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Slowing of Sound Waves in Powdered Media

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Abstract

The velocities of longitudinal sound waves in powdered media are considerably less than the corresponding value in bulk material. To understand this further, an examination has been made of how the sound velocity varies with pressure for the particular case of ultrasonic wave propagation in fine-grain sand. An analysis of the data has been made by treating the particles as linear chains of coupled oscillators.

1. Introduction

Over the years a considerable amount of work has been done on ultrasonic wave propagation in polycrystalline aggregates (see e.g. the review by Anderson 1965). Furthermore, Leach *et al.* (1977, 1978) have used acoustic emissions to determine particle size and size distribution in powders made up of rigid particles. There have also been some acoustic studies of anharmonic echoes in piezoelectric powders (reviewed by Kajimura 1982) and memory echoes in piezoelectric powders (reviewed by Melcher and Shiren 1982). However, the problem of wave propagation in powders does not appear to have enjoyed the same attention.

In an attempt to remedy this, the present work represents an extension of some preliminary studies of elastic wave propagation in powders (Brettell 1987), in which it can be shown that materials in powder form have longitudinal sound velocities which are one to two orders of magnitude lower than when in bulk form. In particular, the present measurements on raw fine-grain sand yield sound velocities in the range 230–460 $\mathrm{m\,s^{-1}}$ depending on pressure. This compares with a longitudinal sound velocity of $6\cdot6\times10^3\ \mathrm{m\,s^{-1}}$ in bulk SiO_2.

To explain these low velocities the powders have been treated as linear chains of coupled oscillators in which the interparticle contact regions are 'springs' which have force constants which increase with increasing pressure.

2. Theory

It has been previously shown (Brettell 1987) that if the particles can be regarded as linear arrays of identical spheres, each of radius R_0 and density ρ, then the spheres may be treated as though they are coupled by springs of stiffness coefficient:

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$$K = \frac{Ea}{1-\mu^2}, \tag{1}$$

where E is Young's modulus, a is the contact radius between the spheres and μ is Poisson's ratio. The contact radius a is determined by the value of a compressive force applied along the length of an array. The total sample consists of an arrangement of parallel arrays leading to simple cubic stacking of the spherical particles.

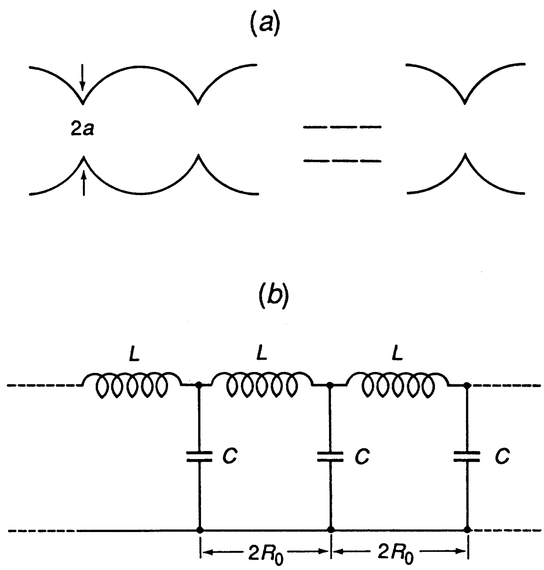

Fig. 1. Linear array of identical spheres, each of radius R_0, and with interparticle contact radius a. (*b*) Equivalent electrical transmission line.

Table 1. Acoustical quantities with their electrical analogues

Acoustical	Electrical
Mass $m = \frac{4}{3}\pi R_0^3 \rho$	Inductance L
Compliance $\frac{1-\mu 2}{Ea}$	Capacitance C
Phase velocity $c = G(\mu)\, V_B \left(\frac{a}{R_0}\right)^{1/2}$ $(\omega \ll \omega_0)$	$\frac{2R_0}{(LC)^{1/2}}$
Cut-off frequency $\frac{\omega_0}{2\pi} = \frac{V_B\, G(\mu)}{2\pi R_0}\left(\frac{a}{R_0}\right)^{1/2}$	$\frac{1}{(\pi^2 LC)^{1/2}}$

When an array is subject to an additional longitudinal wave, assumed simple harmonic, the wave travels along the array with a velocity c according to

$$\frac{c}{V_B} = \frac{\omega G(\mu)}{\omega_0 \sin^{-1}(\omega/\omega_0)}\left(\frac{a}{R_0}\right)^{\frac{1}{2}}, \tag{2}$$

where V_B is the bulk longitudinal velocity of sound waves through a polycrystalline (i.e. isotropic) medium composed of the same material as that of the spheres, ω is the the angular frequency of the wave, ω_0 is the cut-off angular frequency and $G(\mu)$ is given by

$$G(\mu) = \left(\frac{3}{\pi} \frac{1-2\mu}{(1-\mu)^2} \right)^{\frac{1}{2}} . \tag{3}$$

The result represented by equation (2) was derived (Brettell 1987) by combining the theory of linear coupled oscillators (Morse and Ingard 1968) with elasticity theory (Timoshenko and Goodier 1970). Since the electrical analogue of an array may be regarded as the transmission line shown in Fig. 1, the acoustical and electrical analogues are those given in Table 1.

Let us now assume that the powder is packed, in simple cubic form, in a cylinder of cross-sectional area πR^2, and that a uniform pressure acts perpendicular to this cross section. The maximum cross-sectional area of a sphere is πR_0^2 but this will be contained in a cross-sectional area $4R_0^2$, so that the number of contacts n in one monolayer is $\pi R^2/4R_0^2$.

For a total force F' on πR^2, the force per contact then becomes

$$F = \frac{F'}{n} = F' \frac{4R_0^2}{\pi R^2} ,$$

from which the pressure p is given by

$$p = \frac{F'}{\pi R^2} = \tfrac{1}{3} \frac{E}{1-\mu^2} \frac{a^3}{R_0^3} .$$

Writing the $\omega \ll \omega_0$ expression for c in Table 1 in terms of pressure, we then obtain

$$c = G(\mu)\, V_B \left(\frac{3(1-\mu^2)}{E} \right)^{1/6} p^{1/6} . \tag{4}$$

3. Experimental Method

The measurements were made using an ultrasonic pulser/receiver SPIKE 150 PR from PAR Scientific Instruments. The instrument provided a 150 volt pulse of width 100 ns and rise time 5 ns, which in turn excited a 100 kHz PZT transducer. A separate identical transducer was used as receiver.

The raw fine grain sand, 125–300 μm diameter, was packed in a cylindrical container of $2\cdot6$ cm^2 internal cross-sectional area. The container was mounted vertically with the receiver transducer in contact with the sand at the bottom and the transmitter transducer in contact with the sand at the top. The as-supplied transmitter transducer came in a steel casing which had the same diameter as the sample and which supplied a minimum pressure to the sample of $1\cdot7$ kPa. The pressure p is that defined in the previous section. Increasing the pressure was achieved by adding weights onto the steel casing of the transmitter.

The velocity of sound was determined from transit time measurements of the first received signal in samples of length $1\cdot0$ cm.

4. Results and Discussion

The results of measurements of velocity of sound as a function of pressure are shown in Fig. 2. The straight line through the upper points is an attempt to correlate the observed data with the long wavelength formula given by equation (4). This line is represented by $c = \alpha p^{1/6}$, where $\alpha = 73\ \mathrm{m\,s^{-1}\,Pa^{-1/6}}$, with α being chosen to give the best fit to the experimental data. Taking $E = 10^{11}\ \mathrm{N\,m^{-2}}$ and $\mu = 0 \cdot 04$, the theoretical value of α is $1 \cdot 1 \times 10^{2}\ \mathrm{m\,s^{-1}\,Pa^{-1/6}}$. The discrepancy between the experimental and theoretical value is attributed to the existence of a distribution in particle size and a packing structure that is other than simple cubic.

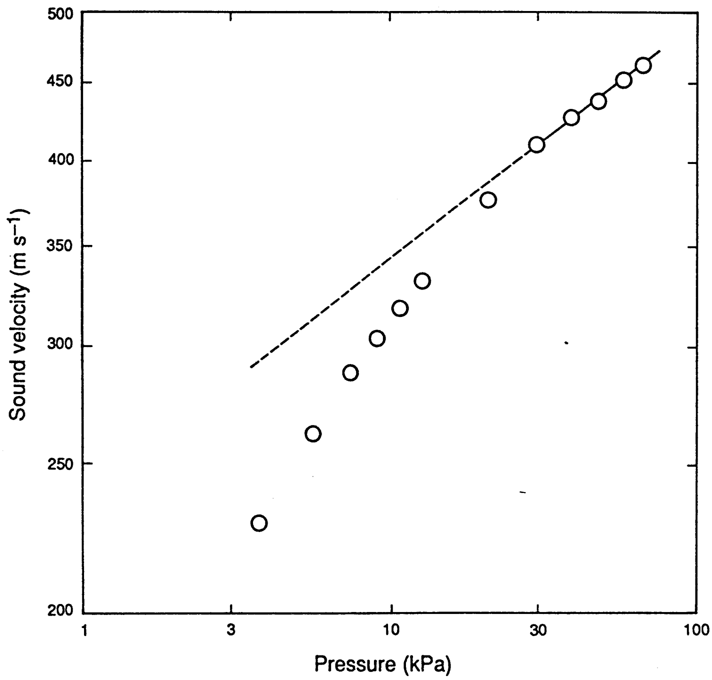

Fig. 2. Velocity of longitudinal sound waves in fine-grain sand plotted as a function of pressure. The pressure is applied in the same direction as that of the propagated wave.

The long wavelength approximation at the higher pressures gives a good fit to the experimental data with regard to the $p^{1/6}$ variation, but at the lower pressures the sound velocities are less than predicted. This suggests that as the pressure is lowered then we approach the critical cut-off frequency $\omega_0/2\pi$. Here the dispersion relation given by equation (2) predicts a lowering of the sound velocity with increasing ω/ω_0. The transducer frequency is fixed but from Table 1 we see that ω_0 decreases with decreasing a/R_0 and hence decreasing p.

For a distribution of particle size the cut-off frequency would presumably, by analogy with the acoustical branch of the one-dimensional diatomic lattice, be determined by the radius of the larger particles. If we therefore use the value $R_0 = 150\ \mu\mathrm{m}$, and calculate c for a cut-off frequency of 100 kHz, i.e. the frequency of the transducer, we have $c = (2/\pi)\omega_0 R_0 = (2/\pi)\omega R_0 = 60\ \mathrm{m\,s^{-1}}$.

Alternatively, if we determine the velocity at the cut-off frequency from the actual velocity of sound measurements, the theoretical dispersion relation predicts a value for c, at this frequency, which is $2/\pi$ times that predicted in the long wavelength approximation ($\lambda \gg 4R_0$). If in Fig. 2 therefore, we extrapolate both (1) the curve through the experimental points and (2) the long wavelength approximation line, bringing them both to that pressure which corresponds to this $2/\pi$ ratio, we obtain a value for c of about 150 m s^{-1} at the point of cut-off ($\lambda = 4R_0$).

With the present equipment, a direct measurement of the cut-off frequency proved difficult since the received signal became weaker the lower the pressure. Although from a transmission line point of view we would expect this to occur near the cut-off frequency, the signal output did in fact show a continual decline in amplitude in going from the highest pressure down to the lowest. In the most part this was attributed to the acoustic impedance mismatch between transducer and powder.

For $\omega \ll \omega_0$ the acoustic impedance of the powder is given by $Z = \rho' c$, where ρ' is the powder density. For simple cubic packing we can write $\rho' = \pi\rho/6$ where ρ is the bulk density, although, for the fine-grain sand used in the present studies, the poured density is $\rho' = 0{\cdot}60\rho$. If we assume that ρ' does not vary significantly with the applied pressure, the predominant variation in Z will come from the sound velocity variation, so the lower the value of c, the greater the impedance mismatch and the weaker the output signal.

5. Conclusions

The results show that, although the velocity of a longitudinal sound wave in fine grain sand is pressure dependent, the observed sound velocities are considerably less than the corresponding velocity in bulk SiO_2. This is believed to be typical of powdered media, and the low velocities appear to be well explained using the model of linear coupled oscillators.

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