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Magnetic Susceptibility of Neutron Matter Using Skyrme Forces

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Abstract

The energy per particle and magnetic susceptibility in neutron matter are calculated using the Skyrme and modified Skyrme interactions. Results show a kind of ferromagnetic transition beyond the Fermi momentum $k_n \sim 1\cdot0\ \mathrm{fm}^{-1}$. It is concluded that such behaviour is a general phenomenon instead of a result due to the choice of a given effective interaction.

1. Introduction

It has been shown by Haensel (1975) and Moszkowski (1970) that the ratio of magnetic susceptibility of free neutron matter to that for neutron matter, χ_F/χ, is an increasing function of density when calculated with realistic two-body interactions such as the Reid (1968) hard core potential and Sawada–Wong (1972) potential. More recently Behera and Satpathy (1979) calculated χ_F/χ with their own effective interactions and found that χ_F/χ exhibits a decreasing tendency beyond the Fermi momentum $k_n = 1\cdot5\ \mathrm{fm}^{-1}$, similar to the results with a non-local separable potential by Mongan (1969). The purpose of the present paper is to see whether such a tendency is due to the inherent structure of the particular effective nucleon–nucleon interaction by Behera and Satpathy (1979), or whether it is also true with other effective interactions currently in use.

In recent years the Skyrme (1959) interaction and its various sets have been widely used to study the problems of nuclear matter, finite nuclei and heavy-ion scattering due to its simplicity in structure for the inclusion of δ-function terms. In view of this we have chosen this interaction, especially set III and the modified Skyrme interaction due to Tondeur (1983), to calculate χ_F/χ at various Fermi momenta k_n. In doing so, the energy per particle and the single particle potential are also calculated as required. In Section 2 we deduce the expressions for the energy per particle, the single particle potential and χ_F/χ with the Skyrme forces in the framework of the energy density formalism (EDF) of Negele and Vautherin (1972). The numerical results and the conclusions are discussed in Sections 3 and 4.

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2. Energy per Particle

In neutron matter, the density ρ_n is related to the Fermi momentum k_n by the relation

$$\rho_n = \frac{1}{3\pi^2} k_n^3 . \tag{1}$$

In first order perturbation theory, the potential energy of neutron matter is given by

$$\langle V \rangle = \tfrac{1}{2} \sum_{i,j<n} [\langle ij|\, v\,|ij\rangle - \langle ij|\, v\,|ji\rangle] ,$$

where the sum is over momenta as well as spin and isospin. In terms of the density matrix $\rho(\boldsymbol{r}_1, \boldsymbol{r}_2)$ and the product of densities $\rho(\boldsymbol{r}_1)$ and $\rho(\boldsymbol{r}_2)$, the direct part of the potential energy is given by

$$\begin{aligned} \langle V \rangle_D &= \tfrac{1}{2} \sum_{i,j<n} \langle ij\,|\,v\,|\,ij\rangle \\ &= \tfrac{1}{2} \int \rho_n(\boldsymbol{r}_1)\, \rho_n(\boldsymbol{r}_2)\, v(\boldsymbol{r}_1 - \boldsymbol{r}_2)\, d^3r_1\, d^3r_2 , \end{aligned} \tag{2}$$

and the exchange part by

$$\begin{aligned} \langle V \rangle_E &= \tfrac{1}{2} \sum_{i,j<n} \langle ij\,|\,v\,|\,ji\rangle \\ &= \tfrac{1}{2} \int \rho_n^2(\boldsymbol{r}_1, \boldsymbol{r}_2)\, v(\boldsymbol{r}_1 - \boldsymbol{r}_2)\, d^3r_1\, d^3r_2 . \end{aligned} \tag{3}$$

Following Negele and Vautherin (1972), we expand as

$$\begin{aligned} \rho_n^2(\boldsymbol{r}_1, \boldsymbol{r}_2) &= \rho_{sl}^2(k_n r) \rho_n^2(\boldsymbol{R}) + \rho_{sl}(k_n r)\, g_{sl}(k_n r) r^2 \\ &\quad \times [\tfrac{1}{2} \rho_n(\boldsymbol{R})\, \nabla^2 \rho_n(\boldsymbol{R}) - 2\rho_n(\boldsymbol{R})\, \mathcal{T}_n(\boldsymbol{R}) + \tfrac{6}{5} k_n^2 \rho_n^2(\boldsymbol{R})] , \end{aligned} \tag{4}$$

$$\begin{aligned} \rho_n(\boldsymbol{r}_1)\, \rho_n(\boldsymbol{r}_2) &= \rho_n^2(\boldsymbol{R}) + g_{sl}(k_n r) r^2 \\ &\quad \times [\tfrac{1}{2} \rho_n(\boldsymbol{R})\, \nabla^2 \rho_n(\boldsymbol{R}) - \tfrac{1}{2} \{\nabla \rho_n(\boldsymbol{R})\}^2] , \end{aligned} \tag{5}$$

where

$$\boldsymbol{R} = \tfrac{1}{2}(\boldsymbol{r}_1 + \boldsymbol{r}_2), \qquad \boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2 ,$$

$$\rho_{sl}(k_n r) = \frac{3 j_1(k_n r)}{k_n r} ,$$

$$g_{sl}(k_n r) = \tfrac{35}{2} j_3(k_n r)/(k_n r)^3 ,$$

and where $j_1(k_n r)$ and $j_3(k_n r)$ are the spherical Bessel functions, and $\mathcal{T}_n(\boldsymbol{R})$ is the kinetic energy density. In neutron matter having uniform density, we can write $\rho_n(\boldsymbol{R}) = \rho_n$ and derivatives of ρ_n are zero. With the Skyrme force given by Vautherin and Brink (1972),

$$\begin{aligned} v(\boldsymbol{r}_1, \boldsymbol{r}_2) = {} & t_0(1 + x_0 P_\sigma)\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \\ & + \tfrac{1}{2} t_1 [\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) k^2 + k'^2 \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)] \\ & + t_2 \boldsymbol{k}' \cdot \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \boldsymbol{k} + \tfrac{1}{6} t_3 (1 + P_\sigma) \\ & \times \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \rho((\boldsymbol{r}_1 + \boldsymbol{r}_2)/2) \\ & + \mathrm{i} W_0 (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \boldsymbol{k}' \times \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \boldsymbol{k}, \end{aligned}$$

the potential energy $\langle V \rangle$ through equations (2) and (3) becomes

$$\langle V \rangle = \frac{t_0(1 - \chi_0)\rho_n^2}{4} + \tfrac{1}{8} \rho_n \mathcal{T}_n (t_1 + 3t_2). \tag{6}$$

Adding the kinetic energy term to equation (6), we get the total energy density of neutron matter as

$$\begin{aligned} H_n^{SK}(\boldsymbol{r}) = H_n^{SK} = {} & \frac{3\hbar^2}{10m} k_n^2 \rho_n + \frac{t_0(1 - x_0)\rho_n^2}{4} \\ & + \tfrac{1}{8} \rho_n \mathcal{T}_n (t_1 + 3t_2), \end{aligned} \tag{7}$$

where we have used $\mathcal{T}_n = \frac{3}{5} k_n^2 \rho_n$. The energy per particle for neutron matter is

$$\begin{aligned} E^{SK}(k_n) & = \frac{H_n^{SK}}{\rho_n} \\ & = \frac{3\hbar^2}{10m} k_n^2 + \frac{t_0(1 - x_0)\rho_n}{4} + \tfrac{1}{8} \mathcal{T}_n (t_1 + 3t_2). \end{aligned} \tag{8}$$

The gradient terms make no contribution in neutron matter.

The modified Skyrme (MSK) interaction due to Tondeur (1983) is given by

$$\begin{aligned} v(\boldsymbol{r}_1, \boldsymbol{r}_2) = {} & t_0(1 + x_0 P_\sigma)\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \\ & + \tfrac{1}{2} t_1 (1 + x_1 P_\sigma)[\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) k^2 + k'^2 \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)] \\ & + t_2 (1 + x_2 P_\sigma) \boldsymbol{k}' \cdot \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \boldsymbol{k} \\ & + \tfrac{1}{6} t_3 (1 + x_3 P_\sigma) \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \rho^\nu \\ & + \mathrm{i} W_0 (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \boldsymbol{k}' \times \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \boldsymbol{k}, \end{aligned} \tag{9}$$

where P_σ is the spin-exchange operator having a value of ± 1 for odd and even states respectively, and ν is a density dependence parameter taken to be $\frac{1}{3}$.

The values of other constants are as in Tondeur (1983). On evaluation in the same manner as for the SK potential, we get

$$H_{\rm n}^{\rm MSK}(\boldsymbol{r}) = \frac{3\hbar^2}{10m}k_{\rm n}^2\rho_{\rm n} + \frac{t_0\rho_{\rm n}}{12\pi^2}(1-x_0)k_{\rm n}^3 + \frac{k_{\rm n}^5\rho_{\rm n}}{40\pi^2}[t_1(1-x_1)+3t_2(1+x_2)] + \frac{t_3k_{\rm n}^4\rho_{\rm n}}{24(3\pi^2)^{\frac{4}{3}}}, \tag{10}$$

and the expression for the energy per particle is

$$E^{\rm MSK}(k_{\rm n}) = \frac{3\hbar^2k_{\rm n}^2}{10m} + \frac{t_0}{12\pi^2}(1-x_0)k_{\rm n}^3 + \frac{k_{\rm n}^5}{40\pi^2}[t_1(1-x_1)+3t_2(1+x_2)] + \frac{t_3k_{\rm n}^4}{24(3\pi^2)^{\frac{4}{3}}}. \tag{11}$$

It may be noted that the contribution of the term containing the parameter t_3 is zero in the case of the SK potential, whereas there is a finite contribution from such terms for the MSK potential. This is also reflected in all further calculations below.

(a) Expression for Single Particle Potential

Following Vautherin and Brink (1972) the single particle wavefunction ψ_i satisfies the Schrödinger equation

$$\left(-\nabla\cdot\frac{\hbar^2}{2m}\nabla\psi_i + u_i\psi_i\right) = \epsilon_i\psi_i, \tag{12}$$

with

$$u_i = \frac{\partial H}{\partial\rho_i} - \nabla\cdot\frac{\partial H}{\partial(\nabla\rho_i)}.$$

The energy density $H(\boldsymbol{r})$ is usually expressed in terms of density functionals $A(\rho)$, $B(\rho)$ and $C(\rho)$ as

$$H(\boldsymbol{r}) = \frac{\hbar^2}{2m}\mathcal{T} + A(\rho) + B(\rho)\mathcal{T} + C(\rho)(\nabla\rho)^2 = \left(\frac{\hbar^2}{2m} + B(\rho)\right)\mathcal{T} + A(\rho) + C(\rho)(\nabla\rho)^2. \tag{13}$$

In neutron matter we have

$$H_{\rm n}(\boldsymbol{r}) = \frac{\hbar^2}{2m}\mathcal{T}_{\rm n} + A(\rho_{\rm n}) + B(\rho_{\rm n})\mathcal{T}_{\rm n}, \tag{14}$$

$$\frac{\partial H(\boldsymbol{r})}{\partial\mathcal{T}_{\rm n}} = \frac{\hbar^2}{2m} + B(\rho_{\rm n}). \tag{15}$$

The effective mass m^* is defined as

$$\frac{\hbar^2}{2m^*} = \frac{\partial H}{\partial \mathcal{T}_\mathrm{n}} . \tag{16}$$

Substituting the values of $\hbar^2/2m^*$ in equation (12) using equations (15) and (16), we get the single particle potential

$$u(k_i) = u_i + B(\rho_\mathrm{n})k_i^2 , \tag{17}$$

with

$$u_i = \partial H/\partial \rho_\mathrm{n} .$$

For the SK interaction, the single particle potential becomes

$$u^{\mathrm{SK}}(k_i, k_\mathrm{n}) = \tfrac{1}{2}t_0(1-x_0)\rho_\mathrm{n} + \tfrac{1}{4}\mathcal{T}_\mathrm{n}(t_1+3t_2) + \tfrac{1}{8}\rho_\mathrm{n}(t_1+3t_2)k_i^2 , \tag{18}$$

and for the MSK interaction it becomes

$$u^{\mathrm{MSK}}(k_i, k_\mathrm{n}) = \tfrac{1}{2}t_0(1-x_0)\rho_\mathrm{n} + \tfrac{1}{4}\mathcal{T}_\mathrm{n}[t_1(1-x_1)+3t_2(1+x_2)] + \tfrac{7}{48}t_3(1-x_3)\rho_\mathrm{n}^{\frac{4}{3}} + \tfrac{1}{8}\rho_\mathrm{n}[t_1(1-x_1)+3t_2(1+x_2)]k_i^2 . \tag{19}$$

(b) Magnetic Susceptibility of Neutron Matter

Following the notation of Haensel (1975), the total energy of a system of N neutrons in the presence of an external magnetic field H can be written as

$$E_H(N,\alpha) = E_{\mathrm{kin}}(N,\alpha) + E_{\mathrm{pot}}(N,\alpha) - \mu_\mathrm{n} HN\alpha , \tag{20}$$

where the first two terms on the right-hand side are the kinetic and nuclear potential energy of the system and α is the spin excess parameter defined as

$$\alpha = (N\uparrow - N\downarrow)/N . \tag{21}$$

Here $N\uparrow$ and $N\downarrow$ are the number of neutrons with spin up and spin down with respect to the direction of the applied field. Assuming that α is a small parameter, the energy per particle $E_H(N,\alpha)/N$ is expanded up to terms containing α^2 and we get

$$E_H(N,\alpha)/N = \epsilon_0 + \tfrac{1}{2}\epsilon_\sigma \alpha^2 - \mu_\mathrm{n} H\alpha . \tag{22}$$

For a fixed value of the spin excess parameter α_0, the total energy of the system reaches the minimum value in the ground state and we have from equation (22)

$$\alpha_0 = \frac{\mu_\mathrm{n} H}{\epsilon_\sigma} . \tag{23}$$

For a neutron excess, α_0, the magnetic moment per unit volume is $\alpha_0 \rho_n \mu_n$ and this gives

$$\chi = \mu_n^2 \rho_n / \epsilon_\sigma . \tag{24}$$

For a Fermi gas of free neutrons we have

$$\chi_F = 3\mu_n^2 \rho_n / 2\epsilon_n , \tag{25}$$

where $\epsilon_n = (\hbar^2/2m)k_n^2$. From (24) and (25) we get

$$\chi_F/\chi = \tfrac{3}{2}\epsilon_\sigma/\epsilon_n , \tag{26}$$

where the kinetic energy part of ϵ_σ is given by

$$\epsilon_\sigma^{\mathrm{kin}} = \tfrac{2}{3}\epsilon_n , \tag{27}$$

and the potential energy part is

$$\epsilon_\sigma^{\mathrm{pot}} = \epsilon_\sigma^{(0)\mathrm{pot}} + \Delta\epsilon_\sigma . \tag{28}$$

Here $\epsilon_\sigma^{(0)\mathrm{pot}}$ and $\Delta\epsilon_\sigma$ denote the non-rearrangement and rearrangement parts respectively.

Following the procedure of Haensel (1975), with the SK interaction we get

$$\epsilon_\sigma^{(0)\mathrm{pot}} = -\tfrac{1}{2}\rho_n t_0(1-x_0) , \tag{29}$$

$$\Delta\epsilon_\sigma = 0 , \tag{30}$$

and with the MSK interaction we get

$$\epsilon_\sigma^{(0)\mathrm{pot}} = -\tfrac{1}{2}\rho_n t_0(1-x_0) - \tfrac{5}{36}\mathcal{T}_n[\tfrac{4}{5}t_1(1-x_1) - \tfrac{24}{5}t_2(1+x_2)] - \tfrac{1}{12}t_3(1-x_3)\rho_n^{\frac{4}{3}} , \tag{31}$$

$$\Delta\epsilon_\sigma = -\frac{t_3(1-x_3)k_n^4}{216(3\pi^2)^{\frac{4}{3}}} . \tag{32}$$

Here $\Delta\epsilon_\sigma \neq 0$ is due to the non-vanishing of the t_3 term as stated earlier.

Using equations (27)–(32) in equation (26), we get

$$\left(\frac{\chi_F}{\chi}\right)^{\mathrm{SK}} = 1 - \frac{3}{2}\frac{\rho_n}{\epsilon_n}\left(\frac{t_0(1-x_0)}{2} + \frac{k_n^2}{15}(t_1 - 6t_2)\right), \tag{33}$$

$$\left(\frac{\chi_F}{\chi}\right)^{\mathrm{MSK}} = 1 - \frac{3}{2}\frac{\rho_n}{\epsilon_n}\left(\frac{t_0(1-x_0)}{2} + \frac{k_n^2}{15}\{t_1(1-x_1) - 6(1+x_2)t_2\} + \frac{19}{216}t_3(1-x_3)\rho_n^{\frac{1}{3}}\right). \tag{34}$$

Table 1. Energy per particle ϵ_n (in MeV) and ratio χ_F/χ as a function of Fermi momentum k_n for three potentials

k_n (fm^{-1})	SEI[A]	Present calculations SK	MSK
	(*a*) ϵ_n		
0·2	0·3809	0·456	0·4287
0·4	1·1958	1·658	1·5136
0·6	2·1560	3·368	3·120
0·8	3·2467	5·369	5·366
1·0	4·7123	7·477	8·473
1·2	7·0445	9·549	13·066
1·4	10·9550	10·498	19·871
1·6	17·3366	13·298	29·868
1·8	27·2206	15·001	44·261
2·0	41·7343	16·742	64·765
2·2	62·0695	18·755	92·161
2·4	89·4554	21·379	129·189
2·6	125·1469	25·071	177·654
2·8	170·4187	30·416	239·875
3·0	226·5637	38·140	318·391
	(*b*) χ_F/χ		
0·2	1·419	1·150	1·2600
0·4	1·716	1·293	1·4581
0·6	1·939	1·421	1·5694
0·8	2·092	1·526	1·5692
1·0	2·175	1·601	1·4327
1·2	2·188	1·638	1·1353
1·4	2·132	1·630	0·6521
1·6	2·007	1·569	
1·8	1·813	1·448	
2·0	1·554	1·259	
2·2	1·233	0·99	

[A] Behera and Satpathy (1979).

3. Results and Discussion

The results of our calculation for the energy per particle ϵ_n in neutron matter for different values of the Fermi momentum k_n are given in Table 1*a* for the two SK (set III) and MSK interactions, along with those for the simple effective interaction SEI of Behera and Satpathy (1979). These results are compared in Fig. 1 with the results obtained with the local realistic interactions of Sawada and Wong (SW) (1972). It is found that the ϵ_n versus k_n curve has a slow rise up to $k_n = 1\cdot5\ \mathrm{fm}^{-1}$ for all four potentials, but beyond $k_n = 1\cdot5\ \mathrm{fm}^{-1}$, the slopes of the curves steepen and are similar in nature for all potentials, except SK. The reason is that for the SK potential, the density dependence part involving the t_3 term, with $t_3 = 14000\ \mathrm{MeV\,fm}^6$, makes no contribution whereas the same is not true for MSK. This is also manifested in the effective mass calculations giving a large value of $m^*/m = 0\cdot623$ for SK compared with $0\cdot583$ for SEI at $k_n = 3\cdot0\ \mathrm{fm}^{-1}$.

The results of our present calculation for χ_F/χ using the SK and MSK potentials are given in Table 1*b* and also plotted in Fig. 2 as a function of k_n, together with those for the Reid hard core (RHC) potential, the Mongan

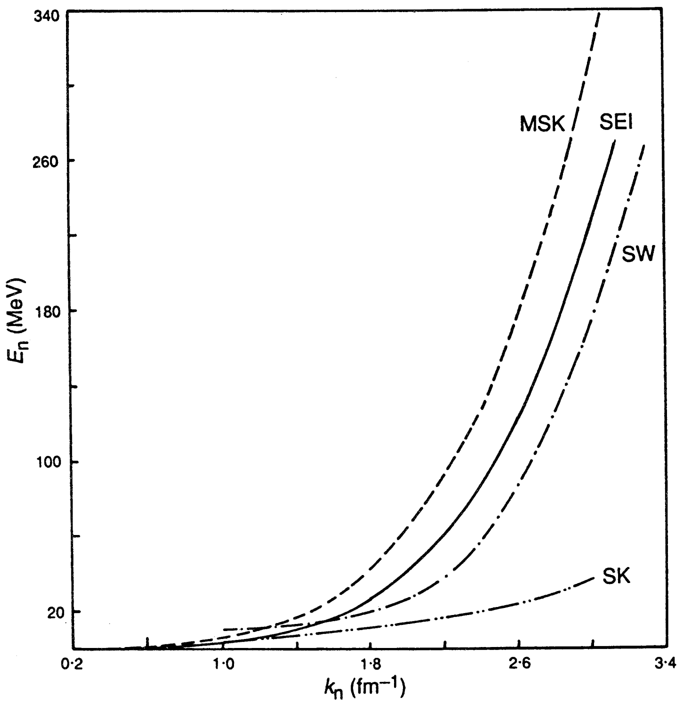

Fig. 1. Energy per particle as a function of Fermi momentum. The symbols SK and MSK are defined in Section 2, and SEI and SW in Section 3.

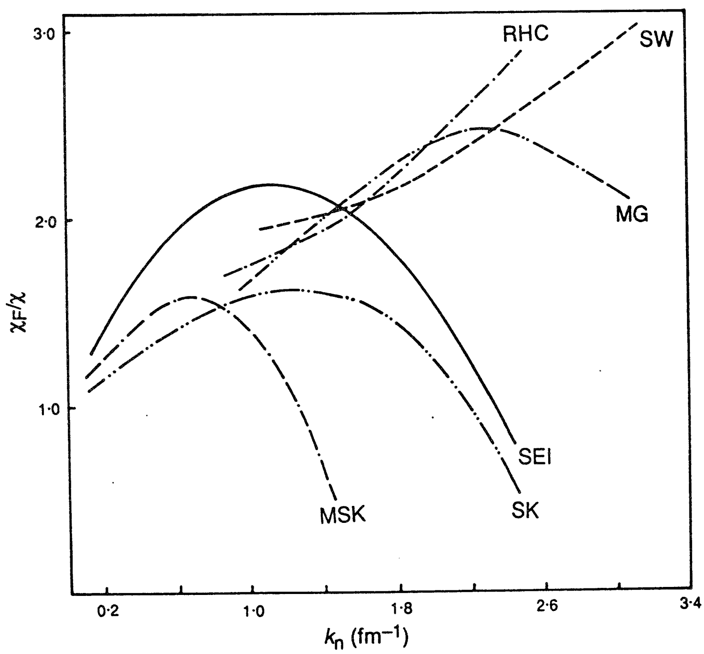

Fig. 2. Ratio of magnetic susceptibilities as a function of Fermi momentum. The symbols SK and MSK are defined in Section 2, and SEI, SW, RHC and MG in Section 3.

(MG) non-local potential and the Sawada-Wong (SW) potential. The results for χ_F/χ by Behera and Satpathy (1979) using their SEI are also shown in Fig. 2 for comparison. It is observed that χ_F/χ is an increasing function of k_n for the RHC and SW interactions above $k_n \approx 1 \cdot 0\ \mathrm{fm}^{-1}$. Beyond $k_n \approx 1 \cdot 0\ \mathrm{fm}^{-1}$, however, for the SK and MSK interactions we find that χ_F/χ decreases steadily showing the tendency for the ferromagnetic transition in conformity with the SEI of Behera and Satpathy (1979) and also with the MG potential due to Haensel (1975). Previously it was thought that the decreasing tendency of χ_F/χ with the SEI at higher densities was due to the particular choice of the potential, i.e. a single density dependence δ-function repulsion and a single gaussian-type attractive term, with the latter dominating at higher densities.

4. Conclusions

From our investigation we find that a decrease in χ_F/χ at higher densities is definitely present when calculations are made with the widely used Skyrme interactions consisting of several δ-function terms, and we thereby conclude that this tendency of the ferromagnetic transition in neutron matter is a general result rather than one due to a particular choice of the effective interaction.

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