Aust. J. Phys., 1986, **39**, 1–13

Superallowed Fermi β Decays, CVC and Universality*

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Abstract

A review is given of the present status of superallowed Fermi β decays including a short discussion of the radiative corrections of order α, $Z\alpha^2$, $Z^2\alpha^3$ and of nuclear effects. We also point out that the interaction of the emitted positron with the magnetostatic field of the spectator nucleons induces additional corrections of about 0·1% to the *ft* values. Despite refinements in the calculation of the theoretical corrections, there is a remaining discrepancy between the *ft* values which is even larger than the theoretical uncertainties. This behaviour of the *ft* values is not understood at present.

1. Introduction

In 1963 Cabibbo implemented the V–A (vector–axial) theory of weak interactions for semileptonic processes by a connection between strangeness conserving and strangeness changing weak currents. This connection is adequately described by a new phenomenological parameter, the Cabibbo angle θ. It appears that the universal weak interaction constant G for current–current coupling can be measured in μ decay, while $G\cos\theta$ can be measured in the $0^+ \rightarrow 0^+$ superallowed (i.e. $\Delta T = 0$) β decays of atomic nuclei. One can then calculate $\cos\theta$ and the coupling constant $G\sin\theta$ for the strangeness changing semileptonic decays. If $G\sin\theta$ coincides with the corresponding value extracted from the strangeness changing hyperon and K_{l3} decays, we have successfully tested the universality concept in Cabibbo's theory.

As has been stressed by Gaillard and Sauvage (1984) and most recently in the addendum by Bohm to Garcia and Kielanowski (1985), the Cabibbo model meets with remarkable success in fitting the most recent and complete data on hyperon semileptonic decays, showing that the effects of both SU(3) breaking and the mixing of higher generations of quarks are small. For the present purpose we therefore disregard the generalization of the original Cabibbo model and the problems connected with the Kobayashi–Maskawa matrix.

Even this reduced program of testing universality is not as straightforward to perform as it appears. As always, there are experimental and theoretical difficulties. In the following discussion we will concentrate on parts of the theoretical aspects of the problem, namely the evaluation of all the corrections that come into play. Since μ and e are charged, it is certainly necessary to consider the modifications due to the

* This paper is based on two lectures given by one of us (G.R.) at the Seventh NUPP Summer School, Canberra, 4–8 February 1985.

0004-9506/86/010001$02.00

electromagnetic radiative corrections of μ decay. Since the nuclei and electrons in nuclear β decay are also charged, one has to consider the electromagnetic radiative corrections to β decay also. In addition, there are the strong interactions which influence β decay; they change, for example, the weak current which at the quark level is proportional to

$$\bar{q}\gamma_\mu(1-\gamma_5)q'.$$

The vector part of this weak current is not altered by the strong interactions; theoretically this is explained by the conserved vector current hypothesis (CVC). The axial current is not conserved and the strong interactions renormalize the axial coupling constant:

$$G_A/G_V = 1{\cdot}25\pm0{\cdot}01.$$

Thus the weak current for nucleons is proportional to

$$\overline{N}\gamma_\mu\{1-(G_A/G_V)\gamma_5\}N'.$$

For the $0^+ \to 0^+$ decays only the vector current contributes. In this case CVC means that meson exchange effects do not affect the matrix element of the vector current except via the electromagnetic corrections; thus the so-called *ft* values of different nuclei should be the same after correction for these electromagnetic effects.

The axial current induces Gamow–Teller transitions which are strongly influenced by meson exchange effects; moreover these vary from one nucleus to another. We will not consider such transitions in the following discussion.

In Section 2 we briefly introduce the phenomenological current–current V–A theory, explaining the concepts of universality and CVC. In Section 3 we briefly introduce μ decay and nuclear $0^+ \to 0^+$, $\Delta T = 0$ decay. In Section 4 we discuss the radiative corrections to these transitions. Further electromagnetic and nuclear corrections to the β decays are discussed in Section 5. Numerical results concerning universality are given in Section 6.

2. Current–Current V–A Coupling, Cabibbo Mixing and CVC

In the following discussion we give a brief description of V–A theory for current–current coupling; we will omit modifications due to the existence of intermediate vector bosons and the more basic gauge theories connected with them, providing they do not alter the conclusions. In fact, the current–current coupling is the low energy limit of the underlying gauge field theory. The weak interaction density is

$$\mathscr{H}_w = (G/\sqrt{2})J^\mu J_\mu^\dagger, \qquad J_\mu = J_\mu(x),$$

where J_μ can be written as the sum of a leptonic part and a hadronic part:

$$J_\mu = J_\mu^{(L)} + J_\mu^{(H)}.$$

The leptonic part is the sum of contributions of all leptons (we omit the contribution due to the τ lepton):

$$J_\mu^{(L)} = L_\mu^{(e)} + L_\mu^{(\mu)},$$

with

$$L_\mu^{(e)} = \bar{e}\gamma_\mu(1-\gamma_5)\nu_e,$$

where the letter for the particle is shorthand for its spinor field in an obvious way. The $L_\mu^{(\mu)}$ has the same form as $L_\mu^{(e)}$, which is a manifestation of e–μ universality.

The hadron current will have contributions from both the strangeness conserving and strangeness changing processes:

$$J_\mu^{(H)} = \cos\theta\, J_\mu^{(0)} + \sin\theta\, J_\mu^{(1)},$$

where θ is the Cabibbo angle. We disregard the strangeness changing part here, because we will not treat those decays in detail. The strangeness conserving current, like the lepton current, is the sum of a vector current V_μ and an axial current A_μ:

$$J_\mu^{(0)} = V_\mu + A_\mu, \qquad J_\mu^{(0)\dagger} = V_\mu^\dagger + A_\mu^\dagger .$$

At this point it is useful to introduce the electromagnetic current $j_\mu^{(H)}$ of the hadrons. Corresponding to the familiar decomposition of the charge operator

$$\tfrac{1}{2}(1+\tau_3)$$

in the isotopic spin notation of nuclear physics, we have

$$j_\mu^{(H)} = s_\mu + V_{\mu 3}.$$

Here $V_{\mu 3}$ transforms as the third component of an isospin vector under isospin transformations and s_μ transforms as a scalar. In fact, all the currents mentioned are associated with an octet representation of SU(3). For our purpose we need only the more restricted assumption that V_μ, $V_\mu^\dagger$ and $V_{\mu 3}$ form an isospin vector, the cartesian components being defined by

$$V_\mu = V_{\mu 1} - \mathrm{i}\, V_{\mu 2}, \qquad V_\mu^\dagger = V_{\mu 1} + \mathrm{i}\, V_{\mu 2}.$$

In fact, we have

$$\int V_{0i}\, \mathrm{d}^3 x = T_i,$$

the ith component of the total isospin operator. Because isospin is conserved up to electromagnetic corrections, we see that $V_{\mu i}$ is a conserved current:

$$\partial^\mu V_{\mu i} = 0 + \mathrm{O}(\alpha) \qquad \text{(CVC)}.$$

3. The μ Decay and Nuclear $0^+ \to 0^+$, $\Delta T = 0$ Transitions

The weak interaction density $\mathscr{H}_w$ contains a term

$$(G/\sqrt{2}) L^{(e)\dagger} \cdot L^{(\mu)}$$

responsible for $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$. Here we have suppressed the Lorentz indices in the Lorentz product and indicated this product by a dot. The $\mathscr{H}_w$ also contains a term

$$(G\cos\theta/\sqrt{2}) L^{(e)\dagger} \cdot J^{(0)} = (G\cos\theta/\sqrt{2}) L^{(e)\dagger} \cdot (V+A)$$

responsible for nuclear β^+ decay. We are interested in nuclear transitions $0^+ \to 0^+$, which certainly are Fermi transitions. One can always think of the well-known decay

$$ {}^{14}_{8}\mathrm{O} \to {}^{14}_{7}\mathrm{N}^* + \mathrm{e}^+ + \nu_\mathrm{e} $$

from the ground state of ${}^{14}_{8}\mathrm{O}$ to the first excited state of ${}^{14}_{7}\mathrm{N}$. The ${}^{14}_{8}\mathrm{O}$, ${}^{14}_{7}\mathrm{N}^*$ and ${}^{14}_{6}\mathrm{C}$ nuclei form an isotopic spin triplet; so for this decay we have $\Delta T = 0$ and $\Delta T_3 = -1$. Our results will apply to all decays of this type.

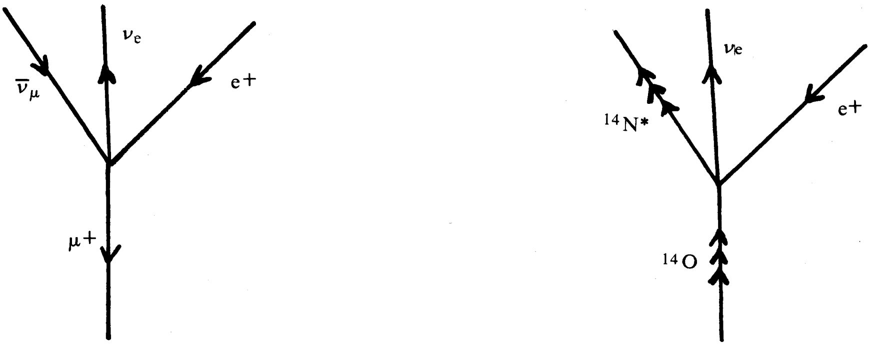

Fig. 1. Feynman graphs for μ^+ and ^{14}O decay.

The Feynman graphs corresponding to μ^+ and ^{14}O decay are shown in Fig. 1. The vector current contribution to the nuclear matrix element can be evaluated exactly for allowed transitions; apart from kinematic factors it becomes

$$ \mathring{\mathscr{M}}^{\mathrm{V}}_{\mathrm{F}} = \langle {}^{14}\mathrm{N}^* | \, T_- \, | {}^{14}\mathrm{O} \rangle = \sqrt{2}, $$

where T_- is the charge lowering total isospin operator. The axial current contribution to a Fermi matrix element is

$$ \mathring{\mathscr{M}}^{\mathrm{A}}_{\mathrm{F}} = 0 . $$

The circle over $\mathscr{M}$ reminds us that we are in lowest order perturbation theory (no radiative corrections yet), with

$$ \mathring{\mathscr{M}}_{\mathrm{F}} = \mathring{\mathscr{M}}^{\mathrm{V}}_{\mathrm{F}} + \mathring{\mathscr{M}}^{\mathrm{A}}_{\mathrm{F}} = \mathring{\mathscr{M}}^{\mathrm{V}}_{\mathrm{F}} . $$

4. Radiative Corrections

(a) Current–Current V–A Coupling

The electromagnetic radiative corrections are due to the electromagnetic interaction density

$$ \mathscr{H}_{\mathrm{EM}} = -(4\pi\alpha)^{\frac{1}{2}} j_\alpha \, a^\alpha . $$

Here the total electromagnetic current j_α is the sum of a lepton part and a hadron part:

$$ j_\alpha = j_\alpha^{(\mathrm{L})} + j_\alpha^{(\mathrm{H})} , $$

where the hadron part was introduced in Section 2 and the lepton part is

$$j_\alpha^{(L)} = \bar{e}\gamma_\alpha e + \bar{\mu}\gamma_\alpha \mu ;$$

a_α is the quantized electromagnetic field.

The electromagnetic radiative corrections to μ decay have been calculated to order α and the results are not only free of ultraviolet divergences but are also in excellent agreement with the electron spectrum and polarization data.

Hadron decay and, in particular, nuclear β decay is considerably more complicated because of the effect of strong interactions. The strong interaction effects can be investigated by current algebra techniques following Abers *et al.* (1968). It was shown that the electromagnetic radiative corrections to the β decay amplitude due to the vector current are independent of strong interaction details, while the corrections due to the axial current depend on the model for the strong interactions. The contribution of the axial current was approximately evaluated for various models of the strong interactions by Abers *et al.* (1968). One problem, however, cannot be solved in the V–A theory; the electromagnetic radiative corrections have ultraviolet logarithmic divergences. This difficulty was resolved by Sirlin (1978, 1982), who demonstrated that these divergences cancel within the framework of the standard SU(2)×U(1) model of electro-weak interactions. The radiative corrections for β decay are finite, as are those for μ decay.

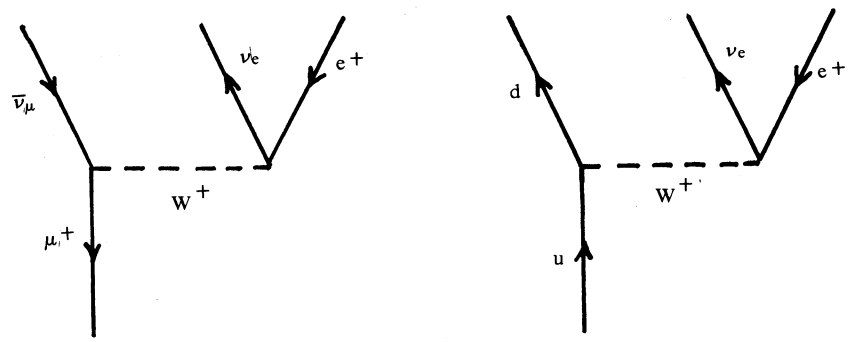

Fig. 2. Lowest order graph for μ^+ and u decay.

(b) Final Result for SU(2)× U(1) Gauge Theories

In contrast to the phenomenological current–current coupling theories, the gauge theories are renormalizable. Hence all ultraviolet divergent terms can be absorbed by redefining observable quantities. The main result is that the divergent integrals in the phenomenological current–current coupling result have to be cut off at the mass of the intermediate vector bosons. Without describing details, we explain the procedure.

The lowest order graphs for the decay of μ^+ and a u quark are shown in Fig. 2. The coupling constant for each vertex is g, with

$$G/\sqrt{2} = g^2/8m_W^2 ,$$

where m_W is the mass of $W^\pm$. Of the radiative correction graphs, first one systematically ignores those which renormalize g for μ decay and for β decay in the same way; that is sufficient for testing universality. Fig. 3 shows one of the many remaining graphs for each of the two processes. Both graphs show a Z exchange as a reminder that now we have 'nonphotonic' as well as 'photonic' contributions to the radiative corrections (termed *electromagnetic* radiative corrections in Section 4*a*).

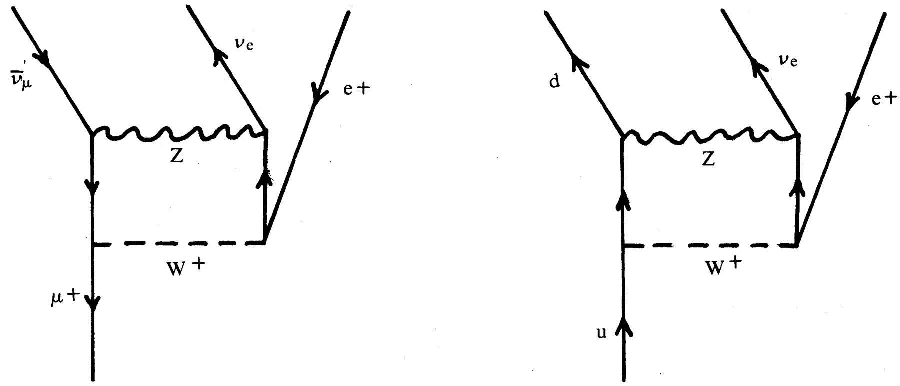

Fig. 3. Typical example of radiative correction graphs to μ^+ and u decay.

Structure effects due to the confinement of quarks in nucleons (and nuclei) are treated by the methods of current algebra. By means of this powerful tool it was established that the radiative corrections to Fermi β decay are rather independent of the details of the strong interactions and are fairly small. We give the final result expressed in terms of G^2, or better $\hat{G}^2$, where $\hat{G}$ is the universally renormalized weak coupling constant applying to μ decay and nuclear β decay:

$$\frac{1}{\tau_\mu} = \frac{\hat{G}^2 m_\mu^5}{192\pi^3}\left(1 - \frac{8m_e^2}{m_\mu^2}\right)\left(1 + \frac{3m_\mu^2}{5m_W^2} + \frac{\alpha}{2\pi}\left(\tfrac{25}{4} - \pi^2\right)\right),$$

where m_e is the mass of $e^\pm$, m_μ is the mass of $\mu^\pm$ and τ_μ is the lifetime of $\mu^\pm$.

For the superallowed Fermi transitions we first give the spectrum of e^+ as

$$P\,d^3p = \tilde{P}\,d^3p\left\{1 + \frac{\alpha}{2\pi}\left(3\ln\frac{m_Z}{m_p} + g(E, E_m) + 6\bar{Q}\ln\frac{m_Z}{m_{A_1}} + 2C + \mathscr{A}\right)\right\}.$$

Here p is the momentum of e^+; E is the energy of e^+ including rest energy; E_m is the end-point energy of the spectrum; m_{A_1} is the approximate mass of the A_1 meson; $g(E, E_m)$ is a known function which does not contain m_W, m_Z or the Weinberg angle θ_W (the analytical form can be found in Sirlin 1967); $\bar{Q} = \frac{1}{6}$ is the average charge of u and d quarks; $\tilde{P}\,d^3p$ is the spectrum without radiative corrections and by neglecting any other corrections it is equal to the statistical spectrum $\mathring{P}d^3p$, where

$$\mathring{P} = \frac{\hat{G}^2\cos^2\theta}{8\pi^4}|\mathring{\mathscr{M}}_F|^2(E_m - E)^2.$$

Also, the term C is a structure-dependent contribution due to the axial current (estimates indicate that $2|C| \approx 1$, which would be negligible); the term $\mathscr{A}$ is induced by the strong interactions and is estimated to be even smaller than $|C|$. Different results by Tóth are mentioned in Bourquin *et al.* (1983).

5. Further Electromagnetic and Nuclear Corrections

In addition to the radiative corrections there are a number of other corrections to the β-decay spectrum that we will treat here.

(a) Fermi Function, Shape Factor and Full Radiative Corrections

It is important to take into account the fact that the outgoing positron is not free but moves in the electrostatic field of the daughter nucleus with charge Z. This leads to a correction factor $F(Z, E)$ multiplying the statistical spectrum. This factor is called the Fermi function. It results in a much larger correction than the radiative corrections and is intimately connected with them. It can be expanded with respect to $Z\alpha$ as

$$F(Z, E) = 1 + \mathrm{O}(Z\alpha) + \mathrm{O}(Z^2\alpha^2) + \dots .$$

Strictly speaking, the term of order $Z\alpha$ should be contained in the radiative corrections. In fact this is so, but it has been omitted in the formula for radiative corrections for the same reason that it is taken into account in the Fermi function! The necessity then arises to calculate the remaining terms of order $Z\alpha^2$, $Z^2\alpha^3, \dots$, which are not contained in the product of $F(Z, E)$ with the radiative corrections. This has been done by Jaus and Rasche (1970) and Jaus (1972). We will call these corrections $\delta_2(Z, E)$ and $\delta_3(Z, E)$ for order $Z\alpha^2$ and $Z^2\alpha^3$ respectively. The correction factor $F(Z, E)$ also takes into account the screening of the nuclear charge by the atomic electrons. This effect, however, is known to be very small and cannot be determined accurately since one does not know the degree of ionization and the atomic state after β decay has occurred.

Another factor, $C(E)$, takes into account the possibility of second forbidden corrections, which are due to the variation of the lepton wavefunctions over the nuclear volume. This factor is called the shape factor, since it affects the shape of the β spectrum. Of course it is very closely connected to $F(Z, E)$. A consistent treatment of all problems involving $F(Z, E)$ and $C(E)$ has been given by Behrens and Bühring (1982). The comparison between theory and experiment has reached a very high level of precision and it seems appropriate to point out that the evaluation of the second-forbidden nuclear matrix elements is not unique in the literature. Szybisz *et al.* (1983) have shown that the disagreement for the integrated statistical rates increases with Z and ranges from $0 \cdot 07\%$ for ^{42}Sc to as much as $0 \cdot 13\%$ for ^{54}Co. An alternative derivation of $C(E)$ in terms of the nuclear isovector form factor has been given by Jaus (1971), with results in agreement with those of Hardy (1975).

With all these corrections it is customary to write the spectrum in the form

$$(1+\Delta_{\mathrm{R}})\mathring{P}\, \mathrm{d}^3p\, F(Z, E)\, C(E)(1+\delta_1+\delta_2+\delta_3+\text{other corrections to be discussed});$$

$$\Delta_{\mathrm{R}} = \frac{\alpha}{2\pi}\left(3 \ln \frac{m_{\mathrm{Z}}}{m_{\mathrm{p}}} + \ln \frac{m_{\mathrm{Z}}}{m_{\mathrm{A}_1}} + \begin{matrix}\text{estimated terms}\\ \text{of order 1}\end{matrix}\right) \approx (2 \cdot 1 \pm 0 \cdot 1)\%,$$

$$\delta_1 = \frac{\alpha}{2\pi}\, g(E, E_{\mathrm{m}}), \qquad \delta_2 = Z\alpha^2 \ln \frac{m_{\mathrm{p}}}{m_{\mathrm{e}}} + \text{small energy-dependent terms},$$

$$\delta_3 = \frac{Z^2\alpha^3}{\pi}(3 \ln 2 - \tfrac{3}{2} + \tfrac{1}{3}\pi^2) \ln \frac{m_{\mathrm{p}}}{m_{\mathrm{e}}} + \text{small energy-dependent terms}.$$

Here Δ_R is independent of the specific Fermi β decay under consideration and therefore is called the 'inner radiative correction'. It is important for the check of universality, but not for a check of CVC. The corrections δ_1, δ_2 and δ_3 depend on the specific decay via E and Z, and are called 'outer radiative corrections'.

Although the other corrections to be discussed here depend on the specific β decay under consideration, they do not depend on E. Therefore we can now discuss the integration over the spectrum. The half-life t of the nucleus under consideration is given by

$$\frac{\ln 2}{t} = (1+\Delta_R)\int \mathring{P}\mathrm{d}^3p\, F(Z,E)\, C(E)(1+\delta_1+\delta_2+\delta_3)\theta(E_m-E)$$

$$= (1+\Delta_R)\frac{\hat{G}^2}{2\pi^3}\cos^2\theta\,|\mathring{\mathscr{M}}_F|^2\int_1^{E_m} S(E)\, F(Z,E)\, C(E)(1+\delta_1+\delta_2+\delta_3)\,\mathrm{d}E,$$

with the kinematic factor

$$S(E) = (E-E_m)^2(E^2-1)^{\frac{1}{2}}E \qquad (m_e = 1).$$

We define

$$f = \int_1^{E_m} S(E)\, F(Z,E)\, C(E)\,\mathrm{d}E,$$

$$\bar{\delta}_i = f^{-1}\int_1^{E_m} S(E)\, F(Z,E)\, C(E)\,\delta_i(Z,E)\,\mathrm{d}E.$$

Since it is a correction, we can put $C(E) = 1$ in the expression for $\bar{\delta}_i$,

$$\bar{\delta}_i = \int S(E)\, F(Z,E)\,\delta_i(Z,E)\,\mathrm{d}E \Big/ \int S(E)\, F(Z,E)\,\mathrm{d}E,$$

so that

$$f_R\, t = ft(1+\bar{\delta}_1+\bar{\delta}_2+\bar{\delta}_3) = \frac{2\pi^3\ln 2}{(1+\Delta_R)|\mathring{\mathscr{M}}_F|^2\hat{G}^2\cos^2\theta}.$$

Table 1. Outer radiative corrections (%) for the eight most accurately measured superallowed Fermi transitions

Nucleus	Z	$\bar{\delta}_1$	$\bar{\delta}_2$	$\bar{\delta}_3$	$\bar{\delta}_1+\bar{\delta}_2+\bar{\delta}_3$
^{14}O	7	1·30	0·26	0·02	1·58
^{26m}Al	12	1·12	0·45	0·05	1·62
^{34}Cl	16	1·01	0·59	0·09	1·69
^{38m}K	18	0·98	0·66	0·11	1·75
^{42}Sc	20	0·95	0·73	0·15	1·83
^{46}V	22	0·92	0·80	0·18	1·90
^{50}Mn	24	0·88	0·87	0·21	1·96
^{54}Co	26	0·85	0·93	0·24	2·02

In fact, ft describes the decay corrected for screening and nuclear size effects, but without radiative corrections. Apart from the radiative and further corrections, it should be the same for all $0^+ \to 0^+$, $\Delta T = 0$ decays as the right-hand side of the last equation shows. To calculate ft one has to measure the half-life t and the maximum decay energy E_m for each of the β decays. The average $\bar{\delta}_i$ has to be computed numerically; for $\bar{\delta}_1$ this has been done by Wilkinson and Macefield

(1970) and for $\bar{\delta}_2$ and $\bar{\delta}_3$ by Jaus (1972). Table 1 shows the relative importance of the radiative corrections for the eight most precisely measured $0^+ \to 0^+$, $\Delta T = 0$ nuclear decays. It should be remembered that $ft \approx 3100$ s for all these decays and that the experimental error in some cases is less than $0 \cdot 1\%$ (see Table 2), so that the corrections should be calculated to an accuracy of better than $0 \cdot 1\%$, if possible. In the case of ^{54}Co it is clearly seen that, with increasing experimental precision, terms of order $Z^3\alpha^4$ might have to be calculated. The quantity $\bar{\delta}_1$ has been interpolated from the tables given in Wilkinson and Macefield (1970), while $\bar{\delta}_2$ and $\bar{\delta}_3$ have been taken from Jaus (1972). For the numerical precision of values in Table 1 it would in fact be sufficient to take

$$\bar{\delta}_i \approx \int S(E)\,\delta_i(Z, E)\,\mathrm{d}E \Big/ \int S(E)\,\mathrm{d}E\,.$$

This excellent numerical approximation has been discussed by Jaus and Rasche (1970).

(b) Recoil Correction and Competing Processes

There is a very minor correction which takes into account the kinetic recoil energy of the final nucleus. It involves multiplying the spectrum by (see e.g. Behrens and Bühring 1982)

$$R(E) = 1 + 2E/M\,,$$

where M is the mass of the nucleus. Since this correction depends on E, it should have been taken into account before averaging over the spectrum. In the values quoted later, this recoil correction is included, if necessary.

Another correction has to take into account the competing K-shell capture, for example

$$^{14}_{8}\mathrm{O} + \mathrm{e}^- \to {}^{14}_{7}\mathrm{N}\,.$$

This can be calculated for each decay and the measured total half-lives have to be corrected for this effect; its contribution is of the order of $0 \cdot 1\%$.

In the case of ^{14}O there is also a branching correction due to the competing process

$$^{14}\mathrm{O} \to {}^{14}\mathrm{N} + \mathrm{e}^+ + \nu_\mathrm{e}\,,$$

where ^{14}N is in its $(1^+, T = 0)$ ground state.

All these corrections, however, are uncontroversial at the present level of experimental accuracy and we do not discuss them further. In the values given later they have tacitly been included.

(c) Isospin Mismatch

Due to Coulomb repulsion inside the nucleus, SU(2) symmetry is broken. Otherwise the members of an SU(2) multiplet would all have the same energy and a superallowed Fermi β decay could not occur at all.

Because of this SU(2) breaking, the parent and the daughter nucleus are no longer eigenstates of isospin. Consequently the Fermi matrix element does not have its charge-independent value of $\surd 2$ but has to be corrected, for example

$$\langle {}^{14}\mathrm{N}^* |\, T_- \,| {}^{14}\mathrm{O} \rangle = \surd 2(1 - \tfrac{1}{2}\delta_\mathrm{c})\,,$$

where δ_c is positive and of order of magnitude 0·5%. It must be adjusted semiempirically (Wilkinson 1977) or calculated microscopically, and it differs from nucleus to nucleus.

The semiempirical method of correction does not reproduce any nuclear shell effects which are expected at the experimental level of accuracy reached now. Thus the microscopic calculation is preferred. It has been performed by Wilkinson (1976) and Towner and Hardy (1978). Because different schemes of approximations are involved for the nuclear wavefunctions and the number of excited states to be taken into account, the two calculations do not give the same result. Therefore both calculations are included in Table 2 and discussed in Section 6.

(d) Magnetostatic Correction

The Fermi function takes account of the fact that the positron moves in the *electrostatic field* of the daughter nucleus. But it is not sufficient to assume a pure Z/r dependence for this potential; the actual charge distribution inside the nucleus has to be taken into account.

The *magnetostatic correction* is a similar effect, which should be calculated at the present stage of experimental accuracy. The outgoing positron moves also in the magnetostatic field of the daughter nucleus. The 0^+ states do not have an overall magnetic moment, but a closer examination of the nucleus shows that the actual magnetic moment distribution inside has to be taken into account.

We have considered this magnetic moment distribution by a microscopic approach. By omitting, for convenience, the contribution due to the orbital motion of the protons, the electromagnetic vector potential due to the nucleons can be written as

$$A(x) = \sum_{i=1}^{A} \frac{\mu^{(i)} \times (x - x^{(i)})}{|x - x^{(i)}|^3}.$$

Here $x^{(i)}$ is the position of nucleon i, $\mu^{(i)}$ is its magnetic moment and x is the position of the positron. Taking the Fourier transform with respect to x,

$$\tilde{A}(k) = -\frac{\mathrm{i}}{2\pi^2} \sum_{i=1}^{A} \frac{\mu^{(i)} \times k}{k^2} \exp(-\mathrm{i}kx^{(i)}),$$

one can then calculate the first order correction to the positron wavefunction and insert this correction into the lepton current of the weak interaction Hamiltonian for the nucleus. Taking the Fermi part of the nuclear matrix element of the correction we obtain a factor $(1+\delta_M)$ for the ft values, with

$$\delta_M = \frac{4\alpha}{3\pi} \frac{G_A}{G_V} \frac{3}{\sqrt{2}m_p} \int_0^\infty \Omega(k)\,\mathrm{d}k,$$

where $\Omega(k)$ is defined by

$$\langle 0^+ | \sum_{i,j} \exp\{-\mathrm{i}k(x^{(i)} - x^{(j)})\} \mu^{(i)} \sigma_l^{(i)} \sigma_m^{(j)} t_-^{(j)} | 0^+ \rangle = \Omega(k) \delta_{lm} + \Gamma(k) k_l k_m .$$

Here $\sigma^{(i)}$ is the spin of the ith nucleon and $t_-^{(j)}$ the isospin operator acting on the jth nucleon; $\mu^{(i)}$ is the magnetic moment of the ith nucleon measured in units of the nuclear magneton μ_B. The decomposition of the matrix element into a part containing $\Omega(k)$ and a part containing $\Gamma(k)$ follows from the rotational symmetry.

In a first order of magnitude estimate one can approximate $\Omega(k)$ by any form factor that approximates the nucleon distribution, with the result

$$\left| \int_0^\infty \Omega(k)\, \mathrm{d}k \right| \approx \frac{2}{r},$$

where r is the r.m.s. radius of the nucleus with mass number A,

$$r \approx 1 \cdot 03 A^{\frac{1}{3}} \times 10^{-13}\ \mathrm{cm}.$$

It follows that

$$|\delta_M| \approx 3 \cdot 5 A^{-\frac{1}{3}} \times 10^{-3}.$$

This gives a correction of about 0·1% to the ft values and is of the order of magnitude of the experimental and theoretical effects discussed earlier. It is to be expected that δ_M, like δ_c, will show pronounced shell effects in addition to the global behaviour indicated in the above estimate. So it might be helpful to bring all the $\mathscr{F}t$ values (defined in Section 6*a*) closer together and obtain an acceptable statistical fit. In addition, it will change the average of all $\mathscr{F}t$ values, but certainly not significantly. A more detailed investigation of magnetostatic corrections will be published separately.

Table 2. Numerical results for the eight most precisely investigated superallowed Fermi β decays

(1) Nucleus	(2) f	(3) t	(4) ft	(5) $f_R\,t$	(6) $\delta_c^{[W]}$	(7) $\delta_c^{[H]}$	(8) $\mathscr{F}t^{[W]}$	(9) $\mathscr{F}t^{[H]}$
^{14}O	42·709	71134·0	3038·1	3086·1	0·46	0·33	3071·9	3075·9±3·9
^{26m}Al	477·83	6350·5	3034·4	3083·6	0·42	0·34	3070·7	3073·1±3·7
^{34}Cl	1997·80	1527·7	3052·0	3103·6	0·80	0·85	3078·8	3077·2±4·7
^{38m}K	3292·8	924·78	3045·1	3098·4	0·50	0·70	3082·9	3076·7±4·6
^{42}Sc	4467·6	682·4	3048·7	3104·5	0·44	0·48	3090·8	3089·6±7·5
^{46}V	7199·2	422·78	3043·7	3101·5	0·35	0·40	3090·7	3089·1±4·3
^{50}Mn	10727·8	283·37	3039·9	3099·5	0·59	0·43	3081·2	3086·2±5·7
^{54}Co	15740·8	193·43	3044·7	3106·2	0·71	0·60	3084·2	3087·6±4·4

6. Numerical Results

(a) Superallowed Nuclear β Decays and π Decay

At present the ft values of 18 superallowed nuclear β decays have been carefully investigated experimentally. The results for the eight most accurately measured transitions are listed in Table 2, which are based on the work of Koslowsky *et al.* (1984). We quote only the errors of the final results for $\mathscr{F}t$. Columns (2) and (3) give f (as calculated from the measured end-point energy) and the relevant partial half-life t (corrected for branching and electron capture) in ms, while columns (4) and (5) give ft and $f_R\,t$. Columns (6) and (7) give the percentage corrections δ_c as calculated by Wilkinson (1976) and Towner and Hardy (1978) respectively, while columns (8) and (9) give the corresponding values (in s)

$$\mathscr{F}t = f_R\,t(1-\delta_c)$$

for the two cases. For $\mathscr{F}t^{(H)}$ we list the uncertainties given by Koslowsky *et al.* (1984). These also include an allowance for the theoretical uncertainties in δ_c. The minute numerical difference between the values listed in column (9) and the $\mathscr{F}t$ values given by Koslowsky *et al.* (1984) arises from the slightly different radiative corrections applied in each case. It can be traced back to the interpolation of δ_1 from the tables given in Wilkinson and Macefield (1970).

It is obvious from Table 2, and the discussion by Towner and Hardy (1984), that the $\mathscr{F}t$ values of the high-Z nuclei are not quite compatible with those of the low-Z nuclei. At the moment no explanation for this discrepancy is available. From Table 2 we deduce as an average value

$$\mathscr{F}t = 3080{\cdot}4 \pm 2{\cdot}4\ \mathrm{s}.$$

Hence, one obtains

$$(1+\tfrac{1}{2}\Delta_R)\,\hat{G}\cos\theta = (1{\cdot}41333\pm0{\cdot}00055)\times10^{-49}\ \mathrm{erg\,cm^3}$$

or

$$(\hbar c)^{-3}(1+\tfrac{1}{2}\Delta_R)\,\hat{G}\cos\theta = (1{\cdot}14805\pm0{\cdot}00045)\times10^{-5}\ \mathrm{GeV^{-2}}.$$

Taking Δ_R from Section 5*a* we have

$$1+\tfrac{1}{2}\Delta_R = 1{\cdot}0105\pm0{\cdot}0005,$$

so that

$$\hat{G}\cos\theta = (1{\cdot}3986\pm0{\cdot}0012)\times10^{-49}\ \mathrm{erg\,cm^3},$$

or

$$(\hbar c)^{-3}\,\hat{G}\cos\theta = (1{\cdot}1361\pm0{\cdot}0009)\times10^{-5}\ \mathrm{GeV^{-2}}.$$

The decay $\pi^+ \to \pi^0 + e^+ + \nu_e$ is also superallowed. Taking the above value for $(1+\frac{1}{2}\Delta_R)\hat{G}\cos\theta$ one obtains for the inverse partial mean lifetime (for details see Towner and Hardy 1984)

$$\tau_\pi^{-1} = 0{\cdot}4031\pm0{\cdot}0016\ \mathrm{s^{-1}}.$$

This result has been derived under the assumption that the inner radiative corrections for nuclear β decay and π decay are the same. A future and more careful investigation must take into account the difference between the structure-dependent terms. The theoretical error is dominated by the uncertainty in the mass difference between π^0 and π^+. Precision measurements of this mass difference are in progress at SIN (see Crawford *et al.* 1985). A recent experiment by McFarlane *et al.* (1983) gave

$$\tau_\pi^{-1} = 0{\cdot}398\pm0{\cdot}015\ \mathrm{s^{-1}}.$$

(b) The μ Decay and Determination of θ

Combining the latest measurements of the mean lifetime of μ (for details see Towner and Hardy 1984) we obtain

$$\tau_\mu = (2{\cdot}197030\pm0{\cdot}000047)\times10^{-6}\ \mathrm{s},$$

which leads to

$$\hat{G} = (1{\cdot}435858\pm0{\cdot}000016)\times10^{-49}\ \mathrm{erg\,cm^3}$$

or

$$(\hbar c)^{-3}\hat{G} = (1\cdot 166347 \pm 0\cdot 000013)\times 10^{-5}\,\mathrm{GeV}^{-2}\,.$$

Combining this with the value of $\hat{G}\cos\theta$ we obtain

$$\cos\theta = 0\cdot 9740 \pm 0\cdot 0009\,,$$

so that

$$\sin\theta = 0\cdot 226 \pm 0\cdot 004\,.$$

This compares well with the value one obtains from hyperon and K_{l3} decays. As a representative value we quote from the Particle Data Group (1984),

$$\sin\theta = 0\cdot 23 \pm 0\cdot 01\,.$$

The large error is due to an estimate of the still uncertain effects of SU(3) breaking (see also Gaillard and Sauvage 1984; García and Kielanowski 1985).

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Manuscript received 4 July, accepted 29 October 1985