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Tumbling for Exotic Fermionic Representations in Grand Unified Models

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Abstract

We consider dynamical symmetry breaking through a tumbling mechanism for exotic representations of fermions in unified models. Possible ways to introduce U(1) gauge symmetry are also discussed. It is shown that the most attractive channel (MAC) hypothesis does not predict physically interesting results unless the peculiar assumption of the maximal preservation of the global SU(3)×U(1) symmetry is made. In such a case the model with two fermionic generations is obtained.

1. Introduction

Dynamical symmetry breaking (Kaptanoĝlu and Pak 1982) has been proposed in the framework of grand unified theories (Georgi and Glashow 1974; Nanopoulous 1980; Ellis 1980; Langacker 1981) in order to explain the appearance of the Higgs sector. The first attempts to apply dynamical symmetry breaking to physical models of interactions were undertaken by Cornwell and Norton (1973) and then by Weinberg (1976, 1979). That led to the technicolour (TC) and extended technicolour (ETC) hypothesis (Dimopoulos and Susskind 1979; Fahri and Susskind 1981) and it provided the only description of dynamical breaking of the standard electroweak $SU(2)_L \otimes U(1)_Y$ group (Glashow 1961; Weinberg 1967; Salam 1969). However, technicolour and extended technicolour suggestions do not unify all elementary interactions.

Tumbling gauge theories based on the Raby–Dimopoulos–Susskind (RDS) rules have raised new hopes. These theories were investigated also by Srednicki (1980), Bais and Frére (1981) and King (1981*a*). In these considerations the exotic fermionic representations (King 1981*b*) were neglected without providing sufficient reasons (King 1981*a*). There is only one such representation of the SU(5) group which satisfies all the fundamental axioms (King 1981*b*; Georgi 1979) of the grand unified theory:

(i) anomaly free (King 1981*b*; Georgi and Glashow 1972; Banks and Georgi 1976; Okubo 1977);
(ii) asymptotically free (King 1981*b*; Gross and Wilczek 1973*a*, 1973*b*, 1974; Politzer 1973, 1974);
(iii) complex with respect to the subgroup $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ (King 1981*b*; Georgi 1979);

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(iv) real with respect to the strong electromagnetic subgroup $SU(3)_c \otimes U(1)_Q$ (King 1981*b*; Georgi 1979), namely

$$f^{\mathrm{ex}}_{\mathrm{SU(5)}} = \mathbf{5} \oplus \mathbf{15} \oplus \mathbf{45} \oplus \mathbf{40}^* . \tag{1}$$

[The second and the last representation at SU(5) level that satisfies the conditions (i)–(iv) is the conventional one,

$$f^{\mathrm{conv}}_{\mathrm{SU(5)}} = \mathbf{5} \oplus \mathbf{10}^* , \tag{2}$$

but it is not exotic.] The exotic representations for other groups are excluded because they do not satisfy condition (iv) (King 1981*b*).

Raby *et al.* (1980) considered tumbling for $f^{\mathrm{conv}}_{\mathrm{SU(5)}}$ and have shown that in this case the SU(5) group breaks to SU(4) only. Our question is—what will occur if we apply the tumbling hypothesis of Raby *et al.* (1980) to the representation $f^{\mathrm{ex}}_{\mathrm{SU(5)}}$?

2. Tumbling Mechanism in Exotic Case

According to RDS rules a condensate will form in a channel for which the potential defined as

$$V(R_1 \otimes R_2 \to R) = \tfrac{1}{2}\alpha^2 (C - C_1 - C_2) \tag{3}$$

has a minimum and is negative, where C, C_1, C_2 denote the quadratic Casimir operators for the composite (C) and constituent systems (C_1, C_2), R_1,R_2 are irreducible representations for constituent fermions and R is an irreducible representation of the composite state. In the case of $f^{\mathrm{ex}}_{\mathrm{SU(5)}}$ this occurs in the cross product:

$$\mathbf{45} \otimes \mathbf{40}^* \to \mathbf{5} , \tag{4}$$

where quadratic Casimir values for these representations are

$$C(\mathbf{5}) = \tfrac{24}{10} , \qquad C(\mathbf{40}^*) = \tfrac{66}{10} , \qquad C(\mathbf{45}) = \tfrac{64}{10} , \tag{5}$$

with the normalisation convention

$$\mathrm{Tr}\{t_a, t_b\} = \delta_{ab} \tag{6}$$

for generators t_a in the fundamental representation. It gives us the binding potential

$$V \sim -\tfrac{53}{10}\,\alpha^2 . \tag{7}$$

The condensate (4) could break

$$\mathrm{SU(5)} \to \mathrm{SU(4)} \otimes \mathrm{U(1)} \tag{8}$$

because we have the branching rules

$$\mathbf{5}_{|\,SU(5)} = \mathbf{1}(4n) \oplus \mathbf{4}(-n)_{|\,SU(4)\otimes U(1)},$$

$$\mathbf{45}_{|\,SU(5)} = \mathbf{6}(6n) \oplus \mathbf{15}(-4n) \oplus \mathbf{4}^*(n) \oplus \mathbf{20}(n)_{|\,SU(4)\otimes U(1)},$$

$$\mathbf{40}^*_{|\,SU(5)} = \mathbf{4}^*(-7n) \oplus \mathbf{6}(-2n) \oplus \mathbf{10}^*(-2n) \oplus \mathbf{20}^*(3n)_{|\,SU(4)\otimes U(1)}. \tag{9}$$

In the parentheses we have noted the values of the charges of the U(1) gauge group; n describes the normalisation of the U(1) generator.

We see that the singlet $\mathbf{1}(4n)$ in SU(4)⊗U(1) is formed from $\mathbf{20}(n)$ and $\mathbf{20}^*(3n)$. They should then acquire a mass of order M_0, where M_0 is defined from equation (7) as

$$\alpha^2(M_0) = \tfrac{10}{53}. \tag{10}$$

However, $\mathbf{20}(n)\otimes\mathbf{20}^*(3n)$ is not a real representation with respect to SU(4)⊗U(1), because the U(1) factor destroys this reality. So $\mathbf{20}(n)\otimes\mathbf{20}^*(3n)$ cannot be massive. We conclude that breaking (8) is impossible. The tumbling scheme only allows the breaking

$$SU(5) \to SU(4). \tag{11}$$

The following branching rules are valid:

$$\mathbf{5}_{|\,SU(5)} = \mathbf{1} \oplus \mathbf{4}_{|\,SU(4)},$$

$$\mathbf{45}_{|\,SU(5)} = \mathbf{6} \oplus \mathbf{15} \oplus \mathbf{4}^* \oplus \mathbf{20}_{|\,SU(4)},$$

$$\mathbf{40}^*_{|\,SU(5)} = \mathbf{6} \oplus \mathbf{4}^* \oplus \mathbf{10}^* \oplus \mathbf{20}^*_{|\,SU(4)},$$

$$\mathbf{15}_{|\,SU(5)} = \mathbf{10} \oplus \mathbf{4} \oplus \mathbf{1}_{|\,SU(4)}. \tag{12}$$

We see that the singlet obtained from **5** is formed from **20** and **20*** and these states become massive of order M_0, as defined in (10). Also, nine vector bosons connected with broken SU(5) generators get the same mass.

The next step of breaking is realised through the channel

$$\mathbf{10} \otimes \mathbf{10}^* \to \mathbf{1}, \tag{13}$$

as the most attractive channel (MAC), with C values

$$C(\mathbf{1}) = 0, \qquad C(\mathbf{10}) = C(\mathbf{10}^*) = \tfrac{45}{10}, \tag{14}$$

and the binding potential

$$V \sim -\tfrac{45}{10}\alpha^2. \tag{15}$$

Because the condensate in (13) is a singlet with respect to SU(4), it does not break it. The only result of this step is to give a mass of order M_1 for **10** and $\mathbf{10}^*$, where M_1 is such that

$$\alpha^2(M_1) = \tfrac{10}{45}\,. \tag{16}$$

Now we have an effective theory with massless fermion content

$$(\mathbf{1},\mathbf{2})\oplus(\mathbf{6}^r,\mathbf{2})\oplus(\mathbf{4},\mathbf{2})\oplus(\mathbf{4}^*,\mathbf{2})\oplus(\mathbf{15}^r,\mathbf{1}), \tag{17}$$

with respect to $SU(4)_{loc}\otimes SU(2)_{glob}$ symmetry, where an additional global symmetry is obvious. The index r denotes that the representation is a real one.

The MAC hypothesis says now that the condensate should be formed in the channel

$$(\mathbf{15}^r,\mathbf{1})\oplus(\mathbf{15}^r,\mathbf{1}) \to (\mathbf{1},\mathbf{1}), \tag{18}$$

with the binding potential $V \sim -4\alpha^2$ $[C(\mathbf{15}^r) = 4]$. It does not break $SU(4)_{loc}$ symmetry and leads to the mass for $\mathbf{15}^r$ of order M_2 where

$$\alpha^2(M_2) = \tfrac{1}{4}\,. \tag{19}$$

After that the effective theory contains the following massless particles in obvious notation:

$$(\mathbf{1},\mathbf{2})\oplus(\mathbf{6}^r,\mathbf{2})\oplus(\mathbf{4},\mathbf{2})\oplus(\mathbf{4}^*,\mathbf{2})\,. \tag{17$'$}$$

The MAC hypothesis predicts

$$(\mathbf{6}^r,\mathbf{2})\otimes(\mathbf{6}^r,\mathbf{2}) \to (\mathbf{1},\mathbf{3}), \tag{20}$$

with the binding potential $V \sim -\frac{5}{2}\alpha^2$ because of the Casimir value $C(\mathbf{6}^r) = \frac{5}{2}$, and then

$$(\mathbf{4},\mathbf{2})\otimes(\mathbf{4}^*,\mathbf{2}) \to (\mathbf{1},\mathbf{3}), \tag{21}$$

with $V \sim -\frac{15}{8}\alpha^2$, because $C(\mathbf{4}) = \frac{15}{8}$. As a result $\mathbf{6}^r$ get a mass of order M_3 defined by

$$\alpha^2(M_3) = \tfrac{2}{5}\,, \tag{22}$$

and the mass of the quartets is $M_4 < M_3$ where

$$\alpha^2(M_4) = \tfrac{8}{15}\,. \tag{23}$$

Finally, we have unbroken $SU(4)_{loc}$ symmetry with two massless singlets of $SU(4)_{loc}$ (see equation 17$'$) and two triplets of pseudo-goldstones connected with broken $SU(2)_{glob}$ symmetry of sextets (20) and quartets (21). Such a result is not interesting from a physical point of view and it is similar to the one obtained for the conventional representation of SU(5) in equation (2) by Raby *et al.* (1980). Nevertheless, if we assume that for some reason the global SU(2) symmetry should not be broken (for example because of the hypothesis of maximal preservation of global symmetry in dynamical breaking), all condensates must be formed as singlets of $SU(2)_{glob}$. For $(\mathbf{6}^r,\mathbf{2})\otimes(\mathbf{6}^r,\mathbf{2})$ this is

$$(\mathbf{6}^r,\mathbf{2})\otimes(\mathbf{6}^r,\mathbf{2}) \to (\mathbf{1}_s,\mathbf{1}_a)\oplus(\mathbf{15}_a,\mathbf{1}_a)\oplus(\mathbf{20}_s,\mathbf{1}_a)\,. \tag{24}$$

The first condensate on the right side is excluded now because of Fermi statistics [it is symmetric in $SU(4)_{loc}$ indices, antisymmetric in $SU(2)_{glob}$ and antisymmetric in spin indices]. The third condensate in (24) cannot be formed because the binding potential is positive, so the only admissible channel is

$$(\mathbf{6}^r,\mathbf{2})\otimes(\mathbf{6}^r,\mathbf{2}) \to (\mathbf{15}_a,\mathbf{1}_a)\,, \tag{25}$$

with the binding potential $V \sim -\frac{1}{2}\alpha^2$, because $C(\mathbf{15}) = 4$.

However, the MAC for fermion content (17′) is not (25) but

$$(\mathbf{4},\mathbf{2})\otimes(\mathbf{4}^*,\mathbf{2}) \to (\mathbf{1},\mathbf{1})\,, \tag{26}$$

with $V \sim -\frac{15}{8}\alpha^2$. It gives for $\mathbf{4}$ and $\mathbf{4}^*$ a mass of order M_4 defined in (23), but it does not break $SU(4)_{loc}$.

After that the binding (25) can be realised. It breaks

$$SU(4) \to SU(3)\otimes U(1) \tag{27}$$

because of the branching rules

$$\begin{aligned}\mathbf{15}_{|\,SU(4)} &= \mathbf{1}(0)\oplus\mathbf{3}(-\tfrac{4}{3})\oplus\mathbf{3}^*(\tfrac{4}{3})\oplus\mathbf{8}(0)_{|\,SU(3)\otimes U(1)}\,,\\ \mathbf{6}_{|\,SU(4)} &= \mathbf{3}(\tfrac{2}{3})\oplus\mathbf{3}^*(-\tfrac{2}{3})_{|\,SU(3)\otimes U(1)}\,.\end{aligned} \tag{28}$$

The singlet in $\mathbf{15}^r$ is formed from $\mathbf{3}(\frac{2}{3})$ and $\mathbf{3}^*(-\frac{2}{3})$ so these states get a mass of order M_5, defined by

$$\alpha^2(M_5) = 2\,. \tag{29}$$

Because of the additional branching rules

$$\begin{aligned}\mathbf{4}_{|\,SU(4)} &= \mathbf{1}(1)\oplus\mathbf{3}(-\tfrac{1}{3})_{|\,SU(3)\otimes U(1)}\,,\\ \mathbf{4}^*_{|\,SU(4)} &= \mathbf{1}(-1)\oplus\mathbf{3}^*(\tfrac{1}{3})_{|\,SU(3)\otimes U(1)}\,,\end{aligned} \tag{30}$$

and remembering that for each representation λ,

$$\lambda^*(-m)_L \equiv \lambda(m)_R\,, \tag{31}$$

where the number in parentheses denotes the U(1) value, we have the following fermion content in the effective low-energy theory:

$$2 \times \mathbf{3}(\tfrac{2}{3})_{\mathrm{L,R}} \quad \text{with mass } M_5\,,$$

$$2 \times \mathbf{3}(-\tfrac{1}{3})_{\mathrm{L,R}} \quad \text{with mass } M_4 > M_5\,,$$

$$2 \times \mathbf{1}(-1)_{\mathrm{L,R}} \quad \text{with mass } M_4\,.$$

3. Conclusions

The results we found above can describe two families with $SU(3)_c \otimes U(1)_Q$ properties:

$$\begin{array}{ll}
\text{(i)}\quad \text{e-family}: & \text{(ii)}\quad \mu\text{-family}: \\
\mathbf{3}(\tfrac{2}{3})^{M_5}_{\mathrm{L,R}} \leftrightarrow u_{\mathrm{L,R}} & \mathbf{3}(\tfrac{2}{3})^{M_5}_{\mathrm{L,R}} \leftrightarrow c_{\mathrm{L,R}} \\
\mathbf{3}(-\tfrac{1}{3})^{M_4}_{\mathrm{L,R}} \leftrightarrow d_{\mathrm{L,R}} & \mathbf{3}(-\tfrac{1}{3})^{M_4}_{\mathrm{L,R}} \leftrightarrow s_{\mathrm{L,R}} \\
\mathbf{1}(-1)^{M_4}_{\mathrm{L,R}} \leftrightarrow e^-_{\mathrm{L,R}} & \mathbf{1}(-1)^{M_4}_{\mathrm{L,R}} \leftrightarrow \mu^-_{\mathrm{L,R}} \\
\mathbf{1}(0)^{0}_{\mathrm{L}} \leftrightarrow \nu_{e\mathrm{L}} & \mathbf{1}(0)^{0}_{\mathrm{L}} \leftrightarrow \nu_{\mu\mathrm{L}}
\end{array}$$

where the electric charge is given in parentheses. All other states including the exotic part are very massive and can be neglected in low-energy theory described by $SU(3)_c \otimes U(1)_Q$. This fact is in agreement with experiment. The same mass for the e and μ family is not very intriguing because the tumbling mechanism only gives us information about the order of the mass. It is worth remarking that neutrinos are massless and particles d and e or s and μ are of the same order of mass, also a characteristic feature of the SU(5) model by Georgi and Glashow (1974). However, there is no place here for the weak interactions described by the Salam–Weinberg model. Finally, one can conclude that the above toy model is an example of dynamical unification of strong and electromagnetic forces with two families by the assumption of maximal preservation of a global symmetry.

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