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Consistent Description of Unbound States Observed in Scattering and Reactions

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Abstract

The unbound states of a nucleus C may be studied in elastic scattering A+a → C → A+a, and in reactions of the type B+b → C+c, in which C is an unstable product nucleus. Expressions for the scattering cross sections or phase shifts, and for the dependence of the reaction cross section on E_C, have been derived from *R*-matrix theory, and it is argued that consistent values of the *R*-matrix parameters should be used in describing the two types of data. This requirement has been disregarded, however, in a recent paper concerned with the 2^+ states of ^{8}Be. On the other hand, support for the consistency argument comes from theoretical and experimental work done recently in Canberra on the low-lying states of ^{5}He and ^{5}Li.

1. Introduction

Unbound states of a nucleus C may be studied in different ways. In elastic scattering

$$\mathrm{A+a \rightarrow C \rightarrow A+a}, \tag{1}$$

C is formed as an intermediate nucleus, and a state of C may be indicated by a peak in the cross section or more specifically, for a state of given J^π, by a rapid increase in the nuclear phase shift δ_J, expressed as a function of the bombarding energy E_a or of the excitation energy E_C of the nucleus C. For simplicity, it is assumed that there is only one open decay channel for C. In reactions of the type

$$\mathrm{B+b \rightarrow C+c} \qquad (\mathrm{C \rightarrow A+a}), \tag{2}$$

C is a product nucleus and, for fixed bombarding energy, the yield N_J attributable to states of definite J^π may show a peak as a function of the energy E_c of the emitted nucleus or of E_C. This paper is concerned with the problem of obtaining a consistent description of $\delta_J(E_C)$ and $N_J(E_C)$.

As an example, which will be useful later on, the case of ^{8}Be is considered. The accepted energy level diagram is shown in Fig. 1, for levels below the ^{7}Li+p threshold. All levels are unbound with respect to breakup into two alphas. Level energies, widths and J^π values are shown, together with the scattering channels and some of the reactions by which ^{8}Be has been studied. Some information about the

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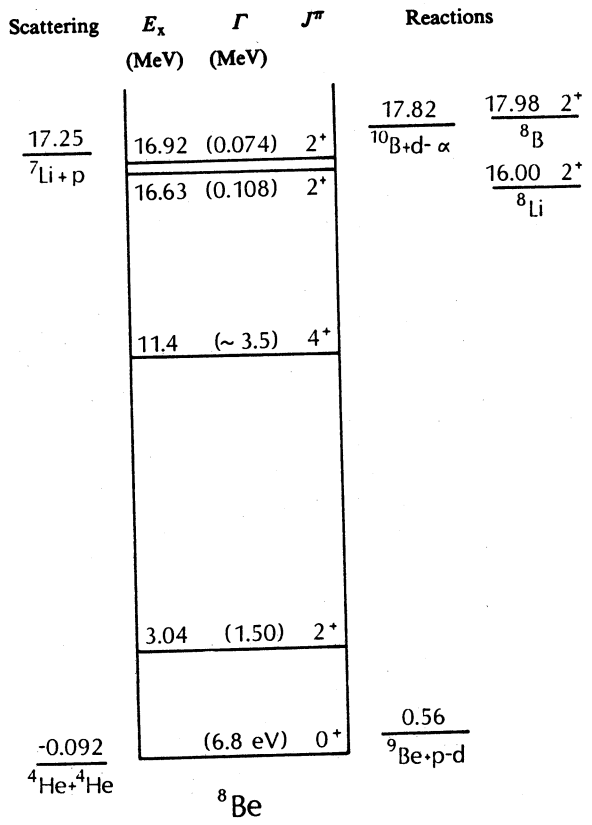

Fig. 1. Low-lying energy levels of ^{8}Be (from Ajzenberg-Selove 1984).

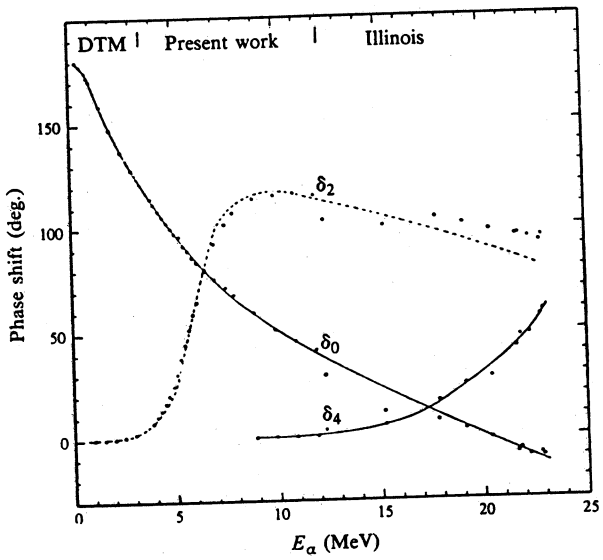

Fig. 2. The α–α nuclear phase shifts δ_J as functions of α-particle energy (from Tombrello and Senhouse 1963).

levels come from elastic scattering

$$\alpha+\alpha \rightarrow {}^{8}\mathrm{Be} \rightarrow \alpha+\alpha, \tag{3}$$

and the nuclear phase shifts $\delta_J(E_\alpha)$ are shown in Fig. 2. Due to the small width of the 0^+ ground state, the rapid rise of δ_0 through 180° could not be measured as such; however, the energy and width of the level were determined from the interference pattern between the s-wave nuclear and the Coulomb scattering amplitudes (Benn *et al.* 1966, 1968). The 2^+ level at about 3 MeV is associated with the rapid rise of δ_2 at $E_\alpha \approx 6$ MeV. The slow rise in δ_4 is attributed to the broad 4^+ level at about 11 MeV. The 2^+ levels at 16·6 and 16·9 MeV (not shown in Fig. 2) each contribute a rise in δ_2 of about 180° (Bacher *et al.* 1972).

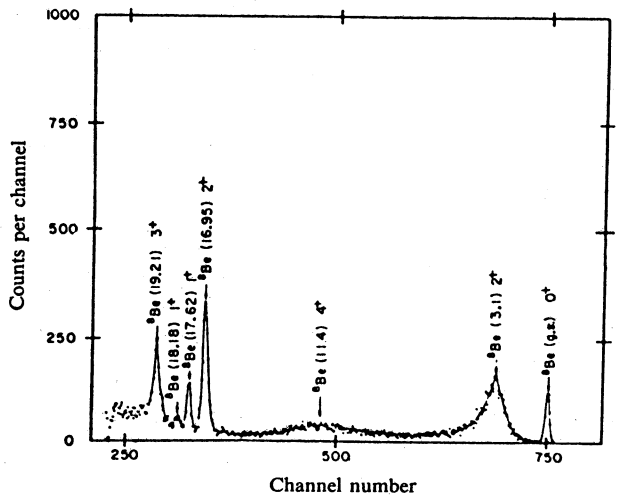

Fig. 3. A deuteron spectrum from $^{9}\mathrm{Be(p, d)}^{8}\mathrm{Be}$ with $E_p = 33{\cdot}6$ MeV and $\theta_{lab} = 60°$ (from Kull 1967).

As typical of the reactions producing ^{8}Be, we consider $^{9}\mathrm{Be(p, d)}^{8}\mathrm{Be}$:

$$^{9}\mathrm{Be}+\mathrm{p} \rightarrow {}^{8}\mathrm{Be}+\mathrm{d} \qquad ({}^{8}\mathrm{Be} \rightarrow \alpha+\alpha). \tag{4}$$

A measured deuteron spectrum is shown in Fig. 3, where the observed peaks are each attributed to definite ^{8}Be levels.

The yield of the reaction (2), as a function of E_C, is often separated into two factors,

$$N_J(E_C) = G^J_{bc}(E_C)\rho_J(E_C), \tag{5}$$

where $\rho_J(E_C)$ is a generalised density-of-states function (Phillips *et al.* 1960) depending only on the properties of the nucleus C [for sharp states $\rho(E) = \Sigma_\lambda\, \delta(E-E_\lambda)$], and $G^J_{bc}(E_C)$ is a feeding function that depends on the reaction producing C. An early way of presenting the problem of a consistent description of scattering and reaction data was by posing the question: 'Can $\rho(E)$ be simply expressed in terms of $\delta(E)$?'

2. One-level Approximation

(a) Sequential Decay

Based on the assumption that reaction (2) proceeds sequentially as indicated, formulae have been proposed and used for $\rho(E)$ in the one-level, one-channel (for decay) approximation. These include

$$\rho(E) \propto \frac{\Gamma^2}{(E_r-E)^2+(\frac{1}{2}\Gamma)^2}, \qquad \text{(Bethe 1937)} \qquad (6)$$

$$\rho(E) \propto \frac{\Gamma}{(E_r-E)^2+(\frac{1}{2}\Gamma)^2}, \qquad \text{(Wheeler 1941)} \qquad (7)$$

$$\rho(E) \propto \frac{P(E)}{(E_r-E)^2+(\frac{1}{2}\Gamma)^2}, \qquad \text{(Bonner } et\ al. \text{ 1948)} \qquad (8)$$

$$\rho(E) \propto \frac{\Gamma_1(E)}{\{E_1+\Delta_1(E)-E\}^2+\{\frac{1}{2}\Gamma_1(E)\}^2}. \qquad \text{(Treacy 1953)} \qquad (9)$$

In the formulae (6)–(8), E_r is the resonance energy and Γ the (constant) width, so that as far as energy dependence is concerned, (6) and (7) are the same. The energy-dependent penetration factor $P(E)$ in (8) was introduced in order to give a better fit to data. Treacy (1953) proposed the form (9) by analogy with the R-matrix formulae of Wigner and Eisenbud (1947) for two-stage reactions in which C is an intermediate nucleus, and only later was it derived rigorously by Lane and Thomas (1958). Here, the width $\Gamma_1(E)$ and level shift $\Delta_1(E)$ are given by

$$\Gamma_1(E) = 2P(E)\gamma_1^2, \qquad \Delta_1(E) = -\{S(E)-B\}\gamma_1^2, \qquad (10)$$

where $P(E)$ and $S(E)$ are the energy-dependent penetration factor and shift factor, and B the constant boundary condition parameter; E_1 is the eigenenergy and γ_1^2 the reduced width. Each of these is a real quantity in R-matrix theory (Lane and Thomas 1958).

The nuclear phase shift can be written in terms of the resonant phase shift (β) and the hard-sphere phase shift ($-\phi$),

$$\delta(E) = \beta(E)-\phi(E), \qquad (11)$$

and in the one-level, one-channel approximation of R-matrix theory, one has

$$\beta(E) = \arctan \frac{\frac{1}{2}\Gamma_1(E)}{E_1+\Delta_1(E)-E}, \qquad (12)$$

where E_1, Γ_1 and Δ_1 are the same as in equation (9) (Lane and Thomas 1958). Then one can write

$$\rho(E) \propto \frac{\sin^2\beta(E)}{\Gamma_1(E)} \propto \frac{\sin^2\beta(E)}{P(E)} = \frac{\sin^2\{\delta(E)+\phi(E)\}}{P(E)}. \qquad (13)$$

In this approximation, $\rho(E)$ can therefore be expressed in terms of $\delta(E)$, together

with the calculable functions $P(E)$ and $\phi(E)$. The latter can be written

$$P(E) = \frac{ka}{F^2(a)+G^2(a)}, \qquad \phi(E) = \arctan\frac{F(a)}{G(a)}, \tag{14}$$

where $F(r)$ and $G(r)$ are the energy-dependent regular and irregular Coulomb radial wavefunctions, k is the wave number and a the channel radius. The latter is a distance beyond which there is no polarising interaction between the particles A and a. The dependence on a is significant and is discussed below.

In some applications, Griffy and Biedenharn (1960) and Paul *et al.* (1977) used a modified form of equation (13),

$$\rho(E) \propto \sin^2\delta(E)/P(E), \tag{15}$$

on the grounds that it gave a better fit to the data, but other justification for this formula is lacking (Fowler and Preist 1961; Alburger *et al.* 1963).

A somewhat different derivation by Phillips *et al.* (1960) led to

$$\rho(E) \propto \mathrm{d}\beta(E)/\mathrm{d}E, \tag{16}$$

which reduces to equation (9) in the one-level approximation (12) under certain restrictive conditions, but equation (16) was criticised by Barker and Treacy (1962) as being unsuitable for general use.

(b) Final-state Interactions

An alternative approach to the problem of relating ρ and δ was based on the assumption that the reaction (2) proceeds essentially by three-body breakup,

$$\mathrm{B}+\mathrm{b} \rightarrow \mathrm{A}+\mathrm{a}+\mathrm{c}, \tag{17}$$

but with strong final-state interactions between particles A and a. These interactions modify the phase-space distribution of energies that would otherwise occur. In this way Watson (1952) and Migdal (1955) derived

$$\rho_l(E) \propto \sin^2\delta_l(E)/k^{2l+1} \tag{18}$$

for a channel with relative orbital angular momentum l, but several approximations were made that were particularly appropriate to the systems in which they were interested (including low energies and no Coulomb interaction in the decay channel). In the same approach but more generally, Hamburger and Cameron (1960) found exactly the relation (13). They explicitly assumed an energy-independent shape of the interior wavefunction, and this is equivalent to the one-level approximation.

Relations of the type (13), (15) or (18) are often referred to as the Watson–Migdal relation; they essentially involve the one-level approximation.

(c) Inadequacy of the One-level Approximation

There is considerable evidence that the one-level approximation of R-matrix theory given by (9), (12) and (13) is not adequate to account for much experimental data involving states of ^{8}Be, in which we are particularly interested:

16·9 MeV), its nonzero width for decay into two alphas being due to some $T = 0$ admixture.

(iii) $^{10}B(d, \alpha)^{8}Be$. A measured α-spectrum from $^{10}B(d, \alpha)^{8}Be$ is shown in Fig. 6 for the ^{8}Be excitation energy range from about 16 to 18 MeV. The spectrum in the region of the 16·6 and 16·9 MeV levels cannot be explained by an incoherent sum of two one-level approximations, but indicates that coherence is necessary with destructive interference in the region between the levels.

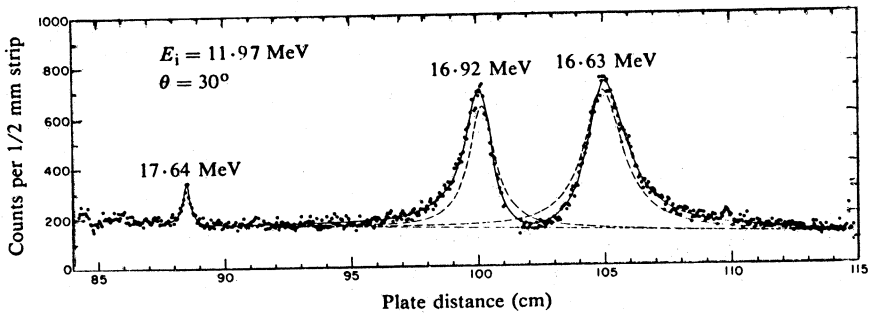

Fig. 6. An α-spectrum from $^{10}B(d, \alpha)^{8}Be$ (from Browne *et al.* 1966). The solid curve is a two-level fit to the experimental points (see Section 3). The dashed curves show the shape and position that would have been observed for each level in isolation.

3. Many-level Approximation

The fit to the $^{10}B(d, \alpha)^{8}Be$ data shown in Fig. 6 is a suggested two-level approximation designed to produce interference (Browne *et al.* 1966; Barker 1966):

$$N(E) \propto \left| \frac{G_1^{\frac{1}{2}} \Gamma_1^{\frac{1}{2}}}{E_1 - E - \frac{1}{2}\mathrm{i}\,\Gamma_1} + \frac{G_2^{\frac{1}{2}} \Gamma_2^{\frac{1}{2}}}{E_2 - E - \frac{1}{2}\mathrm{i}\,\Gamma_2} \right|^2, \tag{19}$$

i.e., the square of the sum of two R-matrix one-level approximation amplitudes (with neglect of the level-shift terms), with real feeding amplitudes $G_\lambda^{1/2}$ ($\lambda = 1, 2$). The yield is no longer separated into the factors G and ρ as in equation (5). A formula with some of the features of (19) had previously been proposed by Griffy and Biedenharn (1960). In the case of n levels of given J^π, and with the inclusion of level shifts, the formula (19) would generalise to

$$N(E) \propto \left| \sum_{\lambda=1}^{n} \frac{G_{\lambda bc}^{\frac{1}{2}} \Gamma_\lambda^{\frac{1}{2}}}{E_\lambda + \Delta_\lambda - E - \frac{1}{2}\mathrm{i}\,\Gamma_\lambda} \right|^2 \tag{20}$$

What seems to be a more reasonable R-matrix form for the yield in the many-level approximation has been obtained as follows (Barker 1967). For the two-stage reaction

$$A' + a' \rightarrow C \rightarrow A + a \tag{21}$$

proceeding through n levels of the compound nucleus C with the same J^π, the R-matrix formula for the cross section (in the many-channel approximation) can be

written as (Lane and Thomas 1958)

$$\sigma_{a'a} \propto \left| \sum_{\lambda,\mu=1}^{n} \Gamma_{\lambda a'}^{\frac{1}{2}} \Gamma_{\mu a}^{\frac{1}{2}} A_{\lambda\mu} \right|^2, \tag{22}$$

where

$$(\mathbf{A}^{-1})_{\lambda\mu} = (E_\lambda - E)\delta_{\lambda\mu} - \sum_c (S_c - B_c + \mathrm{i}\, P_c)\gamma_{\lambda c}\gamma_{\mu c}, \tag{23}$$

the summation over c being over all decay channels (including a and a′). In the one-level approximation this becomes

$$\sigma_{a'a} \propto \frac{\Gamma_{1a'}\Gamma_{1a}}{(E_1 + \Delta_1 - E)^2 + (\frac{1}{2}\Gamma_1)^2}. \tag{24}$$

For the reaction (2), the yield in the one-level R-matrix approximation, given by equations (5) and (9), can be written as (Lane and Thomas 1958)

$$N(E) \propto \frac{G_{1bc}\Gamma_{1a}}{(E_1 + \Delta_1 - E)^2 + (\frac{1}{2}\Gamma_1)^2} \tag{25}$$

The procedure that leads from equation (24) to (25) in the one-level case, namely replacement of $\Gamma_{1a'}$ by G_{1bc}, can be followed in the n-level case, namely replacement of $\Gamma_{\lambda a'}^{1/2}$ in (22) by $G_{\lambda bc}^{1/2}$, to give the n-level R-matrix formula for the yield of reaction (2) that was proposed by Barker (1967), i.e.

$$N(E) \propto \left| \sum_{\lambda,\mu=1}^{n} G_{\lambda bc}^{\frac{1}{2}} \Gamma_{\mu a}^{\frac{1}{2}} A_{\lambda\mu} \right|^2 \tag{26}$$

Note that $A_{\lambda\mu}$ is unchanged, and is still given by equation (23).

If only the one channel a is contributing to the decay of C, as was assumed above, then the yield can be written as (Barker 1967)

$$N(E) \propto \left| \left(\sum_{\lambda=1}^{n} G_{\lambda bc}^{\frac{1}{2}} \Gamma_{\lambda}^{\frac{1}{2}} / (E_\lambda - E) \right) \Big/ \left(1 + \sum_{\lambda=1}^{n} (\Delta_\lambda - \tfrac{1}{2}\mathrm{i}\,\Gamma_\lambda)/(E_\lambda - E) \right) \right|^2 \tag{27}$$

From the many-level form for the resonant phase shift (Lane and Thomas 1958)

$$\beta(E) = \arctan \left\{ \left(\tfrac{1}{2} \sum_{\lambda=1}^{n} \Gamma_\lambda/(E_\lambda - E) \right) \Big/ \left(1 + \sum_{\lambda=1}^{n} \Delta_\lambda/(E_\lambda - E) \right) \right\}, \tag{28}$$

one then has (Barker 1967)

$$N(E) \propto \sin^2\beta(E) \left| \left(\sum_{\lambda=1}^{n} G_{\lambda bc}^{\frac{1}{2}} \Gamma_{\lambda}^{\frac{1}{2}} / (E_\lambda - E) \right) \Big/ \left(\sum_{\lambda=1}^{n} \Gamma_\lambda/(E_\lambda - E) \right) \right|^2 \tag{29}$$

A significant difference between the formulae (20) and (27) (or 29) is that, if a particular level λ is not fed in the reaction, so that $G_{\lambda bc} = 0$, then that level has no effect on $N(E)$ given by (20) but does affect $N(E)$ given by (27). Some evidence in

favour of the latter form concerns the ground state of ^{8}Be, which was observed in both scattering and reactions at the same energy within experimental errors (Benn *et al.* 1966, 1968; Reichart *et al.* 1966). This is what is expected from (27), but follows from (20) only if $B = S(E_1)$, which is not a reasonable requirement for a many-level approximation. Fits to the ^{10}B(d, α)^{8}Be data of Fig. 6 with the appropriate two-level form of (26) are of comparable or better quality than those given by (19) (Callender and Browne 1970).

It is to be noted that $N(E)$ given by equation (27) or (29) involves feeding amplitudes $G_{\lambda bc}^{1/2}$ ($\lambda = 1, \ldots, n$), in addition to the R-matrix parameters that occur in the description of the phase shift $\delta(E)$. These feeding amplitudes are real and may be energy-dependent; e.g. for β-decay, $G_{\lambda bc}$ is proportional to the integrated Fermi function.

A two-level approximation with the form of equation (19), but with $G_2^{1/2}/G_1^{1/2}$ possibly complex, was derived by Kirilyuk *et al.* (1970) on the basis of complex-eigenvalue theory. For consistency, this should be used in conjunction with the complex-eigenvalue expression for the phase shift.

4. Application to ^{8}Be

As some justification for the repeated reference to ^{8}Be, it may be pointed out that each of the formulae (6)–(9), (15), (16), (19), (27) and (29) was first proposed or applied in connection with levels of ^{8}Be. Similar reasons presumably led Louis Brown to give the subtitle 'A History of Nuclear Physics' to a book that he wrote (but never

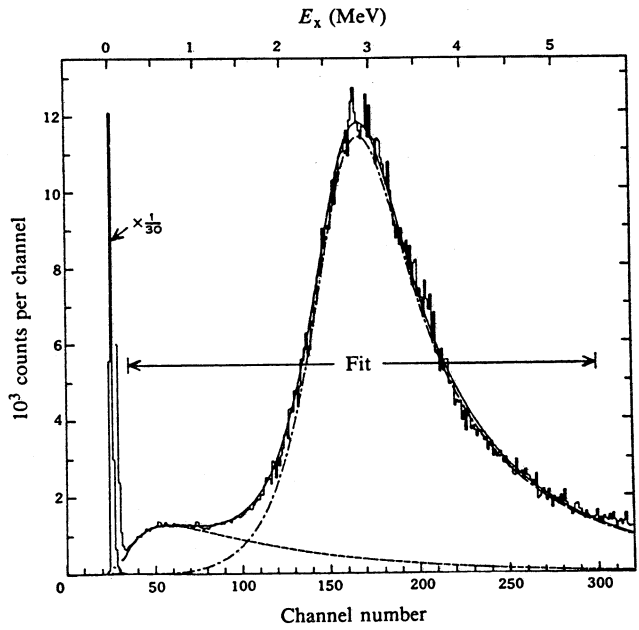

Fig. 7. Calculated (solid curve) and measured (histogram) spectra of deuterons at 10° from ^{9}Be(p, d)^{8}Be with $E_p = 39{\cdot}91$ MeV (from Barker *et al.* 1976). The $l = 0$ and 2 components of the calculated curve are plotted as dashes and dot–dashes respectively. The $l = 0$ component is normalised to the ground state main peak.

published) on 'Beryllium-8'. In particular, he said 'We can hope to see the growth of nuclear physics through studies of a particular isotope or mass system. Given this point of view, ^{8}Be is a worthy choice, very likely the best.'

Table 1. Dependence of properties of the second 0^+ state of ^{8}Be on the channel radius a_0 (from Barker *et al.* 1968)

a_0 (fm)	$E_x(0_2^+)$ (MeV)	$\Gamma_{1/2}(0_2^+)$ (MeV)	$\theta^2(0_2^+)$
6·5	8	11	1·4
7·0	6	9	1·4

(a) 0^+ States of ^{8}Be

In the one-level R-matrix approximation, δ_0 for α–α scattering cannot be well fitted for $E \gtrsim 3$ MeV (Barker and Treacy 1962).

In the three-level approximation, Barker *et al.* (1968) fitted δ_0 for $E \lesssim 17$ MeV by equations (11) and (28) for a wide range of values of the channel radius a_0. When ^{9}Be(p, d)^{8}Be data (Hay *et al.* 1967) were simultaneously fitted using equation (27), with physically-reasonable constraints imposed on the feeding amplitudes for the upper two levels, the best overall fit was obtained with $a_0 \approx 7$ fm (Barker *et al.* 1968). More recent ^{9}Be(p, d)^{8}Be data obtained at higher beam energies, which confer certain advantages, are shown in Fig. 7. The 0^+ ground state of ^{8}Be contributes not only the sharp peak at zero excitation energy (with natural width about 6 eV), but also the peak at $E_x \approx 0{\cdot}6$ MeV, which is referred to as the ghost of the ground state (Barker and Treacy 1962) (the peak at $E_x \approx 3$ MeV is due to the 2^+ first-excited state). For parameter values that fit δ_0, the predicted magnitude and position of the ghost peak depend on the value of a_0. As a_0 increases, the ghost peak moves to lower energies and becomes smaller. The best simultaneous fit to δ_0 and the ghost peak was obtained for $a_0 \approx 6{\cdot}5$ fm (or a larger value if a nonzero background contribution was included) (Barker *et al.* 1976). The strong correlations between the value of a_0 and the excitation energy and FWHM of the second 0^+ state are shown in Table 1. The dimensionless reduced width θ^2 of this state is about 1·4 for all values of a_0; this large value indicates that the state must belong to higher shell model configurations, and therefore be classified as an intruder state.

Table 2. Properties of 0^+ intruder states in light even nuclei (from Ajzenberg-Selove 1984, 1985, 1986*a*, 1986*b*)

Nucleus	$E_x(0_2^+)$ (eV)	$\Gamma_{1/2}(0_2^+)$ (eV)	$\theta^2(0_2^+)$
^{10}Be	6·18		
^{12}C	7·65	8·5	1·4
^{14}C	6·59		
^{14}O	5·92		
^{16}O	6·05		

Table 2 lists similar 0^+ intruder states occurring in neighbouring even nuclei, also at excitation energies of about 6–8 MeV. Of these, only the ^{12}C state is α-unstable; the value $\theta^2 = 1{\cdot}4$ given for it (Barker and Treacy 1962) is the same as for the ^{8}Be state, even though the FWHM of the states differ by six orders of magnitude!

Thus the properties of the 0^+ intruder state in ^{8}Be deduced from the *R*-matrix fits seem reasonable when compared with those in other light nuclei.

(b) 2^+ States of ^{8}Be

Combined three-level *R*-matrix fits to the d-wave α–α phase shift δ_2 up to 17 MeV and to various reaction data have been made by Barker (1969). Fits involving the deuteron spectrum from ^{9}Be(p, d)^{8}Be gave $a_2 \approx 7{\cdot}1$ fm, while more recently similar fits to the 2^+ contribution shown in Fig. 7 required $a_2 \approx 6{\cdot}0$ fm (or greater with a nonzero background) (Barker *et al.* 1976). Fits involving the ^{8}Li(β^-)^{8}Be(α)^{4}He α-spectrum of Alburger *et al.* (1963), with contributions from the 16·6 and 16·9 MeV levels included in the fit, led to $a_2 \approx 6{\cdot}7$ fm (Barker 1969), while Clark *et al.* (1969) obtained $a_2 \approx 6{\cdot}0$ fm from similar fits to their α-spectrum from ^{8}B(β^+)^{8}Be(α)^{4}He. These values of a_2 imply a low-lying broad second 2^+ state with the properties given in Table 3. The large values of θ^2 identify this as an intruder state; similar 2^+ intruder states are found in other light even nuclei, about 1–2 MeV above the 0^+ intruder state (Ajzenberg-Selove 1984, 1985, 1986*a*, 1986*b*; Barker 1969).

Table 3. Dependence of properties of the second 2^+ state of ^{8}Be on the channel radius a_2

a_2 (fm)	$E_x(2_2^+)$ (MeV)	$\Gamma_{1/2}(2_2^+)$ (MeV)	$\theta^2(2_2^+)$
6	12	14	1·7
7	8	9	1·6

(c) A Recent Paper on ^{8}Li(β^-)^{8}Be(α)^{4}He and ^{8}B(β^+)^{8}Be(α)^{4}He

A paper published recently by Warburton (1986) is now discussed at some length. There are two main features of this paper. First, Warburton made available previously unpublished data of Wilkinson and Alburger (1971) on the α-spectra from ^{8}Li(β^-)^{8}Be(α)^{4}He and ^{8}B(β^+)^{8}Be(α)^{4}He. These data are shown in Fig. 8—they have far better statistics and apparent accuracy than previous spectra. Second, Warburton fitted these α-spectra and also the d-wave α–α phase shift, using the many-level *R*-matrix formulae discussed above. He used a channel radius $\alpha_2 = 4{\cdot}5$ fm, and concluded that satisfactory fits to the data can be obtained without introducing intruder states below 26 MeV excitation. The essential difference between his approach and the earlier work was that he *did not require the R-matrix parameters to have the same values in the fits to the α-spectra as in the fit to the phase shift.* Because of his small value of a_2, he found values for the eigenenergy and reduced width amplitude of the 3 MeV level that differed appreciably in the fits to the different data.

It seems of some importance to decide which approach is better, not only in connection with the structure of ^{8}Be, but also because the requirement of consistency in fitting scattering and reaction data has been imposed in other cases, including the calculation of the ^{12}C(α, γ)^{16}O cross section at low energies (Barker 1971, 1987), and a study of the low-lying levels of ^{5}He and ^{5}Li (Barker and Woods 1985; Woods *et al.* 1988), which is discussed in the next section. Consistency is also implied in the widely-used one-level relation (13), which directly relates ρ and δ. This relation

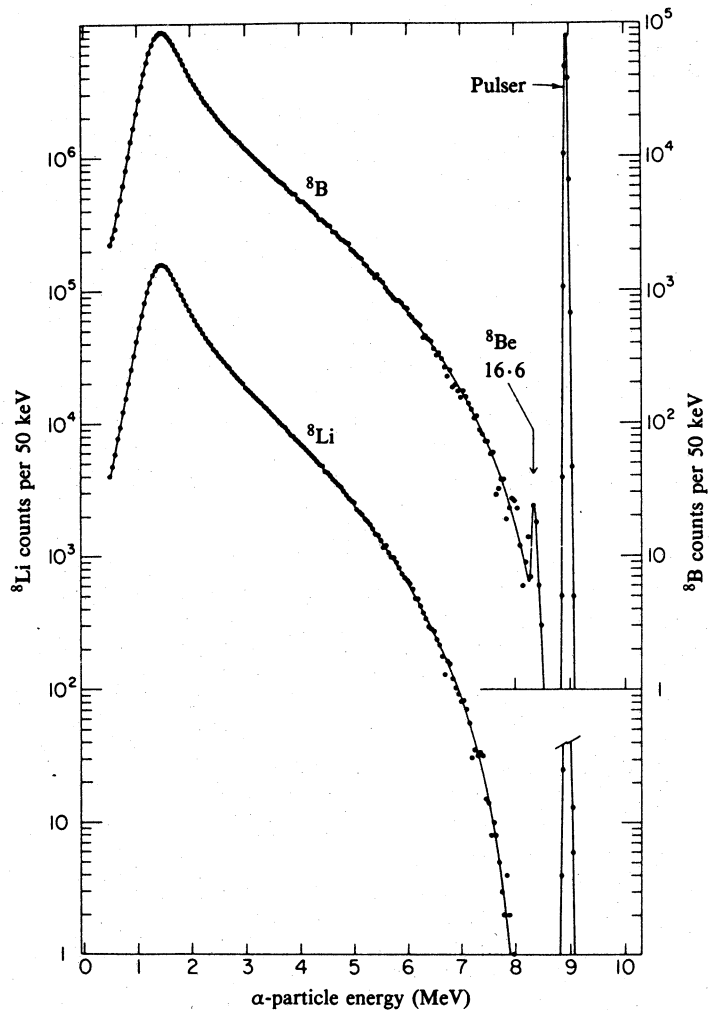

Fig. 8. The α-spectra from $^8\mathrm{Li}(\beta^-)^8\mathrm{Be}(\alpha)^4\mathrm{He}$ and $^8\mathrm{B}(\beta^+)^8\mathrm{Be}(\alpha)^4\mathrm{He}$ measured by Wilkinson and Alburger (1971). The solid curves are drawn to guide the eye. [From Warburton (1986).]

would be unjustified in the approach advocated by Warburton. Presumably he would have to regard as a coincidence the observed equality of the energies of the ^{8}Be ground state as seen in α–α scattering (Benn *et al.* 1966, 1968) and in the $^9\mathrm{Be(p, d)}^8\mathrm{Be}$ reaction (Reichart *et al.* 1966).

Warburton gives various arguments against the large channel radii (low-lying intruder states) found earlier (Barker *et al.* 1968; Barker 1969), and in favour of his small channel radius (high-lying intruder states). He implies (by his use of the words 'adopted', 'assumed', 'postulation') that the large values of a_0 and a_2 in the earlier work were an arbitrary choice, rather than a consequence of the requirement of consistency. He points out that the usual prescription for the channel radius $a = r_0(A_1^{1/3} + A_2^{1/3})$, with $r_0 = 1{\cdot}4$ fm, gives $a \approx 4{\cdot}5$ fm, and that this choice of r_0 is suggested by electron scattering; it is the r.m.s. charge radius, however, that one

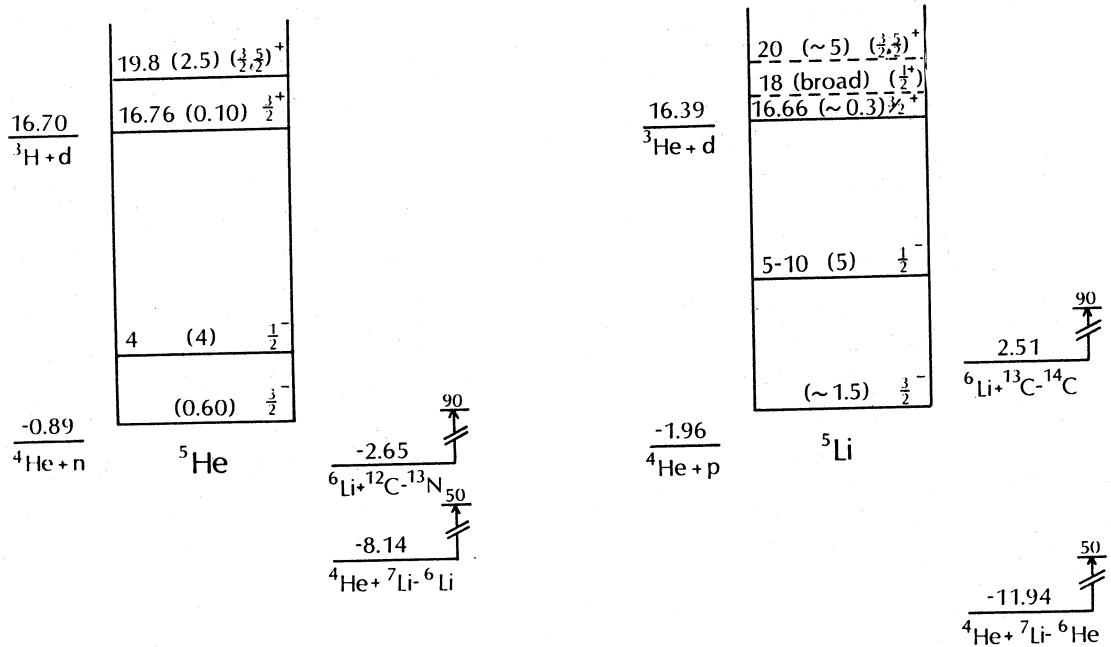

Fig. 9. Energy levels of ^{5}He and ^{5}Li (from Ajzenberg-Selove 1984).

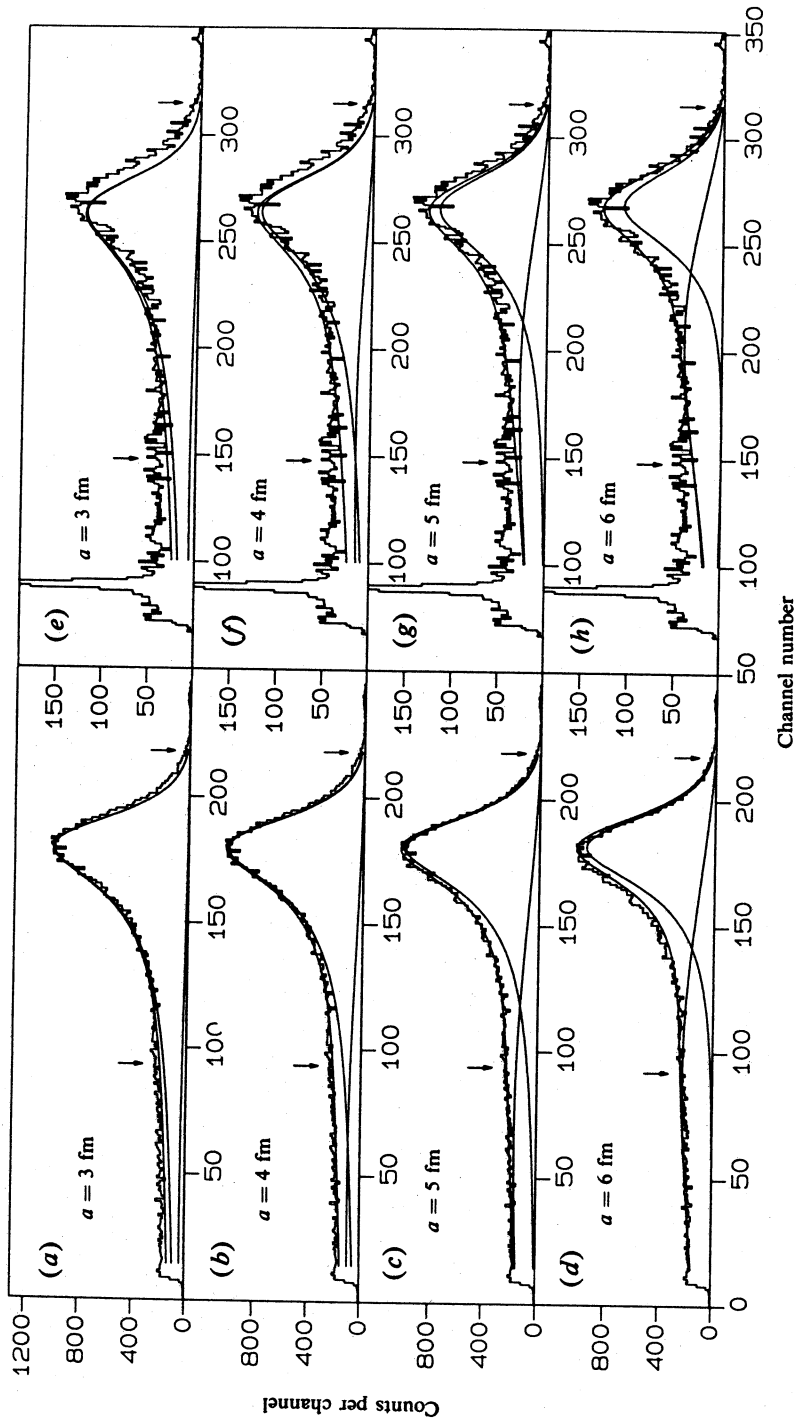

Fig. 10. Dependence on channel radius a of R-matrix fits to stripping data: (*a*)–(*d*) for the ^{4}He(^{7}Li, ^{6}Li)^{5}He reaction and (*e*)–(*h*) for the ^{4}He(^{7}Li, ^{6}He)^{5}Li reaction (from Woods *et al.* 1988). The $\frac{3}{2}^-$ and $\frac{1}{2}^-$ components and the total calculated lineshapes are shown, as well as the experimental spectra (histograms). Limits to the fitting regions are indicated by arrows.

With a channel radius of $a = 5{\cdot}5$ fm, the $s_{1/2}$ n–α and p–α phase shifts require $\frac{1}{2}^+$ levels of ^{5}He and ^{5}Li at about 5·4 and 5·1 MeV respectively (Barker and Woods 1985). The reason why previous fits to the phase shifts had located the $\frac{1}{2}^+$ level above the deuteron threshold was that they had assumed small channel radii ($a \approx 3$ fm). Similarly $\frac{3}{2}^+$ and $\frac{5}{2}^+$ levels are required at about 12 MeV for $a = 5{\cdot}5$ fm, but at 25 MeV or above for $a = 3$ fm. These positive-parity levels are too broad to be directly observed in reactions. Nevertheless, a check is possible by comparing these results from the phase-shift fits with the results of shell model calculations for the $A = 5$ levels. Van Hees and Glaudemans (1983, 1984) performed a $(0+1)\hbar\omega$ calculation, fitting properties of both normal and nonnormal parity levels of nuclei throughout the 1p shell and, in general, obtained good agreement with observed energies and electromagnetic properties. For ^{5}He (and ^{5}Li), they predicted the lowest $\frac{1}{2}^+$ state at an excitation energy of 7·4 MeV, with $\frac{3}{2}^+$ and $\frac{5}{2}^+$ states at about 15 MeV. An interaction chosen specifically to fit the properties of light nuclei gave somewhat lower energies (Barker and Woods 1985). Thus the shell model calculations support the value of the channel radius found by requiring a consistent fit to the scattering and reaction data on the low-lying $A = 5$ levels. Additional support for the large channel radius comes from a comparison of the excitation energy of the $\frac{1}{2}^-$ level, and of the spectroscopic factors of all the levels (Barker and Woods 1985).

6. Summary

This paper has attempted to show that there are no substantial arguments against, and that there is empirical evidence for, the reasonable proposition that the values of the R-matrix parameters used to describe the unbound states of a nucleus should be the same when it is formed as a product nucleus in reactions as when it is the intermediate nucleus in elastic scattering (or two-stage reactions).

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