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Abnormal Parity Solutions of the Wess–Zumino Consistency Conditions for Chiral SU(L)×SU(L) Symmetry

G. A. Christos

Department of Physics, University of Western Australia,
Nedlands, W.A. 6009.

Abstract

We obtain the general abnormal parity solutions of the Wess–Zumino consistency conditions for chiral SU(L)×SU(L) symmetry. We find an additional non-trivial solution which cannot be obtained from the usual Bardeen non-abelian anomaly through the addition of a local Lagrangian counter-term.

1. Introduction

In general a quantum field theory with fermions contains so-called 'anomalies' which arise from renormalisation ambiguities in certain fermion loop diagrams (Adler 1969; Bell and Jackiw 1969). These anomalies have direct physical relevance, for example $\pi^0 \to \gamma\gamma$ decay, resolution of the U(1) problem, anomaly bound-state matching conditions, quark–lepton duality, Skyrme soliton quantisation, magnetic monopole induced baryon-number decay, and the limitations on possible grand unified, supersymmetric and superstring theories.

In a properly regularised theory these anomalies satisfy consistency conditions (Wess and Zumino 1971) obtained by applying the 'gauge' generator commutators to the vacuum generating functional (see below). These consistency conditions have given anomalies a topological underpinning through the mathematical connection with cohomology theory.

It is widely believed that because of their non-linearity, the solution to the Wess–Zumino (WZ) consistency conditions is unique up to an overall normalisation constant and possible Lagrangian counter-terms which correspond to alternative renormalisations of the fermion loop diagrams. We show here that this is not generally true by finding the explicit solution of the WZ consistency conditions for chiral SU(L)×SU(L) symmetry.

2. WZ Consistency Conditions

We consider the connected generating functional

$$\exp(\mathrm{i}\, W[V,A]) = \int \mathscr{D}q\, \mathscr{D}\bar{q}\, \mathscr{D}G_\mu \times \exp\left(\mathrm{i}\int_x [\mathscr{L}(x) + \bar{q}(x)\gamma_\mu\{V^\mu(x) + \gamma_5 A^\mu(x)\}\, q(x)]\right), \quad (1)$$

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where $\mathscr{L}$ is the usual QCD Lagrangian density with L massless quarks, the quark field q is a colour triplet and flavour L-vector, G_μ are the gluon octet gauge fields, and V_μ and A_μ are $L\times L$ traceless flavour matrix source fields.

Performing a *change of variables* in the functional integral (1), which corresponds to a *local* $\mathrm{SU}(L)\times\mathrm{SU}(L)$ chiral transformation

$$q(x) \rightarrow \{1 + \mathrm{i}\,\alpha(x) + \mathrm{i}\,\beta(x)\gamma_5\}\, q(x), \tag{2}$$

where α and β are space–time dependent $L\times L$ traceless flavour matrices ($\alpha = \alpha_i \frac{1}{2}\lambda^i$), gives

$$\exp(\mathrm{i}\, W[V, A]) = \exp\left(-\mathrm{i} \int_x \{\beta_i(x)\, G_i(x) + \alpha_i(x)\, F_i(x)\}\right) \exp(\mathrm{i}\, W[V', A']), \tag{3}$$

where

$$V'_\mu = V_\mu - \mathrm{i}[\alpha, V_\mu] - \mathrm{i}[\beta, A_\mu] - (\partial_\mu \alpha),$$

$$A'_\mu = A_\mu - \mathrm{i}[\alpha, A_\mu] - \mathrm{i}[\beta, V_\mu] - (\partial_\mu \beta).$$

The non-abelian anomalies are contained in the first exponential term on the RHS of (3) which corresponds to the Jacobean for the transformation (2). The anomaly functionals $G_i = G_i[V, A]$ and $F_i = F_i[V, A]$ can be determined in principle (Balachandran *et al.* 1982; Einhorn and Jones 1984) but are left as unspecified here since we seek solutions of the WZ consistency conditions for these functionals below. The usual non-abelian anomalies can be obtained from (3) by functional differentiation with respect to α, β, V_μ and A_μ: for example, the AAA anomaly is specified by the A^2 term in G_i.

More generally one can extend (2) to $\mathrm{U}(L)\times\mathrm{U}(L)$ chiral transformations with α, β, V_μ and A_μ not traceless. This leads to an additional class of current–current anomalies (Christos, in preparation) as well as the usual U(1) axial gluonic anomaly (Fujikawa 1979; Christos 1983). Our discussion is however limited to the non-abelian $\mathrm{SU}(L)\times\mathrm{SU}(L)$ anomalies.

For small α and β the connected vacuum functional $W[V', A']$ can be written as

$$W[V', A'] = W[V, A] + \int_x \alpha_i(x)\, X_i(x)\, W[V, A]$$

$$+ \int_x \beta_i(x)\, Y_i(x)\, W[V, A] + \mathrm{O}(\alpha^2, \beta^2, \alpha . \beta), \tag{4}$$

where

$$X_i(x) = -f_{ilm}\, V^l_\omega(x) \frac{\delta}{\delta V^m_\omega(x)} - f_{ilm}\, A^l_\omega(x) \frac{\delta}{\delta A^m_\omega(x)} - \partial^x_\mu \frac{\delta}{\delta V^i_\omega(x)}, \tag{5a}$$

$$Y_i(x) = -f_{ilm}\, V^l_\omega(x) \frac{\delta}{\delta A^m_\omega(x)} - f_{ilm}\, A^l_\omega(x) \frac{\delta}{\delta V^m_\omega(x)} - \partial^x_\mu \frac{\delta}{\delta A^i_\omega(x)}. \tag{5b}$$

Substituting (4) into (3) and functionally differentiating with respect to α and β gives

$$X_i\, W[V, A] = F_i[V, A], \qquad Y_i\, W[V, A] = G_i[V, A]. \tag{6a, b}$$

The infinitesimal 'gauge' operators X_i and Y_i can be shown to satisfy the commutation relations of the underlying group (Wess and Zumino 1971), here SU(L)×SU(L),

$$[X_i(x), X_j(x')] = f_{ijk}\, \delta^4(x-x')X_k(x), \tag{7a}$$

$$[Y_i(x), Y_j(x')] = f_{ijk}\, \delta^4(x-x')X_k(x), \tag{7b}$$

$$[X_i(x), Y_j(x')] = f_{ijk}\, \delta^4(x-x')\, Y_k(x). \tag{7c}$$

Applying these relations to the connected vacuum functional we derive the WZ consistency conditions

$$X_i(x)\, F_j(x') - X_j(x')\, F_i(x) = f_{ijk}\, \delta^4(x-x')\, F_k(x), \tag{8a}$$

$$Y_i(x)\, G_j(x') - Y_j(x')\, G_i(x) = f_{ijk}\, \delta^4(x-x')\, F_k(x), \tag{8b}$$

$$X_i(x)\, G_j(x') - Y_j(x')\, F_i(x) = f_{ijk}\, \delta^4(x-x')\, G_k(x). \tag{8c}$$

3. Solution

From equations (6), (5) and (1) it follows that

$$F_i = \partial^\mu J^i_\mu + \dots, \qquad G_i = \partial^\mu J^i_{\mu 5} + \dots, \tag{9a, b}$$

where $J^i_\mu = \bar{q}\gamma_\mu \frac{1}{2}\lambda^i q$ and $J^i_{\mu 5} = \bar{q}\gamma_\mu \gamma_5 \frac{1}{2}\lambda^i q$ are the quark vector and axial-vector currents respectively. Therefore, under parity and charge conjugation (see also equation 1)

$$\begin{aligned} F_i &\xrightarrow{\mathrm{P}} F_i \\ &\xrightarrow{\mathrm{C}} -F_i \qquad \text{(with } \lambda_i \to \lambda_i^{\mathrm{T}}), \end{aligned} \tag{10a}$$

$$\begin{aligned} G_i &\xrightarrow{\mathrm{P}} -G_i \\ &\xrightarrow{\mathrm{C}} G_i \qquad \text{(with } \lambda_i \to \lambda_i^{\mathrm{T}}), \end{aligned} \tag{10b}$$

$$\begin{aligned} V_\mu &\xrightarrow{\mathrm{P}} (V^0, -V) \\ &\xrightarrow{\mathrm{C}} -V_\mu^{\mathrm{T}}, \end{aligned} \tag{10c}$$

$$\begin{aligned} A_\mu &\xrightarrow{\mathrm{P}} (-A^0, A) \\ &\xrightarrow{\mathrm{C}} A_\mu^{\mathrm{T}}. \end{aligned} \tag{10d}$$

From the parity transformation properties it follows that the abnormal parity terms (those proportional to $\epsilon_{\mu\nu\alpha\beta}$) of F and G are respectively odd and even in A, with the reverse for the normal parity terms. These conditions together with the fact that F and G must have dimension 4 severely restricts the form of these functionals. The most general expressions for the abnormal parity* pieces of F and G are given by

$$
\begin{aligned}
G_i = \epsilon^{\mu\nu\sigma\tau}\mathrm{Tr}[\tfrac{1}{2}\lambda^i\{ & g_1(\partial_\mu V_\nu)(\partial_\sigma V_\tau)+g_2(\partial_\mu A_\nu)(\partial_\sigma A_\tau) \\
& +g_3(\partial_\mu V_\nu)V_\sigma V_\tau+g_3 V_\mu V_\nu(\partial_\sigma V_\tau)+g_4 V_\mu(\partial_\nu V_\sigma)V_\tau+g_5(\partial_\mu V_\nu)A_\sigma A_\tau \\
& +g_5 A_\mu A_\nu(\partial_\sigma V_\tau)+g_6 A_\mu(\partial_\nu V_\sigma)A_\tau+g_7(\partial_\mu A_\nu)A_\sigma V_\tau+g_7 V_\mu A_\nu(\partial_\sigma A_\tau) \\
& +g_8(\partial_\mu A_\nu)V_\sigma A_\tau+g_8 A_\mu V_\nu(\partial_\sigma A_\tau)+g_9 V_\mu(\partial_\nu A_\sigma)A_\tau+g_9 A_\mu(\partial_\nu A_\sigma)V_\tau \\
& +g_{10} V_\mu V_\nu V_\sigma V_\tau+g_{11} V_\mu V_\nu A_\sigma A_\tau+g_{11} A_\mu A_\nu V_\sigma V_\tau+g_{12} V_\mu A_\nu V_\sigma A_\tau \\
& +g_{12} A_\mu V_\nu A_\sigma V_\tau+g_{13} A_\mu V_\nu V_\sigma A_\tau+g_{14} V_\mu A_\nu A_\sigma V_\tau+g_{15} A_\mu A_\nu A_\sigma A_\tau\}] \\
& +g_{16}\,\epsilon^{\mu\nu\sigma\tau}\mathrm{Tr}[\tfrac{1}{2}\lambda^i\{V_\mu A_\nu-A_\mu V_\nu\}]\mathrm{Tr}[V_\sigma A_\tau],
\end{aligned}
\tag{11a}
$$

$$
\begin{aligned}
F_i = \epsilon^{\mu\nu\sigma\tau}\mathrm{Tr}[\tfrac{1}{2}\lambda^i\{ & f_1(\partial_\mu V_\nu)(\partial_\sigma A_\tau)+f_1(\partial_\mu A_\nu)(\partial_\sigma V_\tau) \\
& +f_2(\partial_\mu V_\nu)A_\sigma V_\tau+f_2 V_\mu A_\nu(\partial_\sigma V_\tau)+f_3(\partial_\mu V_\nu)V_\sigma A_\tau+f_3 A_\mu V_\nu(\partial_\sigma V_\tau) \\
& +f_4 V_\mu(\partial_\nu V_\sigma)A_\tau+f_4 A_\mu(\partial_\nu V_\sigma)V_\tau+f_5(\partial_\mu A_\nu)V_\sigma V_\tau+f_5 V_\mu V_\nu(\partial_\sigma A_\tau) \\
& +f_6 V_\mu(\partial_\nu A_\sigma)V_\tau+f_7(\partial_\mu A_\nu)A_\sigma A_\tau+f_7 A_\mu A_\nu(\partial_\sigma A_\tau)+f_8 A_\mu(\partial_\nu A_\sigma)A_\tau \\
& +f_9 A_\mu V_\nu V_\sigma V_\tau+f_9 V_\mu V_\nu V_\sigma A_\tau+f_{10} V_\mu A_\nu V_\sigma V_\tau+f_{10} V_\mu V_\nu A_\sigma V_\tau \\
& +f_{11} V_\mu A_\nu A_\sigma A_\tau+f_{11} A_\mu A_\nu A_\sigma V_\tau+f_{12} A_\mu V_\nu A_\sigma A_\tau+f_{12} A_\mu A_\nu V_\sigma A_\tau\}].
\end{aligned}
\tag{11b}
$$

Inserting these expressions for F and G into the WZ consistency conditions (8) and equating coefficients of similar terms leads to a set of equations relating the coefficients f and g. (In arriving at these equations it proves useful to re-express the SU(L) f_{ijk} symbol in terms of a λ commutator by $f_{ijk}\lambda_k = -\frac{1}{2}\mathrm{i}[\lambda_i, \lambda_j]$. Also in some cases one obtains different equations for the coefficients depending on whether one integrates over x or x'.) Solving these equations in terms of a minimal set of

* The normal parity pieces are uninteresting because it is generally considered that they can be eliminated by Lagrangian counter-terms (Bardeen 1969). The case for $F_i = 0$ was considered by Xiong and Zhu (1985). In the present case ($F_i \neq 0$) the calculation becomes intractable with their inclusion, involving hundreds of additional parameters.

parameters leads to the general solution of the WZ consistency conditions

$$
\begin{aligned}
G_i = \epsilon^{\mu\nu\sigma\tau}\mathrm{Tr}[\tfrac{1}{2}\lambda^i\{ & (g_2 - \mathrm{i}\, g_5 + \mathrm{i}\, g_7)(\partial_\mu V_\nu)(\partial_\sigma V_\tau) + g_2(\partial_\mu A_\nu)(\partial_\sigma A_\tau) \\
& + (-g_4 - \tfrac{3}{2} g_5 + \tfrac{3}{2} g_7)(\partial_\mu V_\nu) V_\sigma V_\tau + (-g_4 - \tfrac{3}{2} g_5 + \tfrac{3}{2} g_7) V_\mu V_\nu (\partial_\sigma V_\tau) \\
& + g_4 V_\mu (\partial_\nu V_\sigma) V_\tau + g_5 (\partial_\mu V_\nu) A_\sigma A_\tau + g_5 A_\mu A_\nu (\partial_\sigma V_\tau) \\
& + (\tfrac{3}{2} g_5 - \tfrac{3}{2} g_7 - g_8) A_\mu (\partial_\nu V_\sigma) A_\tau + g_7 (\partial_\mu A_\nu) A_\sigma V_\tau \\
& + g_7 V_\mu A_\nu (\partial_\sigma A_\tau) + g_8 (\partial_\mu A_\nu) V_\sigma A_\tau + g_8 A_\mu V_\nu (\partial_\sigma A_\tau) \\
& - \tfrac{1}{2}(g_5 + g_7) V_\mu (\partial_\nu A_\sigma) A_\tau - \tfrac{1}{2}(g_5 + g_7) A_\mu (\partial_\nu A_\sigma) V_\tau \\
& + \mathrm{i}(2 g_4 + \tfrac{5}{2} g_5 - \tfrac{3}{2} g_7 + g_8) V_\mu V_\nu V_\sigma V_\tau \\
& - \mathrm{i}(g_4 + g_5) V_\mu V_\nu A_\sigma A_\tau - \mathrm{i}(g_4 + g_5) A_\mu A_\nu V_\sigma V_\tau \\
& + \mathrm{i}(g_4 + g_5) V_\mu A_\nu V_\sigma A_\tau + \mathrm{i}(g_4 + g_5) A_\mu V_\nu A_\sigma V_\tau \\
& - \mathrm{i}(2 g_4 + \tfrac{5}{2} g_5 - \tfrac{3}{2} g_7 + g_8) A_\mu V_\nu V_\sigma A_\tau \\
& + \mathrm{i}(\tfrac{1}{2} g_5 - \tfrac{3}{2} g_7 + g_8) V_\mu A_\nu A_\sigma V_\tau \\
& - \mathrm{i}(\tfrac{1}{2} g_5 - \tfrac{3}{2} g_7 + g_8) A_\mu A_\nu A_\sigma A_\tau \}],
\end{aligned}
\tag{12a}
$$

$$
\begin{aligned}
F_i = \epsilon^{\mu\nu\sigma\tau}\mathrm{Tr}[\tfrac{1}{2}\lambda^i\{ & (g_2 + \tfrac{1}{2}\mathrm{i}\, g_5 - \tfrac{1}{2}\mathrm{i}\, g_7)(\partial_\mu V_\nu)(\partial_\sigma A_\tau) \\
& + (g_2 + \tfrac{1}{2}\mathrm{i}\, g_5 - \tfrac{1}{2}\mathrm{i}\, g_7)(\partial_\mu A_\nu)(\partial_\sigma V_\tau) - g_4 (\partial_\mu V_\nu) A_\sigma V_\tau - g_4 V_\mu A_\nu (\partial_\sigma V_\tau) \\
& + (g_4 + g_5 + g_8)(\partial_\mu V_\nu) V_\sigma A_\tau + (g_4 + g_5 + g_8) A_\mu V_\nu (\partial_\sigma V_\tau) \\
& - (g_4 + g_5 + g_8) V_\mu (\partial_\nu V_\sigma) A_\tau - (g_4 + g_5 + g_8) A_\mu (\partial_\nu V_\sigma) V_\tau \\
& + (g_4 + g_5 + g_8)(\partial_\mu A_\nu) V_\sigma V_\tau + (g_4 + g_5 + g_8) V_\mu V_\nu (\partial_\sigma A_\tau) \\
& + g_4 V_\mu (\partial_\nu A_\sigma) V_\tau + (\tfrac{1}{2} g_5 - \tfrac{1}{2} g_7 + g_8)(\partial_\mu A_\nu) A_\sigma A_\tau \\
& + (\tfrac{1}{2} g_5 - \tfrac{1}{2} g_7 + g_8) A_\mu A_\nu (\partial_\sigma A_\tau) \\
& - (\tfrac{1}{2} g_5 - \tfrac{1}{2} g_7 + g_8) A_\mu (\partial_\nu A_\sigma) A_\tau \}].
\end{aligned}
\tag{12b}
$$

For reasons that will become clear below we define $\tilde{g}_2$, c_1, c_2 and c_3 by

$$
g_2 = \tfrac{1}{2}\mathrm{i}\, g_5 + \tfrac{1}{2}\mathrm{i}\, g_7 + \mathrm{i}\, g_8 + \tilde{g}_2, \qquad g_4 = -3\mathrm{i}\, g_2(c_2 + 2\mathrm{i}\, c_1),
$$

$$
g_5 = \mathrm{i}\, g_2(1 + 3c_3 + 6\mathrm{i}\, c_1), \qquad g_7 = \mathrm{i}\, g_2(3c_3 - 1).
$$

We can then rewrite our solutions to the WZ consistency conditions as

$$
\begin{aligned}
G_i = (-3\mathrm{i}\, g_2)\epsilon^{\mu\nu\sigma\tau}\mathrm{Tr}[\tfrac{1}{2}\lambda^i\{&(-2c_1+\mathrm{i})(\partial_\mu V_\nu)(\partial_\sigma V_\tau)\\
&+\tfrac{1}{3}\mathrm{i}(\partial_\mu A_\nu)(\partial_\sigma A_\tau)+(-c_2+\mathrm{i}\, c_1+1)(\partial_\mu V_\nu)V_\sigma V_\tau\\
&+(-c_2+\mathrm{i}\, c_1+1)V_\mu V_\nu(\partial_\sigma V_\tau)+(c_2+2\mathrm{i}\, c_1)V_\mu(\partial_\nu V_\sigma)V_\tau\\
&-(c_3+2\mathrm{i}\, c_1+\tfrac{1}{3})(\partial_\mu V_\nu)A_\sigma A_\tau-(c_3+2\mathrm{i}\, c_1+\tfrac{1}{3})A_\mu A_\nu(\partial_\sigma V_\tau)\\
&+(c_3-4\mathrm{i}\, c_1-\tfrac{4}{3})A_\mu(\partial_\nu V_\sigma)A_\tau-(c_3-\tfrac{1}{3})(\partial_\mu A_\nu)A_\sigma V_\tau\\
&-(c_3-\tfrac{1}{3})V_\mu A_\nu(\partial_\sigma A_\tau)+(c_3+\mathrm{i}\, c_1+\tfrac{1}{3})(\partial_\mu A_\nu)V_\sigma A_\tau\\
&+(c_3+\mathrm{i}\, c_1+\tfrac{1}{3})A_\mu V_\nu(\partial_\sigma A_\tau)+(c_3+\mathrm{i}\, c_1)V_\mu(\partial_\nu A_\sigma)A_\tau\\
&+(c_3+\mathrm{i}\, c_1)A_\mu(\partial_\nu A_\sigma)V_\tau+\mathrm{i}(2c_2-1)V_\mu V_\nu V_\sigma V_\tau\\
&+\mathrm{i}(c_3-c_2+\tfrac{1}{3})V_\mu V_\nu A_\sigma A_\tau+\mathrm{i}(c_3-c_2+\tfrac{1}{3})A_\mu A_\nu V_\sigma V_\tau\\
&-\mathrm{i}(c_3-c_2+\tfrac{1}{3})V_\mu A_\nu V_\sigma A_\tau-\mathrm{i}(c_3-c_2+\tfrac{1}{3})A_\mu V_\nu A_\sigma V_\tau\\
&+\mathrm{i}(2c_3-\tfrac{1}{3})V_\mu A_\nu A_\sigma V_\tau-\mathrm{i}(2c_2-1)A_\mu V_\nu V_\sigma A_\tau\\
&-\mathrm{i}(2c_3-\tfrac{1}{3})A_\mu A_\nu A_\sigma A_\tau\}]\\
&+\tilde{g}_2\,\epsilon^{\mu\nu\sigma\tau}\mathrm{Tr}[\tfrac{1}{2}\lambda^i\{(\partial_\mu V_\nu)(\partial_\sigma V_\tau)+(\partial_\mu A_\nu)(\partial_\sigma A_\tau)\}],
\end{aligned}
\tag{13a}
$$

$$
\begin{aligned}
F_i = (-3\mathrm{i}\, g_2)\epsilon^{\mu\nu\sigma\tau}\mathrm{Tr}[\tfrac{1}{2}\lambda^i\{&c_1(\partial_\mu V_\nu)(\partial_\sigma A_\tau)+c_1(\partial_\mu A_\nu)(\partial_\sigma V_\tau)\\
&-(c_2+2\mathrm{i}\, c_1)(\partial_\mu V_\nu)A_\sigma V_\tau-(c_2+2\mathrm{i}\, c_1)V_\mu A_\nu(\partial_\sigma V_\tau)\\
&+(c_2+\mathrm{i}\, c_1)(\partial_\mu V_\nu)V_\sigma A_\tau+(c_2+\mathrm{i}\, c_1)A_\mu V_\nu(\partial_\sigma V_\tau)\\
&-(c_2+\mathrm{i}\, c_1)V_\mu(\partial_\nu V_\sigma)A_\tau-(c_2+\mathrm{i}\, c_1)A_\mu(\partial_\nu V_\sigma)V_\tau\\
&+(c_2+\mathrm{i}\, c_1)(\partial_\mu A_\nu)V_\sigma V_\tau+(c_2+\mathrm{i}\, c_1)V_\mu V_\nu(\partial_\sigma A_\tau)\\
&+(c_2+2\mathrm{i}\, c_1)V_\mu(\partial_\nu A_\sigma)V_\tau+c_3(\partial_\mu A_\nu)A_\sigma A_\tau\\
&+c_3 A_\mu A_\nu(\partial_\sigma A_\tau)-c_3 A_\mu(\partial_\nu A_\sigma)A_\tau\}]\\
&+\tilde{g}_2\,\epsilon^{\mu\nu\sigma\tau}\mathrm{Tr}[\tfrac{1}{2}\lambda^i\{(\partial_\mu V_\nu)(\partial_\sigma A_\tau)+(\partial_\mu A_\nu)(\partial_\sigma V_\tau)\}].
\end{aligned}
\tag{13b}
$$

We now aim to show that up to an overall multiplicative constant (which cannot be determined from the commutator based WZ consistency conditions), the solution above with $\tilde{g}_2 = 0$ can be obtained from the usual Bardeen non-abelian anomaly by adding a local Lagrangian counter-term. The solution proportional to $\tilde{g}_2$ cannot be obtained in this way.

There is a certain arbitrariness in the specification of the anomaly which is associated with renormalisation. One is free to redefine the generating functional $W[V, A]$ through the addition of a local Lagrangian counter-term depending only on the external source fields V_μ and A_μ (Bardeen 1969; Gasser and Leutwyler 1984). If we set

$$W[V,A] \rightarrow W[V,A] + \int_x \Delta\mathscr{L}(V,A;\, x), \tag{14}$$

we obtain in place of (3)

$$W[V,A] = W[V',A'] + \int_x \{\Delta\mathscr{L}(V',A';\, x) - \Delta\mathscr{L}(V,A;\, x)\}$$
$$- \int_x \{\beta_i(x)\, G_i(x) + \alpha_i(x)\, F_i(x)\}\,. \tag{15}$$

Note that in this redefinition we are supposed to treat the new generating functional as if it generated the same Green's functions as in (1). Expanding $\Delta\mathscr{L}(V', A') - \Delta\mathscr{L}(V, A)$ in lowest order in α and β (see equation 4) leads to a new form of the anomaly specified by the functionals

$$\tilde{G}_i(y) = G_i(y) - Y_i(y) \int_x \Delta\mathscr{L}(V,A;\, x), \tag{16a}$$

$$\tilde{F}_i(y) = F_i(y) - X_i(y) \int_x \Delta\mathscr{L}(V,A;\, x). \tag{16b}$$

The most general Lagrangian counter-term which is consistent with parity and charge conjugation invariance is

$$\int_x \Delta\mathscr{L}(V,A) = \int_x \epsilon^{\mu\nu\sigma\tau}\mathrm{Tr}[c_1(\partial_\mu V_\nu)(V_\sigma A_\tau - A_\sigma V_\tau)$$
$$+ c_2 V_\mu V_\nu V_\sigma A_\tau + c_3 V_\mu A_\nu A_\sigma A_\tau]\,. \tag{17}$$

These counter-terms have a direct association with the renormalisation prescription ambiguities in the *VVA*, *VVVA* and *VAAA* triangle and square diagrams. Adding the effect of this counter-term to the usual Bardeen form of the non-abelian anomaly

$$F_i = 0\,,$$

$$G_i = -\frac{1}{4\pi^2}\epsilon^{\mu\nu\sigma\tau}\mathrm{Tr}[\tfrac{1}{2}\lambda^i\{\tfrac{1}{4} V_{\mu\nu} V_{\sigma\tau} + \tfrac{1}{12} A_{\mu\nu} A_{\sigma\tau}$$
$$+ \tfrac{2}{3}\mathrm{i}(A_\mu A_\nu V_{\sigma\tau} + A_\mu V_{\nu\sigma} A_\tau + V_{\mu\nu} A_\sigma A_\tau) - \tfrac{8}{3} A_\mu A_\nu A_\sigma A_\tau\}]\,,$$

where

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - \mathrm{i}[V_\mu, V_\nu] - \mathrm{i}[A_\mu, A_\nu],$$

$$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}[V_\mu, A_\nu] - \mathrm{i}[A_\mu, V_\nu],$$

leads to a new form of the anomaly which corresponds to (13) with $\tilde{g}_2 = 0$ and the overall normalisation $g_2 = -1/12\pi^2$. Consequently the WZ consistency conditions do *not* uniquely determine the form of the non-abelian anomaly up to an overall normalisation constant.

In passing we note that the Bardeen anomaly cannot be eliminated by a Lagrangian counter-term. This is immediately apparent since the counter-term (17) cannot generate an $\epsilon\,\mathrm{Tr}[\lambda(\partial A)(\partial A)]$ term in G.

The solution of the WZ consistency conditions (8) with $F_i = 0$ is (uniquely) the Bardeen anomaly up to an overall multiplicative constant (see also Wess and Zumino 1971; Gottlieb and Marculescu 1972; Xiong and Zhu 1985). This however presupposes that the theory can be renormalised so that $F_i = 0$.

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