Aust. J. Phys., 1985, **38**, 845–54

The Influence of Magnetic Fields on Convective Motions in the Outer Layers of the Sun*

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Abstract

In this paper we make a preliminary investigation of the nonlinear equations of compressible convection under the influence of solar-type magnetic fields. A polytropic model of the basic structure is used and, although the model is somewhat restrictive, good agreement is obtained with general observations in both strong and weak field cases. The value and influence of the turbulent magnetic resistivity is investigated and the depth dependence of the vertical velocity within a given period is used to study the way in which the overstable oscillations change their direction of flow from positive to negative.

1. Introduction

Convective motions occur somewhere in most stars and considerable uncertainty exists about the accuracy of the transport equation when the temperature gradient is super-adiabatic. The mixing-length formalism (Vitense 1953; Böhm-Vitense 1958) or its non-local extensions (Parsons 1969; Nordlund 1974) are generally used, but the formulation of the convective processes has unfortunately not reached the degree of sophistication of radiative transfer theory. It would therefore be of considerable interest to obtain additional information about the structure of such a convective layer and ultimately test the validity and accuracy of the various theoretical models that have been proposed. The outer layers of the Sun form such a region and can therefore be used to gather information about the physical characteristics and structure of the layer from detailed observations of surface motions such as the 3 and 5 minute oscillations, granular and supergranular motions, overstable motions in the umbra of sunspots, and so on (Moore 1981; Bray *et al.* 1984).

In recent years the theory of non-radial oscillations has been used to interpret the various oscillatory motions that have been observed, in particular the 5 minute oscillation, and this has given rise to solar seismology which has yielded some very important information about the structure of the Sun's outer layers (Gough 1983; Provost 1984).

Details are also available about other phenomena such as granulation and supergranulation in non-active regions and the 3 minute oscillations in active regions.

* Paper presented at the R. G. Giovanelli Commemorative Colloquium, Part I, Sydney, 26–29 November 1984.

0004-9506/85/060845$02.00

If it were possible to develop a satisfactory hydrodynamic model of these motions additional information about the structure and hydrodynamic properties of the convective region could be obtained by comparing the theoretical predictions with observations.

Solar seismology has made much progress in recent years as the theory of small non-radial oscillations is well understood and, since periods of oscillation are only marginally affected by nonlinear effects, a linear theory is likely to yield some fairly accurate estimates. On the other hand, the theory of deep convection in a highly stratified and turbulent medium is still in its early stages of development mainly due to our inadequate understanding of turbulent processes under stellar conditions and the difficulty in solving numerically complicated systems of highly nonlinear differential equations.

To solve the basic hydrodynamic equations some information is required regarding the thermal diffusivity κ, the eddy viscosity ν, the molecular weight $\bar{\mu}$, the specific heat at constant pressure C_p, the internal energy per unit volume E and, if a magnetic field is present, the eddy resistivity η and the permeability μ^* of the medium. Some of these quantities such as $\bar{\mu}$, C_p and E can be computed at each level by including the effects of ionization but others, such as κ, ν and η, could only be determined if an adequate and generally accepted theory of turbulence were available.

In the absence of a magnetic field some progress has been made in the development of a hydrodynamical model of solar granulation. In an earlier study (Van der Borght and Fox 1983*a*) the basic characteristics of the solar convection zone were approximated by a polytropic model in which the various constants such as ratio of specific heats γ, C_p, $\bar{\mu}$ and κ were chosen in such a way as to yield a model as close as possible to the mixing-length model. This allowed us to obtain an estimate of the average Prandtl number σ in the Sun's outer layers and it was shown that for $\sigma = 0{\cdot}24$ the theoretical values of the velocities, flux modulation and e-folding time were close to those observed. It was also shown that the value of $\sigma = 0{\cdot}5$ was an upper limit since convective motions ceased for such a high value of the eddy viscosity. A better attempt, in which C_p, κ, $\bar{\mu}$ and E were allowed to be depth dependent, confirmed the results of previous investigations and, in addition, gave information about the turbulent properties of the convective layers and the degree of overshooting into the upper stable layer (Van der Borght and Fox 1984).

When a magnetic field is present additional parameters enter the problem such as η and μ^*, although the latter is usually assumed to be close to unity. In this paper we shall consider both the action of a weak field and of a strong field on convective cells of granular size.

Outside active regions a mean magnetic field of about 2–10 G (1 G $\equiv 10^{-4}$ T) seems to exist (Stenflo 1976), but does not appear to influence to any great extent the observed granular motions. This can only be the case if η is fairly large. Within active regions, in particular the umbra of sunspots, strong magnetic fields of the order of 2–3 kG exist and the granular convective pattern may become oscillatory providing evidence for the existence of overstable motions (Giovanelli 1972; Moore 1981; Thomas 1981).

2. Basic Equations and Boundary Conditions

In this first attempt at investigating the effects of a magnetic field on deep convection in a compressible medium we shall again approximate the basic characteristics of the solar convection zone by a polytropic model and select the various constants γ, C_p, $\bar{\mu}$ and κ to yield a model as close as possible to the mixing-length model of Böhm-Vitense (1958). We shall also assume that the two free parameters η and μ^* are not depth dependent. In what follows μ^* will be taken equal to unity. This model is then placed under the influence of an initially uniform vertical magnetic field.

The averaging process involved in the modal analysis has been outlined elsewhere (Van der Borght 1977) and the basic equations, when a vertical uniform magnetic field is present, can be written as

$$\partial P/\partial t + \mathrm{D}(\rho_0 \psi) - \rho_0 W = 0, \tag{1}$$

$$\frac{\partial}{\partial t}\left(P\psi\right) + H\sigma \mathrm{D}(\rho_0 T_0 + PF) + \mathrm{D}(\psi^2\rho_0 + 2C\psi^2 P)$$
$$+ H\sigma\rho_0 + (\sigma\tau Q \mathrm{D} h/a^2)(\mathrm{D}^2 - a^2)h = 0, \tag{2}$$

$$\frac{\partial}{\partial t}\left(\rho_0\psi + 2CP\psi\right) - CW(\psi\rho_0) - \overline{E}P\psi W + \sigma a^2\psi$$
$$+ \tfrac{1}{3}\sigma \mathrm{D} W + H\sigma \mathrm{D}(\rho_0 F + T_0 P + 2CPF) + \mathrm{D}(2C\psi^2\rho_0 + 3\overline{E}\psi^2 P)$$
$$- \tfrac{4}{3}\sigma \mathrm{D}^2\psi + H\sigma P + (\sigma\tau C Q \mathrm{D} h/a^2)(\mathrm{D}^2 - a^2)h = 0, \tag{3}$$

$$\frac{\partial}{\partial t}\left(\rho_0 W + CPW\right) + H\sigma a^2(T_0 P + \rho_0 F + 2CPF)$$
$$+ \tfrac{1}{2}PW^2(\overline{H} - \overline{E}) - \tfrac{1}{2}CW^2\rho_0 - \tfrac{1}{3}\sigma a^2 \mathrm{D}\psi$$
$$- \sigma \mathrm{D}^2 W + \tfrac{4}{3}a^2\sigma W + \mathrm{D}(CW\rho_0\psi + \overline{E}W\psi P) - \sigma\tau Q(1 + Ch)(\mathrm{D}^2 - a^2)h = 0, \tag{4}$$

$$\frac{\partial}{\partial t}\left(\frac{H\sigma}{\gamma - 1}(T_0\rho_0 + PF) + \frac{\rho_0 W^2}{2a^2} + \frac{CPW^2}{2a^2} + \tfrac{1}{2}\rho_0\psi^2 + CP\psi^2\right)$$
$$+ \frac{H\sigma\gamma}{\gamma - 1}\mathrm{D}(T_0 P\psi + \rho_0\psi F + 2CP\psi F)$$
$$+ \mathrm{D}\left(\frac{\overline{E}W^2 P\psi}{2a^2} + \frac{3\overline{E}P\psi^3}{2} + C\rho_0\psi^3 + 3CP\psi^2 W_0\right)$$
$$+ H\sigma P\psi + \frac{C}{2a^2}\mathrm{D}(\rho_0\psi W^2) - \frac{H\sigma}{\gamma - 1}\mathrm{D}^2 T_0$$
$$+ \sigma\tau Q\{W(1 + Ch) - C\psi \mathrm{D} h\}(\mathrm{D}^2 - a^2)h/a^2 = 0, \tag{5}$$

$$\frac{\partial}{\partial t}\left(\frac{H\sigma}{\gamma-1}(\rho_0 F+2CPF+T_0 P)+\frac{C\rho_0 W^2}{2a^2}+\frac{\overline{E}PW^2}{2a^2}\right.$$
$$\left.+\rho_0 W\psi+C\rho_0\psi^2+\tfrac{3}{2}\overline{E}P\psi^2\right)$$
$$+\frac{H\sigma\gamma}{\gamma-1}\Big(-W\rho_0 T_0+\mathrm{D}(\rho_0\psi T_0)-CW(T_0 P+\rho_0 F)+2C\mathrm{D}(\psi T_0 P+\rho_0\psi F)$$
$$-\overline{E}PWF+3\overline{E}\mathrm{D}(P\psi F)\Big)+\overline{H}\left(-\tfrac{1}{2}WP\psi^2+\frac{1}{a^2}\mathrm{D}(W^2P\psi)+2\mathrm{D}(P\psi^3)\right)$$
$$+\overline{E}\left(-\tfrac{1}{2}W\rho_0\psi^2+\frac{1}{2a^2}\mathrm{D}(W^2\rho_0\psi)+\tfrac{3}{2}\mathrm{D}(\rho_0\psi^3+P\psi^3)\right)$$
$$-\frac{\overline{H}\rho_0 W^3}{2a^2}-\frac{JPW^3}{2a^2}+\frac{H\sigma}{\gamma-1}(-\mathrm{D}^2+a^2)F+2CH\sigma P\psi+H\sigma\rho_0\psi$$
$$-\sigma\tau Q\{W(C+\overline{E}h)-\psi\overline{E}\mathrm{D}h\}(a^2-\mathrm{D}^2)h/a^2=0, \tag{6}$$

$$\partial h/\partial t-C(-\psi\mathrm{D}h+Wh)-W-\tau(\mathrm{D}^2-a^2)h=0, \tag{7}$$

$$\mathrm{D}m=\rho_0, \tag{8}$$

where $\mathrm{D}\equiv\partial/\partial z$. We note that in these equations the viscous dissipation terms have been neglected but all nonlinear terms have been retained since it was shown elsewhere (Van der Borght and Fox 1983*b*) that the use of the anelastic approximation could lead to inaccurate values of some flux variables at the upper boundary.

In deriving the equations the following expansions for density ρ, temperature T, velocity $\boldsymbol{u}$ and scaled magnetic field strength $\boldsymbol{H}'$ were used, and m is the mass contained within the layer:

$$\rho(x,y,z,t)=\rho_0(z,t)+P(z,t)f(x,y), \tag{9}$$

$$T=T_0+Ff, \tag{10}$$

$$\boldsymbol{u}=\left(\frac{W}{a^2}\frac{\partial f}{\partial x},\frac{W}{a^2}\frac{\partial f}{\partial y},\psi f\right), \tag{11}$$

$$\boldsymbol{H}'=\left(\frac{\mathrm{D}h}{a^2}\frac{\partial f}{\partial x},\frac{\mathrm{D}h}{a^2}\frac{\partial f}{\partial y},1+hf\right), \tag{12}$$

where ρ_0, P, T_0, F, W, ψ and h are functions of z and t which, together with m, are to be determined by numerical integration of equations (1)–(8). In these equations:

(i) $C=\surd\frac{1}{6}$, $\overline{E}=\frac{5}{6}$, $\overline{H}=\frac{3}{2}$ and $J=\surd\frac{2}{3}$ are constants resulting from the averaging process;

(ii) $\sigma = \nu/\kappa$ and $\tau = \eta/\kappa$ are the Prandtl and magnetic Prandtl numbers respectively;

(iii) $Q = H_0^2 \mu^* d^2/4\pi\eta\mu$ is the Chandrasekhar number, where μ is the viscosity and H_0 the imposed vertical magnetic field;

(iv) $H = gd^3/\kappa_0 \nu_0$ is the Rayleigh parameter, where $\nu_0 = \mu_0/\rho_{00}$ is the kinematic viscosity, ρ_{00} the density and κ_0 the thermal diffusivity at the top of the convective layer;

(v) the planform function $f(x, y)$ may represent rolls, squares, hexagons or other geometric shapes. In this paper only hexagons will be considered due to the three-dimensional nature of the flow and the similarity of convective patterns on the Sun with hexagons/polygons. For hexagonal cells

$$f(x, y) = (\tfrac{2}{3})^{\frac{1}{2}}\{\cos\, ay + \cos\, a(\tfrac{1}{2}\sqrt{3}x+\tfrac{1}{2}y) + \cos\, a(\tfrac{1}{2}\sqrt{3}x-\tfrac{1}{2}y)\}\,,$$

$$a = \tfrac{8}{3}\pi(\text{depth}/\text{width})\,.$$

Some of the parameters appearing in equations (1)–(8) have been chosen to give the closest possible fit between the polytropic model, considered in this study, and the mixing-length model (Böhm 1963; Kohl 1966):

(i) The ratios of mean temperature and mean density at the upper and lower part of the convective layer, in the polytropic model, are given by

$$(T_0)_l/(T_0)_u = 1 + 1/\phi_0\,, \tag{13}$$

$$(\rho_0)_l/(\rho_0)_u = (1 + 1/\phi_0)^s\,, \tag{14}$$

where s is the polytropic index. The best fit is obtained by choosing

$$s = 1{\cdot}409\,, \qquad \phi_0 = 0{\cdot}597\,. \tag{15a, b}$$

(ii) The ratio of specific heats is given by

$$\gamma = \{(\nabla - \nabla_{\mathrm{ad}}) + s/(s+1)\}^{-1} \tag{16}$$

and must lie between the limits

$$1 < \gamma < (1+s)/s = 1{\cdot}7097\,. \tag{17}$$

We have chosen $\gamma = 1{\cdot}25$ which corresponds to the value $\nabla - \nabla_{\mathrm{ad}} = 0{\cdot}21511$ which, according to the mixing-length theory, occurs at a depth of 150 km.

(iii) The thermal diffusivity at the top κ_0 is given by

$$\kappa_0 = \text{flux} \times (\gamma - 1)(s+1)/\rho_{00}\, g = 4{\cdot}64 \times 10^{12} \tag{18}$$

in this formulation.

(iv) The horizontal wave number a has been set equal to $\pi/\sqrt{2}$. This corresponds to maximum instability and maximum efficiency in convective energy transport. With an average extent of 2000 km for the granules this corresponds to a depth of 530 km.

(v) The Prandtl number $\sigma = \nu_0/\kappa_0$ and the eddy resistivity are to a certain extent free parameters which can be chosen to give the best possible fit to the observed granular motions but it should be kept in mind that: (*a*) Theoretical studies, as mentioned earlier, have already given an average value of $\sigma = 0{\cdot}24$ outside active regions; (*b*) η should lie between well defined limits—an upper limit which ensures the existence of overstable motions in strong fields and a lower limit if granular motions are not to be interfered with by small magnetic fields outside active regions.

The numerical results depend of course to a certain extent on the assumed boundary conditions. In this study we have applied the following boundary conditions:

$$
\begin{array}{rl@{\qquad\qquad}rl}
z = & 0 & z = & 1 \text{ (top)} \\[4pt]
\psi = & 0 & \psi = & 0 \\
W = & 0 & \mathrm{D}W = & 0 \\
T_0 = & (\phi_0+1)/(s+1) & T_0 = & \phi_0/(s+1) \\
F = & 0 & \mathrm{D}F + F/\phi_0 = & 0 \\
m = & 0 & m = M = & \{(\phi_0+1)^{s+1} - \phi_0^{s+1}\}/(s+1)\phi_0^s \\
\mathrm{D}h - ah = & 0 & \mathrm{D}h + ah = & 0
\end{array}
$$

We have therefore assumed:

(i) No overshooting at the boundaries.

(ii) A free boundary at the top and a rigid one at the bottom. This will leave a residual horizontal stress at the bottom and perhaps the formation of a counter cell.

(iii) Average temperatures at top and bottom as close as possible to those given by the mixing-length formalism.

(iv) No temperature fluctuation at the lower boundary but a Newton law of cooling at the top as suggested by the polytropic law.

(v) Conservation of mass during the convective process.

3. Results

The basic equations have been integrated, by means of a method outlined elsewhere (Van der Borght 1980), for various values of H_0 and η with σ being set to $0{\cdot}2$ to facilitate comparison with the results in the absence of a magnetic field (Van der Borght and Fox 1983*a*).

A linear analysis, over the parameter space (H_0, η and σ), is essential for a detailed investigation of the various types of instability which may occur and may give a good first approximation of the period of oscillation of overstable motions. On the other hand, a nonlinear analysis is required to evaluate, in addition to the true period, the other characteristics such as velocity amplitudes and flux modulation. In the present paper we give some results of the nonlinear integrations to illustrate the dynamic effects of magnetic field interaction with convection.

Firstly, in the weak field case where convection is purely unstable and growing with this choice of σ, we may determine some range on η so that the normal granular pattern is left mostly unaffected. The field strength H_0 is taken as 10 G. Table 1 gives the characteristics of granulation for the weak field case for various values of η. Also shown are our previous results in the absence of a magnetic field. Recalling that the molecular value of η is $\eta_m \approx 4\times10^6$ cm^2 s^{-1} (Danielson 1963) and that the turbulent value η_t is less than κ ($\approx 4\cdot64\times10^{12}$ cm^2 s^{-1}) for overstable motions to occur, we can see a relative insensitivity to the value of η down to about 3×10^9 cm^2 s^{-1}. Below this value, changes in the time development of the system occur.

Table 1. Comparison of granular characteristics for values of η for the weak field case

$\sigma = 0\cdot2$, $t \approx 1135$ s

η (cm^2 s^{-1})	Max. vert. vel. (km s^{-1})	r.m.s. hor. vel. (km s^{-1})	Int. fluctuation (%)
	$H_0 = 10$ G		
4×10^{11}	1·200	0·9345	15·316
3×10^{10}	1·198	0·9319	15·218
3×10^{9}	1·202	0·9354	15·345
	$H_0 = 0$[A]		
	1·208	0·9416	15·427

[A] From Van der Borght and Fox (1983*a*).

In the strong field case ($H_0 \geqslant 2000$ G), where motions are usually overstable (oscillatory), the growth (or decay) of any disturbance depends on the properties of the medium, such as buoyancy, and the ability to combine with a given magnetic field to produce oscillations. The value of η can greatly affect the rate of growth (or decay) once the motion is established. The characteristics are now of a different nature, the intensity fluctuation and energy carried are usually lower but the velocities can become quite high, oscillating with small periods. Small time steps are often required to resolve these rapid changes. The field strengths of 2000 and 3000 G, which largely determine the periods of oscillation, are a good representation of typical fields encountered in sunspot umbrae. Values of η, which are quite uncertain, range between the limits as suggested by the weak field case, i.e. between 3×10^9 and 3×10^{11} cm^2 s^{-1}.

Table 2. Characteristics of strong magnetic field interaction with convection

$\gamma = 1\cdot25$, $\sigma = 0\cdot2$

H_0 (G)	η (cm^2 s^{-1})	Av. period of oscillation (s)	Vert./hor. vel. amp. (km s^{-1})
2000	3×10^{10}	68·0	$\pm0\cdot3/\pm0\cdot25$
3000	3×10^{11}	48·0	$\pm0\cdot5/\pm0\cdot4$

Since we are using a polytropic model we will not attempt to model in detail features such as the 3 minute oscillation, but merely demonstrate how the interaction of magnetic fields and convection on the granular scale can model the observed characteristics to a fair degree. Table 2 contains the relevant characteristics for strong field cases, while Figs 1*a–d* show some of the time evolution of vertical and horizontal velocities.

The oscillatory motions reverse their direction of flow within one period so it is of some interest to investigate the depth dependence of the vertical velocity as a function

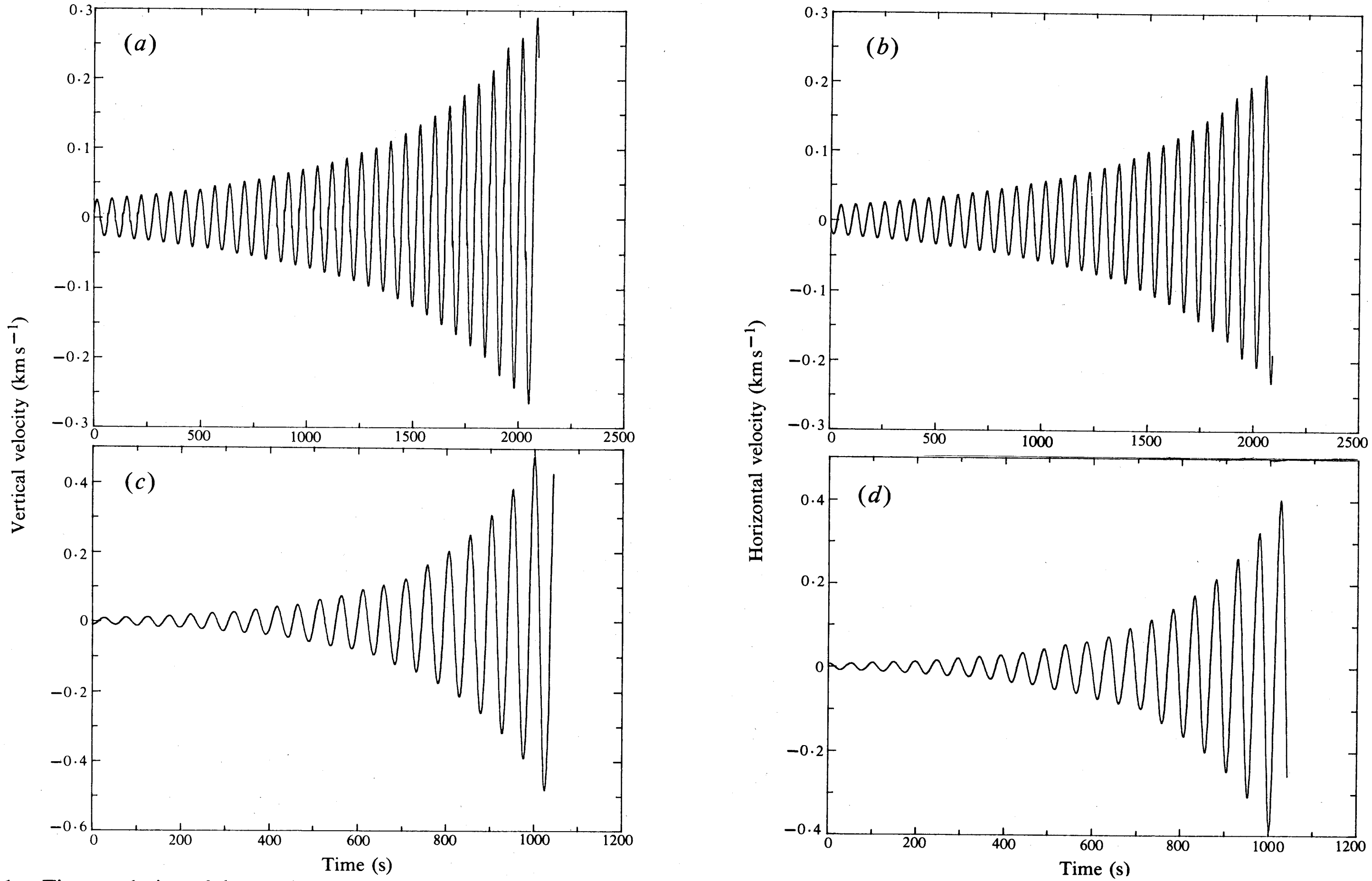

Fig. 1. Time evolution of the maximum vertical velocity (*left*) and of the r.m.s. horizontal velocity (*right*) at the top ($z = 1$) for two cases: $H_0 = 2000$ G, $\eta = 3\times10^{10}$ cm^2 s^{-1} (*a* and *b*); $H_0 = 3000$ G, $\eta = 3\times10^{11}$ cm^2 s^{-1} (*c* and *d*).

of time. This is illustrated in Fig. 2 in the case of a magnetic field $H_0 = 2000$ G and $\eta = 3\times10^{10}$ cm^2 s^{-1}, for a number of time steps. It can be seen that a reversal in the vertical velocity starts as a small counter cell at the bottom of the convective layer ($z = 0$) which gradually grows in vertical extent over half a period; the process is then repeated from the bottom of the layer again but in the opposite direction (i.e. positive amplitude of the counter cell instead of negative). Two other points are: (*a*) Until the velocity becomes significant (>0·1 km s^{-1}) the magnetic field oscillates about its initial value with small amplitude. (*b*) The periods of oscillation tend to increase slightly as the motions develop.

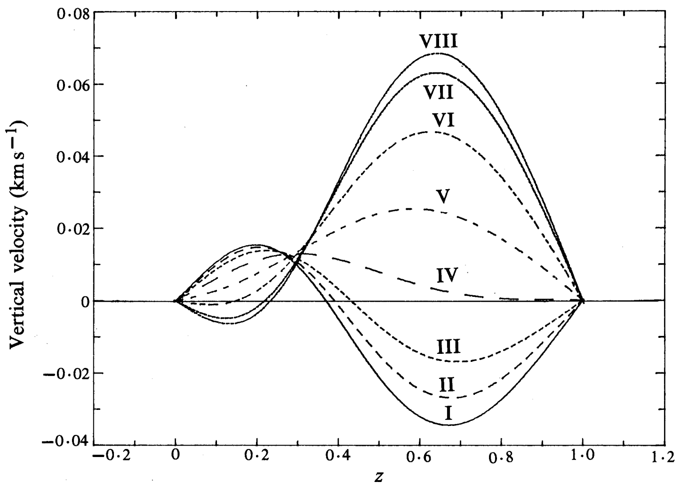

Fig. 2. Depth dependence of vertical velocity at various times (s) labelled by I (1783·47), II (1784·68), III (1786·19), IV (1788·91), V (1791·94), VI (1795·27), VII (1798·29) and VIII (1800·0) for the case $H_0 = 2000$ G and $\eta = 3\times10^{10}$ cm^2 s^{-1}. Note how the nodal point moves in time.

It can be seen from Table 1 that the convective motions in non-active regions, i.e. for small mean magnetic fields of the order of 10 G, are fairly insensitive to the value of η over a fairly large range (3×10^9–3×10^{11} cm^2 s^{-1}). In active regions where the magnetic field is of the order of 2000–3000 G, the motions are periodic and much more sensitive to the value of η. In fact smaller values of η than those given in Table 2 produce very rapidly growing oscillations that were difficult to resolve accurately and have been omitted in this preliminary study.

4. Conclusions

Even with the restriction of a basic polytropic model using constant parameters such as $\gamma = 1·25$ and average Prandtl number $\sigma = 0·2$, it can be seen how the equations of compressible convection under the influence of varying magnetic fields may be used to model various phenomena observed in the Sun's outer layers. The convective motions have a complex dependence on magnetic field strength, resistivity and the general properties of the layer. Although not complete, these integrations indicate that (i) in the weak mean field case, limits may be obtained on the eddy resistivity η, and (ii) in the strong mean field case, the field strength influences the period of oscillation whilst η may be adjusted within limits as given in (i) to study the rate of growth or decay of the motions.

Detailed numerical integrations of more general equations based on a more accurate model of the Sun's outer layers are currently in progress. This will generalize previous work (Van der Borght and Fox 1984) to the case where strong magnetic fields are present.

These nonlinear calculations should provide valuable information for interpreting and modelling phenomena associated with magnetic fields in the Sun's outer layers and permit an investigation of the role that overstable oscillations play in relation to the 3 minute and other umbral oscillations present in strong solar magnetic fields.

References

Böhm, K. H. (1963). *Astrophys. J.* **137**, 881.
Böhm-Vitense, E. (1958). *Z. Astrophys.* **46**, 108.
Bray, R. J., Loughhead, R. E., and Durrant, C. J. (1984). 'The Solar Granulation', 2nd edn (Cambridge Univ. Press).
Danielson, R. E. (1963). *In* 'Stellar and Solar Magnetic Fields', IAU Symp. No. 22 (Ed. R. Lüst), p. 314 (North-Holland: Amsterdam).
Giovanelli, R. G. (1972). *Sol. Phys.* **27**, 71.
Gough, D. O. (1983). *Nature* **304**, 689.
Kohl, K. (1966). *Z. Astrophys.* **64**, 472.
Moore, R. L. (1981). *In* 'The Physics of Sunspots' (Eds L. E. Cram and J. H. Thomas), p. 259 (Sacramento Peak Observatory: New Mexico).
Nordlund, A. (1974). *Astron. Astrophys.* **32**, 407.
Parsons, S. B. (1969). *Astrophys. J. Suppl.* **18**, 127.
Provost, J. (1984). *In* 'Observational Tests of the Stellar Evolution Theory' (Eds A. Maeder and D. Renzini), p. 47 (IAU–Reidel: Dordrecht).
Stenflo, J. O. (1976). *In* 'Basic Mechanisms of Solar Activity', IAU Symp. No. 71 (Eds V. Bumba and J. Kleczek), p. 69 (Reidel: Dordrecht).
Thomas, J. H. (1981). *In* 'The Physics of Sunspots' (Eds L. E. Cram and J. H. Thomas), p. 345 (Sacramento Peak Observatory: New Mexico).
Van der Borght, R. F. E. (1977). *Proc. Astron. Soc. Aust.* **3**, 91.
Van der Borght, R. F. E. (1980). *J. Comput. Appl. Math.* **6**, 283.
Van der Borght, R. F. E., and Fox, P. A. (1983*a*). *Proc. Astron. Soc. Aust.* **5**, 168.
Van der Borght, R. F. E., and Fox, P. A. (1983*b*). *Proc. Astron. Soc. Aust.* **5**, 170.
Van der Borght, R. F. E., and Fox, P. A. (1984). *In* 'Astrophysical and Geophysical Convection' (Eds W. Unno and T. Asai), p. 11 (Univ. Tokyo Press).
Vitense, E. (1953). *Z. Astrophys.* **32**, 135.

Manuscript received 11 March, accepted 12 July 1985