Aust. J. Phys., 1989, **42**, 573–80

Measurements of the Radio Refractive Index Structure Parameter C_n^2 with a Microwave Refractometer in Tropical Latitudes

S. B. S. S. Sarma and P. K. Pasricha

Radio Science Division, National Physical Laboratory,
New Delhi 110012, India.

Abstract

The spatial and temporal variations of the radio refractive index lead to scattering of electromagnetic energy. Modern communication systems are subject to vagaries of the turbulence due to the radio refractive index fluctuations. Vertical resolution of radiosondes is insufficient to resolve the structures of clear air turbulence in the atmosphere. In a tropical country like India, aircraft measurements of radio refractive index fluctuations have been non-existent. Measurements on the fine structure information of radio refractivity have now been made at tropical latitudes, using an airborne microwave refractometer. The atmospheric turbulence structure parameter C_n^2, deduced from such height profiles of radio refractivity, has a lower value in local winter than in summer. These values of C_n^2 are large compared with the values obtained using routine radiosonde observations. Spectra of radio refractive index fluctuations at different heights and the scale size of the inhomogeneities have been estimated.

1. Introduction

The scattered power in a troposcatter system depends upon the spatial spectral distribution of radio refractive index fluctuations (inhomogeneities) appropriate to microwaves. The troposcatter system usually makes use of spectral wave numbers K of the inhomogeneities in the inertial sub-range of the turbulence spectrum. The (three-dimensional) spatial spectral distribution of the presumed isotropic turbulence in the inertial sub-range may be represented by $\phi(K) \sim C_n^2 K^m$ $(m = -11/3)$. The Tatarskii (1961, 1971) structure parameter C_n^2 is a measure of the severity of refractive index fluctuations. A one-dimensional turbulence spectrum $E(K)$ is given by $E(K) = 4\pi K^2 \phi(K)$. Thus, the refractive index spectrum $E(K)$ has the simple power-law dependence on K of $E(K) \sim K^{-5/3}$. The one-dimensional exponent $m = -5/3$ is the (spatial) spectral slope to the high wave number region of $E(K)$. There is large variability in the shape of the spatial spectra of radio refractive index fluctuations $\phi(K)$, deduced from a variety of radio propagation experiments. Such spatial spectra may take on the shapes of a gaussian or an exponential or a power-law distribution at higher wave numbers. The spatial spectra $\phi(K)$ are usually described by a simple power-law as $\phi(K) \sim K^m$. A range of values of the spatial spectral slope m from $-5/3$ to $-22/3$ has been reported from radio measurements with a median value close to $-11/3$ (Gjessing 1969; Gjessing *et al.* 1969; Gjessing and McCormick 1974). The higher values of the exponent m ($\sim -22/3$) probably

0004-9506/89/050573$03.00

indicate the presence of 'sharp' inhomogeneities or thin strata in the turbulent medium. A Kolmogorov spatial spectral slope m of $-11/3$ is generally adopted in electromagnetic scattering problems. It may be noted that the definition of the structure parameter C_n^2 is strictly applicable to the radio refractive index spectrum $\phi(K)$ with the slope of $-11/3$. The structure parameter C_n^2 may be estimated from the variance of radio refractive index fluctuations appropriate to the scattering volume (Gossard 1977; Van Zandt *et al. et al.* 1978). The radio refractive index variance is known to decrease with height (Lane 1968; Borresen and Gjessing 1969; Crane 1980; Gage and Balsley 1980). Values of C_n^2 from troposcatter observations at a height of 1·5 km lie in the range 10^{-12} to $10^{-16}\ \mathrm{m}^{-2/3}$ (Gossard 1977; Ecklund *et al.* 1977; Van Zandt *et al.* 1978; Chadwick and Moren 1980). From the literature, a particular choice for a C_n^2 value in a troposcatter system is not clear. For instance, Gjessing (1969) used a C_n^2 value of $10^{-12}\ \mathrm{m}^{-2/3}$ and Wickerts and Nilsson (1973) used $10^{-16}\ \mathrm{m}^{-2/3}$. In general, the C_n^2 values show a diurnal pattern and also decrease with height.

The atmospheric turbulence parameters (C_n^2 and the spatial spectral slope m) may be determined by in-situ measurements of radio refractive index fluctuations using an airborne microwave refractometer. This paper presents an initial analysis of a limited set of observations taken during the local summer and winter conditions in the northern plains of India. The height profiles of the variance of radio refractive index fluctuations $\langle \Delta n^2 \rangle$ are obtained. A power spectral analysis of the temporal radio refractive index data is carried out to obtain the spatial spectral slopes (to the high frequency region of the spectra) at different heights in the turbulent medium. A 'break-point' frequency in the measured temporal spectrum of radio refractive index fluctuations gives an estimate of the scale of the inhomogeneities. Finally, from the formal definition of the structure parameter C_n^2, height profiles of C_n^2 are obtained using the measured values of $\langle \Delta n^2 \rangle$. A representative set of height profiles of C_n^2 for the local summer and winter conditions are thus obtained. These are compared with the values derived from the simultaneous radiosonde observations, using the mixing length turbulence theory.

2. Theory

The radio refractive index structure parameter can be given by

$$C_n^2 = K_n \langle \Delta n^2 \rangle L_0^{-2/3}, \tag{1}$$

where $\langle \Delta n^2 \rangle$ is the variance of the radio refractive index fluctuations at a given height, L_0 is the outer scale of turbulence (Tatarskii 1971) characteristic of the inertial subrange and K_n a normalisation constant. For all practical purposes, $K_n L_0^{-2/3}$ is just an empirical constant and the 2/3 law holds good until the vertical length scale reaches L_0. The effective size of the inhomogeneities responsible for the scattering is given by the first Fresnel zone at the receiver, i.e. $(\lambda L)^{1/2}$, λ being the observing wavelength and L the radio pathlength. The troposcatter experiments in the S-band and at higher frequencies are characterised by an outer scale of turbulence such that $L_0 \gg (\lambda L)^{1/2}$. This condition is very much satisfied for millimetre waves, even at 10 GHz. However, at UHF and lower frequencies we have $(\lambda L)^{1/2} \approx L_0$ (de Wolf 1975). The value of L_0 can be approximately assumed to be of the order of the thickness of the turbulent layer. However, there are no direct observations on this thickness in general and very little is known about the thickness distribution. A choice of the value of L_0 has been quite varied, ranging from 1 km (Ishimaru 1972) to 10 m (Van Zandt *et al.* 1978).

The normalisation constant K_n 'adjusts' the outer scale to an appropriate 'effective' scale size of the inhomogeneities. The outer scale of turbulence in optical scattering (close to the ground) may be taken as ~ 2 m. The value of $K_n = 2$ is found to be satisfactory for line-of-sight optical wave propagation (close to the ground) through the turbulent atmosphere (Goldstein *et al.* 1965; Strohbehn 1968). For a troposcatter system, Gjessing (1969) used a value of $K_n \approx 2$. [Actually Gjessing defined C_n^2 with respect to the variance of permittivity fluctuations $\langle \Delta\epsilon^2 \rangle$, which may be shown through numerical computations to be $\approx 4\langle \Delta n^2 \rangle$. The constant $K_n = 5 \cdot 3$ of Gjessing (1969) therefore becomes ≈ 20.]

In the present work the basic definition of the structure parameter is used:

$$C_n^2 = \frac{\langle [n(r_1) - n(r_2)]^2 \rangle}{r_{12}^{2/3}}, \qquad (2)$$

where the numerator is the variance $\langle \Delta n^2 \rangle$ of radio refractive index fluctuations and r_{12} the scale size of the inhomogeneities within the inertial subrange of the spatial turbulence spectrum. An airborne microwave refractometer makes an in-situ measurement of the variance $\langle \Delta n^2 \rangle$ in the transverse direction (at about every 15 m in the present case). The temporal power spectrum of the measured radio refractive index fluctuations shows a 'break-point' frequency f_c between the low and high frequency regions. The computed value of f_c defines a scale size l of the inhomogeneities through the relation $l = 2\pi/k = U/f_c$, where U is the transverse velocity of the aircraft. A representative (fixed) value of $r_{12} = l$ is used in the calculations of C_n^2 through relation (2).

3. System Description and Method of Analysis

In order to obtain radio refractive index fine structure information over the Indian subcontinent (a tropical zone) an airborne solid-state digital microwave refractometer has been designed, fabricated and flight tested (Sarma *et al.* 1989). In the present system, the radio refractivity is directly measured by comparing the resonance frequency of the sampling cavity (exposed to the atmosphere) with that of a reference cavity. Digital techniques are used to determine and convert the changes occuring in the resonant frequencies of the sampling cavity to the corresponding radio refractivity N values. The radio refractive index n is simply related to N by $N = (n-1) \times 10^6$. The N values are obtained by printer at the rate of 5 samples per second.

The major system elements of the microwave refractometer are the cavity stabilised solid-state varactor-tuned Gunn oscillator for the generation of microwave power; a pair of high Q ($\sim 10^4$) resonant cavities, one for use as a reference cavity (with provision for evacuation and sealing), the other as a sampling cavity (sensor) through which the atmosphere under investigation can pass through freely; and the electronic circuitry. The output power of the microwave source is divided equally into the reference and sampling channels. As the Gunn oscillator is swept, the cavities produce pulses and these pulses are then applied to amplifier and peak detector circuits. The phase difference between these two channel pulses (which is a measure of the radio refractive index of the atmosphere) is measured using flip–flop circuits and counters. It is ensured that the microwave source is locked with the reference cavity frequency which is highly stable ($\sim 10^7$). In the interface and display circuits the phase difference between the sampling channel and the reference channel is converted to the corresponding N values using digital circuitry. This refractometer has a sensitivity of refractivity better than $0 \cdot 5N$ and the response time of the sampling cavity is of the order of microseconds. It should be pointed out that the sensor of the refractometer was mounted in such a position that the air slipstream does not affect the measurements. To collect the radio refractivity vertical profile information the flight pattern consisted of spiral ascents and descents with a 500 m diameter and a uniform climb rate. The transverse velocity of the aircraft was $100\,\mathrm{m\,s^{-1}}$.

Atmospheric turbulence represents a three-dimensional spatially varying random process. The time variations result due to the transverse velocity of the aircraft. The length of a radio refractivity sample analysed is about 100 s, corresponding to 512 data points. From the statistical theory of turbulence, the mean lifetime of the inhomogeneities is about 250 s (Edmonds 1960). The measured temporal variations in radio refractivity may thus be regarded as due to spatial variations alone (Gossard 1960). The variance $\langle \Delta n^2 \rangle$ was computed from 512 N values. Since the sampling rate was 5 samples per second, and the aircraft ascent/descent rate was $2 \cdot 54\,\mathrm{m\,s^{-1}}$, the height resolution of $\langle \Delta n^2 \rangle$ was 250 m. Such $\langle \Delta n^2 \rangle$ values were computed at different heights for both the up and down legs of the aircraft sorties. In an initial exercise, six such sorties were carried out during the daytime in June 1983, and another six in January 1985. A total number of 24 samples was analysed. The mean of the June and of the January height profiles represent typical summer and winter conditions in the northern plains. The winter height profile of C_n^2 has been compared with the (mean) profile obtained from the simultaneous radiosonde flights conducted at a nearby meteorological station (Lucknow). The summer height profile has, however, been compared with the (mean) profile of routine radiosonde flights at $17 \cdot 30$ hrs local time.

4. Results and Discussion

(a) Height Profiles of C_n^2

The summer and winter height profiles of the radio refractive index variance factor $\langle \Delta n^2 \rangle$ obtained by airborne microwave refractometer are given in Fig. 1. The mean value of the scale (size) $l = r_{12}$ of the inhomogeneities is later shown to be about 200 m. The factor $r_{12}^{-2/3}$ therefore becomes $\approx 0 \cdot 03$. The abscissa in Fig. 1 can therefore be multiplied by $0 \cdot 03$ to obtain the corresponding values of C_n^2. Thus, the height profiles of C_n^2 for the 'summer' and 'winter' periods are obtained. The computed C_n^2 values lie in the range 6×10^{-13} to $1 \cdot 5 \times 10^{-12}$ $\mathrm{m}^{-2/3}$ at $1 \cdot 5$ km. The summer C_n^2 values are higher than in the winter period, a result also reported by Gossard (1977). Further, the mixing length theory in conjunction with Tatarskii's theoretical formulation permits evaluation of C_n^2

from the mean meteorological fields (Van Zandt *et al.* 1978, 1981; Warnock and Van Zandt 1985; Warnock *et al.* 1985). The values so obtained are normalised to a value of $5\times10^{-15}\,m^{-2/3}$ at $1\cdot5$ km (Tatarskii 1961; Gossard 1977; Van Zandt *et al.* 1978; Pasricha *et al.* 1983) and also shown in Fig. 1. Thus, it can be concluded that the radiosonde observations underestimate the C_n^2 values for both seasons. The computed C_n^2 values ranging from 10^{-13} to $10^{-12}\,m^{-2/3}$ seem to be on the high side. The values of C_n^2 have been shown to be in the range 10^{-15} to $10^{-13}\,m^{-2/3}$ (Bean *et al.* 1971; Sengupta *et al.* 1987). This discrepancy may be due to the fact that the present measurements are for tropical latitudes.

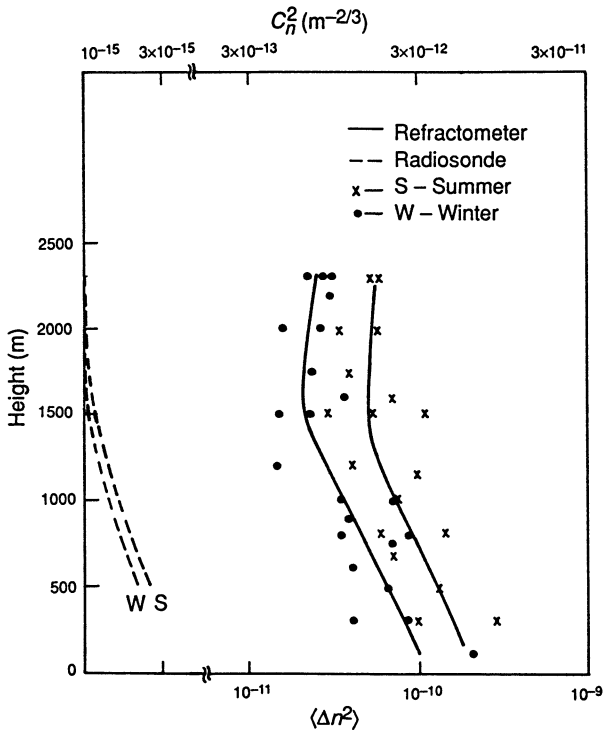

Fig. 1. Height profiles of the variance $\langle \Delta n^2 \rangle$ of refractive index fluctuations measured with the refractometer for summer and winter conditions (solid curves). The height profiles of C_n^2 are obtained after appropriate normalisations. Corresponding C_n^2 profiles obtained from radiosonde observations are given by the dashed curves.

(b) Spectral Analysis of Radio Refractive Index Fluctuations

Power spectral analyses of the temporal variations in radio refractive index fluctuations are performed to compute the slope of the high frequency fall-off. An estimate of the scale size of the inhomogeneities involved in the turbulent

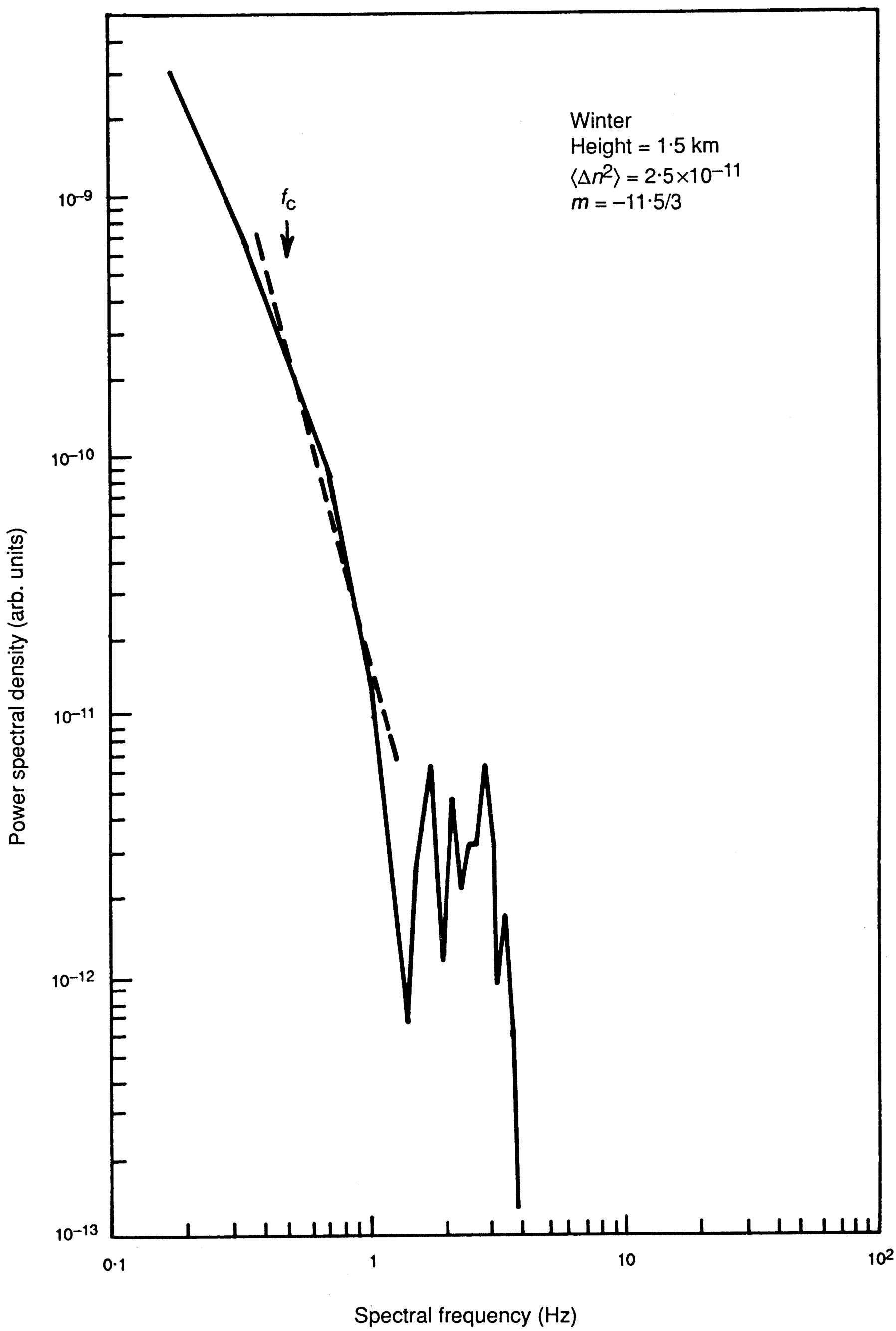

Fig. 2. Power spectra of refractive index fluctuations at $1 \cdot 5$ km. The spectral slope is $-11 \cdot 5/3$.

medium is also made. The linear trend of each selected radio refractivity profile was removed leaving fluctuations about the mean (Blackman and Tukey 1958). The power spectra are computed using the fast Fourier transform (FFT) algorithm with a time sequence of roughly 100 s of sampling time.

Smoothed spectral estimates are obtained by averaging a spectrum over five frequency points and through cumulative addition of the two spectra of upleg and downleg radio refractive index measurements. Before using the radio refractive measurements to compute power spectra, spurious points due to aircraft and other types of interference were removed by adopting an empirical editing procedure. These spurious radio refractivity values often occur as large single or multiple spikes which can be successfully detected and removed. A representative plot of power spectral density at 1·5 km height, obtained from one of the sorties in January 1985, is presented in Fig. 2. The statistical uncertainty in the estimation of the 'power' P in the computed spectrum is given by $\Delta P/P \approx 30\%$. The (mean) values of spectral slope were found to lie in the range −9/3 to −13/3 for samples in the height range 500 m to 2 km. However, a value of power spectral slope −15/3 was consistently obtained at 500 m. With the limited amount of data, no clear-cut variation of the spectral slope with height or season is evident.

A typical value of the scale size l ($=r_{12}$) of the inhomogeneities may be given in terms of the break-point frequency f_c marked in Fig. 2. A representative value of $f_c = 5\times10^{-1}$ Hz is adopted for the computations of structure parameter. Thus, the scale size becomes $l = U/f_c = 200$ m. Now a mathematically convenient (but an inadequate) Gaussian description of the (temporal) turbulence spectrum may be given by $\phi(f) \sim \exp(-f^2/f_c^2)$, and so a correlation size may be given as $r_0 = U/\pi f_c$. A typical value of r_0 is $\sim l/3$ or ~65 m in the radio refractive index measurements with the airborne microwave refractometer.

5. Conclusions

An airborne microwave refractometer gives an in-situ measurement of the radio refractive index distribution in turbulent atmosphere. These measurements are more accurate than the routine radiosonde observations. A continuous usage of such a system is inhibited by the large expenditure involved. The temporal variations in radio refractive index, due to the aircraft motion, have been analysed to compute the structure parameter C_n^2 and the spectral slope of radio refractive index fluctuations in the atmosphere. The computed values C_n^2 lie in the range 6×10^{-13} to $1\cdot5\times10^{-12}$ $m^{-2/3}$ at a typical height of 1·5 km. Atmospheric turbulence theory using routine radiosonde observations underestimates C_n^2 values for both seasons. The mean value of the scale size of the inhomogeneities is shown to be about 200 m. The corresponding value of the correlation size in the turbulent medium is about 65 m. There is a need to obtain additional measurements of the structure parameter C_n^2 in the tropics under a variety of climatic conditions.

Acknowledgments

The authors are grateful to an anonymous referee for his constructive criticism in improving the quality of the paper.

References

Bean, B. R., McGavin, R. E., Chadwick, R. B., and Warner, B. D. (1971). *Boundary Layer Meteorol.* **1**, 466.
Blackman, R. B., and Tukey, J. W. (1958). 'The Measurement of Power Spectra' (Dover: New York).
Borresen, J. A., and Gjessing, D. T. (1969). *Radio Sci.* **4**, 1195.
Chadwick, R. B., and Moren, K. P. (1980). *Radio Sci.* **15**, 355.
Crane, R. K. (1980). *Radio Sci.* **15**, 177.
de Wolf, D. A. (1975). *Radio Sci.* **10**, 53.
Ecklund, W. L., Carter, D. A., and Gage, K. S. (1977). *J. Geophys. Res.* **82**, 4969.
Edmonds, F. N. (1960). 'Statistical Methods in Radiowave Propagation', p. 197 (Pergamon: New York).
Gage, K. S., and Balsley, B. B. (1980). *Radio Sci.* **15**, 243.
Gjessing, D. T. (1969). *Radio Sci.* **4**, 1195.
Gjessing, D. T., Kjelass, A. G., and Nordo, J. (1969). *J. Atmos. Sci.* **26**, 462.
Gjessing, D. T., and McCormick, K. S. (1974). *Trans. IEEE* **AP-22**, 1325.
Goldstein, I., Miles, P. A., and Chabot, A. (1965). *Proc. IEEE* **53**, 1172.
Gossard, E. E. (1960). *Trans. IRE* **AP-7**, 186.
Gossard, E. E. (1977). *Radio Sci.* **12**, 89.
Ishimaru, A. (1972). *Trans. IEEE* **AP-20**, 10.
Lane, J. A. (1968). *Proc. IEE* **115**, 1227.
Pasricha, P. K., Mahendra Mohan, Tiwari, D. K., and Reddy, B. M. (1983). *Ind. J. Radio Space Phys.* **12**, 23.
Sarma, S. B. S. S., Reddy, B. M., Rao, E. B., Rao, M. V., Sarebahi, K. N., and Verma, A. K. (1989). *J. Phys.* E **22** (in press).
Sengupta, N., Warnock, J. M., Gossard, E. E., and Strauch, R. G. (1987). NOAA Tech. Rep. ERL 432-WPL 61, 17.
Strohbehn, J. W. (1968). *Proc. IEEE* **56**, 1301.
Tatarskii, V. I. (1961). 'Wave Propagation in a Turbulent Atmosphere' (McGraw–Hill: New York).
Tatarskii, V. I. (1971). 'The Effects of the Turbulent Atmosphere on Wave Propagation' (NTIS: VA).
Van Zandt, T. E., Green, J. L., Gage, K. S., and Clark, W. L. (1978). *Radio Sci.* **13**, 819.
Van Zandt, T. E., Gage, K. S., and Warnock, J. M. (1981). Proc. 20th Conf. on Radio Meteorology, Boston, p. 129.
Warnock, J. M., and Van Zandt, T. E. (1985). NOAA Tech. Rep. ERL AL-10, 175.
Warnock, J. M., Van Zandt, T. E., and Green, J. L. (1985). Seventh Symp. on Turbulence and Diffusion, p. 156 (Am. Meteorological Soc.: Boston).
Wickerts, S., and Nilsson, L. (1973). 'Modern Topics in Microwave Propagation and Air–Sea Interaction', p. 217 (Reidel: Dordtrecht).

Manuscript received 24 January, accepted 30 June 1989