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Meson Mass Spectrum from First Order Static Cavity Wavefunctions

Lloyd C. L. Hollenberg and Bruce H. J. McKellar

Research Centre for High Energy Physics,
School of Physics, University of Melbourne,
Parkville, Vic. 3052, Australia.

Abstract

Large-basis $O(g)$ static cavity wavefunctions, containing bremsstrahlung and quark-sea states, previously fitted to the light quark meson sector, are applied to the ground-state meson spectrum in general. The only parameters of the model in the heavy quark sector are the quark masses m_c and m_b which we fit to D and B states. Predictions for the meson mass spectrum are given and are found to be a significant improvement over the predictions of the original MIT model.

1. Introduction

The MIT concept of non-interacting valence quarks confined to a static cavity (DeGrand *et al.* 1975; Lee 1979; Close and Horgan 1980, 1981) has recently been extended to include the effects of higher order states in the wavefunction due to quark–gluon interactions (Hollenberg and McKellar 1989*a*). Features of the resulting large-basis model (LBM) include an expanded $O(g)$ ground-state wavefunction consisting of $j=\frac{1}{2}$ quarks and $l=1$ gluons coupled to a simple prescription for dealing with centre-of-mass (CM) corrections and plane wave projection. After fitting parameters to the pion charge radius and π, ρ and K masses the model gives reasonable values for K* and ϕ masses. The kaon charge radius is also described well by the model.

In this paper we determine the charm and bottom quark masses for the model by fitting to D(1867) and B(5273) meson states and compute the remaining meson masses for the ground-state spectrum.

2. Large-basis Model

The basic starting point of the LBM is the construction of large-basis wavefunctions (Hollenberg and McKellar 1989*b*) which are (schematically) of the form

$$|\psi\rangle = |q\bar{q}\rangle + \sum_{\{n\}}^{N_B}\sum_{\alpha} a(\{n\},\alpha)\,|q\bar{q}G\rangle_{\{n\},\alpha} + \sum_{\{m\}}^{N_B}\sum_{\beta} b(\{m\},\beta)\,|q\bar{q}q\bar{q}G\rangle_{\{m\},\beta}\,, \tag{1}$$

where $|q\bar{q}\rangle$ is the usual valence state from which the higher states evolve. For the bremsstrahlung ($|q\bar{q}G\rangle$) and vacuum fluctuation states ($|q\bar{q}q\bar{q}G\rangle$) respectively, the sets $\{n\}$ and $\{m\}$ contain the mode numbers of the quarks and gluons in terms of the basis of states in the static cavity, whilst the remaining quantum

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numbers (flavour, orbital, spin and colour) are denoted by α and β. The amplitudes $a(\{n\},\alpha)$ and $b(\{m\},\beta)$ are calculated from QCD in the static cavity approximation for which the quark and gluon fields are known. The basis size N_B serves as the cut-off parameter; i.e. a given calculation at basis size N_B includes all modes $n_i, m_j \le N_B$ for $n_i \in \{n\}$ and $m_j \in \{m\}$. The wavefunction is renormalised by fitting to physical quantities such as masses and charge radii, which are independent of the cut-off scheme, thereby allowing the model parameters to aquire a basis size dependence. When calculating the ground-state energy of the wavefunction (1) the inclusion of self-energy terms, transverse and Coulomb, is implicit. Furthermore, only those states giving rise to connected energy shifts are considered. The single gluon exchange contributions provide the mechanism for the splitting of scalar and vector states.

Associated with the static cavity approximation are the problems of centre-of-mass corrections and the construction of plane wave states. To combat these difficulties the bound state mass is approximated by

$$M^2 = E_\psi^2 - \langle P^2 \rangle , \tag{2}$$

where E_ψ is the ground-state energy of (1) and $\langle P^2\rangle$ is given by a sum over shell momenta weighted according to the Fock state probabilities. Plane wave states are constructed using a wavepacket projection

$$|\psi\rangle = \int d^3p \, \frac{\phi(p)}{2E(p)} \, |\psi(p)\rangle , \tag{3}$$

with a Gaussian parametrisation of the distribution amplitude

$$\phi(p) = \left(\frac{2}{\pi \Lambda^2}\right)^{\frac{3}{4}} \left(\frac{2E(p)}{(2\pi)^3}\right)^{\frac{1}{2}} e^{-p^2/\Lambda^2} . \tag{4}$$

The parameter Λ is fixed by the consistency condition that $\phi(p)$ should give the same value of $\langle P^2\rangle$ as used in (2), i.e.

$$\langle P^2\rangle = (2\pi)^3 \int d^3p \, \frac{|\phi(p)|^2}{2E(p)} \, p^2 = \tfrac{3}{4}\Lambda^2 . \tag{5}$$

LBM parameters such as the effective quark–gluon coupling $\alpha = g^2/4\pi$, the confinement pressure B and the strange quark mass are fitted to π, ρ and K masses. The remaining degree of freedom, contained in the zero-point parameter, is taken up by fitting to the pion charge radius.

3. Results

After fitting charm and bottom quark masses to D and B states, the rest of the ground-state meson masses are calculated and given in Table 1, including previous results for the light mesons (Hollenberg and McKellar 1989*a*). The errors quoted for the LBM masses are due to the experimental uncertainty in the pion charge radius in the fitting procedure. For comparison, we have included results from the potential model and the MIT model. The potential model results quoted are from Godfrey and Isgur (1985); their model appears to be the most successful at fitting the entire spectrum

Table 1. Theoretical calculations of the ground-state meson spectrum and comparison with experiment (all values in MeV)

Potential model (PM) results given are those of Godfrey and Isgur (1985). Experimental values for K, K*, D, D* and B are taken to be the average of charged and neutral states. LBM mass values, together with the percentage dependence of these quantities on the basis size (in parenthesis), are at $N_B = 16$

Particle	Experiment	LBM		PM	MIT
π	139	139	(0%)	150	139
ρ	770±3	770	(0%)	770	770
K	496	496	(0%)	470	496
K*	894	904·4±0·1	(−0·019%)	900	917
ϕ	1019·5±0·1	1040·6±0·2	(−0·036%)	1020	1071
D	1867	1867	(0%)	1880	1867
D*	2009	2009·4±2·1	(−0·006%)	2040	2030
D_s	1970·5±2·5	1999·1±0·5	(−0·010%)	1980	2043
η_c	2981·0±2·0	3007·2±2·9	(−0·048%)	2970	3151
J/ψ	3096·9±0·1	3081·9±0·4	(−0·046%)	3100	3241
B	5273	5273	(0%)	5310	5273
Υ	9460·0±0·2	9402·4±16·9	(−0·050%)	9460	9689

Table 2. Some predictions (MeV) in the ground-state charm and bottom quark sector, for states which have not yet been established experimentally, and a comparison with results of the potential and MIT models

Connected LBM mass values, together with the percentage dependence of these quantities on the basis size (in parenthesis), are at $N_B = 16$

Particle	Experiment	LBM		PM	MIT
D_s^*	2111·2±1·8	2128·5±1·9	(−0·015%)	2130	2186
η_b	—	9393·0±19·3	(−0·050%)	9400	9653
B*	5330±5	5313·6±1·5	(−0·004%)	5370	5331
B_s	5376?	5397·8±1·4	(−0·004%)	5390	5440
B_s^*	—	5435·1±0·3	(−0·003%)	5450	5492
B_c	—	6288·4±7·3	(−0·033%)	6270	6464
B_c^*	—	6312·6±5·4	(−0·032%)	6340	6507

Table 3. Squared mass differences (GeV²) and a comparison with results of the potential and MIT models

LBM values are at $N_B = 16$

Particle	Experiment	LBM	PM	MIT
$(\rho)^2-(\pi)^2$	0·57	0·57	0·57	0·57
$(K^*)^2-(K)^2$	0·55	0·57	0·59	0·59
$(D^*)^2-(D)^2$	0·55	0·55	0·63	0·64
$(D_s^*)^2-(D_s)^2$	0·58?	0·53	0·62	0·60
$(B^*)^2-(B)^2$	0·55?	0·43	0·64	0·62
$(B_s^*)^2-(B_s)^2$	—	0·40	0·65	0·57
$(B_c^*)^2-(B_c)^2$	—	0·31	0·88	0·56
$(J/\psi)^2-(\eta_c)^2$	0·71	0·45	0·79	0·58
$(\Upsilon)^2-(\eta_b)^2$	—	0·18	1·13	0·70

generally while incorporating relativistic effects, be it with a large number of parameters, for the light mesons. A direct comparison with the MIT model, fitted in exactly the same manner as the LBM, is also given.

Godfrey and Isgur also calculated a plethora of states, many of which have not been observed. In Table 2 we compare predictions for some of these states. Values for squared mass differences are presented in Table 3. Quark masses for the two models are given in Table 4.

Table 4. Quark masses (MeV) in the connected LBM, potential and MIT models

LBM values are at $N_B = 16$

Quark	LBM	PM	MIT
Up/down	0	220	0
Strange	215	419	303
Charm	1245	1628	1573
Bottom	4347	4977	4906

It can be seen immediately, from Table 1, that the LBM version of the meson spectrum is consistently more accurate than that of the MIT, and is at the same level of precision as the potential model. The basis size dependence shows that the non-fitted masses are essentially independent of N_B (to within about 0·05%). It is interesting that the LBM and potential model are in good agreement in Table 2. Where possible we have indicated recent experimental values but note that, according to the 1988 review by the Particle Data Group (Yost *et al.* 1988), the states $D_s^*(2110)$ and $B^*(5325)$ are not well established. The result (Côté *et al.* 1988) for $B_s(5376)$ has not been reviewed by the Particle Data Group. Since the model errors for these predictions should be comparable with those of Table 1, these results lend credibility to the assignments of $D_s^*(2110)$, $B^*(5325)$ and $B_s(5376)$.

The squared mass differences across the spectrum in Table 3 show that the LBM produces reasonable values for the spin splittings up to about the charm and bottom sectors where the disagreement with the potential model becomes evident. Even the MIT splittings, due to single gluon exchange only, appear to follow a more favourable trend for the heavy states. The reason that the LBM and MIT splittings differ is most likely due to the fact that the Fock state normalisation, present in the calculation of $\langle P^2 \rangle$, depends on J resulting in an extra source of spin splitting.

Overall, however, these results demonstrate that the implicitly relativistic LBM, with its scope for treating $O(g^2)$ QCD effects, does remarkably well in describing masses of the heavy as well as light meson systems.

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