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Baryon Structure and QCD: Nucleon Calculations

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Abstract

We present numerical calculations for the structure and mass of the $\frac{1}{2}^+$ nucleon in the chiral limit, using a covariant, QCD based formalism developed previously. The three-body problem of quarks interacting via gluon exchange is treated as a quark-diquark two-body problem. The nucleon mass and a nucleon-quark-diquark form factor are determined as a function of the one parameter, the diquark form factor normalisation, which can be determined by functional methods. The constituent mass of the unpaired quark within the nucleon is estimated to be about 0.44 GeV.

1. Introduction

Despite the successes of quantum chromodynamics (QCD) as a description of hadron dynamics, calculation of one of the most basic properties of hadrons, the low energy baryon spectrum, remains a problem. Calculations to date have generally suffered from being highly model dependent, or often they severely restrict many of the important symmetries present in QCD. For instance, bag models and potential models, as well as relying heavily on phenomenological modelling to produce desired physical effects, tend to obscure any link with the important $U_L(N_F) \otimes U_R(N_F)$ chiral symmetry of the QCD action. Another candidate for the study of hadron physics is lattice gauge theory (LGT), which has proved a useful numerical tool for studying gluon dynamics. However, LGT maintains Lorentz and chiral symmetry only as small remnant subgroups, hopefully to be restored to the full symmetries in the continuum limit. As a result, an accurate lattice study of the low energy hadron spectrum is proving to be computationally formidable.

Cahill *et al.* (1989, present issue p. 129) present a treatment of baryon structure aimed specifically at addressing many of the shortcomings plaguing the conventional treatments of low energy hadron physics. The paper sets up a three-body formulation of baryons as qqq colour singlet states bound by gluon exchange. Systematically derived from QCD with well defined approximations, the treatment maintains those aspects of QCD which are important to the low energy hadron spectrum, namely the colour algebra, Lorentz covariance and hidden chiral symmetry. The purpose of the current paper is to present the results of numerical calculations applying the formulation of Cahill *et al.* to

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the chiral limit $\frac{1}{2}^+$ baryon. This will provide a basis for subsequent studies of the baryon octet, including the nucleon, treating quarks with small bare current masses perturbatively.

The treatment is based on summing ladder diagrams representing gluon exchange between three valence quarks. The quarks acquire a dynamical mass via gluon dressing. The colour algebra ensures binding of two quarks into a $\overline{3}$ colour diquark state, allowing the reduction of a three-body problem to a non-local covariant two-body problem, in which the baryon is treated as a bound quark-diquark state. The picture of baryons as quark-diquark bound states began with the work of Ida and Kobayashi (1966) and independently Lichtenberg and Tassie (1967). Skytt and Fredriksson (1988) have prepared a compilation of the subsequent diquark literature. For the $\frac{1}{2}^+$ baryon an integral equation is obtained in terms of the nucleon-quark-diquark form factor. Consistency of this equation determines the chiral nucleon mass. In Section 2 we analyse the nucleon integral equation. Exploiting the spatial $O(3)$ invariance of the equation, it is reduced to a numerically managable form. We discover unexpected singularities in the quark propagators which affect the analytic structure of the integral kernel. Our numerical results are given in Section 3. At this stage the calculations are resricted to unphysical values of certain input parameters to avoid problems with the quark propagator singularities, though there is nothing in principle to prevent an extension of the method to include the physical nucleon. An estimate of the constituent mass of the quark within the nucleon is also given. Conclusions are drawn in Section 4.

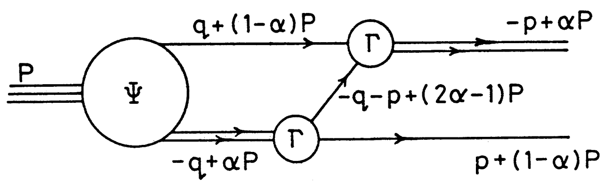


Fig. 1. Feynman diagram for the nucleon integral equation (1): Γ is the diquark form factor, Ψ the nucleon-quark-diquark form factor and α an arbitrary momentum partitioning parameter.

2. The Baryon Integral Equation

Our starting point for the current calculation is the integral equation derived in Cahill *et al.* (1989) describing the nucleon as a bound state of a quark and a diquark. In a Euclidean metric, the equation for the spin $\frac{1}{2}^+$ nucleon form factor (a spinor) Ψ, viz.

$$\Psi(p) = \frac{1}{f^2}\int \frac{d^4q}{(2\pi)^4}\Gamma((p+\tfrac{1}{2}q+\frac{2-3\alpha}{2}P)^2)\Gamma((q+\tfrac{1}{2}p+\frac{2-3\alpha}{2}P)^2)$$
$$\times G((2\alpha-1)P-p-q)G((1-\alpha)P+q)d((\alpha P-q)^2)\Psi(q), \qquad (1)$$

where Ψ is defined in terms of an arbitrary momentum partitioning parameter α, is shown pictorially in Fig.1. It is derived from an approximation to QCD which retains an effective two-point gluon exchange between dressed quarks,

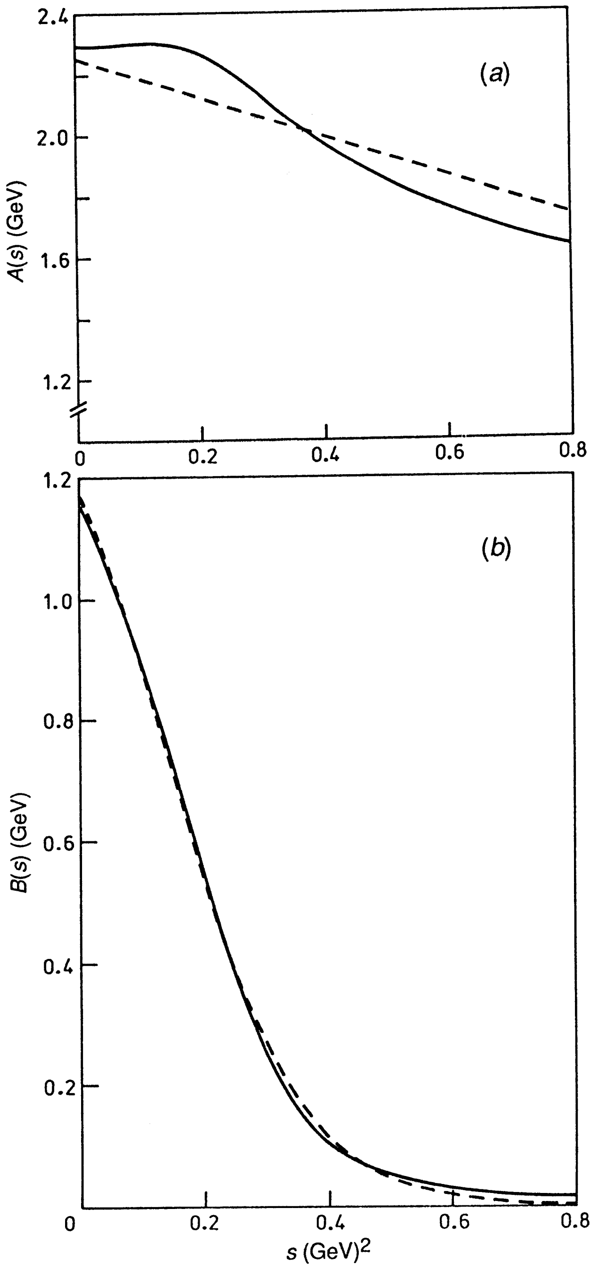


Fig. 2. (*a*) The function $A(s)$ in the dressed quark propagator equation (2). The solid curve is the numerical solution of the simplified Schwinger-Dyson equation and the dashed curve the straight line approximation (6). (*b*) Same as Fig. 2*a* for the function $B(s)$; the dashed curve is the analytic fit (7).

and sums all ladder diagrams. In the chiral limit, the dressed quark propagator takes the form

$$G(q;V)=\frac{1}{i\not{q}A(q^2)+VB(q^2)}, \tag{2}$$

where the matrix $V=\exp(i\sqrt{2}\pi^a F^a\gamma_5)$ reflects a vacuum degeneracy parametrised by the arbitrary real constants π^a. The global $U_L(N_F)\otimes U_R(N_F)$ chiral flavour symmetry present at the classical level is broken in the quantum theory to $H=U_V(N_F)$ with a vacuum manifold $Ga/H=U_A(N_F)$. As shown in Cahill *et al.* (1989), physical quantities such as the nucleon mass are independent of the choice of V, and the quark propagator used in (1) is $G(q)\equiv G(q;\mathbf{1})$. The functions A and B derive from a simplified Schwinger-Dyson equation (Praschifka *et al.* 1988) which describes quark dressing by an effective two-point gluon function. Numerical solutions for A and B on the positive real q^2-axis are shown in Figs 2*a* and 2*b*. We will say more about the analytic structure of these functions later. Note also that the propagator (2) implies a running effective quark mass $m(q^2)=B(q^2)/A(q^2)$ which $\to 0$ as Euclidean $q^2\to\infty$.

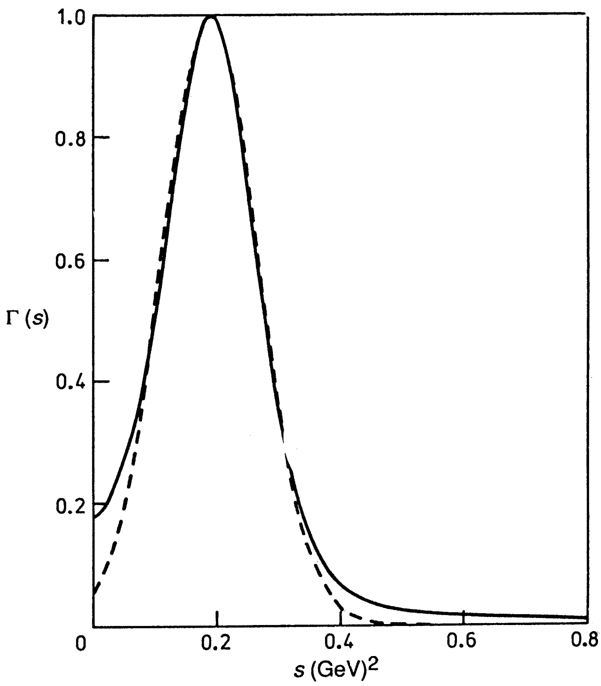


Fig. 3. The diquark form factor $\Gamma(s)$. The solid curve is the numerical solution from Praschifka *et al.* (1988) normalised to peak at $\Gamma=1$, and the dashed curve the gaussian fit (9).

The diquark form factor $\Gamma(q^2)$ is determined in Cahill *et al.* (1987) and Praschifka *et al.* (1988) from an approximate homogeneous Bethe-Salpeter equation. A plot of the numerical solution for $\Gamma(q^2)$ for positive real q^2 is given in Fig. 3. The solution assumes an $O(4)$ invariant diquark form factor calculated in the diquark rest frame. Strictly speaking, to maintain complete Lorentz covariance allowance should be made for the dependence of Γ on the relative momentum of the diquark within the nucleon, which is assumed here to be sufficiently small to have little effect on Γ. Determination of the diquark propagator normalisation from the treatment given in Cahill *et al.* (1989) is a non-trivial problem. For the purposes of this paper it is left as a free parameter which we incorporate into f, the diquark form factor normalisation. Fortunately, f can also be determined by a full functional treatment of QCD diquarks in baryons. Work in this direction is currently underway. The mass functional technique for calculating Γ also gives a dynamically generated diquark mass which for the scalar 0^+ diquark is $m_d = 0.568$ GeV. In (1) we keep only the pole part of the diquark propagator d, viz. $d(Q^2) = 1/(Q^2 + m_d^2)$. The vector 1^+ diquark has a somewhat higher dynamically generated mass of 1.1 GeV, and it is unlikely to make a significant contribution to nuclear structure.

Solutions to M and Ψ in (1) are sought in the rest frame of the nucleon. Accordingly we set $P = (\mathbf{0}, iM)$. With the above choices of Γ, G, d and P, (1) enjoys a spatial $O(3)$ symmetry. A direct calculation shows that the integral operator commutes with the angular momentum operator $\mathbf{J} = \mathbf{L} + \mathbf{S} = i(\partial/\partial\mathbf{p}) \times \mathbf{p} + \frac{1}{2}\boldsymbol{\sigma}$, so we take Ψ to be either of the general $L = 0$, $S = \frac{1}{2}^+$ states

$$\Psi_\uparrow = \begin{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(p) \\ \frac{\boldsymbol{\sigma}.\mathbf{p}}{|p|} \begin{pmatrix} 1 \\ 0 \end{pmatrix} v(p) \end{pmatrix}, \qquad \Psi_\downarrow = \begin{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(p) \\ \frac{\boldsymbol{\sigma}.\mathbf{p}}{|p|} \begin{pmatrix} 0 \\ 1 \end{pmatrix} v(p) \end{pmatrix},$$

where u and v are functions only of p_4 and $|\mathbf{p}|$. The Euclidean Dirac matrix representation used is

$$\boldsymbol{\gamma} = \begin{pmatrix} 0 & -i\boldsymbol{\sigma} \\ i\boldsymbol{\sigma} & 0 \end{pmatrix}, \qquad \gamma_4 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

Equation (1) then becomes

$$\begin{pmatrix} u(p) \\ v(p) \end{pmatrix} = \frac{1}{f^2} \int \mathrm{d}^4 q K_M(p_4, |\mathbf{p}|\,; q_4, |\mathbf{q}|\,; \mathbf{p}.\mathbf{q}) \begin{pmatrix} u(q) \\ v(q) \end{pmatrix}, \tag{3}$$

where

$$K_M(p_4, |\mathbf{p}|\,; q_4, |\mathbf{q}|\,; \mathbf{p}.\mathbf{q})$$
$$= \frac{1}{(2\pi)^4} \Gamma\left((p_4 + \tfrac{1}{2} q_4 + \frac{2 - 3\alpha}{2} iM)^2 + |\mathbf{p} + \tfrac{1}{2}\mathbf{q}|^2 \right) \Gamma(p \leftrightarrow q) \tilde{G}_1 \tilde{G}_2 d(s_3),$$

$$\tilde{G}_1 = \frac{1}{s_1 A^2(s_1) + B^2(s_1)}$$
$$\times \begin{pmatrix} ((2\alpha - 1)M + i(p_4 + q_4))A(s_1) + B(s_1) & (|\mathbf{q}| + \frac{\mathbf{p}.\mathbf{q}}{|\mathbf{q}|})A(s_1) \\ -(|\mathbf{p}| + \frac{\mathbf{p}.\mathbf{q}}{|\mathbf{p}|})A(s_1) & (\tilde{G}_1)_{22} \end{pmatrix},$$

where $(\tilde{G}_1)_{22} = [-((2\alpha - 1)M + i(p_4 + q_4))A(s_1) + B(s_1)]\mathbf{p}.\mathbf{q}/|\mathbf{q}||\mathbf{p}|$,

$$\tilde{G}_2 = \frac{1}{s_2 A^2(s_2) + B^2(s_2)} \times \begin{pmatrix} ((1-\alpha)M - iq_4)A(s_2) + B(s_2) & -|\mathbf{q}|A(s_2) \\ |\mathbf{q}|A(s_2) & -((1-\alpha)M - iq_4)A(s_2) + B(s_2) \end{pmatrix},$$

and the arguments of the quark and diquark propagators are

$$\left.\begin{aligned} s_1 &= (p_4 + q_4 - (2\alpha - 1)iM)^2 + |\mathbf{p} + \mathbf{q}|^2 \\ s_2 &= (q_4 + (1-\alpha)iM)^2 + |\mathbf{q}|^2 \\ s_3 &= (q_4 - i\alpha M)^2 + |\mathbf{q}|^2 . \end{aligned}\right\} \tag{4}$$

The kernel K_M satisfies

$$K_M(-p_4^*, |\mathbf{p}|; -q_4^*, |\mathbf{q}|; \mathbf{p.q}) = K_M(p_4, |\mathbf{p}|; q_4, |\mathbf{q}|; \mathbf{p.q})^* .$$

It follows that

$$u(-p_4^*, |\mathbf{p}|) = u(p_4, |\mathbf{p}|)^*,$$

$$v(-p_4^*, |\mathbf{p}|) = v(p_4, |\mathbf{p}|)^*.$$

The functions u and v depend on the momentum partitioning parameter α: a change in α from α_1 to α_2 effects a shift of the functions u and v by a distance $(\alpha_1 - \alpha_2)M$ along the imaginary p_4 axis. Alternatively, a shift in α can be thought of as a change of integration contour in the q_4-plane in (3), and care must be taken to avoid ambiguities which may arise from shifting the contour across singularities in the integral kernel K_M.

Evaluation of the kernel requires knowledge of the dressed quark propagator functions $A(s)$ and $B(s)$ in the subset of the complex s-plane defined by

$$x > \frac{y^2}{4M^2(1-\alpha)^2} - (1-\alpha)^2 M^2 \qquad \text{if} \quad \alpha < \tfrac{2}{3},$$

$$x > \frac{y^2}{4M^2(1-2\alpha)^2} - (1-2\alpha)^2 M^2 \qquad \text{if} \quad \alpha > \tfrac{2}{3}, \tag{5}$$

where $s = x + iy$. To extend A and B into the required part of the complex plane, approximate analytic fits are made to the numerical solutions of the Schwinger-Dyson equation obtained previously for real positive s. Our fits,

$$A(s) = 2.25 - 0.65s, \tag{6}$$

$$B(s) = 0.7336(1 - \tanh 4.779(s - 0.1435)), \tag{7}$$

are plotted in Fig. 2. A computer search for zeros of the quantity $sA^2(s) + B^2(s)$ revealed unexpected poles in the resulting quark propagator at s_P and s_P^* where

$$s_P = 0.1737 + 0.2062i.$$

The position of the poles is reasonably stable for several choices of analytic fits to A and B (see Table 1), suggesting that the dressed quark propagator does contain a genuine singularity in the vicinity of s_P. We note in passing a possibly related phenomenon for the dressed electron propagator in massless QED (Atkinson and Blatt 1979) which has a pair of conjugate branch points in the complex plane. The fits to A and B in Table 1 also produce poles on the negative real s-axis which are clearly model dependent and which are sufficiently far away not to interfere with the regions defined by the inequalities (5). Approximate forms for A and B used in earlier works (Cahill *et al.* 1987) were free of poles for negative real s, reflecting the confining nature of QCD. However these forms were not complex analytic functions and not suitable for the current calculation.

Table 1. Position of pole s_P in the quark propagator $G(s)$ for various fits to the functions A and B

The linear and tanh fits are given by equations (6) and (7). The gaussian fit is a fit to the function $B(s) = K e^{[(s-\mu)/\sigma]^2}$ and the power law fit is a fit to $A(s) = K + D[s + B/(s+C)]^{-\gamma}$

$A(s)$	$B(s)$	s_P
2	gaussian fit	$0.1462 + 0.2324i$
2	tanh fit	$0.1764 + 0.1968i$
linear fit	gaussian fit	$0.1378 + 0.2494i$
linear fit	tanh fit	$0.1737 + 0.2062i$
power law	gaussian fit	$0.1372 + 0.2863i$
power law	tanh fit	$0.1732 + 0.2196i$

The diquark propagator is also a source of singularities in the kernel K_M. Our calculation uses a free scalar propagator with a pole at $s = -m_d^2$. This pole is purely an artifact of the approximation used. The diquark belongs to a colour $\overline{3}$ representation and in its fully dressed form should also have a confining propagator which avoids poles on the negative s-axis.

From (4) we see that the integral kernel K_M has poles in the complex q_4-plane arising from poles in the quark and diquark propagators at the following points:

$$\left.\begin{aligned} q_4^{(1,2)} &= -(1-\alpha)iM \pm \surd(s_P - |\mathbf{q}|^2) \\ q_4^{(3,4)} &= -(1-\alpha)iM \pm \surd(s_P^* - |\mathbf{q}|^2) \\ q_4^{(5,6)} &= -p_4 - (1-2\alpha)iM \pm \surd(s_P - |\mathbf{p}+\mathbf{q}|^2) \\ q_4^{(7,8)} &= -p_4 - (1-2\alpha)iM \pm \surd(s_P^* - |\mathbf{p}+\mathbf{q}|^2) \\ q_4^{(9,10)} &= i(\alpha M \pm \surd(|\mathbf{q}|^2 + m_d^2)). \end{aligned}\right\} \qquad (8)$$

Fig. 4 shows the locus of these poles for typical values of α and M. For the case shown, poles from the propagator G_2 cross the real q_4-axis as $|\mathbf{q}|$ varies.

To keep the real q_4-axis free from poles, the following conditions must be met:

$$(\alpha-1)M-\eta<0<(\alpha-1)M+\eta\,,$$

$$(2\alpha-1)M-\eta<0<(2\alpha-1)M+\eta\,,$$

$$\alpha M-m_d<0<\alpha M+m_d\,,$$

where $\eta = \mathrm{Im}(\sqrt{s_P})$. This is possible provided

$$M<\min(3\eta, m_d+\eta)\,. \tag{9}$$

For higher nucleon masses the contour of integration should be deformed to pass beneath the poles 1 and 4 in Fig. 4. For these cases care should be taken to choose a value of α which avoids, if possible, pinching of the contour between poles 1 and 4 of the propagator G_2 and 6 and 7 of the propagator G_1.

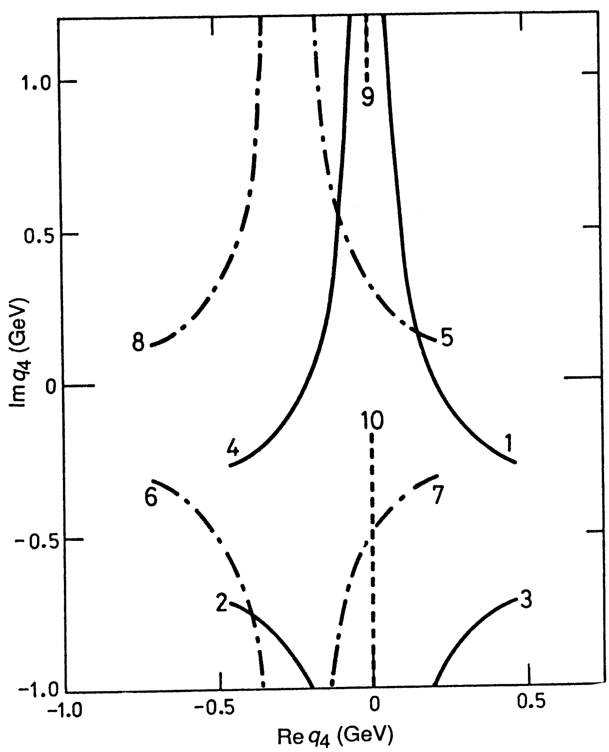


Fig. 4. Loci of the poles (8) in the integral kernel K_M in the complex q_4 plane as $|\mathbf{q}|$ varies. In this example we have chosen a nucleon mass of $M = 0.9$ GeV and momentum partitioning parameter $\alpha = 0.45$. The diquark mass is $m_d = 0.568$ GeV. The poles 1 to 4 arise from the quark propagator $\tilde{G}_2$, poles 5 to 8 from $\tilde{G}_1$ and poles 9 and 10 from the diquark propagator d. As p_4 varies, the loci of poles 5 to 8 translate left and right along the real axis. The loci shown are for $p_4 = 0.25$.

3. Numerical Results

In this section, we examine numerical solutions to the integral equation (3) for the $\frac{1}{2}^+$ nucleon. To avoid singularities along the contour of integration we will restrict ourselves to nucleon mass values M satisfying (9). Using the above values of $m_d = 0.568$ GeV and $s_P = (0.1737 + 0.2062i)$ GeV gives $M \leq 0.65$ GeV, with a corresponding optimal value for the momentum partitioning of $\alpha = \frac{2}{3}$. Unfortunately this mass range excludes the expected chiral nucleon mass of ~ 0.910 GeV, though it should, in principle, be possible to treat higher values of M by leaving the contour of integration in (3) along the real q_4-axis, and compensating for poles on the 'wrong' side of the contour by including residue terms.

The integral is solved as an eigenvalue problem for the wave function (u_p, v_p) and diquark form factor normalisation f. Starting with a trial wave function and input nucleon mass M, we iterate with K_M. The integration is done using Simpson's rule and a cutoff in p_4 and $|\mathbf{p}|$ of 1 GeV. By setting the parameter α to its optimal value of $\frac{2}{3}$, the need to analytically continue the form factor Γ off the real axis has been avoided. The procedure converges rapidly to give the highest eigenvalue f^2 and corresponding wave function. We have also evaluated $\det(f^{-2}K_M - 1)$ at fixed f and variable M in a couple of cases to check that the procedure is giving the lowest mass bound state. In Fig. 5 we plot the nucleon mass M against the calculated diquark form factor normalisation f. As expected on physical grounds, increasing the strength of the diquark-quark coupling by reducing f decreases the nucleon mass. The square of the nucleon form factor $\Psi^\dagger \Psi = u^* u + v^* v$ is plotted in Fig. 6*a* for a nucleon mass of 0.6 GeV. The form factor Ψ shares with the diquark form factor Γ a strong peaking at $p^2 = 0.2$ GeV^2, the height of the ridge becoming more skewed towards the p_4-axis as M increases.

To check that our results are independent of the momentum partitioning parameter α, we have also calculated f for $M = 0.5$ GeV and $\alpha = 0.6, \frac{2}{3}$ and 0.7, using the approximate gaussian fit (see Fig. 3)

$$\Gamma(s) = \exp\left[-\left(\frac{s - 0.19}{0.11}\right)^2\right], \tag{10}$$

to enable an analytic continuation of Γ to the required part of the complex s-plane. The resulting values of f agreed with each other to within 2%, and also agreed with the result using the original function Γ at $\alpha = \frac{2}{3}$ to within 2%. Changing α amounts to a shift of the function Ψ along the imaginary p_4 axis, giving $\Psi(p)$ in a different part of the complex p_4-plane. We find that the plot of $\Psi^\dagger \Psi$ remains qualitatively unchanged as α varies, apart from a slight skewing of the ridge away from the the p_4-axis as α increases.

The strongly peaked behaviour of $\Gamma(s)$ has been used (Praschifka *et al.* 1988) to argue that the constituent mass of the quark within the scalar diquark is about 270 MeV, agreeing well with deep inelastic scattering experiments. We can use the similar behaviour of Ψ to make an estimate of the constituent mass of the third quark within the nucleon. Since the form factors appearing in the integral equation (3) are strongly peaked functions acting almost like δ-functions, we assume that the integral is dominated by a narrow range of

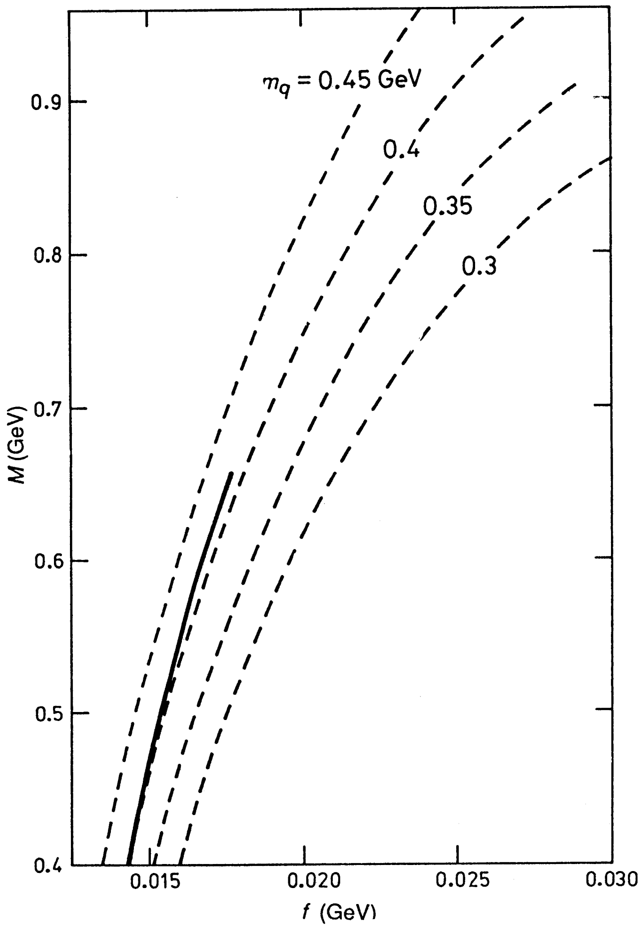
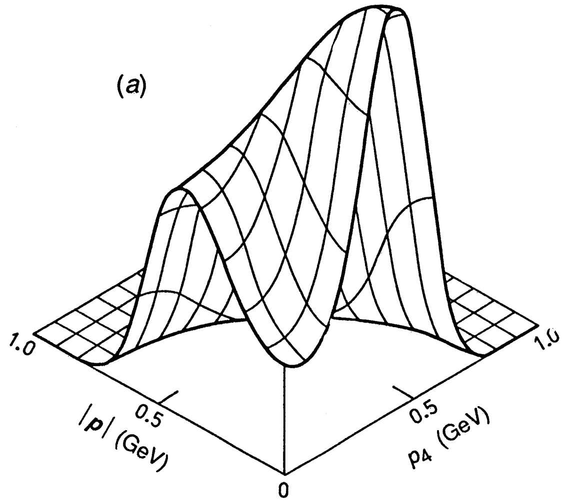


Fig. 5. The $\frac{1}{2}^+$ nucleon mass M plotted against the diquark form factor normalisation f (in GeV). The solid curve is obtained using the analytic fits (6) and (7) to the quark propagator functions A and B, the numerical solution in Fig. 3 for the diquark form factor Γ, and $\alpha = \frac{2}{3}$. The curve is truncated at $M = 0.657$ GeV, at which point the loci of poles 1 to 4 in (8) begin to cut the real q_4-axis. The dashed curves are obtained by replacing the quark propagators in (1) with propagators for free fermions of mass m_q and using the gaussian fit (10) for Γ.

arguments of the quark propagators. The running quark mass can then be replaced by a typical constituent quark mass without seriously affecting the calculation. To this end, we replace the confining quark propagator G by the propagator for a free fermion of mass m_q, by setting $A(s) = 2$ and $B(s) = 2m_q$. The dashed curves in Fig. 5 are the resulting plots of f against the nucleon mass M for various effective quark masses m_q. For these calculations, the nucleon mass must be less than the ionisation energy $m_q + m_d$, otherwise the integral in (3) encounters poles in either the diquark or quark propagators on the negative real s-axis. The calculations were done using the gaussian fit (10) for Γ and with α set equal to an optimal value for the free propagators of $m_d/(m_d + m_q)$, allowing M to take values up to $m_d + m_q$.



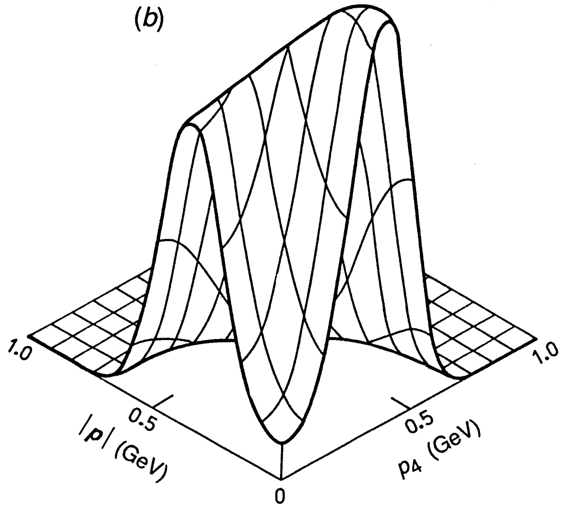


Fig. 6. (*a*) The square $\Psi^{\dagger}\Psi$ of the nucleon form factor at f = 0.0167 GeV, a nucleon mass M = 0.6 GeV and momentum partitioning $\alpha = \frac{2}{3}$. The vertical scale is arbitrary. (*b*) Same as Fig. 6*a*, but with the quark propagator functions A and B set equal to constants to give an effective quark mass of 0.418 GeV.

Comparing with the results of the full calculation, we read off a slowly varying constituent mass of 0.396 to 0.418 GeV over a range of nucleon masses from 0.4 to 0.6 GeV. Finally, we plot in Fig. 6*b* the square of the nucleon form factor for a fixed constituent quark mass of 0.418 GeV, nucleon mass of 0.6 GeV and $\alpha = \frac{2}{3}$ for comparison with the full calculation. The plots are similar, confirming our assumption that the contribution to the integral in (3) from the quark propagator is dominated by only a narrow range of the running quark mass. A naive linear extrapolation gives a constituent quark mass of about 0.45 GeV at a chiral nucleon mass near the physical value of 0.9 GeV. The resulting nucleon mimics a weakly bound quark-diquark state with a binding energy (in terms of equivalent mass, unconfined fermions) of about 0.1 GeV. Confinement then arises through a momentum dependence of the effective quark mass in the full quark propagator. A similar picture of effective weak binding was observed for the quarks within the $J^P = 0^+$ diquark (Praschifka *et al.* 1988).

4. Conclusions

We have made numerical calculations of the $\frac{1}{2}^+$ nucleon mass using a QCD based formalism developed previously. In this formalism, the problem of three valence quarks interacting via gluon exchange is reduced to a quark-diquark two-body problem. The mass calculation then amounts to finding the solution of the integral equation (3) for the nucleon-quark-diquark form factor. The calculation is done in Euclidean 4-space.

At this stage we study the nucleon in the chiral limit, in which the bare current mass of the quark is zero. The quarks acquire a dynamical constituent mass via gluon dressing. Our dressed quark propagator $G(q^2)$ is confining in the sense that it should have no poles on the negative q^2-axis. However, analytic fits to $G(q^2)$, required for the solution of the nucleon integral equation, point consistently to the existence of singularities in G deep in the complex q^2-plane. These singularities appear to be driven by the inflection point of the function $B(s)$ in Fig. 2*b*, suggesting a possible connection with chiral symmetry restoration as Euclidean q^2 increases. Although the precise meaning of these singularities is unclear, it is clear that they have repercussions for choosing an appropriate contour of integration in the nucleon integral equation. The calculated nucleon mass is a function of one free parameter, the diquark form factor normalisation f, which is so far undetermined (see the Postscript below). For sufficiently small nucleon masses, singularities in the kernel $\mathcal{K}_M$ of the nucleon integral equation (3) remain clear of the real q_4-axis, leading to an unambiguous choice of contour. For the higher nucleon mass values, the contour of integration is determined by demanding that the nucleon mass be a continuous function of f.

In this paper we have restricted our numerical calculations to nucleon mass values which do not require a deformation of the contour off the real axis. This mass range does not include the desired chiral nucleon mass of about 1 GeV, though there is nothing to prevent an appropriate extension of the calculation to include this value. For nucleon masses up to 0.657 GeV we have calculated the functional dependence of the nucleon mass on the diquark form factor normalisation f, and find that the nucleon mass is very sensitive to small changes in f. Like the diquark form factor, the nucleon form factor $\Psi(p)$ is sharply peaked about a non-zero value of its argument, suggesting that the quark and diquark within the nucleon propagate with a prefered momentum. We have exploited this idea to estimate the constituent mass of the third quark within the nucleon, obtaining a value of 0.396 to 0.418 GeV for nucleon masses of 0.4 to 0.6 GeV. A linear extrapolation to a more realistic nucleon mass of 0.9 GeV suggests a constituent quark mass of about 0.45 GeV.

It is clear that no exact QCD based calculation of the low energy hadron spectrum is currently possible. However, the approximations to QCD leading to our analysis are mainly well defined, and in certain cases, can in principle be avoided. For instance, the diquark is a $\overline{3}$ colour state and should have a confining propagator analogous to the confining quark propagator. It should be a straightforward matter to correct the diquark propagator with gluon dressing. Also, virtual quark loops could be included as nucleon self-dressing by virtual pions. Other missing diagrams are those including crossed propagators between valence quarks, though it is not clear how one could correct for these, or whether the omission is a serious one. Inclusion of the vector diquark as well as the scalar diquark is also possible with no further technological breakthroughs, but it is unlikely that this would seriously alter our results.

To summarise, we have made initial numerical calculations of the nucleon mass from a fully covariant QCD based treatment of hadron structure, and find that the analysis admits the existence of quark-diquark bound states. Our numerical calculations are extremely economical in terms of computer time compared with the analogous quenched calculations of LGT. Obtaining a single point in Fig. 5, together with the nucleon form factor takes about 15 minutes of CPU time on a Prime-9955, which is equivalent to less than 10 seconds on a CRAY I.

Postscript

Equation (1) has now also been derived using functional integral calculus (FIC) methods (Cahill 1989, present issue p. 171), and we find that $f^2 = f_0[\Gamma]^2/3$, where $f_0[\Gamma]$ is the functional defined in Cahill *et al.* (1987). This then gives $f = 0.057$ GeV for the Γ used herein, and from Fig. 5 we would then expect a bare nucleon chiral mass of ≈ 1.2 GeV. Dressing of such a bare nucleon by pions would lower the mass typically by some 200 MeV (see Pearce and Afnan 1986).

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