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# The First Excited State of $^9$B

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*Abstract*

The energy of the $\frac{1}{2}^+$ first excited state of $^9$B is derived from the measured energy of the analogue state in $^9$Be together with calculated values of the Coulomb displacement energy. The latter include contributions from the internal Coulomb interaction, the electromagnetic spin–orbit interaction and the different external wavefunctions in $^9$Be and $^9$B. The $\frac{1}{2}^+$ excitation energy is predicted to be greater in $^9$B than in $^9$Be, arising from an inverted Thomas–Ehrman shift, due to the $^9$Be state being above the $^8$Be(g.s.) + s-wave neutron threshold. This result is in conflict with a recently published calculation (Sherr and Bertsch 1985), which was based on a single-particle potential model.

## 1. Introduction

The most recent compilation of energy levels for $A = 9$ nuclei (Ajzenberg-Selove 1984) gives the low-lying levels as shown in Fig. 1. The first excited state of $^9$Be is a $\frac{1}{2}^+$ level at 1·685 MeV, just above the threshold for breakup into $^8$Be+n. The analogue state in the mirror nucleus $^9$B has not been identified with certainty. The tentative broad level shown at 1·6 MeV, with a width of about 0·7 MeV, is based on evidence from the $^{10}$B($^3$He, $\alpha$)$^9$B reaction obtained by Kroepfl and Browne (1968). In a more recent study of the $^9$Be($^3$He, t)$^9$B reaction with 90 MeV $^3$He, which is *a priori* more likely to populate the $\frac{1}{2}^+$ level, the results were interpreted in terms of a level with a peak energy of 1·65 MeV and a width of about 1 MeV (Djaloeis *et al.* 1983*a*, 1983*b*). Difficulty in observing the $\frac{1}{2}^+$ level in $^9$B can be attributed to its expected large width, weak production in most reactions, and competition from alternative reaction sequences.

Recently Sherr and Bertsch (1985) calculated the location and shape of the $\frac{1}{2}^+$ level in $^9$B, using a single-particle potential model. They suggested that the level should appear as a broad continuum with a peak energy at about 0·9 MeV and a width of 1·4 MeV. As evidence for the validity of the potential model, they gave results for pairs of analogue states in mirror nuclei with $A = 11$, 13, 15 and 17, finding reasonable agreement with experiment for both energies and widths. In many cases these levels are bound so that there is no ambiguity in the definition of the energy of the level. But for unbound levels, different definitions are possible, and the definition used by Sherr and Bertsch for the $\frac{1}{2}^+$, $A = 9$ levels is different from those that they used in the test cases. They concluded that the much lower excitation energy predicted for the $\frac{1}{2}^+$ level in $^9$B, as compared with that in $^9$Be, is a manifestation of

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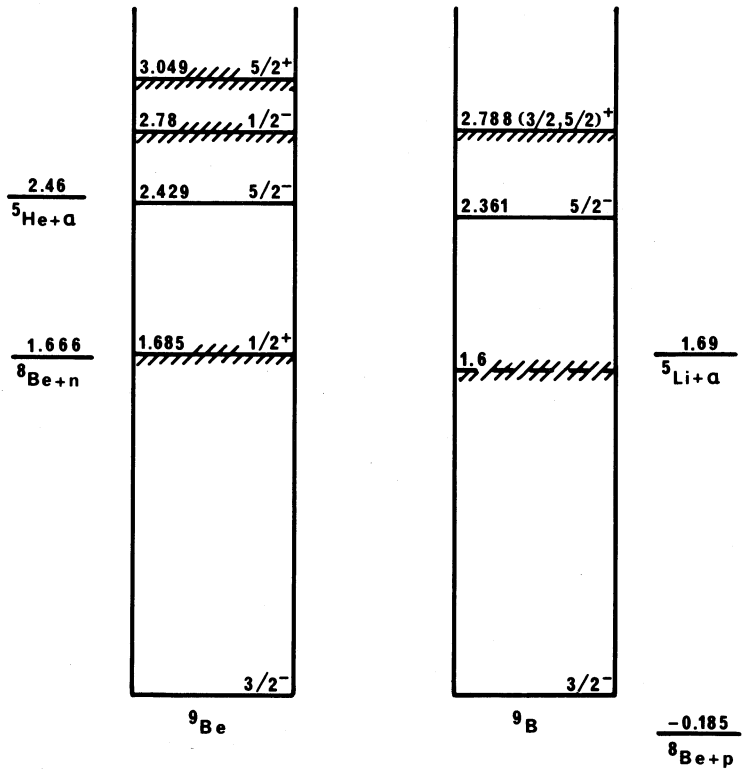


**Fig. 1.** Low-lying energy levels of $^9$Be and $^9$B (from Ajzenberg-Selove 1984).

the Thomas–Ehrman shift, which therefore persists when the s-wave neutron becomes unbound.

The potential parameters that Sherr and Bertsch used for describing the properties of the $\frac{1}{2}^+$ level of $^9$B are the same as they found for the $\frac{1}{2}^+$ level of $^9$Be in order to best fit $^9$Be$(\gamma, n)^8$Be cross section data. Their fit is only moderately good. They stated that the $R$-matrix theory gives a better fit (Barker 1983), but that it introduces parameters that do not allow a prediction of the analogue state energy. The latter part of this statement is, however, not correct. The analogue state energy can be calculated in terms of the parameters that come into the $R$-matrix formulae. Such calculations have been done previously for other light nuclei, in particular for the very similar $^{13}$C–$^{13}$N system (Barker and Ferdous 1980), with which the original Thomas–Ehrman shift was concerned.

The calculation for the $\frac{1}{2}^+$ states and other low-lying states of $^9$Be and $^9$B is given in the next section. The one-level approximation is used throughout. Section 3 contains a discussion of the results, and of the calculation by Sherr and Bertsch.

## 2. Coulomb Displacment Energies for $A = 9$ States

Energies of analogue states of given $J^\pi$ in $^9$Be and $^9$B are related by the Coulomb displacement energy, defined by

$$\Delta E_C(J^\pi) = M(^9\mathrm{B}, J^\pi) - M(^9\mathrm{Be}, J^\pi) + \delta_{np}, \qquad (1)$$

where $\delta_{np}$ is the neutron–proton mass difference and all masses are nuclear masses. Thus the excitation energy of a $^9$B state can be obtained from the measured excitation energy of the analogue state in $^9$Be together with the calculated net displacement, which is the difference between the Coulomb displacement energy for that pair of excited states and the value for the ground states.

We calculate the Coulomb displacement energies for $A = 9$ states using methods, formulae and notation essentially the same as those used previously for the low-lying levels of $^{13}$C and $^{13}$N by Barker and Ferdous (1980). For each $J^\pi$ value, we have

$$\Delta E_C = \Delta H^c + \Delta L, \tag{2}$$

where

$$\Delta H^c = \langle \Psi_{\frac{1}{2}1}(-\tfrac{1}{2}) | H^c | \Psi_{\frac{1}{2}1}(-\tfrac{1}{2}) \rangle - \langle \Psi_{\frac{1}{2}1}(\tfrac{1}{2}) | H^c | \Psi_{\frac{1}{2}1}(\tfrac{1}{2}) \rangle, \tag{3a}$$

$$\Delta L = -\sum_c \frac{\hbar^2}{2m_c a_c} u_{\tilde{c}}^2(a_c) \mathscr{S}_{\tilde{c}} \sum_{m_t} \{ (\tilde{T}\tfrac{1}{2} -\tfrac{1}{2} - m_t\, m_t | \tfrac{1}{2} -\tfrac{1}{2})^2 S_{-\frac{1}{2}}(\tilde{c}m_t) - (\tilde{T}\tfrac{1}{2}\,\tfrac{1}{2} - m_t\, m_t | \tfrac{1}{2}\,\tfrac{1}{2})^2 S_{\frac{1}{2}}(\tilde{c}m_t) \}. \tag{3b}$$

Here $\Psi_{\frac{1}{2}1}(M_T)$ is a state of good isospin $T = \frac{1}{2}$ defined in the internal region of $R$-matrix theory (Lane and Thomas 1958) and satisfying the boundary condition that its logarithmic derivative at the channel radius $r_c = a_c$ is constant, and the same for $^9$B ($M_T = -\frac{1}{2}$) as for $^9$Be ($M_T = \frac{1}{2}$). The spectroscopic factor $\mathscr{S}_{\tilde{c}}$ is related to the reduced width $\gamma_{\lambda c}^2$ of $R$-matrix theory by

$$\gamma_{\lambda c}^2 = \mathscr{S}_{\tilde{c}} (\tilde{T}\tfrac{1}{2}\, M_T - m_t\, m_t | \tfrac{1}{2}\, M_T)^2 \frac{\hbar^2}{2m_c a_c} u_{\tilde{c}}^2(a_c) \quad (c \equiv \tilde{c}m_t), \tag{4}$$

where $m_t = +\frac{1}{2}, -\frac{1}{2}$ for neutron, proton channels respectively, and $u_{\tilde{c}}(r_c)/r_c$ is the radial wavefunction normalised by (Barker 1978, equation 17)

$$\int_0^{a_c} u_{\tilde{c}}^2(r)\, \mathrm{d}r = 1. \tag{5}$$

Also $S_{M_T}(\tilde{c}m_t)$ is the shift factor of $R$-matrix theory. The implied definition of the energy $E_r$ of an unbound state is the energy at which the resonant nuclear phase shift passes through $\pi/2$ (cf. Barker and Ferdous 1980, equations 3 and 4).

For $A = 13$, Barker and Ferdous (1980) calculated many contributions to the Coulomb displacement energies, but the only ones that gave significant contributions to the net displacements were the internal (point) Coulomb interaction and the electromagnetic spin–orbit interaction, which contribute to $\Delta H^c$, and the boundary-condition level displacement, which contributes to $\Delta L$. Thus we include only these three contributions here, and use approximations similar to those made previously (Barker 1978; Barker and Ferdous 1980). Shell model wavefunctions for normal parity $A = 8, 9$ states are taken from Kumar (1974), and for non-normal parity $A = 9$ states from Woods and Barker (1984). Matrix elements in the internal contribution $\Delta H^c$, given by equation (3a), are calculated with harmonic oscillator wavefunctions with the length parameter $b = 1{\cdot}65$ fm, as for $A = 8$ (Barker 1978). The radial wavefunctions in the surface contribution $\Delta L$, given by equation (3b), are calculated

for a Woods–Saxon potential with parameter values $r_0 = 1{\cdot}75$ fm and $a = 0{\cdot}65$ fm (Millener *et al.* 1983). We include contributions to $\Delta L$ from the nucleon channels involving the $^8$Be levels $\tilde{J}\tilde{T} = 00, 20, 40, 20^*, 10, 30, 21, 11$ and 31, and their analogues in $^8$Li and $^8$B. The energies of the $^9$Be and $^9$B levels are as given in Fig. 1, except that the $\frac{1}{2}^+$ level of $^9$Be is here taken at 1·733 MeV from the $^9$Be$(\gamma, n)^8$Be fit (Barker 1983) and for consistency with the above definition of level energy; for the time being the same excitation energy is assumed for the $\frac{1}{2}^+$ level of $^9$B. The $\frac{1}{2}^-$ level of $^9$B is taken at 2·6 MeV (Fazely *et al.* 1982).

**Table 1. Calculated Coulomb displacement energies and $^9$B excitation energies**

| $J^\pi$ | $E_r(^9\text{Be})$ (MeV) | $\Delta H^c$ (MeV) Coul. | s.o. | $a_c$ (fm) | $\Delta L$ (MeV) | $\Delta E_C$ (MeV) | $E_r(^9\text{B})$ (MeV) |
|---|---|---|---|---|---|---|---|
| $\frac{3}{2}^-$ | 0·0 | 2·003 | −0·042 | 4 | −0·448 | 1·513 | 0·0 |
| | | | | 5 | −0·277 | 1·684 | 0·0 |
| | | | | 6 | −0·158 | 1·803 | 0·0 |
| | | | | 7 | −0·092 | 1·869 | 0·0 |
| $\frac{1}{2}^+$ | 1·733 | 1·939 | −0·015 | 4 | 0·313 | 2·237 | 2·457 |
| | | | | 5 | 0·151 | 2·075 | 2·124 |
| | | | | 6 | 0·072 | 1·996 | 1·926 |
| | | | | 7 | 0·040 | 1·964 | 1·828 |
| $\frac{5}{2}^-$ | 2·429 | 2·012 | −0·027 | 4 | −0·388 | 1·597 | 2·513 |
| | | | | 5 | −0·251 | 1·734 | 2·479 |
| | | | | 6 | −0·143 | 1·842 | 2·468 |
| | | | | 7 | −0·081 | 1·904 | 2·464 |
| $\frac{1}{2}^-$ | 2·78 | 1·998 | 0·050 | 4 | −0·302 | 1·746 | 3·01 |
| | | | | 5 | −0·183 | 1·865 | 2·96 |
| | | | | 6 | −0·104 | 1·944 | 2·92 |
| | | | | 7 | −0·059 | 1·989 | 2·90 |
| $\frac{5}{2}^+$ | 3·049 | 1·905 | −0·034 | 4 | −0·491 | 1·380 | 2·916 |
| | | | | 5 | −0·342 | 1·529 | 2·894 |
| | | | | 6 | −0·206 | 1·665 | 2·911 |
| | | | | 7 | −0·125 | 1·746 | 2·926 |

**Table 2. Contributions to $\Delta L$ (in MeV) from different channels for $a_c = 6{\cdot}0$ fm**

| $J^\pi$ | $l$ | $\tilde{J}\tilde{T}$ 00 | 20 | 40 | 20*, 10, 30 | 21, 11, 31 |
|---|---|---|---|---|---|---|
| $\frac{3}{2}^-$ | 1 | −0·121 | −0·038 | | −0·001 | 0·002 |
| $\frac{1}{2}^+$ | 0 | 0·094 | | | 0·000 | 0·000 |
| | 2 | | −0·022 | | 0·000 | 0·000 |
| $\frac{5}{2}^-$ | 1 | | −0·143 | −0·002 | −0·001 | 0·003 |
| $\frac{1}{2}^-$ | 1 | −0·031 | −0·072 | | −0·001 | 0·000 |
| $\frac{5}{2}^+$ | 0 | | −0·147 | | 0·000 | 0·000 |
| | 2 | −0·044 | −0·015 | 0·000 | 0·000 | 0·000 |

Because of the approximations involved in the formulae (2) and (3) and in their evaluation, the calculated values of $\Delta E_C$ are dependent on the choice of the channel radius $a_c$. The conventional value is $a_c = 1{\cdot}45(A_1^{1/3} + A_2^{1/3})$ fm $= 4{\cdot}35$ fm. Results are given in Table 1 for a range of $a_c$ values from 4 to 7 fm. The level energies $E_r$ are expressed as excitation energies.

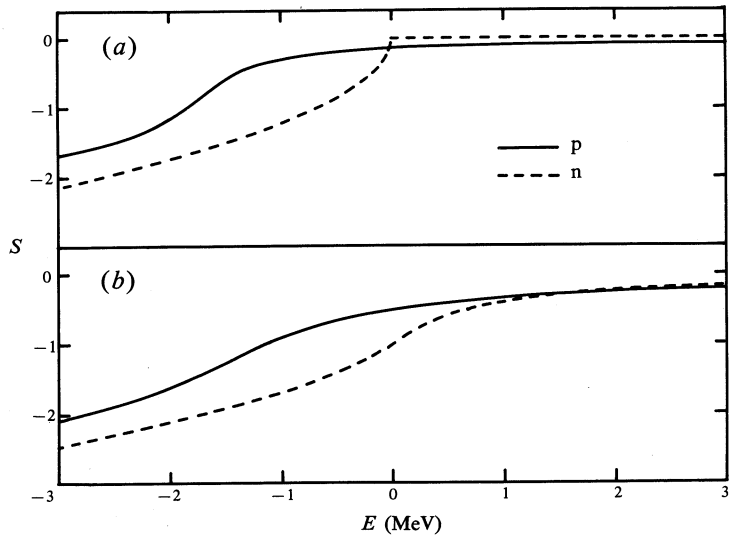


**Fig. 2.** Energy dependence of proton and neutron shift factors for $^8$Be + nucleon channels with $a_c = 6{\cdot}0$ fm, for (*a*) $l = 0$ and (*b*) $l = 1$.

A striking feature of the values in Table 1 is that $\Delta L$ is negative in all cases except for the $\frac{1}{2}^+$ states, where it is positive. To trace the source of this behaviour, Table 2 gives the contributions to $\Delta L$ from the various channels, for the particular channel radius $a_c = 6{\cdot}0$ fm; other values give similar results, but this value was favoured in a fit to the decay strength of the $\frac{1}{2}^+$ state of $^9$Be (Barker 1984). The main contributions are from the channels involving the ground state and first excited state of $^8$Be, and these are all negative except for the $^8$Be(g.s.) + s-wave nucleon channel that dominates for $\frac{1}{2}^+$ states. For these $\tilde{T} = 0$ channels, the dependence of equation (3b) on the shift factors is (with a slight change of notation) through the term $S_p(^9\text{B}) - S_n(^9\text{Be})$. Values of these shift factors are shown in Fig. 2 for $a_c = 6{\cdot}0$ fm and for the two cases of (*a*) a channel with $l = 0$ and (*b*) one with $l = 1$ (typifying channels with $l \neq 0$). For neutrons the abscissa is the channel energy, and for protons it is the channel energy minus the Coulomb displacement energy; then the difference of $S_p$ and $S_n$ at the same abscissa enters equation (3b). It is seen that in most cases $S_p(^9\text{B}) - S_n(^9\text{Be})$ is positive, but that it is negative for $l = 0$ channels with an unbound neutron level ($E > 0$). Thus the anomalous sign of $\Delta L$ for $\frac{1}{2}^+$ states in Table 1 is due to the peculiar energy dependence of the s-wave neutron shift factor in the threshold region. The effect is enhanced by the large value of $u_c^2(a_c)$ for 2s wavefunctions.

The $\Delta L$ contribution to the net displacement for the $\frac{1}{2}^+$ levels of $^9$Be and $^9$B is then positive. In the corresponding $^{13}$C–$^{13}$N case, the value of $\Delta L$ for the $\frac{1}{2}^+$ states is even more negative than for the ground ($\frac{1}{2}^-$) states, due to the $\frac{1}{2}^+$ state in $^{13}$C being bound by nearly 2 MeV; this results in a negative contribution of $\Delta L$ to the net displacement, which is the classic Thomas–Ehrman shift. A similar shift is found in the $^{17}$O–$^{17}$F system, where the $\frac{1}{2}^+$ state of $^{17}$O is bound by over 3 MeV. Thus we predict an inverted Thomas–Ehrman shift for $A = 9$. The positive contribution from $\Delta L$ to the net displacement of the $\frac{1}{2}^+$ levels outweighs the negative contribution from the Coulomb and spin–orbit interactions, leading to an excitation energy for the $\frac{1}{2}^+$ level of $^9$B higher than that for $^9$Be.

The sensitivity of our results to the input parameter values and assumptions has been investigated. Use of the shell model wavefunctions of Barker (1966) for the normal parity states gives little change in either component of $\Delta H^c$ ($\lesssim 10$ keV) or in $\Delta L$ ($\lesssim 5\%$) for all states. The non-normal parity $A = 9$ wavefunctions of Barker (1961) change the Coulomb contributions to $\Delta H^c$ by $+4$ and $-17$ keV for the $\frac{1}{2}^+$ and $\frac{5}{2}^+$ states respectively, and the spin–orbit contributions by $+5$ and $-12$ keV respectively, while $\Delta L$ for the $\frac{1}{2}^+$ states is increased by about 60% and $|\Delta L|$ for the $\frac{5}{2}^+$ states is decreased by about 15%. A value $b = 1{\cdot}75$ fm has been used to fit electron scattering data in $^9$Be (Woods and Barker 1984); the corresponding changes in the values of $\Delta H^c$ are easily obtained since the Coulomb contribution is proportional to $b^{-1}$ and the spin–orbit contribution to $b^{-3}$. The conventional value of the Woods–Saxon radius parameter $r_0 = 1{\cdot}25$ fm (as used by Sherr and Bertsch 1985) gives values of $|\Delta L|$ about 30–40% less than those of Table 1, except that the values for the $\frac{1}{2}^+$ states are little affected. These changes indicate the uncertainties in the quantities listed in Table 1. The values given in Table 1 are our preferred values.

So far we have made use of the value of only one of the three parameters that were determined in the $^9$Be$(\gamma, n)^8$Be fit (Barker 1983), namely the $\frac{1}{2}^+$ level energy $E_R \equiv E_r - E_T$ (where $E_T$ is the threshold energy). Of the other two, the reduced transition probability $B \equiv B(E1; \frac{1}{2}^+ \rightarrow \frac{3}{2}^-)$ is not of direct interest here, since we are concerned primarily with energies. The other parameter $\epsilon_R$ is related to the reduced width $\gamma^2$ by $\epsilon_R = 2m_c a_c^2 \gamma^4/\hbar^2$, and hence by equation (4) to the spectroscopic factor $\mathcal{S}$ of the $\frac{1}{2}^+$ state for the $^8$Be(g.s.)+s-wave nucleon channel. For the best fit value $\epsilon_R = 192$ keV, one finds $\mathcal{S} = 0{\cdot}151$, $0{\cdot}188$, $0{\cdot}248$ and $0{\cdot}324$ for $a_c = 4$, 5, 6 and 7 fm respectively. Acceptable fits to the $^9$Be$(\gamma, n)^8$Be data were obtained for $\epsilon_R$ as large as $0{\cdot}34$ MeV, and a similar best fit to $^9$Be$(p, p)^9$Be data (Tucker *et al.* 1970) gave $\epsilon_R = 0{\cdot}52$ MeV, corresponding to increases of the above $\mathcal{S}$ values by 33% and 64% respectively. These values are all much smaller than the value $\mathcal{S} = 0{\cdot}606$ used in Table 1, which was obtained from a shell model calculation (Woods and Barker 1984). These smaller $\mathcal{S}$ values suggest that the $\Delta L(\frac{1}{2}^+)$ values in Table 1 should be reduced in magnitude, but the precise amount is uncertain because wavefunctions giving the smaller $\mathcal{S}$ values are not known. One would still expect $E_r(^9\text{B}, \frac{1}{2}^+)$ to be greater then $E_r(^9\text{Be}, \frac{1}{2}^+)$.

The level energies used so far are energies $E_r$ at which the resonant nuclear phase shift $\beta$ passes through $\pi/2$, where $\beta$ is defined in terms of the total nuclear phase shift $\delta$ and the hard-sphere phase shift $-\phi$ by $\beta = \delta + \phi$. In the one-level approximation of $R$-matrix theory, which was found to describe adequately the $\frac{1}{2}^+$ contribution to the $^9$Be$(\gamma, n)^8$Be cross section (Barker 1983), one has

$$\tan\beta = \frac{\frac{1}{2}\Gamma}{E_r + \Delta - E} = \frac{\gamma^2 P}{E_r - \gamma^2(S-B) - E}. \tag{6}$$

Here we have included only the $^8$Be(g.s.) channel contribution to the level shift $\Delta$, and $P$, $S$ and $B$ are the penetration factor, shift factor and boundary condition parameter for this channel. The condition $\beta(E_r) = \pi/2$ requires $\Delta(E_r) = 0$ or $B = S(E_r)$. Measurement of $^8$Be+nucleon elastic scattering is, however, not feasible; if it were, it should provide the clearest and cleanest evidence about the properties of the $\frac{1}{2}^+$ states of $^9$Be and $^9$B. If the $\frac{1}{2}^+$ level of $^9$B is to be observed at all, it will be in some reaction such as $^9$Be$(^3\text{He}, t)^9$B in which the $^9$B is formed as an unstable product nucleus. The yield of such a reaction as a function of the $^9$B excitation energy may be

expressed in terms of a density-of-states function or lineshape which, in the one-level approximation of $R$-matrix theory, may be written as (Lane and Thomas 1958; Barker and Treacy 1962)

$$\rho(E) = \frac{\frac{1}{2}\Gamma}{(E_r+\Delta-E)^2+(\frac{1}{2}\Gamma)^2}$$

$$= \frac{\gamma^2 P}{\{E_r-\gamma^2(S-B)-E\}^2+(\gamma^2 P)^2} \propto \frac{\sin^2\beta}{P}. \qquad (7)$$

We define $E_m$ as the excitation energy at which $\rho(E)$ is a maximum, and $\Gamma_{1/2}$ as the FWHM of $\rho(E)$. For the $\frac{1}{2}^+$ state of $^9$Be, the $^9$Be$(\gamma, n)^8$Be best fit (Barker 1983) gives $E_m = 1{\cdot}696$ MeV and $\Gamma_{1/2} = 0{\cdot}130$ MeV. Values of $E_m$ and $\Gamma_{1/2}$ calculated for the $\frac{1}{2}^+$ state of $^9$B are given in Table 3. In case (a), the shell model spectroscopic factor $\mathscr{S} = 0{\cdot}606$ is used and the values are consistent with those in Table 1. Case (b) uses values of $\mathscr{S}$ derived from $\epsilon_R = 0{\cdot}192$ MeV; the values of $E_r$ are estimated by taking the $^8$Be(g.s.) contribution to $\Delta L(\frac{1}{2}^+)$ proportional to $\mathscr{S}$, and leaving all other contributions to $\Delta E_C$ unchanged. Although the values of $E_r$ for case (b) are lower than the corresponding values for case (a), the values of $E_m$ are higher for (b) than (a), due to the level being narrower so that the difference between $E_m$ and $E_r$ is smaller. For $a_c \approx 6$ fm, for which there is some preference (Barker 1984), one might therefore expect the $\frac{1}{2}^+$ level in $^9$B to have a peak excitation energy of about $1{\cdot}8$ MeV and a width of 1–2 MeV.

**Table 3. Calculated values for the $\frac{1}{2}^+$ state of $^9$B**

| Case | $a_c$ (fm) | $E_r$ (MeV) | $\gamma^2$ (MeV) | $E_m$ (MeV) | $\Gamma_{1/2}$ (MeV) |
|---|---|---|---|---|---|
| (a) | 4 | 2·457 | 2·13 | 1·98 | 3·02 |
| | 5 | 2·124 | 1·36 | 1·79 | 2·50 |
| | 6 | 1·926 | 0·862 | 1·71 | 2·02 |
| | 7 | 1·828 | 0·566 | 1·69 | 1·65 |
| (b) | 4 | 2·15 | 0·529 | 2·12 | 0·95 |
| | 5 | 1·99 | 0·423 | 1·96 | 0·95 |
| | 6 | 1·87 | 0·353 | 1·83 | 0·94 |
| | 7 | 1·81 | 0·302 | 1·77 | 0·97 |

## 3. Discussion

The experimental value of $\Delta E_C$ for the $\frac{3}{2}^-$ ground states of $^9$Be and $^9$B is $1{\cdot}851$ MeV (Ajzenberg-Selove 1984), with which the calculated values in Table 1 are in reasonable agreement for $a_c = 6$–7 fm. It should be remembered, however, that some significant contributions to $\Delta E_C$ may be neglected here (cf. Table 6 of Barker and Ferdous 1980), and that the calculated values are sensitive to the choice of $b$. The differences between the calculated (Table 1) and experimental (Fig. 1 and Fazely *et al.* 1982) excitation energies of the $\frac{5}{2}^-$, $\frac{1}{2}^-$ and $\frac{5}{2}^+$ levels of $^9$B should not be taken too seriously, because we have not distinguished between $E_r$ and $E_m$ values in these cases.

The main point is the big difference between the value of about $1{\cdot}8$ MeV that we expect for the excitation energy of the $\frac{1}{2}^+$ first excited state of $^9$B and the prediction

of about 0·9 MeV by Sherr and Bertsch (1985) (henceforth referred to as SB). SB used a single-particle model, in which the $\frac{1}{2}^+$ state is assumed to have the structure $^8$Be(g.s.)+s-wave proton, corresponding to a spectroscopic factor $\mathscr{S} = 1$, whereas we used the much smaller shell model value $\mathscr{S} = 0{\cdot}606$ and even smaller values obtained from fitting $^9$Be$(\gamma, n)^8$Be data. A more important reason for the difference, however, appears to lie in the different definitions used for the energy of an unbound level.

We defined two energies associated with an unbound level; $E_r$ is the energy at which the resonant nuclear phase shift $\beta$ passes through $\pi/2$, and $E_m$ is the energy at which the density-of-state function $\rho$ reaches a maximum. Theoretically $E_r$ is of more significance, as it occurs explicitly in both of the formulae (6) for $\beta$ and (7) for $\rho$, whereas $E_m$ is of more interest experimentally as it is closely related to the observable peak energy.

SB made use of four definitions or prescriptions for the energy of an unbound level:

(a) the real part of the energy of a pole in the scattering matrix;

(b) the energy at the maximum of

$$\rho_1(E) = \int_0^\infty \Psi_E^2(r) \frac{\mathrm{d}V}{\mathrm{d}r} r^2 \, \mathrm{d}r, \tag{8a}$$

with the single-particle continuum wavefunction $\Psi_E(r)$ normalised by

$$\Psi_E(r) \xrightarrow[r\to\infty]{} \frac{\sin(kr+\delta)}{k^{\frac{1}{2}} r}, \tag{8b}$$

where $\delta$ is the total nuclear phase shift;

(c) the energy at the maximum of

$$\rho_2(E) = \mathrm{d}\delta/\mathrm{d}E; \tag{9}$$

(d) and the energy at the maximum of

$$\rho_3(E) = \left| \int \Psi_E(r) r \phi_0(r) \, r^2 \, \mathrm{d}r \right|^2, \tag{10}$$

with $\phi_0(r)$ the single-particle bound-state wavefunction describing the ground state of $^9$Be.

SB considered (a) as the correct definition in principle, but said that it is not easy to apply. They found that the prescriptions (b) and (c) gave roughly equal values for the $A = 11$–$17$ cases that they investigated, but that (c) was to be preferred for describing the very broad $\frac{1}{2}^-$ states of $^9$Be and $^9$B. They claimed that neither (b) nor (c) could be used to compute the energy of the unbound $\frac{1}{2}^+$ state of $^9$Be, because 'there is no potential barrier, and so the resonance does not exist in a rigorous sense'. This is reminiscent of the suggestion by Spencer *et al.* (1960) that this 'is not a state in the usual sense'. It was shown, however, by Barker and Treacy (1962) that the then-available data could be explained adequately by assuming a normal $\frac{1}{2}^+$ state and using *R*-matrix formulae, and subsequent studies (Barker and Fitzpatrick 1968; Tucker *et al.* 1970; Barker 1983; Barker 1984) have tended to confirm this.

Nevertheless, for the particular case of the $\frac{1}{2}^+$ levels of $^9$Be and $^9$B, SB introduced the definition (d), and this led to the prediction of 0·9 MeV for the energy of the $\frac{1}{2}^+$ level of $^9$B.

We now consider the relationship between the energies defined in (a)–(d) and our quantities $E_r$ and $E_m$. For simplicity we assume the one-channel one-level approximation of $R$-matrix theory.

To show the connection between $E_r$ and the pole of the scattering matrix, we assume that $\Delta$ (or $S$) is a linear function of $E$, so that equation (6) can be written as

$$\tan\beta = \tfrac{1}{2}\Gamma^\circ/(E_r - E), \tag{11a}$$

where

$$\Gamma^\circ = \Gamma/(1 - \mathrm{d}\Delta/\mathrm{d}E), \tag{11b}$$

and that $\Gamma^\circ$ (or $P$) is independent of $E$. Then the scattering matrix is

$$\begin{aligned} U \equiv \exp(2\mathrm{i}\,\delta) &= \exp(-2\mathrm{i}\,\phi)\exp(2\mathrm{i}\,\beta) \\ &= \exp(-2\mathrm{i}\,\phi)(E_r - E + \tfrac{1}{2}\mathrm{i}\,\Gamma^\circ)/(E_r - E - \tfrac{1}{2}\mathrm{i}\,\Gamma^\circ), \end{aligned} \tag{12}$$

so that the energy of the pole is $E_r - \frac{1}{2}\mathrm{i}\,\Gamma^\circ$. With these approximations, the definition (a) of SB therefore gives the energy of the unbound level equal to $E_r$, as we have used in Table 1; however, the assumptions that $S$ is a linear function of $E$ and $P$ is constant are not in general good near threshold. In fact $E_r$ in equation (6) is necessarily greater than $E_T$ for an unbound state and less than $E_T$ for a bound state, which seem very reasonable properties for the energy of a state, whereas the real part of the energy of a pole is less than $E_T$ for both a bound state and a virtual (unbound) state (Nussenzveig 1959; Humblet and Rosenfeld 1961).

In prescription (b) of SB, the asymptotic form (8b) is appropriate to an s-wave neutron channel; for the general case, this becomes

$$\Psi_E(r) \xrightarrow[r\to\infty]{} \frac{F_l(r)\cos\delta + G_l(r)\sin\delta}{k^{\frac{1}{2}} r}, \tag{13}$$

where $F_l$ and $G_l$ are the regular and irregular Coulomb functions. The term $\mathrm{d}V/\mathrm{d}r$ in equation (8a) was introduced to simulate a surface-peaked reaction mechanism. If we take the extreme case

$$\mathrm{d}V/\mathrm{d}r \propto \delta(r - a_c), \qquad V(r) = 0 \quad \text{for} \quad r > a_c, \tag{14}$$

then $\rho_1(E) \propto \Psi_E^2(a_c)$, and $\Psi_E(a_c)$ is given by the asymptotic form (13), so that

$$\rho_1(E) \propto \sin^2(\delta+\phi)/P = \sin^2\beta/P, \tag{15}$$

where we have used the relations

$$\tan\phi = F_l(a_c)/G_l(a_c), \qquad P = k a_c/\{F_l^2(a_c) + G_l^2(a_c)\}. \tag{16a, b}$$

Thus, with the approximations (14), $\rho_1(E)$ is proportional to the $R$-matrix density-of-states function $\rho(E)$ given in equation (7), and the level energy defined by prescription (b) is the same as our $E_m$.

We start our discussion of prescription (c) from the density-of-states function derived and used by Phillips *et al.* (1960)

$$\bar{\rho}(E) = \frac{1}{\pi} \mathrm{d}\beta/\mathrm{d}E . \tag{17}$$

Phillips *et al.* noted that $\bar{\rho}(E)$ is approximately the same as $\rho(E)$ defined by equation (7) if it is assumed that $\Gamma$ and $\Delta$ are slowly varying with $E$. Barker and Treacy (1962) pointed out that this assumption may not be good for levels near thresholds, as in the present case, and that $\bar{\rho}(E)$ is not necessarily positive, so that it is not always suitable as a definition of a density-of-states function. Johnson (1973) also took the resonance energy as where $\bar{\rho}(E)$ is a maximum, which he assumed to be the same as $E_r$; he then stated that if $\phi$ varies linearly with energy, this will also be where $\mathrm{d}\delta/\mathrm{d}E$ is a maximum, and this is the basis of prescription (c) of SB. The assumption that $\phi$ is a linear function of $E$ is, however, not good near threshold. Thus there are several arguments against the use of prescription (c) for the low-lying $A = 9$ levels.

SB introduced their prescription (d) particularly in order to describe the $\frac{1}{2}^+$ levels of $^9$Be and $^9$B. For the $^9$Be case, the integral in equation (10) is an electric dipole matrix element between the ground state and the $\frac{1}{2}^+$ state, and SB assumed that the $^9$Be$(\gamma, \mathrm{n})^8$Be cross section $\sigma_{\gamma \mathrm{n}}$ is proportional to $\rho_3(E)$. From equations (13)–(15) of Barker and Ferdous (1980), equation (13) above, and $\sigma_{\gamma \mathrm{n}} \propto (k_\mathrm{n}^2/E_\gamma^2)\sigma_{\mathrm{n}\gamma}$, we find $\sigma_{\gamma \mathrm{n}} \propto E_\gamma \rho_3$. The additional factor $E_\gamma$ would enhance the disagreement between fit and data in Fig. 4(a) of SB (the fact that their model does not allow a good fit to the data is due to their spectroscopic factor $\mathscr{S}$ being much too large). To describe the $\frac{1}{2}^+$ state of $^9$B, SB used $\rho_3$ calculated from equation (10), with $\Psi_E$ appropriate to $^9$B but $\phi_0$ still the ground state of $^9$Be, and found a peak at an excitation energy of about 0·93 MeV. An additional $E_\gamma$ factor would increase the peak energy to about 1·3 MeV. The hybrid nature of the matrix element, and the poor fit to the $^9$Be$(\gamma, \mathrm{n})^8$Be cross section data, however, make this an unsuitable prescription for calculating the energy of the $\frac{1}{2}^+$ state of $^9$B.

We have already mentioned that the triton spectrum from the $^9$Be$(^3$He, t$)^9$B reaction measured by Djaloeis *et al.* (1983*a*, 1983*b*) shows some evidence for the $\frac{1}{2}^+$ state of $^9$B, but its position and width would depend very much on the properties assumed for the background. SB have suggested that the background could be eliminated or minimised by observing triple coincidences between the tritons and the two alphas from the $^8$Be breakup. It seems that a simpler way of greatly reducing the background would be to require triton–proton double coincidences, with the proton energy gated on decay through $^8$Be(g.s.)—this would have the added bonus of effectively removing the peak corresponding to the strongly produced $\frac{5}{2}^-$ state of $^9$B at 2·36 MeV, since this state decays less than 0·5% to $^8$Be(g.s.)+proton (Wilkinson *et al.* 1966).

In summary, an $R$-matrix calculation of the Coulomb displacement energies predicts a higher excitation energy for the $\frac{1}{2}^+$ state of $^9$B than for its analogue state in $^9$Be. This inverted Thomas–Ehrman shift is due to the peculiar energy dependence of the s-wave neutron shift factor in the threshold region. The suggestion by Sherr and Bertsch (1985) of a much lower excitation energy for the $\frac{1}{2}^+$ state of $^9$B seems to be unreliable because the single-particle model and the definition of the energy of an unbound level that they used are both inappropriate for these states.

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