permitted to absorb a photon and, therefore, there is no absorption at frequencies greater than the critical frequency.

(b) Classical Cyclotron Absorption

The expressions derived for χ_{ij} in Section 2 apply also in more conventional plasmas, with lower magnetic fields. Quantum effects are generally neglected in such plasmas, but it is nonetheless interesting to observe the fate of the singularity of the quantum response when conditions become more classical.

Fig. 3 shows the absorption coefficient, averaged over polarizations, in a magnetic field $B = 10^{-6}$ ($=4{\cdot}4\times10^7$ G). At the adopted temperature of 10 keV, over 10^5 Landau levels are populated. Nevertheless, only electrons in the lowest levels can contribute to the absorption at frequencies close to the critical frequency, as can be seen from equation (31). The lower the frequency, the larger the number of electrons able to contribute. Each time a new threshold frequency is reached, the absorption goes through a singularity similar to that of Fig. 2 and emerges at a higher level. The spacing in frequency of these steps is ωB or $10^{-6}\omega$ in the present case, and the singularity is, in practice, a spike with a width, given by the finite lifetime of the excited Landau levels, roughly equal to 1% of the step spacing in the case of the highest frequency spike. For comparison, the absorption obtained with the classical expression for the antihermitian part of the susceptibility tensor is also shown in Fig. 3. Both the curves were obtained under the small gyro-radius approximation (see Section 2*b*).

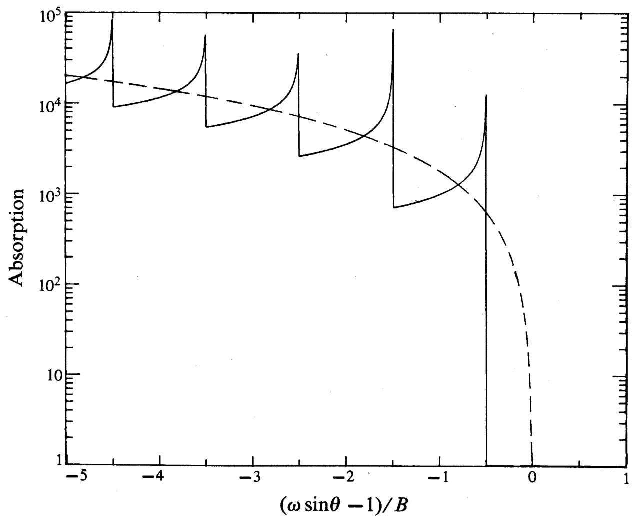

Fig. 3. Imaginary part of the refractive index of a plasma with a magnetic field of $4 \cdot 4 \times 10^7$ G. Shown are the results of a purely classical calculation (dashed curve) and of the exact quantum mechanical calculation, including the first few Landau levels (solid curve). The temperature is 10 keV and the angle of propagation 85°. Along the abscissa, θ is the propagation angle (in this case 85°), ω is the frequency in units of the cyclotron frequency and B (in this case 10^{-6}) is the magnetic field in units of B_c ($= m^2c^3/\hbar e = 4 \cdot 4 \times 10^{13}$ G). In the theory presented here, the plotted quantities in this and all subsequent figures are infinite at the critical frequencies mentioned in Section 3 (the finite values shown are an artefact of the plot software).

(c) Gamma-ray Bursts

Although the hermitian part of the response tensor χ_{ij}^{H} is calculated in Section 2*b* under the restriction $T \ll 1$, the antihermitian part χ_{ij}^{A} is obtained exactly. Since it is χ_{ij}^{A} which is responsible for gyromagnetic absorption, it is possible to obtain exact results for the absorption cross section at all electron temperatures from these calculations; the hermitian part operates only in determining the polarization vectors of the normal modes. To see this, note that the imaginary part of the refractive index given by equation (41) may be averaged over polarizations to obtain

$$\langle \mathrm{Im}(n) \rangle = \mathrm{Im}(n_{\mathrm{I}}), \tag{43}$$

which is completely determined by χ_{ij}^{A}. If polarization dependent cross sections are required, these are also easily obtained from χ_{ij}^{A}. Thus, linear polarization is obtained when the xy and yx components of χ^{H}_{ij} are zero in the cartesian coordinate representation used in equation (42). This corresponds to

$$\mathrm{Im}(n_C) = 0 \tag{44}$$

[cf. equation (42)]. Consequently, the cross sections for linear polarization in the plane containing $\boldsymbol{B}$ and $\boldsymbol{k}$ and perpendicular to it are obtained from

$$\{\mathrm{Im}(n)\}_{||,\perp} = \mathrm{Im}(n_I) \pm \mathrm{Im}(n_L). \tag{45}$$

Of course, these cross sections just reproduce well-known results from the theory of gyromagnetic emission in most parameter ranges. However, there seems to be a range of parameters of interest in the theory of γ-ray bursts for which no accurate results are available. Liang (1982), for example, has used formulae for synchrotron radiation given by Petrosian (1981) to fit the observed spectra of γ-ray bursts between 20 keV and 2 MeV. However, as noted by Liang, these formulae apply only when

$$\omega \gg B \qquad \text{and} \qquad T < 1 \tag{46}$$

and thus cannot be employed near the gyrofrequency. Since many of these bursts display features thought to be cyclotron lines (Mazets *et al.* 1982) an extension of the formulae is desirable. However, standard treatments of the first few harmonics (see Bekefi 1966) make the additional (nonrelativistic) assumption $B \ll 1$ which prohibits their use in the present context. Hameury *et al.* (1984) have calculated the emission without using the restriction $T \ll 1$, but their results do not include the quantum terms which are needed at low harmonics when $B \sim 1$. By using the calculations of Section 2, the polarization averaged cross section can be computed from the following electron distribution function

$$\begin{aligned} f(\epsilon, n) &= A\mathrm{e}^{-\epsilon/T} \qquad (0 \leqslant n \leqslant n_{\max}) \\ &= 0 \qquad (n < 0 \quad \text{or} \quad n > n_{\max}), \end{aligned} \tag{47}$$

with $\epsilon = (1+p^2+2nB)^{\frac{1}{2}}$ and where the normalization constant is given by

$$\begin{aligned} A^{-1} &= \sum_{n=0}^{n_{\max}} \int_{-\infty}^{+\infty} \mathrm{d}p\, g_n \mathrm{e}^{-\epsilon/T} \\ &= 2\sum_{n=0}^{n_{\max}} g_n \epsilon_n \mathrm{K}_1(\epsilon_n/T), \end{aligned} \tag{48}$$

with $\epsilon_n = (1+2nB)^{\frac{1}{2}}$. Thus, for $n_{\max} \gg T/B$ the distribution is close to its equilibrium value, and results are obtained which are applicable to the case $T \sim B \sim \omega$.

Fig. 4 displays the gyromagnetic absorption cross section averaged over polarizations for a magnetic field such that the energy of a photon of the cyclotron frequency is 100 keV. The temperature is also 100 keV, the angle of propagation 60°, and 20 Landau levels were included in the electron distribution function. In these calculations, the small gyro-radius approximation is relaxed. The most notable feature of the cross section is the sharp dip at the frequency

$$\omega = \{(1+2B)^{\frac{1}{2}}-1\}/\sin\theta = 106\ \mathrm{keV}, \tag{49}$$

where the contribution of the fundamental frequency ($n'-n = 1$) cuts out. In addition, the splitting of the fundamental is clearly seen; the difference in frequency between the transitions $n = 0 \rightarrow 1$ and $n = 1 \rightarrow 2$ being

$$\Delta\omega = \{2(1+2B)^{\frac{1}{2}}-(1+4B)^{\frac{1}{2}}-1\}/\sin\theta = 14\ \mathrm{keV}. \tag{50}$$

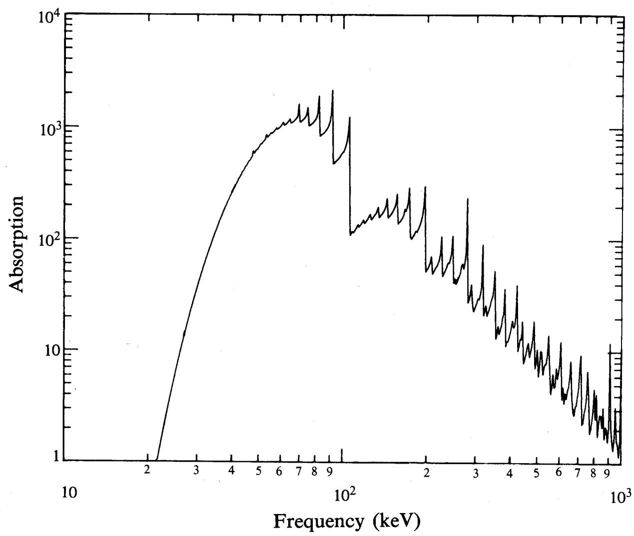

Fig. 4. Gyromagnetic absorption cross section averaged over polarizations for a magnetic field of $8\cdot6\times10^{12}$ G ($\hbar\omega_B = 100$ keV) and a temperature of 100 keV. The angle of propagation is 60°, and the lowest 20 Landau levels are included in the calculation.

In a separate calculation Hameury *et al.* (1984) have evaluated the absorption cross section averaged over polarizations for a gas in which all electrons occupy the lowest Landau level, despite having a velocity dispersion along the field lines which corresponds to a temperature of 500 keV. Only the transition $n = 0 \rightarrow 1$ is included, and the absorption cross section in the rest frame of an electron is taken from the nonrelativistic expression given by Daugherty and Ventura (1977). Following this procedure, one arrives at the result

$$\frac{\sigma}{\sigma_{\mathrm{T}}} = \frac{3\pi B}{16\alpha \mathrm{K}_1(1/T)\omega^2} \sum_{i=1}^{2} \frac{\mathrm{e}^{-\gamma_i/T}(1+\cos^2\theta_i)}{\gamma_i|\beta_i-\cos\theta|}, \tag{51}$$

where σ is the absorption cross section, σ_{T} is the Thomson cross section, α is the fine structure constant, γ_i and β_i are the Lorentz factor and velocity (in units of c) of resonant electrons, and θ_i is the angle of propagation of the photon in the frame of a resonant electron.

Equation (51) provides a good approximation for the cross section of the first harmonic provided $B \ll 1$. Fig. 5 displays both the approximate expression (51) and the exact expression for the distribution function given by equation (47) with $n_{\max} = 0$. For the relatively low magnetic field strength under consideration (corresponding to 10^{12} G) the two curves are indistinguishable at the first harmonic. However, the cross section at the higher harmonics remains considerably in excess of the Thomson cross section up to energies of about 50 keV.

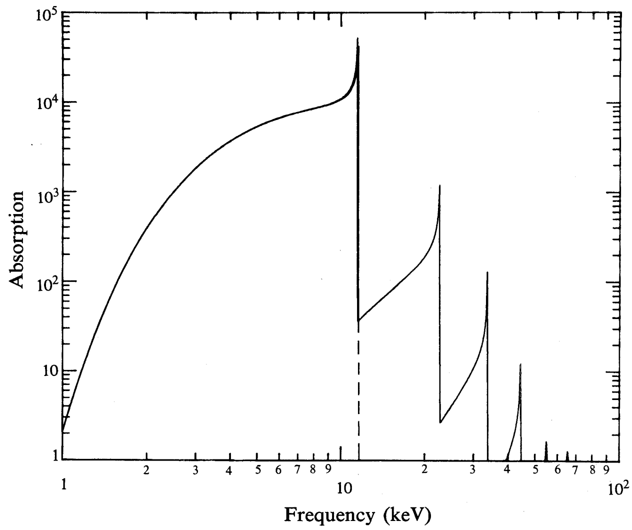

Fig. 5. Gyromagnetic absorption cross section averaged over polarizations for a magnetic field of 10^{12} G and a temperature of 500 keV. The angle of propagation is 89°. Shown are the approximate expression for the first harmonic (dashed line) and the exact expression (solid curve) for all harmonics. In both cases all electrons are assumed to occupy the Landau ground state.

4. Conclusions

In this paper we present a new treatment of the calculation of the dielectric tensor of a magnetized plasma, with particular care taken to ensure validity in the case of an extremely strong field, such as is commonly thought to occur in several astrophysical objects. The advantages of the present calculation over previous work in this field (Kirk 1980; Pavlov *et al.* 1980) are that it employs an accurate numerical scheme for the evaluation of the hermitian part of the tensor, and that the antihermitian part is obtained exactly for arbitrary temperatures. Several applications of this calculation are discussed. The results of Herold *et al.* (1981), in which the response tensor is found to possess a singularity on the real axis, are confirmed. Further investigation of the nature of this behaviour shows that the singularities persist in the classical (high quantum number) parameter regime, but that their integrable nature ensures that non-singular results are obtained in the classical limit itself. Finally, we indicate that the evaluation of the antihermitian part of the dielectric tensor can be used to obtain results in a parameter range which has so far not been extensively investigated and which is of interest in the theory of γ-bursts. As an example, the exact absorption cross section is displayed in two cases and compared with the results obtained from an approximate treatment (Hameury *et al.* 1984).

Acknowledgments

One of us (J.G.K.) wishes to thank the members of the Department of Theoretical Physics, University of Sydney for their generous hospitality during his stay. This work was performed with the support of the Australian Research Grants Scheme. We are also indebted to an anonymous referee for a constructive report on a previous version of this paper.

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Manuscript received 28 February, accepted 12 August 1985