Aust. J. Phys., 1986, **39**, 461–71

A Phenomenological Nucleon–Nucleon Interaction

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Abstract

A phenomenological NN interaction has been developed in momentum space to fit the elastic scattering phase shifts, the deuteron properties and to saturate nuclear matter. The special features of this interaction are gaussian momentum dependent form factors and the use of only three mesons to characterise the phase shifts.

1. Introduction

A momentum-space interaction is developed here, that is very similar to one previously defined (Petris 1981) to fit the phase shifts and the deuteron properties. The specification of that interaction, however, contained a computational error which, for some states, considerably affected the fit to the phase shifts (the fit to the low energy phase shifts and the deuteron properties was not greatly affected). In fact, the inability of the previous interaction to produce saturation of nuclear matter was a direct consequence, specifically because the repulsion in the S states was underestimated considerably.

The 1981 interaction originated from the investigation of a certain functional form (Petris 1971)—a gaussian form factor divided by the usual Yukawa denominator—for fitting the NN data. This functional form was used in conjunction with three ranges which in meson exchange theory correspond to the mass of one pion, two pions and a ρ(765 MeV) or ω(783 MeV) meson. The strengths for each contribution were determined separately for each partial-wave state. Here, we present an interaction which is similar to the 1981 one, but with some additional features, and with which we are able to fit the phase shifts ($L = 0 - 5$, $E_{\text{lab}} = 0$–400 MeV), to obtain the deuteron properties and to saturate nuclear matter close to the correct values of E/N and k_{F}.

2. The Interaction

The interaction form we considered has, for the state $\alpha = [LSJ, T]$, diagonal ($\alpha = \alpha'$) matrix elements given by

$$V_\alpha(k, k') = G\{ V_\pi^\alpha F_L(k, k', m_\pi) + V_{2\pi}^\alpha F_L''(k, k', 2m_\pi) + V_3^\alpha F_L''(k, k', m_3)\}, \quad (1)$$

0004-9506/86/040461$02.00

Table 1. The V_i^a strengths

State	V_π	$V_{2\pi}$	V_3	State	V_π	$V_{2\pi}$	V_3
$^1S_0$	-4	$-1+2z^A$	4	$^3P_0$	$+z$	$2-2z$	$10+10z$
$^1P_1$	6	$3+0{\cdot}5z$	$-3+7z$	$^3P_1$	6	$1{\cdot}5+1{\cdot}5z$	$10+6z$
$^1D_2$	-2	$-1-0{\cdot}5z$	$12-10z$	$^3P_2$	0	$-0{\cdot}6-0{\cdot}2z$	$1-2z$
$^1F_3$	6	$3+1{\cdot}5z$	$-35+25z$	ϵ_2	$+2z$	$1{\cdot}1$	$2-z$
$^1G_4$	-2	$-1-1{\cdot}3z$	$10-25z$	$^3F_2$	-1	$-1{\cdot}4z$	25
$^1H_5$	6	$3+1{\cdot}5z$	$+150z$	$^3F_3$	6	$1+2z$	$60-30z$
				$^3F_4$	0	$-1{\cdot}5$	$15-15z$
$^3S_1$	-6	$-2+4z^A$	$6+6z^A$	ϵ_4	$+2z$	3	$+6z$
ϵ_1	$-4z$	-1	$-6z$	$^3H_4$	-1	-3	-1
$^3D_1$	12	$2+6z$	$25+25z$	$^3H_5$	6	$1+4z$	$-60+60z$
$^3D_2$	-8	$-2-3z$	$7-8z$	$^3H_6$	—	—	—
$^3D_3$	0	$-0{\cdot}7z$	$8-4z$				
ϵ_3	$-6z$	-6	$-14z$				
$^3G_3$	3	$5+2z$	$+50z$				
$^3G_4$	-8	$-2-8z$	$100-90z$				
$^3G_5$	—	—	—				

[A] See Section 2.

where the overall strength G has the value $20{\cdot}4\ \mathrm{fm}^{-1}$ for all states,* the basic ranges and scales are

$$\begin{aligned} m_\pi &= 135{\cdot}0/\hbar c\ \mathrm{fm}^{-1}, & m_\pi^{-1} &= 1{\cdot}462\ \mathrm{fm}, \\ m_{2\pi} &= 279{\cdot}1/\hbar c\ \mathrm{fm}^{-1}, & m_{2\pi}^{-1} &= 0{\cdot}707\ \mathrm{fm}, \\ m_3 &= 774{\cdot}0/\hbar c\ \mathrm{fm}^{-1}, & m_3^{-1} &= 0{\cdot}255\ \mathrm{fm}, \\ M &= 938{\cdot}9/\hbar c\ \mathrm{fm}^{-1}, & \hbar c &= 197{\cdot}33\ \mathrm{MeV\,fm}, \end{aligned} \tag{2}$$

and $E_{\mathrm{lab}} = 2\lambda k^2 = 2\lambda k'^2$ (where $\lambda = 41{\cdot}5\ \mathrm{MeV\,fm}^2$) on the energy shell. The F_L are defined by

$$F_L(k, k', m_i) = (1/kk')Q_L(z_i)(M/m_i)\exp\{-M/m_i-(k^2+k'^2)/2m_i^2\}, \tag{3}$$

in which

$$z_i = (k^2+k'^2+m_i^2)/2kk', \tag{4}$$

and the $Q_L(z_i)$ are Legendre functions of the second kind. The functions F_L'' are defined by

$$F_L''(k, k', m_i) = \{(k^2+k'^2)/2m_i^2\}F_L(k, k', m_i), \tag{5}$$

and the scaling strengths are taken in the form

$$V_i^a = A+Bz_i, \tag{6}$$

* This phenomenological value of G is equal, coincidentally, to $2(2M+m_\pi)$.

where A and B are constants. In the 1981 interaction the strength coefficients V_i^a had constant values. In this version the dependence on z_i represents contributions from operators such as $\boldsymbol{k}'.\boldsymbol{k}$ and $\boldsymbol{S}.\boldsymbol{k}'\,\boldsymbol{S}.\boldsymbol{k}$; a specific set of which will eventually be used to characterise the entire V_i^a set. The V_i^a, given in Table 1, are subject to one proviso: a z_i contribution in the S states is assumed to arise from the operator $\boldsymbol{k}'.\boldsymbol{k}$ (tensor terms do not contribute in the $^1S_0$ state and they produce $Q_1(z)$ contributions in the $^3S_1$ state), and for the partial-wave expansion one has

$$\int_{-1}^{+1} [\boldsymbol{k}'.\boldsymbol{k}/\{(\boldsymbol{k}'-\boldsymbol{k})^2+m^2\}]P_L(\mu)\,\mathrm{d}\mu = zQ_L(z)-\delta_{L0}, \tag{7}$$

where $\mu = \hat{\boldsymbol{k}}'.\hat{\boldsymbol{k}}$ and $P_L(\mu)$ is a Legendre function of the first kind. For $L = 0$ the right-hand side of (7) is simply $Q_1(z)$. Hence, for both the S states (and only the S states) the $\{A+Bz\}Q_L(z)$ part of $V_i^a F_L''$ must be replaced by $AQ_0(z)+BQ_1(z)$. This is important because $zQ_0(z) \gg Q_1(z)$.

For the off-diagonal matrix elements $V_{\alpha\alpha'}(k, k')$ ($\alpha = [L = J\pm1, S, J;\ T]$ and $\alpha' = [L' = J\mp1, S, J;\ T]$), which are due to tensor operators and which determine the coupling parameters ϵ_J, $Q_L(z)$ in equation (3) is replaced by $Q_J(z)$, and F_J is replaced by F_J'' for the one pion term, whence

$$V_{\alpha\alpha'}(k, k') = G\{V_\pi F_J''(k, k', m_\pi) + V_{2\pi} F_J''(k, k', 2m_\pi) + V_3 F_J''(k, k', m_3)\}. \tag{8}$$

3. Fit of NN Data

The matrix inversion technique (Haftel and Tabakin 1970) for solving the Lippman–Schwinger equation was used to obtain the R-matrix elements for the interaction, from which the Blatt–Biedenharn phase shifts were obtained and, by transformation, the nuclear bar phase shifts. The resulting nuclear bar phase shifts for our interaction are compared with the np energy-dependent and single-energy analysis of Arndt *et al.* (1983) for the various channels indicated in Fig. 1.

The strengths V_i^a given in Table 1 have been determined to give a good fit to the energy-dependent analysis of Arndt *et al.* using mainly integer numbers. The exception is the $V_{2\pi}$ strengths which are more sensitive to the data. The important difference between these V_i and those of the 1981 interaction is the greatly enhanced repulsion in the $^1S_0$ state ($V_3 = 4$ while previously $V_3 = 1$) and the $^3S_1$ state ($V_3 = 6+6z$ as opposed to $V_3 = 1$). Hence, the repulsion in the $^1S_0$ state is four times, and that in the $^3S_1$ state is approximately six times, that in the 1981 version. The singlet ($S = 0$) V_π values for $L > 0$ are simply $-2\tau_1.\tau_2$, and such a contribution is similar to the central part of the one-pion exchange potential (OPEP) but modulated by the gaussian form factor $\exp(-k^2/m_\pi^2)$. The $^1S_0$ and $^3S_1$ values of V_π (-4 and -6 respectively) are, however, considerably larger than what would be consistent with the OPEP, but are necessary to reproduce the scattering lengths, deuteron properties and the low energy phase shifts. The remaining V_π quite adequately represent the central plus spin–orbit plus tensor contributions for the triplet ($S = 1$) states.

It is difficult to comment on the values of $V_{2\pi}$ because of the many different ways this very necessary 2π contribution has been included in NN interactions. It suffices to say that with this particular functional form and scaling, the values of $V_{2\pi}$ lie mainly in the range $-3 \rightarrow +3$ (note that $z_2 \approx 1+1/k^2$ on the energy shell),

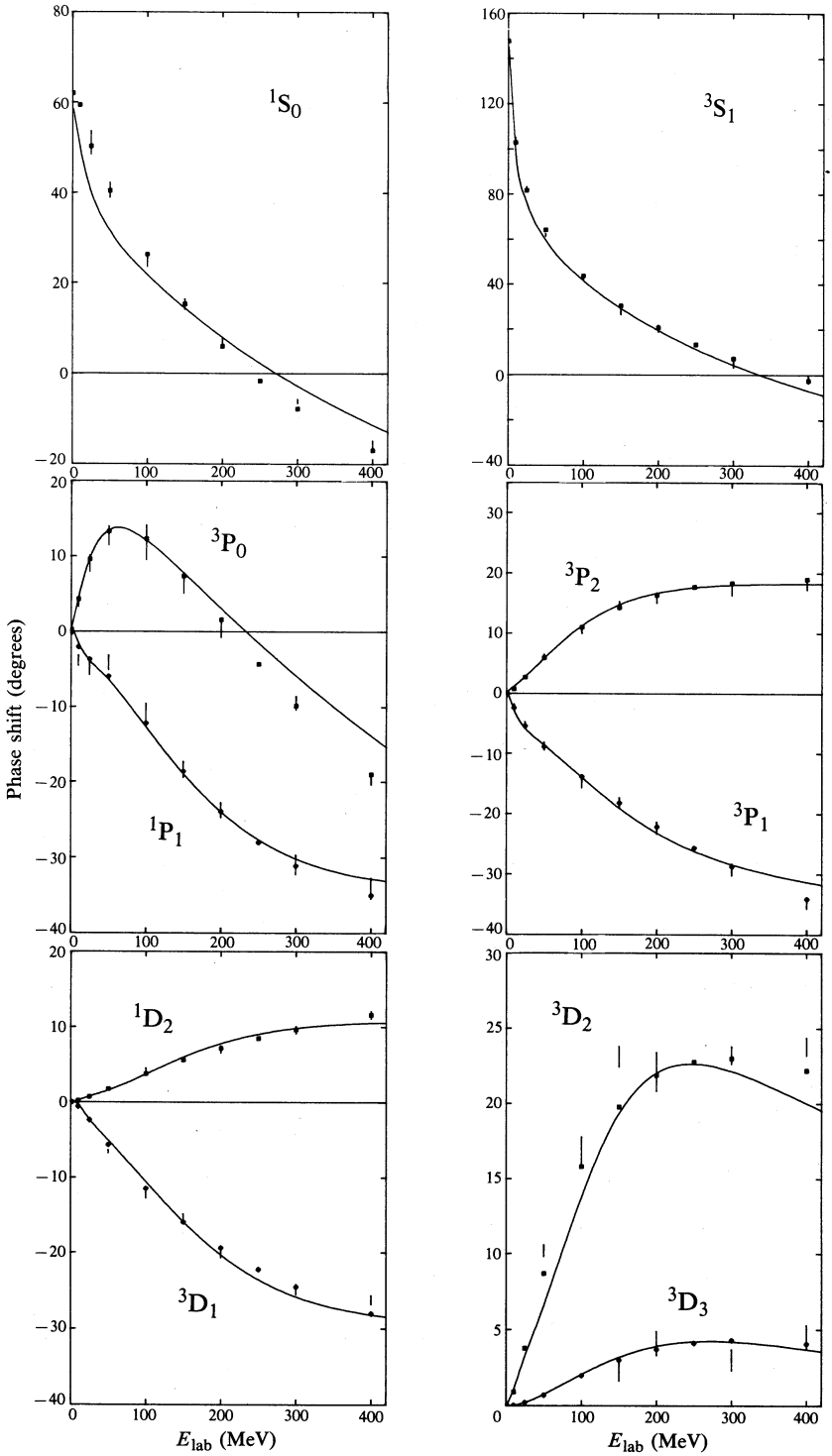

Fig. 1. Curves of the phase shift as a function of lab energy for the present interaction for the various channels indicated. The squares and diamonds denote the np energy-dependent values from Arndt *et al.* (1983), while the vertical bars denote a combined np and pp single-energy analysis by the same authors.

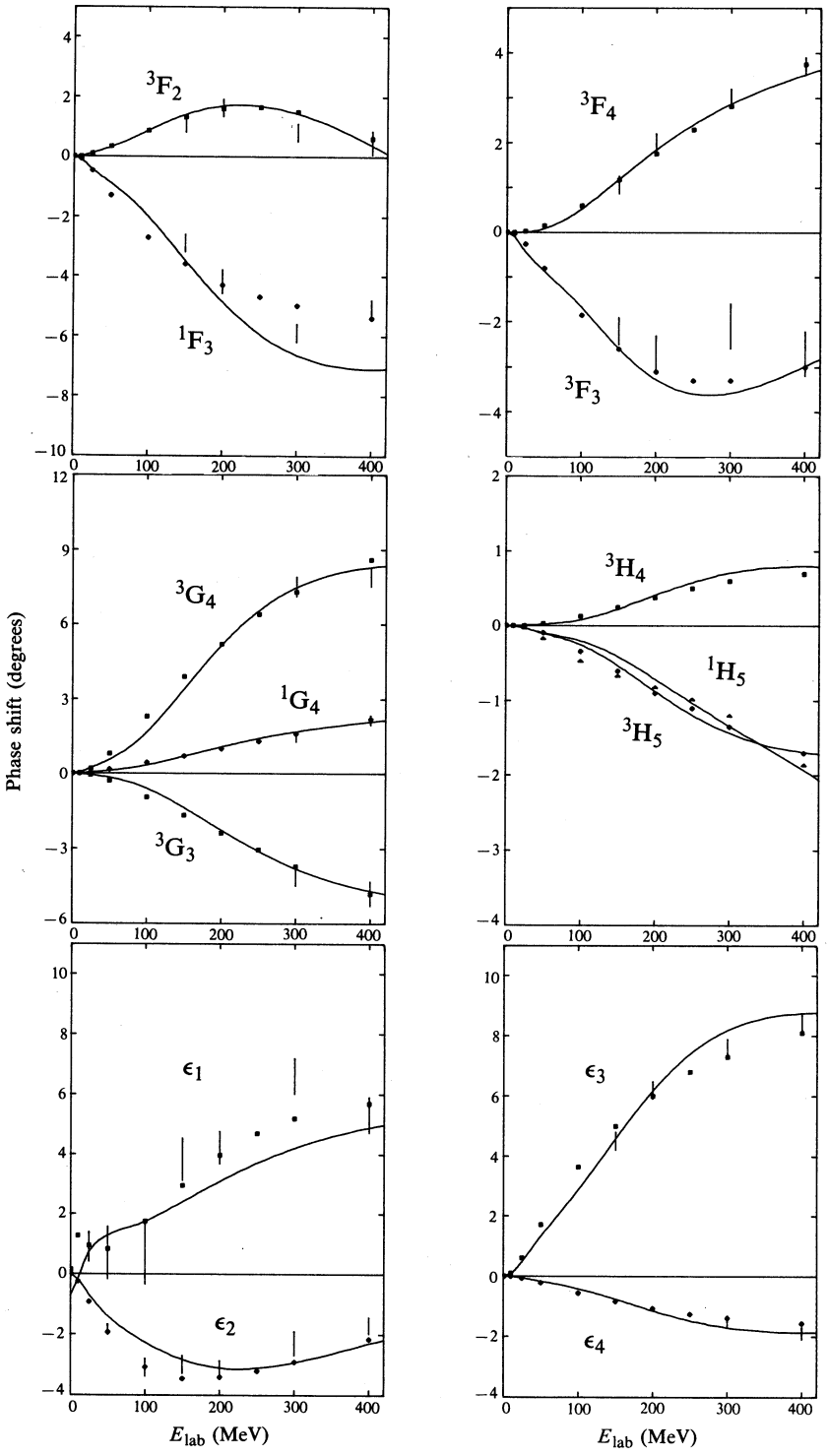

Fig. 1 [*Continued*]

and produce contributions which are ideal for describing the intermediate energy (100–300 MeV) behaviour of the phase shifts.

Although it is generally considered that the fit to the phase shifts need only be good up to 400 MeV (the energy to which the phase shifts are plotted), a comparison has been made up to $E_{\text{lab}} = 1$ GeV for the purpose of getting a better understanding of the (ρ, ω) contribution. Noting that on the energy shell $z_3 \approx 1 + 8/k^2$, so that z_3 has an average value of 2 between 300 and 1000 MeV, this contribution to the phase shifts is not very different from that of the 1981 interaction. The z-dependence of the V_3 appears to blend well with the functional form to describe the phase shifts even above 400 MeV for all except a few states—the unusually behaved $^1D_2$ state and the $^3P_0$ and $^3P_1$ states. A slightly unsatisfactory feature of the (ρ, ω) contribution is this inability to reproduce the strong high-energy repulsion in the $^3P_0$ and $^3P_1$ states. This is due to the off-the-energy-shell character of the functional form, and preliminary calculations indicate that this problem does not arise with a form factor that depends on the square of the momentum transfer $q^2 = (k' - k)^2$. The fact that many of the states have parameters V_3 proportional to $1 - z$ may be further indication of the need for such a dependence since on-shell $q^2 = 2k^2(1 - k' \cdot k) \to 2k^2(1 - z)$. Such a form factor is presently being investigated, but it is felt that even if a q^2 dependence is more appropriate, the present functional form is more tractable for nuclear structure and reaction studies, and thus is, at least, a very appropriate approximation.

Table 2. Low energy results for the np system

Quantity	Exp. value	Interaction	Method of calc.
a_S(np) (fm)	$-23{\cdot}7 \pm 0{\cdot}1$	$-24{\cdot}7$	$R_0(0,0)$
a_T (fm)	$5{\cdot}42 \pm 0{\cdot}03$	$5{\cdot}9$	$R_0(0,0)$
Deuteron ($^3S_1$–$^3D_1$)			
E_D (MeV)	2·2246	2·19	Zero determ.
P_D	4–7%	6·0%	

The scattering lengths a_S and a_T, the deuteron binding energy E_D and the D-state probability P_D are compared with the experimental values in Table 2. All quantities converged to better than 1% using 15 Laguerre points in the matrix inversion calculations, except for E_D for which 32 points were required. In view of the integer values used for the V_i to characterise the S states, the agreement is quite good. It is possible to obtain a much more accurate simultaneous fit to a_T and E_D, and a_S and the low-energy $^1S_0$ phase shifts, by using the off-shell ($k^2 \neq k'^2$) behaviour $k'k/m^2$ instead of $(k^2 + k'^2)/2m^2$ for the z_i terms. This causes only small changes in the S-state potential strengths V_i, and it is felt that until a q^2 dependence is properly investigated, the present fit to these quantities is quite satisfactory.

It is very interesting to note that the value of P_D depends almost entirely on the V_3 in the ϵ_1 row of Table 1. This is required to be z-dependent, and a change in this V_3 from $-4z$ to $-6z$ changes the value of P_D from 3% to 6%, while hardly affecting a_T and E_D.

The deuteron wavefunctions in momentum and coordinate space are presented in Figs 2 and 3 respectively. These bear the strongest resemblance to the deuteron wavefunctions obtained from the Reid hard-core potential, as can be seen from Mathelitsch and Zingl (1978) where the deuteron wavefunctions of the most popular NN interactions are compared.

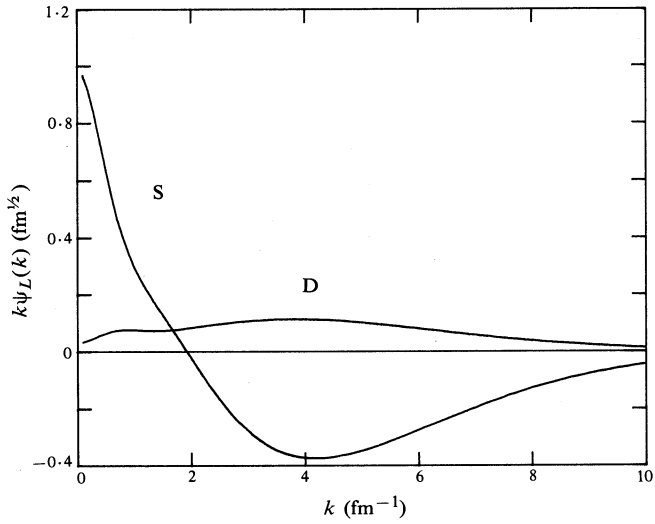

Fig. 2. Deuteron wavefunction in momentum space calculated with the present interaction.

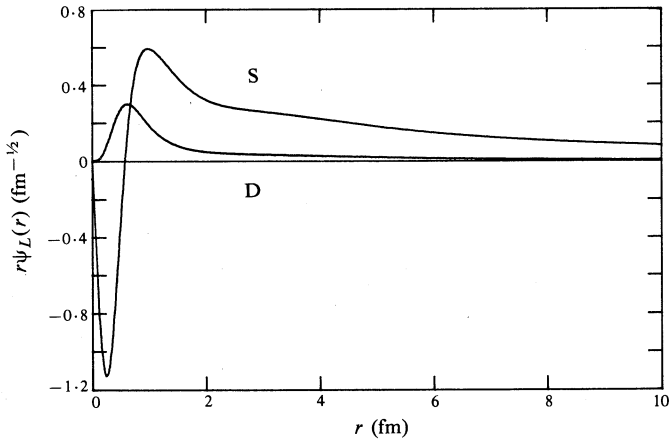

Fig. 3. Deuteron wavefunction in coordinate space calculated with the present interaction.

4. Nuclear Matter

The derivation of the correct nuclear matter energy per particle E/N and saturation density $\rho = 2k_F^3/3\pi^2$, where k_F is the Fermi momentum, is a criterion that a realistic (i.e. fitting the two-body data) NN interaction is expected to satisfy. There are, however, uncertainties in the method of calculating E/N, the conventional one being the Brueckner–Hartree–Fock (BHF), and a satisfactory agreement with the empirical values has not been achieved so far. The explanations put forward for this have included the neglect of higher order diagrams (Day 1981), the need for relativistic corrections (RBHF; see Shakin *et al.* 1980) and approximations made in BHF calculations (see Mahaux *et al.* 1975, 1978; Ma and Kuo 1983*a*, 1983*b*). In this section we present the results of a BHF calculation, making no comment on

Table 3. Nuclear matter results

State	k_F						
	1·2	1·3	1·4	1·5	1·6	1·7	1·8
$^1S_0$	−11·9	−13·8	−15·7	−17·6	−19·5	−21·3	−23·1
$^1P_1$	1·9	2·7	3·7	5·0	6·5	8·4	10·7
$^1D_2$	−1·5	−2·1	−2·8	−3·8	−5·0	−6·4	−8·2
$^1F_3$	0·4	0·6	0·8	1·1	1·5	1·9	2·5
$^1G_4$	−0·2	−0·3	−0·4	−0·6	−0·8	−1·1	−1·5
$^1H_5$	0	0·1	0·1	0·2	0·2	0·3	0·4
$^3S_1$	−21·0	−24·4	−27·9	−31·5	−35·3	−39·1	−42·9
$^3P_0$	−2·4	−3·0	−3·7	−4·4	−5·0	−5·7	−6·3
$^3P_1$	7·6	10·3	13·7	17·8	22·7	28·5	35·2
$^3P_2$	−4·4	−6·0	−7·9	−10·2	−13·1	−16·4	−20·2
$^3D_1$	1·0	1·5	2·2	3·0	4·0	5·2	6·7
$^3D_2$	−1·9	−2·6	−3·4	−4·5	−5·7	−7·2	−9·0
$^3D_3$	−0·3	−0·4	−0·6	−0·8	−1·1	−1·4	−1·8
$^3F_2$	−0·3	−0·4	−0·6	−0·8	−1·1	−1·4	−1·9
$^3F_3$	1·2	1·7	2·2	3·0	3·9	5·0	6·2
$^3F_4$	−0·1	−0·3	−0·4	−0·7	−1·0	−1·5	−2·1
$^3G_3$	0·1	0·1	0·2	0·3	0·5	0·7	0·9
$^3G_4$	−0·3	−0·4	−0·6	−0·9	−1·3	−1·8	−2·4
$^3G_5$	0	0	0	0	0	0	0
$^3H_4$	0	0	−0·1	−0·1	−0·2	−0·3	−0·4
$^3H_5$	0·2	0·3	0·4	0·6	0·8	1·1	1·5
$^3H_6$	0	0	0	0	0	0	0
KE	17·9	21·0	24·4	28·0	31·9	36·0	40·3
E/N (BHF)	−14·0	−15·3	−16·4	−17·0	−17·1	−16·5	−15·3
E/N (RBHF)	−12·5	−12·7	−11·9	−9·7	−5·5	1·4	11·8

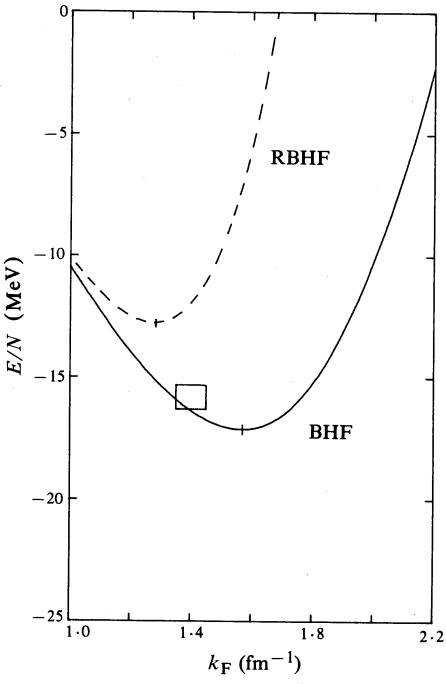

Fig. 4. Energy per particle E/N of nuclear matter as a function of the Fermi momentum k_F. The continuous curve shows the BHF result and the dashed curve the RBHF result. The box represents the area where saturation is expected.

the magnitude of the higher order corrections, and noting that the RBHF correction appears to cancel the effect of the model-space approach (MBHF) of Ma and Kuo; the BHF calculation shows that the present interaction produces saturation close to the expected (E/N, k_F) values.

The matrix inversion technique is again used to solve the Brueckner–Goldstone equation and calculate the G matrix for infinite nuclear matter (Haftel and Tabakin 1970). In the average c.m. momentum approximation, E/N in BHF theory is given by

$$E/N = \tfrac{3}{5}\epsilon_F + \frac{4}{\pi}(\hbar^2/M)\sum_a (2J+1)(2T+1)\int_0^{k_F}(1-\tfrac{3}{2}k/k_F+\tfrac{1}{2}k^3/k_F^3) \times G_{aa}(K_{av};\, k, k)k^2\,\mathrm{d}k, \quad (9)$$

where K_{av} refers to the average value of the c.m. momentum of the two nucleons below the Fermi sea with relative momentum k. To obtain G_{aa}, 10 Laguerre points were used for the matrix inversion and a 10 point Simpson's rule was used to evaluate the integral in equation (9). As found by Haftel and Tabakin, using 15 and 24 Laguerre points changes the results by less than 2%, and increasing the number of Simpson points has almost no effect.

The variation of the partial-wave potential energy contributions, the kinetic energy and E/N with k_F is shown in Table 3. As can be seen in Fig. 4, E/N for BHF has a minimum value consistent with the -16 MeV from the volume term of the semi-empirical mass formula, although this occurs at $k_F = 1{\cdot}56\ \mathrm{fm}^{-1}$ instead of the expected $1{\cdot}36\ \mathrm{fm}^{-1}$. The compressibility of nuclear matter, given at the minimum by

$$\kappa = k_F^2\,\partial^2(E/N)/\partial k_F^2, \quad (10)$$

is 160 MeV for the E/N value of Fig. 4.

In a BHF calculation one simply uses a free particle spectrum for nucleons with $k > k_F$. Such a spectrum has a discontinuity at k_F, and Mahaux *et al.* (1975, 1978) advocated the use of a continuous single-particle spectrum, while more recently Ma and Kuo (1983*a*, 1983*b*) have proposed a better way of defining the latter. But for convenience, and to compare with most previous estimates, a discontinuous spectrum has been used in our calculations, the result of which will be referred to as the BHF result. Nevertheless, we note that Ma and Kuo (1983*b*) showed for the Reid (1968) and Paris (Lacombe *et al.* 1980) potentials, that their model-space approach (MBHF) increases the S-state contribution (mainly the $^3S_1$) by about -3 MeV in the range $k_F = 1{\cdot}2$–$1{\cdot}6\ \mathrm{fm}^{-1}$, and has negligible effect on the other states. This provides an interesting conjecture when one takes into account the other important correction to the nuclear matter result, namely the relativistic one (RBHF) proposed by Shakin *et al.* (1980). This correction has a strong density dependence and is given approximately by

$$\Delta E/N \approx 3{\cdot}6(\rho/\rho_0)^{2{\cdot}4}, \quad (11)$$

where ρ_0 is the density at $k_F = 1{\cdot}36\ \mathrm{fm}^{-1}$. The magnitude of $\Delta E/N$ increases rapidly from 1·5 to 7·3 MeV as k_F increases from 1·2 to $1{\cdot}5\ \mathrm{fm}^{-1}$, and such a contribution may force any interaction to saturate close to the correct k_F.

It is interesting to note that the MBHF and RBHF corrections, if applied to our BHF result, would produce saturation at $k_F = 1{\cdot}3$ fm^{-1} with E/N about -16 MeV. The RBHF result obtained by adding $\Delta E/N$, as given in equation (11), to our BHF result is shown in Fig. 4. It is also interesting to note that, except for the $^3S_1$ and $^3P_1$ states, the potential contributions for the other states are very similar to those obtained by other potentials. If there is any one factor responsible for producing the good value of E/N, then it must be the much larger $^3S_1$ contribution obtained with our interaction.

Since it was not the original intention to demand that nuclear matter be saturated precisely, and in view of the above corrections, let alone the possibility of sizeable higher order corrections, it is not known how much credence should be attached to our result. Of course, it is pleasing to obtain such a result with an interaction determined purely to fit the two-body data.

It should be noted that an ordinary Hartree–Fock (V matrix as opposed to G matrix) calculation yields $E/N > 0$, indicating that the G matrix elements of this interaction should be calculated for nuclear structure calculations. In the 1981 interaction, where the repulsion was underestimated, the HF and BHF results did not differ greatly.

5. Discussion of the Interaction

Two important features of the interaction are to be stressed:

(a) The gaussian form factor has the effect of confining the three contributions in the prediction of phase shifts, to particular energy ranges. The π contribution becomes negligible above 100 MeV. The 2π contribution dominates between 100 and 300 MeV in most states, while the (ρ, ω) contribution which is negligible below 200 MeV, dominates above 400 MeV. This form factor has a far more important effect on the properties of the interaction than the usual Yukawa-type meson–nucleon form factors used by Holinde (1981) and other one-boson exchange potentials (OBEP, e.g. see Erkelenz 1974).

(b) The 2π contribution has no rigorous experimental justification and theoretically its role in the OBEP interactions varies from author to author (see Holinde 1981). Under the guise of the σ meson it plays a varying role in the quark and gluon exchange NN interactions (see e.g. Bakker *et al.* 1982; Faessler 1984), where the long and intermediate range ($r > 0{\cdot}5$ fm) part of the force is still attributed to 'the exchange of color singlet objects which can be identified with physical mesons' (Faessler 1985). The 2π contribution, however, is crucial to this interaction and together with this specific form factor, serves the function of the large collection of mesons used in other potentials.

6. Summary

A phenomenological two-nucleon interaction has been found with attributes in momentum space that not only characterise the deuteron and fit the scattering phase shifts, but also appear to saturate nuclear matter correctly. As far as NN interactions are concerned, this is quite unusual. Because the potential strengths V_i^a are not derived from operators, however, one can justifiably call this a $3n$-parameter interaction, where n is the number of states that have been fitted. For this reason,

until the V_i^a can be derived from a specific operator set, and the dependence of the form factor on the square of the momentum transfer is investigated, this remains as a phenomenological potential. But it is one which is most convenient for use in finite nucleus structure and reaction studies. Such studies are in progress.

Acknowledgments

The authors wish to thank Dr K. Amos for many helpful discussions and a critical reading of this manuscript, and the Physics Department at Melbourne University for making available its facilities.

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Manuscript received 2 October 1985, accepted 28 February 1986