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On the Importance of Self-Interaction in QCD*

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Abstract

The electromagnetic self-energy of charged particles has remained a problem in classical as well as in quantum electrodynamics. In contrast here, in a review of the analysis of the chromodynamic self-energy of quarks in quantum chromodynamics (QCD), we see that the quark self-energy is a finite and a dominant effect in determining the structure of hadrons.

1. Introduction

It is just over one hundred years since the concept of electromagnetic mass was introduced and studied by Abraham, Lorentz, Poincaré and others. As Pais (1982) comments 'All that remains from those early times is that we still do not understand the problem'. The idea was that the mass of a charged particle is to have its origin in self-energy, that is, the energy of its electromagnetic field, as shown in Fig. 1*a* for a static finite-size charged particle. This electromagnetic mass idea preceded, of course, the special relativity ideas of the equivalence of energy and inertial mass. A deep assumption in this approach is that an isolated charged particle has such a field. Experimentally we only know that such a field is useful in describing the force $\mathbf{F}_Q$ acting on another charge Q. The $\mathbf{E}$ field of Fig. 1*a* arises from the assumption that $\mathbf{E} = \mathbf{F}_Q/Q$ has a physically meaningful limit as $Q \to 0$, despite the fact the electric charge is quantised. At a fundamental level, appropriate to the self-energy problem, the classical-physics concepts symbolised in Fig. 1*a* may actually be meaningless. Feynman *et al.* (1965) have given an elegant statement of the electromagnetic mass problem, but implicitly assumed the reality of such an $\mathbf{E}$. Basically the concepts of simple charged particles and the electromagnetic field are inconsistent *even* in the classical theory. The difficulty is with the concept of electromagnetic momentum and energy when applied to a charged particle. If we calculate the energy associated with the electromagnetic field of a charged particle we obtain $U_{elec} = \frac{3}{4} m_{elec} c^2$, whereas from relativity we know that the relationship must be $U = mc^2$—the '$\frac{4}{3}$-problem'. The discrepency between these formulas for the electromagnetic mass is because one neglected the non-electromagnetic forces which must be present to hold the electron together. It was Poincaré who pointed out that these extra forces must be included in the energy and

* Dedicated to Professor Ian McCarthy on the occasion of his sixtieth birthday.

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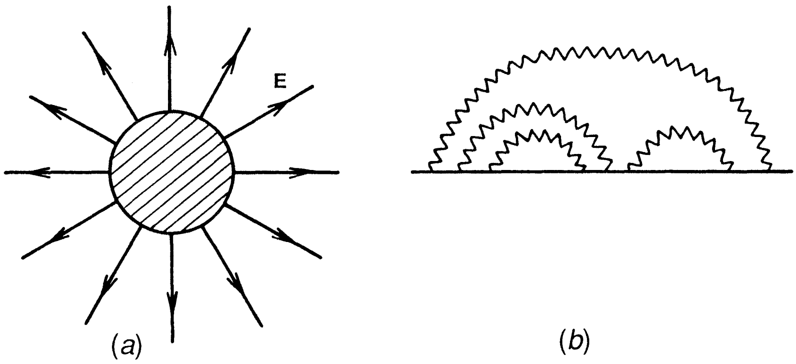

Fig. 1. (*a*) Classical picture of the electron self-energy; (*b*) quantum mechanical self-energy processes.

momentum calculations in order to get consistent results, and they are now known as the 'Poincaré stresses'. Hence in the classical theory it is impossible to get all of the mass from nothing but electromagnetism. The situation was not saved by quantum electrodynamics (QED) which produces an infinity for the self-energy of an electron. Typical self-energy diagrams of QED are shown in Fig. 1*b*. Almost paradoxically we argue here that the study of the quark self-energy problem in the much more complicated quantum chromodynamics (QCD) has yielded, in recent years, a much greater insight into self-interaction processes.

Here we analyse the chromodynamic self-interaction of quarks in QCD and show its supreme importance in determining the structure and interaction of the hadrons, and its probable role in the confinement of quarks and other conceivable quark bound states which carry colour charge. For such an analysis we clearly need a systematic method for proceeding from the fundamental defining action of QCD to the low energy manifestation of QCD—the phenomena of hadronic physics, as described by the hadronic effective action. The numerous phenomenological models for hadrons will clearly be of little use in this analysis, for the dynamical and constructional guesses that comprise these models almost certainly have little relevance to the quark–gluon dynamics as determined by the action and quantisation of chromodynamics.

In Section 2 (Hadronic Laws) we briefly outline the systematic procedure which allows us to derive the laws of hadronic physics from the quark–gluon physics. This involves the powerful but perhaps abstract techniques of functional integral calculus (FIC). An important aspect arising here are questions concerning the colour-charged 'states', such as the $\mathbf{3}_c$ quarks, the $\overline{\mathbf{3}}_c$ diquarks, the $\mathbf{8}_c$ baryons, and other states.

In Section 3 (Quark Self-Interaction) we explain the dynamical content of this hadronisation of QCD, together with truncations of the analysis which are necessary to permit detailed practical computations. Of particular significance will be the manifestations of the hidden chiral symmetry associated with the quark self-interaction. Of course the gluon self-interaction is also very significant to QCD and critical to the finiteness of the quark self-interaction. Because all known hadronic states contain quarks, it is the quark self-interaction which has the most direct relevance to hadronic laws. Of particular significance here is that the quark 'mass function' varies rapidly over the momentum range of relevance to the quarks forming bound states, unlike the previous familiar situations in atomic and nuclear physics. This is a new and important phenomenon associated with self-energy processes and we explore some possible interpretations or 'pictures'. These 'pictures' are used to motivate the idea of the *self-energy colour filter.* This is a possible dynamical processes (associated with the local colour symmetry) which may explain why only colour singlet quark states ($\mathbf{1}_c$ mesons and baryons) arise in the observable QCD spectrum, and how the other colour charged states (mentioned above) are removed from the QCD mass spectrum.

Finally in Section 4 (Conclusions) we argue that QCD, despite its dynamical complexity and the consequent need for truncations of the analysis, is the best phenomenon we have in which to study the fundamental self-interaction process of matter, primarily because the consequences are manifestly large and experimentally accessible.

2. Hadronic Laws

The action defining chromodynamics is

$$S[A_\mu^a,\overline{q},q]=\int d^4x\left(\tfrac{1}{4}F_{\mu\nu}^aF_{\mu\nu}^a+\frac{1}{2\xi}(\partial_\mu A_\mu^a)^2+\overline{q}(\gamma_\mu(\partial_\mu-ig\frac{\lambda^a}{2}A_\mu^a)+\mathcal{M})q\right),$$

which involves the quark and gluon fields and the field strength tensor for the gluon fields. As we discuss later the use of the Euclidean metric as the defining metric of quantum field theories may be more than a matter of convenience. Quantised chromodynamics (QCD) is here defined using the functional integral formulation, in which the generating functional is given by

$$Z[J,\overline{\eta},\eta]=\int D\overline{q}DqDA\exp(-S[A_\mu^a,\overline{q},q]+\overline{\eta}q+\overline{q}\eta+J_\mu^aA_\mu^a), \tag{2·1a}$$

where J, $\overline{\eta}$ and η are source fields. A slightly different formulation is to drop sources and introduce a finite Euclidean time $0\le x_4\le T$, and to introduce (anti-) periodic boundary conditions for the (quark) gluon fields. In this case Z becomes the partition function (up to a multiplicative constant)

$$Z(T)=\sum_n\exp(-E_nT)=\int D\overline{q}DqDA\exp(-S[A_\mu^a,\overline{q},q]), \tag{2·1b}$$

where $\{E_n\}$ is the energy spectrum of QCD, which we believe to be that of the known baryons and mesons, and also their numerous bound states—the nuclei. Of course this energy spectrum is parametrised by the hadronic mass spectrum in the usual way ($E^2=m^2+\mathbf{p}^2$).

The chromodynamic action clearly has two important invariance groups, the local colour symmetry and, for massless quarks, the global chiral symmetry

$G = U_L(N_f) \otimes U_R(N_f)$. The current masses (for $\mathcal{M} = \{m_u, m_d\}$) are so small compared with the hadronic energy scale that any analysis or computational scheme must properly handle the dynamical processes that cause the chiral symmetry to become a hidden symmetry. The scheme reviewed here is ideally suited to this chiral regime, and involves the use of FIC techniques to change variables in $(2 \cdot 1)$ and to induce an effective action for these new variables. Because the change of variables is determined by the dynamics of QCD we find that we must proceed through a meson–diquark bosonisation of QCD and then finally to the meson–baryon variables. In changing variables we must in practice solve various eigenvalue equations, which will be seen to give the masses and relativistic wavefunctions of the meson, diquark and baryon states.

We can write $(2 \cdot 1\mathrm{a})$ in the form

$$Z = \int D\overline{q}DqDA \exp(\overline{q}(\gamma.\partial + \mathcal{M})q - \overline{\eta}q - \overline{q}\eta) \exp(ig\overline{q}\frac{\lambda^a}{2}\gamma_\mu q \frac{\delta}{\delta J^a_\mu}) \exp(W[J]),$$

where $W[J]$ is the generating functional for connected gluon Green's functions which have no internal quark loops:

$$\exp(W[J]) = \int DA \exp\left(-\tfrac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^a_\mu)^2 + J^a_\mu A^a_\mu \right).$$

$W[J]$ has the expansion

$$W[J] = \int\int d^4x d^4y \tfrac{1}{2} D^{ab}_{\mu\nu}(x, y; \xi) J^a_\mu(x) J^b_\nu(y) + W_R[J],$$

$$W_R[J^a_\mu] = \sum_{n=3}^{\infty} \int d^4x_1 \dots d^4x_n \frac{1}{n!} D^{a_1 \dots a_n}_{\mu_1 \dots \mu_n}(x_1 \dots x_n; \xi) \prod_{i=1}^{n} J^{a_i}_{\mu_i}(x_i).$$

Here W_R involves the $n(\geq 3)$-point connected gluon Green's functions. Effects due to 'instantons', 'glue-balls', ... if they are indeed relevant, will appear in these $D^{(n)}$. To simplify the presentation we use the gauge in which the $n = 2$-point function is expressed as,

$$g^2 D^{ab}_{\mu\nu}(x) = \delta^{ab}\delta_{\mu\nu} \int \frac{d^4q}{(2\pi)^4} \frac{4\pi\alpha(q^2)}{q^2} e^{iq.x} = \delta^{ab}\delta_{\mu\nu} D(x).$$

The $\alpha(s)$ is the important 'running' coupling constant which arises because of the gluon self-interactions (Gross and Wilczek 1973; Politzer 1973). This $\alpha(s)$ does not include contributions from quark loops, which are included at a later stage in the formalism.

Using the easily established identity

$$\exp\left(\int f^a_\mu \frac{\delta}{\delta J^a_\mu} \right) \exp(W[J]) = \exp(W[J^a_\mu + f^a_\mu]),$$

Z may be written (we now put $J=0$),

$$Z[0,\overline{\eta},\eta]=\int D\overline{q}Dq\exp(-\overline{q}(\gamma.\partial+\mathcal{M})q+\overline{\eta}q+\overline{q}\eta))\exp\left(W[ig\overline{q}\frac{\lambda^a}{2}\gamma_\mu q]\right),$$

and we can then write Z in the form

$$Z=\exp\left(W_R\left(ig\frac{\delta}{\delta\eta(x)}\frac{\lambda^a}{2}\gamma_\mu\frac{\delta}{\delta\overline{\eta}(x)}\right)\right)\int D\overline{q}Dq\exp\left(-S[\overline{q},q]+\overline{\eta}q+\overline{q}\eta\right),$$

where

$$S[\overline{q},q]=\int d^4xd^4y\Big(\overline{q}(x)(\gamma.\partial_x+\mathcal{M})\delta^4(x-y)q(y)$$

$$+\tfrac{1}{2}g^2\overline{q}(x)\frac{\lambda^a}{2}\gamma_\mu q(x)D^{ab}_{\mu\nu}(x-y)\overline{q}(y)\frac{\lambda^b}{2}\gamma_\nu q(y)\Big). \tag{2·2}$$

Using Fierz identities it is possible to rearrange the quartic term to obtain

$$S[\overline{q},q]=\int d^4xd^4y\Big[\overline{q}(x)(\gamma.\partial+\mathcal{M})\delta^4(x-y)q(y)-\tfrac{1}{2}\overline{q}(x)\frac{M^\theta_m}{2}q(y)D(x-y)$$

$$\times\overline{q}(y)\frac{M^\theta_m}{2}q(x)-\tfrac{1}{2}\overline{q}(x)\frac{M^\phi_d}{2}\overline{q}(y)^{cT}D(x-y)q(y)^{cT}\frac{M^\phi_d}{2}q(x)\Big], \tag{2·3}$$

with $q^c=Cq$, $\overline{q}^c=\overline{q}C$. The Fierz identities are the two Dirac matrix identities

$$\gamma^\mu_{rs}\gamma^\mu_{tu}=K^a_{ru}K^a_{ts};\{K^a\}=\{\mathbf{1},i\gamma_5,\frac{i}{\sqrt{2}}\gamma^\mu,\frac{i}{\sqrt{2}}\gamma^\mu\gamma_5\},$$

$$\gamma^\mu_{rs}\gamma^\mu_{tu}=(K^aC^T)_{rt}(C^TK^a)_{us};\quad\{C=\gamma^2\gamma^4,C^2=-\mathbf{1},C\gamma^\mu C=\gamma^{\mu T}\},$$

and for the colour algebra we use

$$\lambda^a_{\alpha\beta}\lambda^a_{\gamma\delta}=\tfrac{4}{3}\delta_{\alpha\delta}\delta_{\beta\gamma}+\tfrac{2}{3}\sum_{\rho=1}^{3}\epsilon_{\rho\alpha\gamma}\epsilon_{\rho\delta\beta}.$$

As we will see this colour identity leads to the emergence of colour $\mathbf{1}_c$ mesons and colour $\overline{\mathbf{3}}_c$ diquark states. Previous bosonisations of QCD either ignored the colour or proceeded via a bosonisation in terms of $\mathbf{1}_c$ and $\mathbf{8}_c$ Bose fields (Cahill *et al.* 1983, 1985; Roberts and Cahill 1987), which suffers from the major deficiency that the $\mathbf{8}_c$ fields correspond to unbound $\overline{q}q$ states. This is because gluon exchange in such states is repulsive. Thus it had been completely unclear as to what should be done with these unphysical Bose fields. In the bosonisation reviewed here the fields that arise are only the $\mathbf{1}_c$ $\overline{q}q$ fields that arose in the first bosonisation and $\overline{\mathbf{3}}_c$ qq diquark fields (and their $\mathbf{3}_c$ $\overline{q}\overline{q}$ antimatter partners). This result is important for two reasons.

First because gluon exchange between q and q in $\overline{\mathbf{3}}_c$ states is attractive (see Cahill *et al.* 1987 for the dynamical argument) there exists, as we will discuss here, an expansion of the bilocal diquark fields into local diquark fields, with each such local diquark field describing a particular diquark bound state. The masses of these states will be shown to be determined by Bethe–Salpeter type equations. Second, the $\overline{\mathbf{3}}_c$ qq states play a fundamental role in baryon structure because baryons in QCD are three quark colour singlet states and hence (see Cahill *et al.* 1987; Cahill 1989*a*) any two of the quarks are necessarily in $\overline{\mathbf{3}}_c$ states. Hence the diquark boson fields that arise in the new bosonisation of QCD are the components of the baryons, and we are clearly on the path to a meson–baryon effective action description of QCD. Interestingly the diquark $\mathbf{6}_c$ states, for which gluon exchange is repulsive, do not arise in the new bosonisation. Hence it could be said that we have replaced the unphysical $\mathbf{8}_c$ $\overline{q}q$ sector by the physical $\overline{\mathbf{3}}_c$ and $\mathbf{3}_c$ diquark sector.

For $N_f = 3$ the Fierz flavour identities are

$$\delta_{ij}\delta_{kl} = F^c_{il}F^c_{kj}; \qquad \{F^c, c = 0, ..., 8\} = \left\{\frac{1}{\surd 3}\mathbf{1}, \frac{\lambda^1}{\surd 2}, ..., \frac{\lambda^8}{\surd 2}\right\},$$

$$\delta_{ij}\delta_{kl} = H^f_{ik}H^f_{lj}; \qquad \{H^f, f = 1, ..., 9\} = \{F^c, c = 7,5,2,0,1,3,4,6,8\},$$

where $\{\lambda^a/2\}$ are the generators of $SU(3)$ in the usual Gell-Mann representation. We define the tensor products $\{M^\theta_m\} = \{\surd\frac{4}{3}K^aF^c\}$ and $\{M^\phi_d\} = \{i\surd\frac{2}{3}K^a\epsilon^\rho H^f\}$, where $(\epsilon^\rho)_{\alpha\beta} = \epsilon_{\rho\alpha\beta}$. We see that $\overline{q}(y)M^\theta_m q(x)$ are $\mathbf{1}_c$ bilocal $\overline{q}q$ fields with the flavour ($\mathbf{1}_f$ or $\mathbf{8}_f$) determined by the flavour generators ($\{F^0\}$ or $\{F^{1,...,8}\}$) in M^θ_m, while $q(y)^{cT}M^\phi_d q(x)$ are $\overline{\mathbf{3}}_c$ bilocal qq fields with the flavour ($\overline{\mathbf{3}}_f$ or $\mathbf{6}_f$) determined by the flavour generators ($\{H^{1,2,3}\}$ or $\{H^{4,...,9}\}$) in M^ϕ_d respectively. These results follow from the colour and flavour representations of the quark fields. The (integral) spin of these boson fields is determined by the K^a.

We make the first FIC change of variables by noting that the quartic terms in $\exp(-S)$ may be generated by the following bilocal FIC variables:

$$Z = \exp(W_R)\int D\overline{q}DqD\mathcal{B}D\mathcal{D}D\mathcal{D}^\star \exp\left(\iint\left[-\overline{q}(x)(\gamma.\partial + \mathcal{M})\delta^4(x-y)q(y)\right.\right.$$

$$-\frac{\mathcal{B}^\theta(x,y)\mathcal{B}^\theta(y,x)}{2D(x-y)} - \frac{\mathcal{D}^\phi(x,y)\mathcal{D}^\phi(x,y)^\star}{2D(x-y)} - \overline{q}(x)\frac{M^\theta_m}{2}q(y)\mathcal{B}^\theta(x,y)$$

$$\left.\left.-\tfrac{1}{2}\overline{q}(x)\frac{M^\phi_d}{2}\overline{q}(y)^{cT}\mathcal{D}^\phi(x,y)^\star - \tfrac{1}{2}\mathcal{D}^\phi(x,y)q(y)^{cT}\frac{M^\phi_d}{2}q(x)\right] + \int(\overline{\eta}q + \overline{q}\eta)\right),$$

where $\mathcal{B}^\theta(x,y) = \mathcal{B}^\theta(y,x)^\star$ are 'hermitean' bilocal fields. To confirm the above results it is necessary to use FIC identities like

$$\int D\mathcal{D}D\mathcal{D}^\star \exp\left(-\int\mathcal{D}^\star A\mathcal{D} + \int\mathcal{D}^\star J + \int J^\star\mathcal{D}\right) = (DetA)^{-1}\exp\left(\int J^\star A^{-1}J\right).$$

Bilocal fields are necessary in the FIC approach to quantum field theory in order to preserve the internal structure of two-particle (meson or diquark) states. If one attempts to produce a bosonisation using local fields, then the resulting two-particle bound states are forced to have zero size in Euclidean space–time. Such a zero size corresponds to momentum-space form factors which are constants, and this causes a plethora of mathematical UV divergences. The usual ad hoc fix-up is to introduce UV cutoffs, and also to use the same cutoff for every state in order to avoid introducing too many unknown parameters.

Integration over the quark fields completes the change of variables to bilocal meson and diquark fields, giving ($\Theta = (\overline{\eta}, -\eta^T)$)

$$Z[0,\overline{\eta},\eta] = \exp(W_R)\int D\mathcal{B}D\mathcal{D}D\mathcal{D}^\star (Det\mathcal{F}^{-1}[\mathcal{B},\mathcal{D},\overline{\mathcal{D}}])^{\frac{1}{2}}$$

$$\times \exp\left(\int\int -\frac{\mathcal{B}^\theta(x,y)\mathcal{B}^\theta(y,x)}{2D(x-y)} - \int\int \frac{\mathcal{D}^\phi(x,y)\mathcal{D}^\phi(x,y)^\star}{2D(x-y)} + \tfrac{1}{2}\int \Theta\mathcal{F}\Theta^T\right), \qquad (2\cdot4)$$

$$\mathcal{F}^{-1}[\mathcal{B},\mathcal{D},\overline{\mathcal{D}}] = \begin{pmatrix} -\mathcal{D} & G^{-1T} \\ -G^{-1} & -\overline{\mathcal{D}} \end{pmatrix},$$

$$G^{-1}(x,y,[\mathcal{B}]) = (\gamma.\partial + \mathcal{M})\delta^4(x-y) + \mathcal{B}(x,y), \qquad \mathcal{B}(x,y) = \mathcal{B}^\theta(x,y)\frac{M_m^\theta}{2},$$

$$\overline{\mathcal{D}}(x,y) = \mathcal{D}^\phi(x,y)^\star \frac{M_d^\phi}{2} C^T, \qquad \mathcal{D}(x,y) = \mathcal{D}^\phi(y,x) C^T \frac{M_d^\phi}{2}.$$

Using a new determinant identity [where A,B,C and D are $N \times N$ matrices (Cahill *et al.* 1989*b*), the existence of which was in fact suggested by the baryonisation of the diquark sector],

$$Det\begin{pmatrix} A & B \\ C & D \end{pmatrix} = Det(CB)Det(C^{-1}DB^{-1}A - \mathbf{1}),$$

we obtain $Det\mathcal{F}^{-1} = (Det(G^{-1}))^2 Det(\mathbf{1} + \overline{\mathcal{D}}G^T\mathcal{D}G)$, and using $Det(M) = \exp(TrLn(M))$, (2·4) becomes

$$Z = \int D\mathcal{B}D\mathcal{D}D\mathcal{D}^\star \exp\Big(TrLn(G[\mathcal{B}]^{-1}) + \tfrac{1}{2}TrLn(\mathbf{1} + \overline{\mathcal{D}}G[\mathcal{B}]^T\mathcal{D}G[\mathcal{B}])$$

$$- \int\int \frac{\mathcal{B}^\theta\mathcal{B}^{\theta\star}}{2D} - \int\int \frac{\mathcal{D}^\phi\mathcal{D}^{\phi\star}}{2D} - R[\mathcal{B},\mathcal{D},\overline{\mathcal{D}}]\Big), \qquad (2\cdot5)$$

where $\exp(-R[\mathcal{B},\mathcal{D},\overline{\mathcal{D}}]) = \left(\exp(W_R)\exp(\tfrac{1}{2}\int\Theta\mathcal{F}\Theta^T)\right)|_{\overline{\eta},\eta=0}$. This FIC representation for Z is exact, although the functional R is only implicitly defined. Here Z is to be understood as having sources or a finite x_4 range and shows that QCD, defined in terms of the fundamental quark and gluon variables, may be reformulated as a functional integral over colour singlet $\overline{q}q$ and colour triplet qq bilocal fields. That is, once the various n-point gluon propagators are known the quark degrees of freedom of QCD may be completely replaced by the above bilocal fields. The effective action defined by (2·5) is similar to that of the global colour symmetry model (GCM) in Cahill *et al.* (1983, 1985) and subsequent papers, but differs in the important point that G only involves $\mathbf{1}_c$ fields and not $\mathbf{8}_c$, and in the presence of extra terms containing $\overline{\mathbf{3}}_c$ and $\mathbf{3}_c$ diquark fields. The GCM is so defined because it is equivalent to using only the action (2·2) which has global colour symmetry, even though dynamical consequences of the local colour symmetry are incorporated into the running coupling constant. We show later how local fields emerge from the bilocal fields, but the very important point is that diquarks are now shown to play a fundamental role in the rigorous reformulation of QCD. Of course they will ultimately emerge as constituents of the $\mathbf{1}_c$ baryons.

We now show (Cahill 1988, 1989) how the diquark sector of the meson–diquark bosonisation generates the colour singlet baryon states of QCD. The above FIC analysis thus gives us a proper introduction of diquarks into the analysis of QCD. There has in fact been a long history of somewhat phenomenological applications of the quark–diquark approach to the calculation of baryon properties. Initial studies were by Ida and Kobayashi (1966) and by Lichtenberg and Tassie (1967). A literature compilation is available (Skytt and Fredriksson 1988), as well as reviews by Lichtenberg (1988) and Fredriksson (1988). Pervushin and Ebert (1979) have studied a bilocal functional approach to diquarks in 2-D QCD.

At this stage we will not include the R contributions which contain the higher order gluon processes—the significant role of R will become apparent later. Consider the diquark part of Z;

$$Z = \int D\mathcal{D}D\mathcal{D}^\star \exp\left(\tfrac{1}{2}TrLn(\mathbf{1} + \overline{\mathcal{D}}G^T\mathcal{D}G) - \int \frac{\mathcal{D}\mathcal{D}^\star}{2D} + \int (J^\star\mathcal{D} + \mathcal{D}^\star J)\right),$$

where the bilocal diquark source terms facilitate the analysis, and in which the $\mathcal{B}$ dependence of $G[\mathcal{B}]$ will affect the non-trivial vacuum, the mesons and provide the meson–baryon couplings. Consider the $\frac{1}{2}TrLn$ term, which on expansion gives

$$\frac{1}{2}\sum_{n=1}^{\infty} \frac{(-1)^{(n+1)}}{n} Tr(\overline{\mathcal{D}}G^T\mathcal{D}G)^n, \qquad (2\cdot 6)$$

which are single loop processes (Fig. 3*a*), but with the quark lines alternating in direction, in accord with their coupling to the diquark and anti-diquark fields. These loop structures are the key to noticing that Z contains contributions representable in terms of baryonic FIC variables. The derived effective action for these fields is that of all the known baryon states. Using (2·6) the diquark part of the action has the expansion $S[\mathcal{D}^\star, \mathcal{D}] = \sum_n S_n[\mathcal{D}^\star, \mathcal{D}]$, and we write

$S_1 = \int \mathcal{D}^{\Phi *}(\Delta_d^{-1})^{\Phi\Psi}\mathcal{D}^{\Psi}$. Defining $S' = S - S_1$, then we get

$$Z = \exp\left(S'\left[\frac{\delta}{\delta J}, \frac{\delta}{\delta J^*}\right]\right)\int D\mathcal{D}D\mathcal{D}^* \exp\left(-\int \mathcal{D}^*\Delta_d^{-1}\mathcal{D} + \int (J^*\mathcal{D} + \mathcal{D}^*J)\right),$$

$$= \exp\left(S'\left[\frac{\delta}{\delta J}, \frac{\delta}{\delta J^*}\right]\right)\exp\left(-TrLn(\Delta_d^{-1}) + \int J^*\Delta_d J\right),$$

where, keeping only the translation invariant part of $\mathcal{B}$, i.e. $\mathcal{B}_v(x-y)$ the vacuum configuration, Δ_d^{-1} has eigenvalues and eigenvectors (diquark form factors) from

$$\int \frac{d^4q}{(2\pi)^4}(\Delta_d^{-1})^{\Phi\Psi}(p,q;P)\Gamma_k^{\Psi}(q;P) = \lambda_k(P^2)\Gamma_k^{\Phi}(p;P). \qquad (2\cdot 7)$$

We have the orthonormality and completeness relations

$$\int \frac{d^4q}{(2\pi)^4}\Gamma_k(q;P)^*\Gamma_l(q;P) = \delta_{kl}, \quad \sum_k \Gamma_k(q;P)\Gamma_k(p;P)^* = (2\pi)^4\delta^4(q-p),$$

and hence the spectral expansion,

$$(\Delta_d^{-1})^{\Phi\Psi}(p,q;P) = \sum \Gamma_k^{\Phi}(p;P)\lambda_k(P^2)\Gamma_k^{\Psi *}(q;P)^*. \qquad (2\cdot 8)$$

Using the completeness relation we find (Cahill 1989*c*)

$$TrLn(\Delta_d^{-1}) = \sum_k \int d^4x \int \frac{d^4P}{(2\pi)^4} ln(\lambda_k(P^2)) = \sum_k TrLn(\lambda_k(-\partial^2)\delta^4(x-y)),$$

and we may construct the local-diquark-field FIC representation

$$\exp(-TrLn(\Delta_d^{-1})) = \int Dd_k Dd_k^* \exp\left(-\sum_k \int d_k(x)^*\lambda_k(-\partial^2)\delta^4(x-y)d_k(y)\right),$$

which is a fundamental identity as it implements the reduction of the bilocal diquark FIC representation to local FIC variables.

By Fourier transforming (2·7), multiplying by $D(x)$ and then inverse Fourier transforming, we obtain an off-mass-shell Bethe–Salpeter (BSE) type integral equation. This is represented somewhat cryptically in Fig. 2*a*. The $\lambda - D - \Gamma$ term actually represents the convolution $\lambda(P^2)\int[d^4q/(2\pi)^4]D(p-q)\Gamma(q;P)$. The on-mass-shell BSE has $\lambda(-m^2) = 0$, and this condition determines the bound state mass. The FIC analysis provides a special definition of the off-mass-shell BSE, and the resulting $\lambda(P^2)$ will be seen to give the effective action (at 2nd order) for these bound states. It is convenient to write $\lambda(P^2) = (P^2 + m(P^2)^2)f^2$, which defines the mass function $m(P^2)$. The off-mass-shell form factors $\Gamma_k(q;P)$ are needed when determining couplings of these states to other states and also when these states are constituents of more complicated bound states, such as baryons.

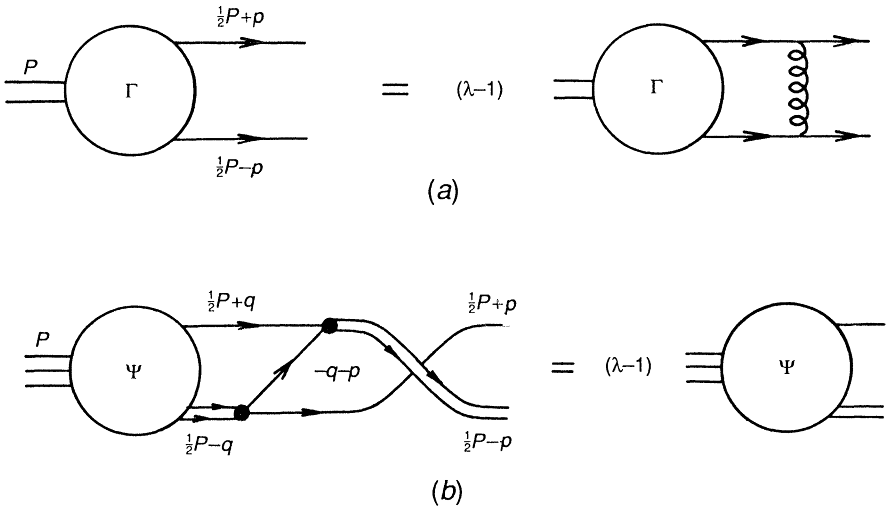

Fig. 2. (*a*) Diquark BSE giving off-mass-shell form factors Γ_k and eigenvalues $\lambda_k(P^2)$; (*b*) corresponding equation (2·11) for baryons in terms of the quark–diquark picture. Shaded vertex is Γ from (*a*). In both the quark lines represent extended quarks due to extensive gluon dressing, analogous to Fig. 1*b*.

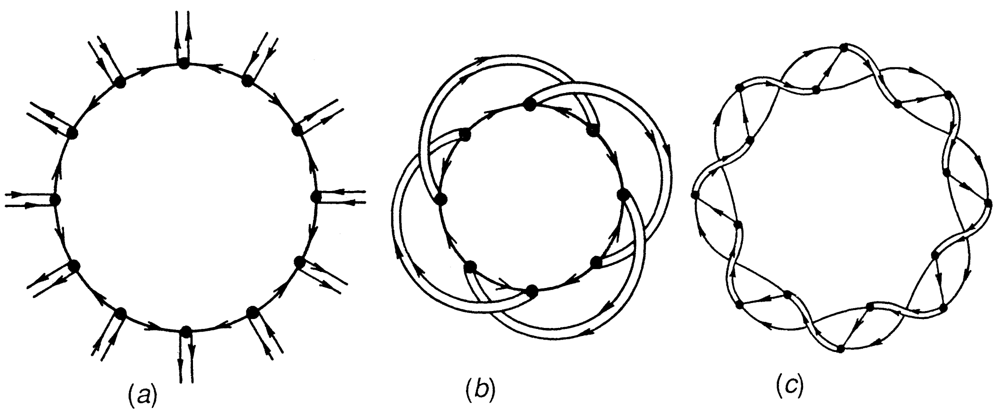

Fig. 3. (*a*) Diagrams from expansion of diquark *TrLn* in (2·6); (*b*) diagrams from (*a*) after diquark field integration; (*c*) similar diagram after redrawing to reveal baryon loop. These loop functionals determine the mass spectrum.

While standard integral equation numerical techniques may be applied to solving (2·7) a special technique has been developed to provide a more direct calculation of the bound state mass and on-mass-shell form factors (Cahill *et al.* 1987). In this technique rather than implicitly determine m by searching for a zero of $\lambda(P^2)$, the on-mass-shell BSE is transformed to define a mass functional $M[\Gamma]$ with the property that $\delta M[\Gamma]/\delta\Gamma(x) = 0$ gives m and the on-mass-shell Γ.

This technique is approximate and is best suited to low mass bound states. For the scalar diquark state 0^+ we find

$$M[\Gamma]^2 = -\frac{24}{f[\Gamma]^2}\int \frac{d^4q}{(2\pi)^4}\frac{\Gamma(q)^2}{A(q)^2q^2+B(q)^2} + \frac{9}{f[\Gamma]^2}\int d^4x \frac{\Gamma(x)^2}{D(x)},$$

where $f[\Gamma]$ is a normalisation functional. This mass functional is very similar to the variational form of the Schrödinger equation for the energy functional $E[\psi]$. It has kinetic energy and potential energy terms. Because of stability arguments (related to the quark self-energy dynamics) the $\lambda(P^2)$ can only have zeros in the time-like region $P^2 < 0$. The diquark masses that arise from $(2\cdot 7)$ are constituent masses, and as we argue in Section 3, they only have meaning if the diquark is part of a colour singlet hadron.

One expects that $(2\cdot 8)$ may be truncated after one or two terms as the higher mass diquark states will not contribute to the lower mass baryon states. It is also important to note that the sum in $(2\cdot 8)$ is purely discrete, that is, there is no continuum contribution. This is because the quark propagators $G[\mathcal{B}_V]$ (where $\mathcal{B}_V$ is the vacuum configuration) are confining, that is, the propagators have no poles on the $q^2 < 0$ (Euclidean metric) axis. Hence there is no 'ionisation' threshold leading to a continuum. To simplify the notation we now only consider one term in the sum $(2\cdot 8)$ and, with the $J^P = \frac{1}{2}^+$ baryon octet $(p, n, \Lambda, \Sigma, ...)$ in mind, we keep the first $J^P = 0^+$ scalar diquark [which has an effective mass (Praschifka *et al.* 1988, 1989) of $\approx 0\cdot 6$ GeV]. The above comments imply that the qq correlations in baryons are influenced by only a few diquark states, and this should have strong implications for the baryon structure functions. In fact the calculated diquark form factors show strong peaking, which causes a quark constituent mass effect in diquarks. The constituent quark mass values are shown in Table 1.

Introducing local sources

$$j_k(X) = \int d^4Y d^4x \Gamma_k^{\phi}(x, X-Y)^* J^{\phi}(x, Y),$$

so that

$$\frac{\delta}{\delta J^{\phi}(x,X)} = \sum_k \int d^4Y \Gamma_k^{\phi}(x, Y-X)^* \frac{\delta}{\delta j_k(Y)},$$

but then keeping only the first scalar diquark, Z may be written

$$Z[j^\star, j] = \exp\left(-S'\left[\frac{\delta}{\delta j}, \frac{\delta}{\delta j^\star}\right]\right)\exp\left(-TrLn(\Delta_d^{-1}) + \int j^\star(X)\lambda_0(-\partial^2)^{-1}j(X)\right).$$

Evaluating the effect of the functional operator we find that $Z[0,0]$ has the form $\exp(-W)$ where W is the sum of connected loop diagrams, with the vertices now joined by the diquark propagators $\lambda_0(P^2)^{-1}$, and with $\Gamma_0(p;P)$ at the vertices. Of particular significance is that infinite subset of diagrams which will be seen to have the form of three-quark (i.e. baryon) loops (Fig. 3*b*). These come with a combinatoric factor of 2 (except for the order $n = 3$ diagram) which cancels

Table 1. Summary of some of the numerical results (1983–90) from the GCM in the chiral limit, based on the simple model for the gluon propagator in equation (3·6)

m_q is the constituent mass of the quark in the meson or diquark

Observable	Theory	Experiment
MIT bag constant (3 quarks)	161 MeV	146 MeV
f_π	74 MeV	93 MeV
Wess–Zumino coeff. λ	1	Witten ($\pi^0 \to 2\gamma$) => 1
$g_{\rho\pi\pi}$	7·51	6·14
Meson masses (chiral limit)	m (m_q)	
δ	390(130) MeV	—
π	0(–) MeV	135 MeV
ρ, ω	745(340) MeV	m_ω =783 MeV
a_1, D	1310(140) MeV	m_D =1283 MeV
$\Delta = (1 - m_\rho^2/m_\omega^2)^{\frac{1}{2}}$	0·29	0·181
Diquark masses (chiral limit)	m (m_q)	
0^+	607(280) MeV	>400 MeV
0^-	948(180) MeV	—
1^-	1950(140) MeV	—
1^+	1170(310) MeV	—
Baryon masses (chiral limit) (quenched approximation)		
$\frac{1}{2}^+$	1200–1300 MeV	Compare future lattice cal.

the $\frac{1}{2}$ coefficient of the *TrLn*. These 3-loops are planar for even order, but non-planar, with one twist, for odd order. To exhibit the three-quark loop structure we show, in Fig. 3*c*, a typical diagram from Fig. 3*b* after deformation, revealing a closed double helix: a diagram of order n is drawn on a Mobius strip of $n-1$ twists.

This infinite series may be summed as the diagrams are generated by the kernel K, defined by the one-twist segment. The weightings are such that all the double helix diagrams may be summed to $TrLn(\mathbf{1}+K)-TrK = W_B - TrK$. Thus $Z[0,0] = \exp(+W_B + W_R)$, in which W_R is the sum of the remaining diagrams. To determine the content of W_B we consider the eigenvalue problem for $\mathbf{1}+K$;

$$(\mathbf{1}+K)\Psi_k = \lambda_k \Psi_k, \tag{2·9}$$

which, for $\mathcal{B} = \mathcal{B}_V$, has the momentum space form (Fig. 2*b*)

$$\int \frac{d^4q}{(2\pi)^4} K(p,q;P)^{\beta j,\rho h}_{\alpha f,\gamma l} \Psi^{\gamma l}_{\rho h}(q;P) = (\lambda(P^2) - 1)\Psi^{\beta j}_{\alpha f}(p;P), \tag{2·10}$$

$$K(p,q;P)^{\beta j,\rho h}_{\alpha f,\gamma l} = \sum_{i\delta} \tfrac{1}{12}\Gamma_0 \epsilon_{\gamma\alpha\delta}\epsilon_{lfi} i\gamma_5 C^T G^T C^T i\gamma_5 \epsilon_{\beta\delta\rho}\epsilon_{jih}\Gamma_0 G \Lambda_0^{-1},$$

in which we keep only the lowest mass scalar diquark state, and where the momentum arguments are given in Fig. 2*b* and (2·12). Equation (2·10) is a bound state equation for a three-quark state in which the paired quarks form a scalar diquark. It is a simple matter to include further diquark states from the spectral expansion (2·8), but in doing so we note that the generalised (coupled) equations still have only one $\int d^4q$ because (2·7) has only a discrete spectrum. In (2·10) Ψ is a Dirac spinor and, as well, a 2nd rank tensor in both colour and flavour.

Previously we reported only the physically significant colour-singlet flavour-octet baryons, but a full colour and flavour multiplet analysis of (2·10) reveals some important results. Decomposing $\Psi^{\gamma l}_{\rho h}$ according to

$$(\mathbf{3}\otimes\overline{\mathbf{3}})_C\otimes(\mathbf{3}\otimes\overline{\mathbf{3}})_F=(\mathbf{1}_C\otimes\mathbf{1}_F)\oplus(\mathbf{1}_C\otimes\mathbf{8}_F)\oplus(\mathbf{8}_C\otimes\mathbf{1}_F)\oplus(\mathbf{8}_C\otimes\mathbf{8}_F),$$

or in detail,

$$\Psi^{\gamma l}_{\rho h}=\tfrac{1}{9}\delta_{\gamma\rho}[\Psi^{\alpha k}_{\alpha k}]\delta_{lh}+\tfrac{1}{3}\delta_{\gamma\rho}[\Psi^{\alpha l}_{\alpha h}-\tfrac{1}{3}\Psi^{\alpha k}_{\alpha k}\delta_{lh}]$$
$$+\tfrac{1}{3}[\Psi^{\gamma k}_{\rho k}-\tfrac{1}{3}\Psi^{\alpha k}_{\alpha k}\delta_{\gamma\rho}]\delta_{lh}+[\Psi^{\gamma l}_{\rho h}-\tfrac{1}{3}\Psi^{\alpha l}_{\alpha h}\delta_{\rho\gamma}-\tfrac{1}{3}\Psi^{\gamma k}_{\rho k}\delta_{fk}+\tfrac{1}{9}\Psi^{\alpha k}_{\alpha k}\delta_{\gamma\rho}\delta_{lh}],$$

and each particular member ($\Psi\equiv[\ldots]$) of one multiplet is then seen to be an eigenvector of

$$\int\frac{d^4q}{(2\pi)^4}K(p,q;P)\Psi(q;P)=(\lambda(P^2)-1)\Psi(p;P),\qquad(2\cdot11)$$

$$K(p,q;P)=-\frac{N[m]}{6}\Gamma_0(\frac{P}{4}+q+\frac{p}{2};\frac{P}{2}-p)\Gamma_0(\frac{P}{4}+p+\frac{q}{2};\frac{P}{2}-q)$$
$$\times\,\lambda_0((\frac{P}{2}-q)^2)^{-1}G(-q-p)G(\frac{P}{2}+q),\qquad(2\cdot12)$$

where the $N[m]$ depend on the multiplet, and we find $N[\mathbf{1}_C\otimes\mathbf{1}_F]=-2, N[\mathbf{1}_C\otimes\mathbf{8}_F]=+1, N[\mathbf{8}_C\otimes\mathbf{1}_F]=+1, N[\mathbf{8}_C\otimes\mathbf{8}_F]=-\frac{1}{2}$.

Now (2·11), with $N=+1$, is the basic equation for the $J^P=\frac{1}{2}^+$ baryon octet ($\mathbf{1}_C\otimes\mathbf{8}_F$) where Ψ is the baryon–quark–diquark form factor, and has been analysed and solved numerically (Burden *et al.* 1989), and was indeed first obtained by summing ladder diagrams in a covariant three-body Faddeev manner (Cahill *et al.* 1989*a*). It has bound state solutions with a mass [from $\lambda(-M^2)=0$] of $\approx 1\cdot2$–$1\cdot3$ GeV (which is indeed the expected value for a bare nucleon before Nambu–Goldstone boson dressing). This result is to be directly compared with lattice model calculations of baryon masses in the quenched approximation (i.e. no quark loops) however, unlike the analysis here, the lattice computations have not yet been made to work in the chiral limit ($\mathcal{M}\to 0$).

The $\mathbf{1}_C\otimes\mathbf{1}_F$ and $\mathbf{8}_C\otimes\mathbf{8}_F$ multiplets have negative values for N and thus the quark rearrangement process is repulsive, and the corresponding $\lambda(-M^2)$ have no zeros in the time-like region. However the colour octet–flavour singlet 'baryons' have an N value which means they are degenerate in mass with the colour singlet–flavour octet baryons. This perhaps surprising result is in fact a valuable indicator to the role of the R term in the meson–diquark intermediate effective action. In this section we continue to extract the effective action for the colour singlet sector of the theory.

Let us now construct an appropriate FIC representation for the colour singlet baryons part of $\exp(TrLn(\mathbf{1}+K))$. To this end note that an eigenvalue for positive energy solutions, with degeneracy 2 (for spin $\uparrow$ and $\downarrow$), has the form

$$\lambda_+^{\uparrow\downarrow}(P^2) = (M(P^2) + ia(P^2)\sqrt{P^2})F$$

[define F so that $a = 1$ when $\lambda = 0$, then $M_k(P^2)$ are baryon off-mass-shell mass functions], while for negative energy solutions (anti-baryons) $\lambda_-^{\uparrow\downarrow} = (\lambda_+^{\uparrow\downarrow})^*$. Thus, from the spectral representation for $\mathbf{1}+K$,

$$\exp(TrLn(\mathbf{1}+K)) = \exp\Big(\sum_k n\int d^4x\int\frac{d^4P}{(2\pi)^4}[ln(\lambda_+^{\uparrow\downarrow}(P^2)^2) + (\lambda_-^{\uparrow\downarrow}(P^2)^2)]\Big),$$

where k sums the ground state and excited baryons states of (2·11), and the squares in the ln terms arise from the spin degeneracy, while $n = 8$ from the flavour degeneracy (the other baryon states are not shown), and with $a = 1$ for simplicity,

$$= \exp\Big(\sum_k n\int d^4x\int\frac{d^4P}{(2\pi)^4}ln[(P^2 + M_k(P^2)^2)^2F_k^4]\Big)$$

$$= \exp\Big(\sum_k nTrLn((\gamma.\partial + M_k(-\partial^2))F_k^2\delta^4(x-y))\Big)$$

$$= \int D\overline{N}_kDN_k \exp\Big(-\sum_k\int d^4x\overline{N}_k(x)(\gamma.\partial + M_k(-\partial^2))F_k^2N_k(x)\Big), \qquad (2\cdot 13)$$

in terms of $\overline{N}_k$ and N_k, each of which is a flavour octet of local baryonic spin-$\frac{1}{2}$ FIC variables. Hence the exponentiated sum of the closed double helix diagrams is representable as a (free) baryon field theory. The F_k may be absorbed with a re-definition of the baryon fields. Other more complicated (including baryon multi-loops) diagrams are present and constitute a wealth of dressings and interactions between these (bare) baryons. Of some interest would be the terms describing non-mesonic nucleon–nucleon interactions, which would affect the short range nucleon–nucleon force.

We now briefly indicate the explicit form of the meson sector of (2·5). The complete Z has the form, with $S[\mathcal{B}]$ the non-diquark part of the effective action,

$$Z = \int D\mathcal{B}\exp\Big(-S[\mathcal{B}] - \sum_{diquarks} TrLn(\lambda_k(-\partial^2;[\mathcal{B}_\nu])\delta^4(x-y))$$

$$+ \sum_{baryons} TrLn(\gamma.\partial + M_k(-\partial^2;[\mathcal{B}_\nu])\delta^4(x-y)) + \ldots\Big). \qquad (2\cdot 14)$$

To extract the content of (2·14) we first determine the 'vacuum' configuration $\mathcal{B}_V$, as the solution of the Euler–Lagrange (EL) equation $\delta[S+...]/\delta\mathcal{B}=0$, where $[S+...]$ is the complete action, which becomes

$$\mathcal{B}_V^{\theta}(x,y)=D(x-y)\left[tr\left(G(x,y,[\mathcal{B}_V])\frac{M_m^{\theta}}{2}\right)+...\right], \qquad (2\cdot 15)$$

a Schwinger–Dyson type equation describing quark self-energy processes (to be discussed in Section 3), where '+...' denotes contributions from the diquark and baryon loops etc. This a nonlinear equation for the $\{\mathcal{B}_V^{\theta}\}$, and only translation invariant solutions, depending only on $x-y$, are known. In the chiral limit ($\mathcal{M}\rightarrow 0$), this equation has degenerate solutions and the analysis in Section 3 shows that G has the form

$$G(q;V)=[iA(q)q.\gamma+VB(q)]^{-1}=\zeta^{\dagger}G(q;\mathbf{1})\zeta^{\dagger};\ \zeta=\sqrt{V},$$

where $V=\exp(i\sqrt{2}\gamma_5\pi^aF^a)$ and $\{\pi^a\}$ are arbitrary real constants $|\,\pi|\in[0,2\pi]$. Thus in the chiral limit the vacuum is degenerate and is the manifold G/H where G is the chiral group and $H=U_V\subset G$. Thus the chiral symmetry is represented as a hidden symmetry. It may be shown that the eigenvalues in (2·10) are chiral invariants, i.e. independent of the values of these π^a. Let us now change variables in (2·14) so that $\mathcal{B}=0$ is now the vacuum. It is convenient here to give the quarks small current masses to avoid dealing with the degenerate vacuum. Expanding $S[\mathcal{B}]$ about its minimum we have

$$S[\mathcal{B}]=\sum_{n=0,2,3\,...}S_n[\mathcal{B}],$$

where S_n is of order n in $\mathcal{B}$ and we write $S_2=\frac{1}{2}\int\mathcal{B}^{\theta}(\Delta_m^{-1})^{\theta\psi}\mathcal{B}^{\psi}$. Introducing bilocal source terms in (2·14) we have, with $S'=S-S_2$, and showing only the meson part of (2·14),

$$\begin{aligned}Z[J]&=\int D\mathcal{B}\exp\left(-S'[\mathcal{B}]-S_2[\mathcal{B}]+\int\mathcal{B}^{\theta}J^{\theta}\right)\\&=\exp\left(-S'[\frac{\delta}{\delta J}]\right)\int D\mathcal{B}\exp\left(-\int\tfrac{1}{2}\mathcal{B}^{\theta}(\Delta_m^{-1})^{\theta\psi}\mathcal{B}^{\psi}+\int\mathcal{B}^{\theta}J^{\theta}\right)\\&=\exp\left(-S'[\frac{\delta}{\delta j}]\right)\int Dm_k\exp\left(-\sum_k\tfrac{1}{2}\int m_k(x)\lambda_k(-\partial^2)m_k(x)+\int j_km_k\right), \qquad (2\cdot 16)\end{aligned}$$

by using techniques similar to that for the diquarks. Here $\{m_k(x)\}$ is an infinite set of local meson fields, each corresponding to one physical meson type, and the λ_k are the eigenvalues of the meson form of (7)—a Bethe–Salpeter equation, which also gives the meson form factors $\Gamma_k(p,P)$. Applying the functional operator $\exp(-S'[\delta/\delta j])$, and with $j\rightarrow 0$, we obtain

$$Z=\int Dm_k\exp\left(-\sum_k\tfrac{1}{2}\int m_k(x)\lambda(-\partial^2)m_k(x)-S'[m_k]\right).$$

By explicit evaluation (Praschifka *et al.* 1987; Roberts *et al.* 1989) of $S'[m_k]$, and identifying the mesons by their quantum numbers, we obtain the full local FIC representation;

$$Z = \int D\pi D\rho D\omega \ldots D\overline{N}DN \ldots \exp\Big(-S[\pi,\rho,\omega,\ldots]$$

$$-\sum \int (\overline{N}_k(\gamma.\partial + M_k(-\partial^2))N_k) + \ldots \Big) \qquad (2\cdot 17)$$

$$S[\pi,\rho,\omega,\ldots] = \int d^4x \Big[\frac{f_\pi^2}{2}[(\partial_\mu\pi)^2 + m_\pi^2\pi^2] + \frac{f_\rho^2}{2}[\rho_\mu\partial^2\rho_\mu$$

$$+(\partial_\mu\rho_\mu)^2 + m_\rho^2\rho_\mu^2] + \frac{f_\omega^2}{2}[\rho \to \omega] - f_\rho f_\pi^2 g_{\rho\pi\pi}\rho_\mu.\pi\times\partial_\mu\pi$$

$$-if_\omega f_\pi^3 \epsilon_{\mu\nu\sigma\tau}\omega_\mu\partial_\nu\pi.\partial_\sigma\pi\times\partial_\tau\pi - if_\omega f_\rho f_\pi G_{\omega\rho\pi}\epsilon_{\mu\nu\sigma\tau}\omega_\mu\partial_\nu\rho_\sigma.\partial_\tau\pi$$

$$+\frac{i\lambda[A,B,\Gamma_\pi]}{80\pi^2}\epsilon_{\mu\nu\sigma\tau}tr(\pi.F\partial_\mu\pi.F\partial_\nu\pi.F\partial_\sigma\pi.F\partial_\tau\pi.F) + \ldots \Big] + S[0,\ldots],$$

where we have written $\lambda_j(P^2) = (P^2 + m_j(P^2)^2)f_j^2$; $m_j(P^2)$ are the off-mass-shell meson mass functions, and only the physical masses [from $\lambda(P^2) = 0$] are shown above. The imaginary terms in this meson action are the chiral anomalies of QCD. All of the 'observable' parameters $(f_\pi, m_\pi, \ldots)$ have been explicitly calculated and the broad ranging agreement with the experimental values [or the phenomenological values in those cases where quantum fluctuations following from the functional integrations in $(2\cdot 16)$ are important] and their critical dependence on the quark self-energy effect (Section 3) indicate that we have identified the dominant dynamical processes in QCD, and that we have a viable approximation scheme to compute their effects. This general result suggests that the consequences of the R term must be small for the colour singlet states described by the above effective action. Of particular interest is the coefficient $\lambda[A,B,\Gamma_\pi]$ of the Wess–Zumino term, in which five NG bosons (π's) are coupled by a single quark loop. Because of the critical role of the quark self-interaction we have $B(s) = \Gamma_\pi(s)$ in the chiral limit and we find (Praschifka *et al.* 1987*b*; Cahill *et al.* 1988*a*; Roberts *et al.* 1988) $\lambda[A,B,B] = 1$ independent of the detailed form of A or B.

The inclusion of the baryon states essentially completes the program of the hadronisation of QCD begun by Kleinert (1976*a*, 1976*b*, 1982*a*, 1982*b*) and by Ebert and Pervushin (1976), and shows how the effective action describing the low energy domain (i.e. nuclear physics) may be derived by FIC methods from the defining action of QCD in a completely systematic procedure. Note that this effective action is an S-type and not a Γ-type effective action (see Cahill 1990 for a discussion of these types in the context of composite systems).

Iannella and Cahill (1990) have shown how one may restore the finite (Euclidean) time interval $(0,T)$ in FI like $(2\cdot 17)$, corresponding to the steps

(for a boson field say)

$$\int D\phi \exp\Big(-\int \phi(x)\lambda(-\partial^2)\phi(x)\Big) = \exp\Big(-\int d^4X \int \frac{d^4P}{(2\pi)^4} ln(\lambda(P^2))\Big)$$

$$\to \exp(-E_{vac}T)\sum_n \exp(-E_n T),$$

in which the energy spectrum $\{E_n\}$ is parametrised by the physical mass from $\lambda(P^2) = 0$, and E_{vac} is a 'vacuum energy'. If there are no such zeros in the time-like region, as for confining quark propagators, then we do not obtain such a spectrum, but merely a contribution to E_{vac}. The baryon loops (Fig. 3*c*) sum to give the baryon *TrLn*, in (2·13), and by re-introducing the finite T the expected baryon contribution to the energy spectrum is obtained. Fermi–Dirac or Bose–Einstein statistics arise from using the appropriate Matsubara 'frequencies'.

When the quarks are massless special techniques are needed to deal with the degenerate vacuum manifold, for in this case the NG boson local fields $\pi(x)$ form homogeneous Riemann coordinates on the vacuum manifold, and we must use the matrix field $U(x) = \exp(i\sqrt{2}\pi^a(x)F^a)$ where $V(x) = P_L U(x)^\dagger + P_R U(x) = \exp(i\sqrt{2}\gamma_5\pi^a(x)F^a)$. The ground state pseudoscalars, in the chiral limit, play a dual role: they are at the same time both the NG bosons associated with the hidden chiral symmetry and also $\bar{q}q$ bound states. To maintain the hidden chiral symmetry necessitates a derivative expansion in $\partial_\mu U(x)$, and we obtain (Roberts *et al.* 1988), for the NG boson sector only,

$$\int d^4x \frac{f_\pi^2}{2}[(\partial_\mu\pi)^2 + m_\pi^2\pi^2] \to \int d^4x\Big(\frac{f_\pi^2}{4}tr(\partial_\mu U\partial_\mu U^\dagger) + \kappa_1 tr(\partial^2 U\partial^2 U^\dagger)$$

$$+\kappa_2 tr([\partial_\mu U\partial_\mu U^\dagger]^2) + \kappa_3 tr(\partial_\mu U\partial_\nu U^\dagger\partial_\mu U\partial_\nu U^\dagger) + \frac{\rho}{4}tr([2\mathbf{1} - U - U^\dagger]\mathcal{M}) + \ldots\Big) \quad (2\cdot 18)$$

Under a chiral transformation $G = U_L \otimes U_R$ we find $U(x) \to U_L U(x) U_R^\dagger$. The coupling of the baryon states to the above mesons requires us to keep the full $\mathcal{B}$ in analysing the baryon sector, and not just the vacuum value $\mathcal{B}_V$. However the long wavelength limit of the NG-boson–baryon coupling may be inferred from the chiral invariance of (2·14). Now

$$TrLn\Big((\gamma.\partial + M(-\partial^2))\delta^4(x-y)\Big) = TrLn\Big((\gamma.\partial + \mathcal{V}M(-\partial^2))\delta^4(x-y)\Big)$$

reflects that invariance in (2·13), where $\mathcal{V} = \exp(i\sqrt{2}\gamma_5\pi^a T^a)$, with $\{T^a\}$ the generators of $SU(3_f)$ **8** representation. Letting $\pi \to \pi(x)$ then gives the long wavelength coupling. Expanding $\mathcal{V}$ to 1st order in $\pi^a(x)$ (2·17) gives

$$Z = \int D\pi \ldots D\overline{N}DN \ldots \exp\Big(-\int\Big[\frac{f_\pi^2}{2}((\partial_\mu\pi)^2 + m_\pi^2\pi^2) + \ldots$$

$$+tr\{\overline{N}(\gamma.\partial + m_N + \Delta m_N + m_N\sqrt{2}i\gamma_5\pi^a\mathcal{T}^a + \ldots)N\} + \ldots\Big]\Big), \quad (2\cdot 19)$$

in which the baryon octet is finally written as a 2nd rank tensor, $N = N^a \mathcal{T}^a$, where the $\{\mathcal{T}^a\}$ are generators (Cahill *et al.* 1989*a*) of the $SU(3_f)$ **3** rep. We have shown m_π and Δm_N which are mass terms from the chiral symmetry breaking quark current masses, while m_N is the 'chiral mass' of the baryons. For nonzero quark current masses the NG boson masses $\{m_\pi\}$ and the baryon octet mass splittings $\{\Delta m_N\}$ are seen to satisfy the Gell-Mann–Okubo and Coleman–Glashow formulae.

Let us now clearly state what the result (2·19) means. First, ignoring the various colour carrying bound states that arise at the level of the GCM truncation, and the effects of the R action term on these (and on the colour uncharged states), we see that we have essentially made a dynamically determined change of integration variables in re-writing (2·1b) as

$$\int D\overline{q}DqDA\, \exp(-S[A^a_\mu,\overline{q},q]) = \int D\pi ... D\overline{N}DN ... \exp(-S[\pi, ...,\overline{N},N]), \qquad (2\cdot 20)$$

in which $\pi(x), N(x), ...$ are local (bare) hadronic fields, basically describing the 'centre-of-mass' motion of these extended and complex bound states, and where we mean that the partition function of QCD may be approximately determined, at least for the low mass states, by the hadronic functional integrals. This just corresponds to the well understood idea that the hadrons are the low energy states of QCD. The detailed form of $S[\pi, ...]$ and the (bare) hadronic parameters it contains, such as masses and coupling constants, are completely determined by the above derivation, although it is obviously necessary to truncate the formulation in practical computations. The appearance of the physically observed particles of nuclear physics in (2·20), or more accurately, the physical states which emerge from carrying out the hadronic functional integrals, follows from the dynamical implications of the colour algebra, which ingeniously can bind $q\overline{q}$, qq, and qqq states, all in the necessary colour states. The hadronic integrations in (2·20) include the well known phenomenon of dressing of the bare states, the most significant being the dressing of the bare baryon states by the NG bosons (principally the pions). They also have been used to generate level splitting, such as the splitting of the ρ from the ω due to the $\rho \to \pi\pi \to \rho$ channel being available only to the ρ (see the Δ parameter in Table 1). However in these computations it is necessary to keep the non-locality of the hadronic couplings, otherwise loop calculations from (2·20) would have the usual, but in the present case, spurious divergences (Roberts *et al.* 1989; Hollenberg *et al.* 1990). These divergences arise whenever the extended nature of the hadronic states is suppressed. Of course the formal derivation of this hadronisation of QCD must be accompanied by detailed computations of the quark, diquark, meson and baryon mass functions and relativistic form factors, all driven by the gluon 2-point functions.

Ball (1990) and Reinhardt (1990) have given adaptions of the '$\mathbf{1}-\overline{\mathbf{3}}-\mathbf{3}$' meson–diquark bosonisation path to the hadronisation of quark physics to a Nambu–Jona-Lasinio (NJL) (1961) type model, which is obtained from the full analysis herein by neglecting all $D^{(n)}, n \geq 3$, as in the GCM, but with the additional extreme (NJL) approximation $D^{(2)} \to d\delta^{(4)}(x-y)$, which introduces spurious UV divergences, and hence does not permit any dynamical appreciation of the quark self-energy effects. These adaptions also overlooked the occurrence of the $\mathbf{8}_C \otimes \mathbf{1}_F$ baryons in GCM (and NJL) type models.

3. Quark Self-Interaction

We saw in the previous section that the effective action needs to be expanded about its minimum in order to extract the hadronic dynamics. Ignoring for simplicity the diquark and baryon loop functionals in (2·14), and keeping only the quark loop functional, the minimum of the action with respect to the bilocal meson fields is determined by (2·15). In general the configurations so determined are called 'vacuum configurations', and unless we expand the effective action about its true minimum we will obtain tachyonic excitations, the presence of which will merely indicate the instability of the false 'vacuum configuration'. However in the hadronisation of QCD these 'vacuum equations' are simultaneously also the quark self-energy dynamics, and in this section we explore this most significant effect. The $\mathcal{B}^\theta_\gamma(x,y)$ fields from (2·15) minimise the action

$$S[\mathcal{B}] = -TrLn(G[\mathcal{B}]^{-1}) + \int d^4x \int d^4y \frac{\mathcal{B}^\theta(x,y)\mathcal{B}^\theta(x,y)^\star}{2D(x-y)}. \qquad (3\cdot 1)$$

Multiplying (2·15) by $M^\theta_m/2$ and using the Fierz identities for the spin and flavour algebras, we obtain

$$\Sigma(x,y) = D(x-y)\gamma_\mu G(x,y)\gamma_\mu,$$

with

$$G(x,y)^{-1} = (\gamma.\partial + \mathcal{M})\delta^4(x-y) + \Sigma(x,y). \qquad (3\cdot 2)$$

These coupled equations may be written as one equation;

$$(\gamma.\partial + \mathcal{M})G(x,z) - \int d^4y D(x-y)\gamma_\mu G(x,y)\gamma_\mu G(y,z) = \delta^4(x-z), \qquad (3\cdot 3)$$

which is a nonlinear equation for the quark propagators, and sums the diagrams of Fig. 2b (with gluons instead of photons). The full dynamical content of $S[\mathcal{B}]$, and its E–L (the Schwinger–Dyson) equation of motion, have long been overlooked. The non-planar diagrams arise through meson (etc.) dressing, i.e. from the functional integrations in (2·17).

Hence we see that the vacuum configuration amounts to the quantum mechanical chromodynamic self-energy of the quarks. Thus the first computation we must perform in implementing the hadronisation is the self-interaction dynamics, and we will see that this is directly related to the mode of realisation of the chiral symmetry and the question of confinement. Remember however that G is a matrix in spin, flavour and colour space, as well as the explicitly indicated space–time dependence. The only known solutions of (3·3), with $\mathcal{M} \neq 0$, are translation invariant, i.e. depend only on $x-y$, and for which Σ is diagonal in flavour and colour, and in spin has the structure $\Sigma(z) = \gamma.\partial(A(z) - \delta^4(z)) + B(z)$, where $A(z)$ and $B(z)$ depend on the individual quark current masses ($m_u, m_d, ...$). The translation invariance means that the equations are most easily solved in momentum space, and we obtain [with $\Sigma(p) = i\gamma.p(A(p^2) - 1) + B(p^2)$]

$$\Sigma(p) = \mathcal{M} + \int \frac{d^4q}{(2\pi)^4} D(p-q)\gamma_\mu \frac{1}{i\gamma.q + \Sigma(q)}\gamma_\mu, \qquad (3\cdot4)$$

which gives two coupled equations, for each A and B pair per flavour, assuming as usual that $\mathcal{M}$ is diagonal in $m_u, m_d, \ldots$.

However in the chiral limit $\mathcal{M} \rightarrow 0$ (3·4) has a new class of solutions (Cahill and Roberts 1985; Roberts and Cahill 1987) which are not diagonal in flavour, namely

$$\Sigma(p) = i\gamma.p(A(p^2) - 1) + VB(p^2), \qquad (3\cdot5)$$

where V is the matrix $V = \exp(i\sqrt{2}\pi^a F^a \gamma_5)$, with $\{\pi^a\}$ arbitrary real constants (the F^a were defined in Section 2), and so G needs to be labelled by V; $G(p; V)$. These new solutions are easily seen from, say (3·4) (with $\mathcal{M} = 0$), in which we use $G(p; V) = \zeta^\dagger G(q; \mathbf{1})\zeta^\dagger$, $\zeta = \sqrt{V}$, and $\{\gamma_\mu, \gamma_5\} = 0$. We see that these extra solutions arise because the coupling is of the vector type. The presence of V in the chiral limit indicates that in this limit the action (3·1) has degenerate minima, i.e we have degenerate vacuum configurations. This vacuum manifold is the coset space $G/H = U_A(N_F)$, where $H = U_V(N_F) \subset G$, the chiral group, in which case we say that the chiral symmetry becomes a hidden symmetry. It is possible to check that this new class of solutions persists even if we keep the complete action in (2·14). They are therefore a key feature of chiral QCD. They are probably also the cause of the difficulties with lattice QCD computations in the chiral limit.

In the chiral limit there also exists a perturbative type solution for which $A(q) \neq 1$ everywhere and $B(q) = 0$, in which case chiral symmetry is maintained as an explicit (Weyl) symmetry, and we have a unique vacuum solution. It is a question of dynamics as to which class of solutions corresponds to the true minimum. Roberts and McKellar (1990) showed that, unless the strength of D is made artificially small, the degenerate solutions are the minima, and the $B = 0$ solution is a maximum of $S[\mathcal{B}]$.

We note that in (3·3) or (3·4) that a factor of $\frac{4}{3}$ is missing (as a multiplier of the integral) compared with the corresponding equation determined in the previous $\mathbf{1}_C - \mathbf{8}_C$ bosonisation, or indeed may be derived from the Feynman rules for QCD. Hence the hadronisation of Section 2 has its own '$\frac{4}{3}$-problem', and which is associated with the colour algebra. However this missing 'strength' indicates that the FI formalism will generate further diagrams of the type indicated in Fig. 1*b*. This interesting effect is discussed elsewhere (Clarke and Cahill 1990), but here and in the next section the numerical results arise from computations with the $\frac{4}{3}$ factor included in (3·3) and (3·4).

Equation (3·4) may be easily solved iteratively once a form for the gluon propagator is known. We illustrate the nature of the solutions by using a simple form which models the infrared slavery and asymptotic freedom behaviour

(Praschifka *et al.* 1988, 1989):

$$D(p) = \frac{3\pi^2\chi^2}{\Delta^2}\exp\left(-\frac{p^2}{\Delta}\right) + \frac{16\pi^2}{11p^2 ln(1+p^2/\Lambda^2+\epsilon)}, \qquad (3\cdot 6)$$

with parameter values $\chi = 1\cdot 14$ GeV, $\Lambda = 0\cdot 19$ GeV, $\Delta = 0\cdot 0002$ (GeV)^2 and $\epsilon = 2\cdot 0$. With this form we obtain the solutions for $A(s)$ and (shown in Fig. 4) $B(s)$ (with $s = q^2$). $B^2(x)$ is shown in Fig. 7 below.

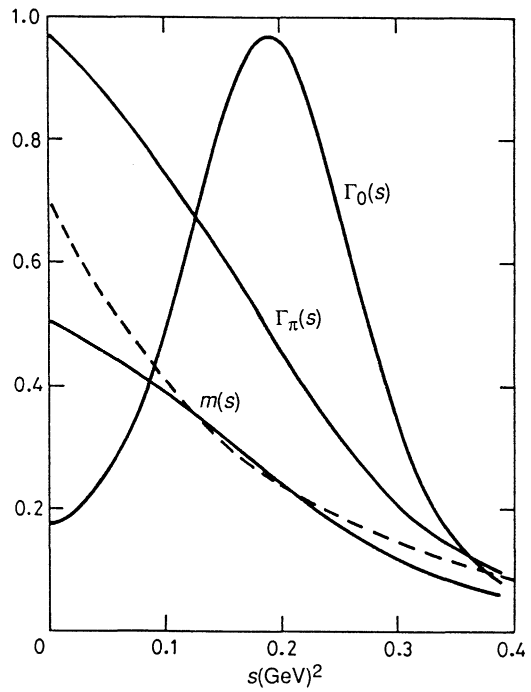

Fig. 4. $\Gamma_0(s)$ and $\Gamma_\pi(s) = B(s)$ are the on-mass-shell diquark and pion form factors (arbitrary scale); $m(s)$ (GeV) (solid curve) is the quark mass function from solving integral equation (3·4), while $m(s)$ (GeV) (dashed curve) is a simple variational form f. əm (3·10). The nucleon multiplet form factors $|\Psi(s)|$ are very similar to $\Gamma_0(s)$.

Also shown in Fig. 4 is $m(s) = B(s)/A(s)$ which is the quark mass function. It is important to note that these $m(s)$ values are not 'on-mass-shell' or 'physical mass' values. The physical mass would be that value of $\sqrt{(-s)}$ which solves the equation $A^2(s)s + B^2(s) = 0$. Whether or not there is any such solution is related to the confinement of quarks. The absence of any solution, i.e. the absence of a pole in $G(q)$, means that we have a confining propagator. However it is important to note that the integral equations for the 2- and 3-quark states, in the attractive channels, have bound state solutions even if the quark propagators do not have the traditional pole. Tandy and Frank (1991, present issue p. 181) show this also for soliton models. We also note that $B(q)$ is also the form factor $\Gamma_\pi(q)$ of the Nambu–Goldstone bosons, whose existence is associated with the hidden chiral symmetry, through the Goldstone theorem. The masslessness of these is directly related to the degeneracy of the 'vacuum equations' or quark-self-energy dynamics.

Let us now consider an important 'picture' associated with this self-energy. We can easily compute the change in the value of the action when we change $B(s)$ from its non-perturbative value to its perturbative value ($B=0$). We can then write from (3·1), using the translational invariance,

$$S[B]-S[0]=-\int d^4X\mathcal{B}, \tag{3·7}$$

where $\mathcal{B}$ is a constant, and given by

$$\mathcal{B}=N_F\frac{6\pi^2}{(2\pi)^4}\int_0^\infty sds\left[ln\left(\frac{A(s)^2s+B(s)^2}{A(s)^2s}\right)-\frac{B(s)^2}{A(s)^2s+B(s)^2}\right], \tag{3·8}$$

where this form of the last term arises from the identity [from (3·4) with $\mathcal{M}=0$ and including the '$\frac{4}{3}$']

$$\int d^4x\frac{B(x)^2}{D(x)}=\tfrac{16}{3}\int\frac{d^4q}{(2\pi)^4}\frac{B(q)^2}{A(q)^2q^2+B(q)^2}.$$

The $\int d^4X$ is merely the space–time volume (here of an infinite system). The constant $\mathcal{B}$ is computed from (3·8) to have the value $(122\text{ MeV})^4$ per quark, and minimises $S[\mathcal{B}]$.

We can illustrate an important aspect of this minimisation by considering some simple deviations from the vacuum manifold. For the case $\mathcal{B}^\theta(x,y)M_m^\theta/2=\sigma B(x-y)+i\gamma_5\pi.\tau B(x-y)$, where on the vacuum manifold $\sigma^2+\pi^2=1$, and because this deviation is translation invariant, the action $S[\mathcal{B}]$ can be explicitly evaluated, and comparing with the perturbative vacuum (induced here by $\sigma=0,\pi=0$), we obtain

$$S(\sigma,\pi)-S(0,0)=-\int d^4X\mathcal{B}(\sigma,\pi),$$

where

$$\mathcal{B}(\sigma,\pi)=N_F\frac{6\pi^2}{(2\pi)^4}\int_0^\infty sds\left[ln\left(\frac{A(s)^2s+(\sigma^2+\pi^2)B(s)^2}{A(s)^2s}\right)\right.$$
$$\left.-(\sigma^2+\pi^2)\frac{B(s)^2}{A(s)^2s+B(s)^2}\right].$$

This $\mathcal{B}(\sigma,\pi)$ is plotted in Fig. 5, and has the 'Mexican-hat' form, which was originally considered by Gell-Mann and Levy (1960) in connection with the σ-model for the nucleon. This model brought together the ideas of hidden chiral symmetry, the partially conserved axial current (PCAC) hypothesis, current algebra, etc. Here the σ model ideas follow from the action $S[\mathcal{B}]$, and the minimum is determined by the SD equation (3·3) or (3·4). If we include the

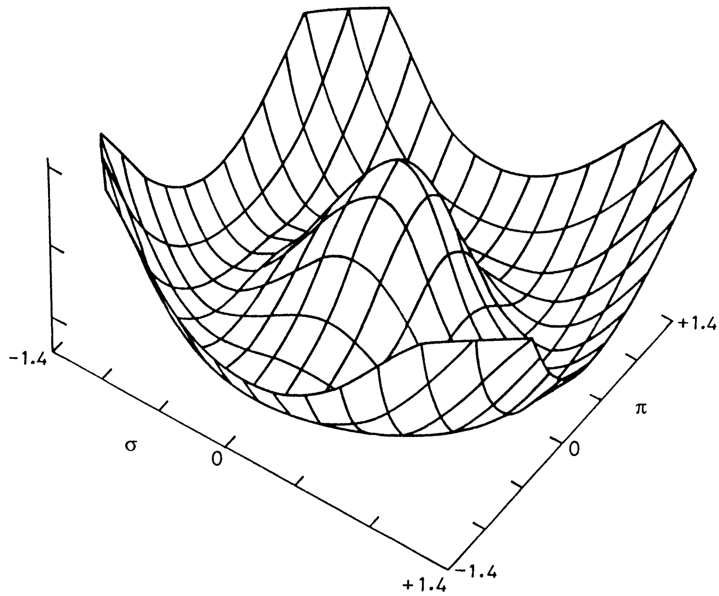

Fig. 5. The 'Mexican-hat' $\mathcal{B}(\sigma, \pi)$, where $\mathcal{B}$ is $(122\ \mathrm{MeV})^4$ per quark at the minimum and 0 at the centre. For the three-quark state $3 \times (122\ \mathrm{MeV})^4 = (161\ \mathrm{MeV})^4$ (see Table 1).

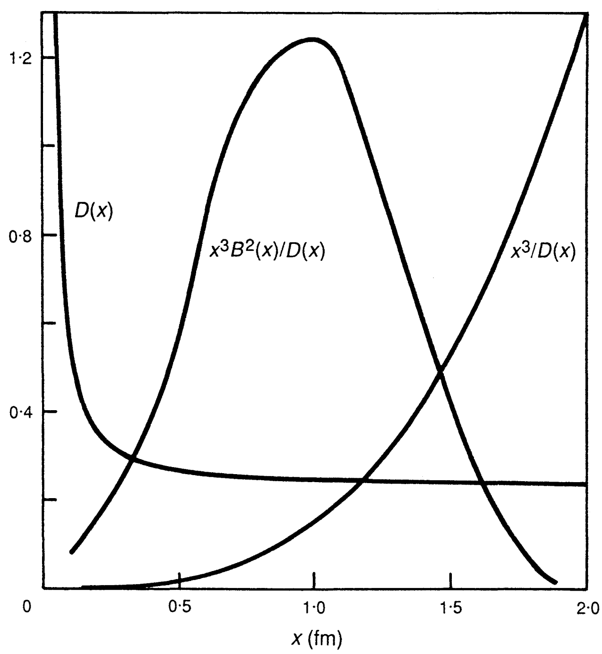

Fig. 6. Forms involving $D(x)$, the gluon propagator, and $B(x)$, the quark structure function, used in (3·9) (arbitrary scale).

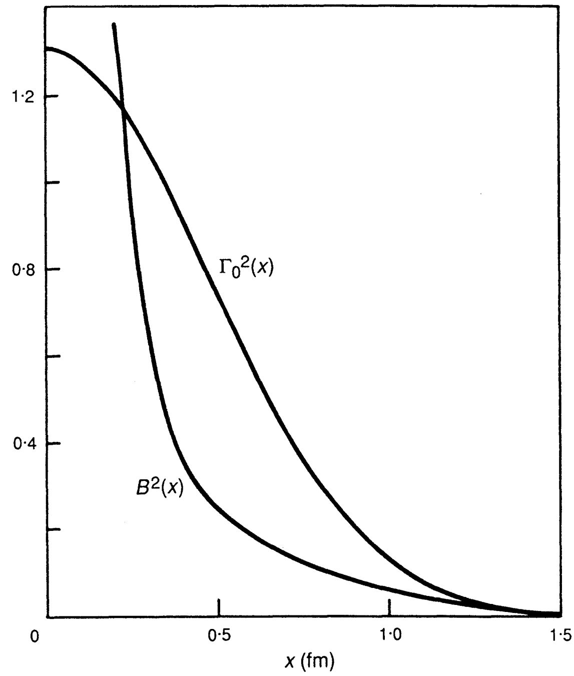

Fig. 7. Fourier transformed pion (and quark structure function) and the diquark form factors (squared) (arbitrary scale).

baryon one-loop functional of (2·14) in $S[\mathcal{B}]$ we see that the nucleon σ–model is indeed contained in QCD. Again we have, an easily computable, finite result completely determined by the 'running' coupling constant $\alpha(s)$. The vacuum degeneracy refers of course to the degenerate minimum in Fig. 5.

We can also associate a size with this quark self-energy. In Fig. 6 we show the Fourier transforms $D(x)$, $x^3/D(x)$, and the integrand $x^3B^2(x)/D(x)$, which arises when, for the translational invariant vacuum, we write the last term in (3·1) for the scalar field

$$\int d^4y\int d^4z\frac{B(y-z)B(y-z)}{D(y-z)}=2\pi^2\int d^4X\int_0^\infty dx\frac{x^3B(x)^2}{D(x)}. \qquad (3\cdot 9)$$

Also, for comparison, we show $B^2(x)$ and $\Gamma_0^2(x)$ in Fig. 7. The minimisation of $S[\mathcal{B}]$ can also be perfomed directly, rather than going through the SD equations (3·3) or (3·4). By considering only the determination of $B(s)$, the minimisation of S becomes a maximisation;

$$\frac{\delta}{\delta B(q)}\left\{\int_0^\infty\frac{q^2dq^2}{(2\pi)^4}ln\left(\frac{A(q)^2q^2+B(q)^2}{A(q)^2q^2}\right)-\tfrac{3}{16}\int_0^\infty x^2dx^2\frac{B(x)^2}{D(x)}\right\}=0. \qquad (3\cdot 10)$$

Indeed by using a 2-parameter form $B(q)=be^{-\beta q^2}$, whose Fourier transform $B(x)$ is easily evaluated, this maximisation now becomes $d\{...\}/db=0$, $d\{...\}/d\beta=0$. The resulting $m(s)=B(s)/A(s)$ is shown in Fig. 4. This result illustrates how practical quark self-energy calculations are in QCD.

Fig. 6 clearly shows that in the minimisation the self-energy of the quarks requires integration over distances of 0–1·6 fm. This value of the maximum propagation distance for the gluon follows from the particularly flat form of $D(x)$ associated with the modelled IR behaviour of $D(q)$ in (3·6). This IR region is where $D(q)$ is not well known, and Hollenberg *et al.* (1990) have suggested, on the basis of a study of $\rho-\omega$ splitting, that this IR form may require some changes. We also note that

$$\langle x^2\rangle = \int_0^\infty x^2 dx^2 x^2 B(x)^2 \Big/ \int_0^\infty x^2 dx^2 B(x)^2 = 1\cdot 0\ \mathrm{fm}^2\,.$$

On the basis of the above we now suggest an interpretation or 'picture' for this dressed quark, which is really a many body 'state'. We have that the self-energy dynamics involve gluon propagation over distances up to the order of 1 fm, that there is a natural energy density of $\mathcal{B} = (122\ \mathrm{MeV})^4$ per quark, and that in the chiral limit $B(q)$ is the pion form factor $\Gamma_\pi(q)$ (see also Delbourgo and Scadron 1979). Significantly the mass function $m(q^2) = B(q^2)/A(q^2)$ varies rapidly from $0\cdot 5 - 0\cdot 0$ GeV over a momentum range $0 < q^2 < 0\cdot 4\ \mathrm{GeV}^2$, typical for quarks in mesons, diquarks and baryons. This situation is very different from QED in which the electron mass is constant, at least over momentum values relevant to atoms. The $B(q)$ is as well $O(4)$ invariant. This suggests that we should picture a dressed quark as a new kind of extended structure with essentially the size of hadrons, namely 1 fm. Such a quark does not even necessarily have a specific mass, i.e. no pole in the propagator.

Hence not only are the self-energy dynamics finite and computable, but indeed appear to be the explanation for the success of the MIT bag model (Chodos *et al.* 1974*a*, 1974*b*) and its derivatives. In fact it seems that we should associate a 'pre-bag' with each individual quark. However since the individual quark-bags are comparable with hadronic sizes, we see that these individual bags overlap almost completely in hadrons (when the effects of the gluon exchanges between different quarks are taken into account), giving us the one-bag MIT 'picture' of the hadrons. The quark 'pre-bag' structure means that the quark is adapted, from the beginning, to be a constituent of hadronic states. Indeed from the full effective action in (2·14) we see that the colour singlet meson and baryon structures feed back on the calculation of the quark structure functions $\mathcal{B}_\nu(x-y)$. We might in fact consider the $G(q)$ as an 'environmental propagator', i.e. it is the propagator only if this quark is part of a colour singlet hadronic state. For 'free' quarks or quarks inside 'free' diquarks or colour octet baryons the quark propagator would be completely different. This relates to the self-energy colour filtering mechanism.

That the MIT bag constant unavoidably arose from $S[\mathcal{B}]$ was noted some time ago (Cahill *et al.* 1983, 1985). Indeed using a very simple one parameter model for the gluon propagator, $D(q^2) = \frac{3}{16}\chi^2\delta^4(q)$ (Munczek and Nemirovsky 1983), which is essentially the first term of (3·6), one obtains $A^2 = 4$, $B(s)^2 = 2m_\rho^2 - 4s$, for $s < m_\rho^2/2$, and $B(s) = 0$ for $s > m_\rho^2/2$. The ρ-meson mass arises when this mass function is used in the Bethe–Salpeter equation for the ρ, and allows the one parameter of the gluon propagator to be replaced by one known experimental meson mass ($\chi^2 \approx 2m_\rho^2$). With this form the MIT bag constant (per flavour) is found to have the value (Cahill and Roberts 1985) $\mathcal{B} = m_\rho^4/128\pi^2 = (129\ \mathrm{MeV})^4$ per quark.

We easily check that the Munczek–Nemirovsky form of the quark propagator has no poles in the time-like region, i.e. it is a confining propagator. A propagator with the same property occurs in massless QED (Fukuda and Kugo 1976), where the propagator arises from minimising an action like (3·1), but in the Landau gauge and for photons (Roberts and Cahill 1986). Atkinson and Blatt (1979) showed that the QED $B(s)$ function has branch points in the complex s-plane. Stainsby (1990*a*, 1990*b*) using a variety of models for the gluon propagator have shown that the quark $B(s)$ function also has (logarithmic) branch points in the complex s-plane. The significance of these results is that they suggest a possible general result that self-interaction can produce a confining 'propagator', i.e. does not allow free particles as there is no mass shell. Nevertheless bound states of these 'particles' do exist.

If these singularities in the complex plane are genuine, and not an artifact of the approximations, then they raise serious questions about the choice of space–time metric in quantum field theories. This is because they prevent the 'Wick rotation', and mean that the choice of Minkowski or Euclidean metric is not a matter of practical convenience. Then, using the Euclidean metric as the defining metric, the hadron masses are determined by $\lambda(P^2) = 0$, which has solutions only for $P^2 < 0$, and so involves an analytic continuation from the Euclidean ($P^2 \geq 0$) to a Minkowski metric.

There is one other fascinating but controversial twist to the story of quark self-energies which we should mention. We have noted that in the chiral limit (3·3) has solutions of the form $G(x,y;V) = [i\gamma.\partial A(x-y) + VB(x-y)]^{-1}$, where the value of the matrix V indicates one element of the degenerate vacuum manifold. Suppose we look for solutions of (3·3) for which different elements of the vacuum manifold are selected at different space–time points, that is we look for solutions of the form $G(x,y;V) = [i\gamma.\partial A(x-y) + V((x+y)/2)B(x-y)]^{-1}$, where $V(x) = \exp(i\sqrt{2}\pi^a(x)F^a\gamma_5)$. Hence we are looking for solutions to the quark-self-energy dynamics in which 'classical type' $\vec{\pi}(x)$ fields are present and have a space–time dependence. This corresponds to choosing a different point in the minima of the Mexican-hat at different space–time points. With this ansatz (3·3) can be used to determine the form of $\pi(x)$. If such solutions were to be found, and with a non-trivial topology (i.e. with proper winding around the minimum), they would be the famous Skyrmions or topological solitons which had been claimed as a QCD inspired model for baryons (see Zahed and Brown 1986 for an extensive review).

The proponents of this baryon model would not recognise the above formulation. To obtain the usual formulation one must substitute the above ansatz into the action and derive a derivative expansion, in terms of $\partial_\mu U(x)$ where $U(x) = \exp(i\sqrt{2}\pi^a(x)F^a)$, and we obtain the RHS of (2·18). Then the Euler–Lagrange equations $\delta S[U]/\delta U(x) = 0$ for this action is equivalent to solving (3·3) for the solutions of the above form (Cahill *et al.* 1985, 1988*a*). However (2·18) is a long wavelength expansion which would be unlikely to be valid over distances of the size of hadrons. Nevertheless by suitably truncating this expansion and in particular dropping those terms which de-stabilise the solutions, it is possible to find topological solitons. There have been many phenomenological attempts, in recent years, to restore stablity to these solitons. However it is now generally believed that there are no true solutions of $\delta S[U]/\delta U(x) = 0$, other than U independent of space–time. Unfortunately (2·18) was usually constructed on phenomenological grounds without any appreciation of the above self-energy dynamics, and the relationship of these supposed solitons to (3·3). We also see from the hadronisation that (2·18) arises together with another part of the effective action accounting for baryons.

We can now bring to a focus the present state of the FIC approach to determining the low energy phenomenon of QCD. We saw the transformation of QCD, by a dynamically determined change of integration variables, from being defined in terms of quark–gluon FI to the meson–diquark–baryon FI. Truncating the effective action at the level of retaining only the gluon 2-point function gave the GCM. The numerous successes of that model, which are dominated by the quark self-interaction via the gluons at the level of the gluon 2-point function, are reviewed in Table 1, and we note that all of these results are determined by only the parameters of the gluon running coupling constant, apart from the perturbative splitting of the various flavour multiplets by the differing quark current masses.

These GCM results actually represent something of a mystery as, although the colour singlet sector corresponds very well with the observed mesons and baryons and their interactions, yet at the same time we have in the GCM colour charged states, namely the colour $\overline{\mathbf{3}}$ diquarks and the colour $\mathbf{8}$ baryons. While the diquarks are well understood as composite constituents of the colour singlet baryons, neither the $\mathbf{8}_C$ baryons nor the diquarks are wanted as free observable states.

However it is here we see the marvel of QCD with its local colour symmetry. Consider the $\mathbf{1}_C$ and the $\mathbf{8}_C$ baryons, which are in fact degenerate in the implementation of the GCM considered above. They do however differ considerably as far as their colour properties are concerned: while the $\mathbf{1}_C$ baryons are colour uncharged, we have also, on the basis of them being composed of three essentially overlapping extended one-quark 'pre-bags', that they are also locally colour neutral. Local colour neutrality is here defined as meaning zero colour charge densities. This to be contrasted with atoms, for example, which, while being electrically uncharged, nevertheless do not have local charge neutrality because the positive charge of the nucleus is spatially separated from the negative charge of the electrons.

For the above reasons we would then expect the $\mathbf{1}_C$ and $\mathbf{8}_C$ baryons to have very different masses when we go beyond the the GCM truncation and include all of the additional interactions involving the $D^{(n)}$ $(n = 3, 4, \ldots)$-point. These are included in the formal $R[\mathcal{B}, \mathcal{D}, \overline{\mathcal{D}}]$ term in the effective action in (2·5), and they arise solely because of the local colour symmetry of QCD. Hence this R will cause further self-interaction of the $\mathbf{8}_C$ baryons, but these same self-interactions being ineffective for the $\mathbf{1}_C$ baryons. This difference is represented in Fig. 8. Of course the self-interaction of $\mathbf{8}_C$ baryons actually involves quark self-interaction as well as quark–quark interaction via these $D^{(n)}$. If these new self-interactions are indeed a significant effect (there are an infinite number of $D^{(n)}$ generated proper self-energy diagrams!) on the mass of the $\mathbf{8}_C$ baryons, which might well make them infinitely massive and remove

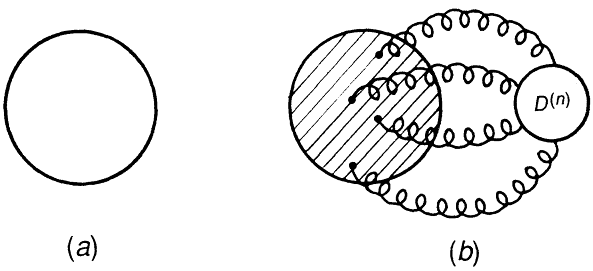

Fig. 8. Colour filter effect: (*a*) representation of colour singlet and locally colour-neutral hadron; (*b*) higher order self-energy processes for colour charged 'state' which, it is proposed, have minimal effect on states in (*a*).

them from the QCD spectrum, these self-interactions might well act to remove any slight residual local-colour non-neutrality of the $\mathbf{1}_C$ baryons to make their local colour neutrality complete. This could be achieved by minor changes to the wavefunctions without large effects on the masses. To implement a study of this effect in any practical and inclusive manner would seem to necessitate some mean gluon field approach.

For the same reasons we would argue that the colour singlet mesons would show little consequences of these extra self-interactions. However the diquarks represent an interesting situation. As a composite constituent of a $\mathbf{1}_C$ baryon they would be shielded from these additional and presumably overwhelming self-interactions by being shielded by the remaining quark—i.e. they are part of a local colour neutral bound state. However a possible isolated diquark, because of its colour charge would experience in full the above additional self-interactions, and also presumably be removed from the mass spectrum. The same argument also applies to single quarks—their propagators being very different depending on whether they are part of a colour singlet state or not.

Hence the R term appears to play a very important role, namely the removal of colour charged quark bound states. It could be described as a *colour filter*, leaving only colourless states in the spectrum. Indeed this *self-energy colour filtering* is the same as the long sought for colour confinement mechanism. Detailed study will be necessary to prove that it does act in the way proposed above. If this is proved to be the case, then in fact it probably plays little role in the colour singlet sector of QCD, and the GCM is then essentially QCD for this sector.

There may be in fact some indirect experimental evidence for the colour filter effect as it is based on the idea of local colour neutrality of colour singlet states, and this appears to be related to the idea of *colour transparency*. This idea has been used to suggest an explanation for the very small cross section with nuclear matter of hadrons involved in hard exclusive scattering (Meuller 1989), although in this case there is the curious assumption that such hadrons are temporarily 'point-like'.

4. Conclusions

Hence we see that the quark self-energy dynamics arises naturally and immediately in the hadronisation, that it is finite and extends over distances and with an energy density that imply it has a significant effect on the structure of hadrons. Indeed it seems that the bag phenomenology is nothing more than the quark self-energy effect. We also saw that, in the chiral limit, the degeneracy of the solutions to the self-energy equations was responsible for the dynamical effect of the hidden chiral symmetry, and the emergence of the NG bosons. The dominant quark self-energy dynamics involve the dressing of each quark by an extensive 'cloud' of gluons. Hence in analysing the deep inelastic scattering data from nucleons it is very important to remember that we are probing targets, for the first time, with a complex quantum field theoretic structure, unlike the previous situations in atomic and nuclear physics. We now begin to realise what a superb opportunity we have in being able to test and develop our understanding of quantum field theory dynamics on such non-trivial but experimentally accessible objects.

Acknowledgment

It is with great pleasure that I dedicate this review to Professor Ian McCarthy who has supported the research work described here over a long period of time. Thank you Ian and happy birthday.

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