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Soliton Matter*

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Abstract

Although quantum chromodynamics is generally accepted as the fundamental theory of the strong interactions, there exist no solutions to the theory which describe the binding of quarks to form the observed particles of nuclear physics. Soliton bag models of hadronic matter attempt to capture the main features of quark confinement, while remaining amenable to calculation. In the context of the non-topological soliton model, nuclear matter is regarded as a collection of quark clusters arranged on a regular lattice in bag-like soliton structures. The conjectured phase transition of nuclear matter to quark plasma at high densities is then modelled as the melting of the soliton crystal under compression. This paper reviews some recent results on multi-soliton solutions of the linear sigma model in one space and one time dimension. Remarkably, exact periodic soliton solutions can be derived in terms of complete Jacobi elliptic integrals for arbitrarily large lattices. The conditions for a phase transition in the model, and implications for more realistic calculations are discussed.

1. Introduction

In principle hadron and nuclear structure should be derivable from quantum chromodynamics (QCD), the basic field theory of strong interactions between quarks and gluons. However, the many efforts to identify unambiguously quark effects in low energy nuclear physics have so far proved fruitless. We can understand the challenge faced by the nuclear theorist in applying QCD to describe nuclear structure by considering the role played by quantum electrodynamics (QED) in calculations of atomic or molecular structure.

Apart from questions of fine detail, atomic and molecular structure and interactions are well predicted by the semiclassical theory, where the fermions are bound by electromagnetic forces described by Maxwell's equations. There is no need to introduce quantum fluctuations of the electromagnetic field, and indeed, if ease of computation is the goal, it is undesirable to do so. Of course with the discovery of the Lamb shift, the corrections to this large scale picture and the subsequent refinement of QED were major achievements in the development of consistent, relativistic quantum field theories.

In QCD we have a renormalisable quantum field theory which has been tested at distances which are small on the nuclear scale, where the couplings between quarks and gluons may be treated perturbatively, but whose 'classical'

* Dedicated to Professor Ian McCarthy on the occasion of his sixtieth birthday.

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limit is unknown. Can we not emulate the successes of atomic physics by treating the gluon field of QCD as a classical field? Unfortunately our analogy with QED breaks down. Gluons, unlike photons which have no electric charge, carry colour charge and interact with each other in a complex way. The coupling between quarks and gluons grows very strong on the long distance (1 fm) scale of nuclear physics and in spite of the huge investment in computing time, lattice simulations have not yet yielded much information about the large distance properties of QCD. The nuclear theorist who sets out to explain nuclear structure in terms of QCD is like an atomic physicist attempting to predict atomic stucture without the help of Coulomb's law. Furthermore, a clear experimental signature of underlying quark or gluon degrees of freedom, analogous to the Lamb shift of atomic physics, has not yet been observed in low energy nuclear physics. Nevertheless there is much information about nuclear interactions in the form of phenomenological meson exchange models, and one might hope to show that this description is consistent with the large scale behaviour of QCD, by demonstrating that mesonic degrees of freedom naturally arise as the long range excitations of systems of quarks and gluons.

In recent years there has been much work on theories which describe hadrons as localised bound state solutions of quantum field theories at the classical level. These are called solitons, in the broad sense that they involve non-dispersive solutions of a set of nonlinear equations. The non-topological solitons which will be discussed here owe their stability to the introduction of explicit fermion degrees of freedom. The nucleon for example is considered as three quarks bound together by a scalar field or fields whose quantum fluctuations are, in the first instance, ignored, just as the photon field is treated classically in the analogous atomic bound state problem. It is hoped that these effective field theories will provide a bridge between the fundamental theory and the established models of nuclear physics. Most work has been directed at exploring nucleon properties predicted by the various versions of the soliton models, but eventually one would like to justify a particular choice of fields and interactions. Some work at deriving effective Lagrangians for 'coarse-grained' fields from QCD is being undertaken (Pirner *et al.* 1987) and is a necessary step in linking particle and nuclear physics (see also the contributions by Cahill 1991 and by Tandy and Frank 1991, present issue pp. 105 and 181).

The aim of the present article is to introduce some of the basic ideas about soliton bag models using a model field theory in one space and one time dimension where much explicit information about soliton solutions has been derived (Campbell and Liao 1976; Dodd *et al.* 1987). In particular exact multi-soliton solutions of the model have been found recently (Dodd and Lohe 1990) and these will be used here to provide a simple picture of the transition of nuclear matter to quark plasma. For comprehensive discussions of applications of non-topological soliton models to nuclear physics, and particularly the problem of nuclear matter, the interested reader is referred to two excellent recent reviews (Wilets 1989; Birse 1990).

In the next section we define the model and show examples of the exact bag-like soliton solutions discovered by Campbell and Liao (1976). The shallow bag solution of these authors is taken to represent an isolated nucleon, the 'quarks' being described by the modes of a Fermi field ψ and the colourless gluon condensate, which takes a non-vanishing value in the physical vacuum where the quark density approaches zero, by a self-interacting scalar field σ.

In Section 3 we describe how periodic multi-soliton solutions representing 'nuclear matter' are constructed. It will be emphasised that an ansatz due to Lohe has enabled a complete solution (Dodd and Lohe 1991) in this case. As well as these crystal-like solutions, gas-like solutions where the σ field is constant and there is no fermion clustering also exist. The energy of the various solutions is compared in Section 4 and the conditions for a deconfining first order phase transition, where under compression the crystal structure dissolves, are discussed. We conclude with some remarks on the possibility of more realistic calculations.

2. The Model

We consider mean field solutions for a system of fermions interacting with a scalar field σ in one space and one time dimension, described by the Lagrangian density

$$\mathcal{L}(x,t) = \overline{\psi}(i\gamma\cdot\partial - g\sigma)\psi + \tfrac{1}{2}\partial^{\mu}\sigma\partial_{\mu}\sigma - U(\sigma)\,. \tag{1}$$

The quarks of our model have three colours and two flavours, up and down, so that ψ stands for a set of six Fermi fields each having two spinor components, and

$$U(\sigma) = \frac{\lambda}{4}(\sigma^2 - f^2)^2 \tag{2}$$

is the self-energy density of the scalar field of the linear sigma model. In the tree approximation the mass of the σ particle is $m_\sigma = \sqrt{2\lambda}f$ and the mass of the quarks is $m_q = gf$. In the following we express all dimensioned quantities in units of the quark mass and rescale the dimensionless scalar field so that $U(\sigma) = 0$ when $\sigma = \pm 1$.

In the mean field approximation the scalar field σ is taken as a classical field satisfying the Euler–Lagrange equations of the Lagrangian (1), while the quark fields ψ are expanded in a set of basis functions satisfying the same field equations as ψ,

$$\psi_i = \begin{pmatrix} u_i\,(x) \\ v_i\,(x) \end{pmatrix} e^{-i\epsilon_i t}\,. \tag{3}$$

The total energy is then minimised under variations of the spinor components u_i, v_i and σ. The energy eigenvalues ϵ_i serve as Lagrange multipliers for the normalisation constraints on the quark states. The resulting quark and sigma field configurations are determined by the competition between $U(\sigma)$, representing the long range order of the gluon condensate, and the quark-σ coupling term, which acts as an effective mass for the quark. In regions where the quark density is small, $U(\sigma)$ is lowered by σ taking its vacuum

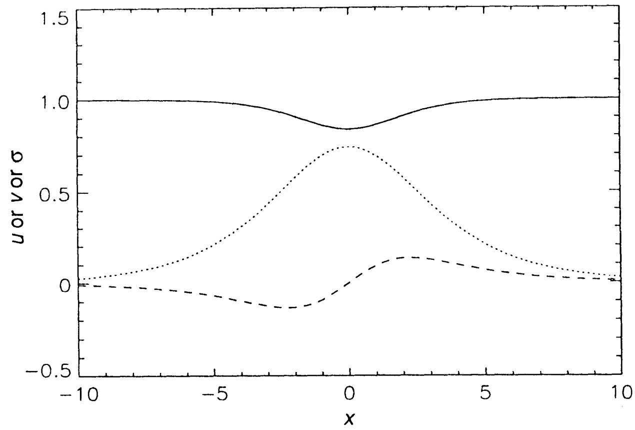

Fig. 1. The isolated shallow bag, representing a single nucleon. The components u_0 (dotted curve) and v_0 (dashed curve) of the quark spinor are localised in a depression in the σ field (solid curve).

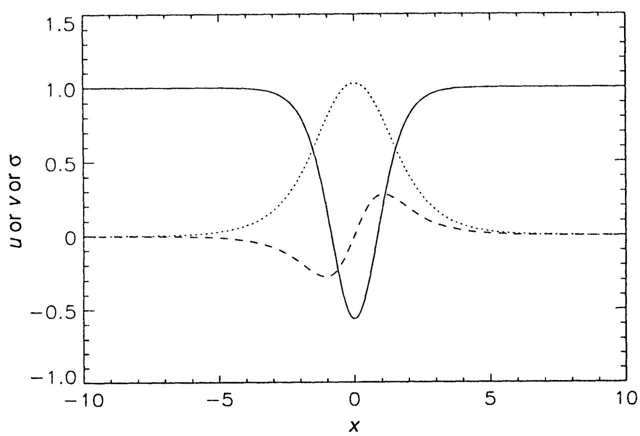

Fig. 2. The deep bag solution, showing the cavity in the σ field and the upper (dotted curve) and lower (dashed curve) components of the quark spinor.

value $\sigma = \pm 1$ and the quark mass is g. In regions of high quark density the total energy is minimised by σ taking a value close to zero and the mass of the quark is small.

We first consider an isolated soliton where all N_q quarks are in the same mode $i = 0$. The mean field equations which result from the above variation are then (the prime denotes differentiation with respect to x)

$$\begin{aligned} u_0' &= -(\epsilon_0 + \sigma)v_0 , \\ v_0' &= (\epsilon_0 - \sigma)u_0 , \end{aligned} \tag{4}$$

$$\sigma'' = u_0^2 - v_0^2 + \frac{dU(\sigma)}{d\sigma} , \tag{5}$$

where the quark states are normalised by

$$2\int_0^\infty (u_0^2+v_0^2)dx = \frac{N_q}{f^2}\,. \tag{6}$$

For the particular ratio of the coupling constants $\lambda/g^2 = 2$, corresponding to $m_\sigma/m_q = 2$, Campbell and Liao (1976) were able to find explicit, exact solutions of the mean field equations. Their solutions are of two types, isolated kinks or antikinks in the σ field, or soliton bags where the fermions are localised in cavities in the field. The latter solutions are simple examples of non-topological solitons (Friedberg and Lee 1977, 1978; Lee 1981). We may write such a solution as

$$u_0 = \kappa\epsilon_0^{\frac{1}{2}}\left[\frac{1}{cosh\kappa(x+x_0)}+\frac{1}{cosh\kappa(x-x_0)}\right],$$

$$v_0 = \kappa\epsilon_0^{\frac{1}{2}}\left[-\frac{1}{cosh\kappa(x+x_0)}+\frac{1}{cosh\kappa(x-x_0)}\right], \tag{7}$$

$$\sigma = 1-\frac{\kappa^2}{\epsilon_0}\frac{1}{cosh\kappa(x+x_0)\,cosh\kappa(x-x_0)}\,. \tag{8}$$

The parameter $\kappa = (1-\epsilon_0^2)^{\frac{1}{2}}$ and x_0 is defined by $\kappa tan\kappa x_0 = 1-\epsilon_0$. The quark energy eigenvalue is determined by

$$64\epsilon_0^2(1-\epsilon_0^2) = \frac{N_q^2}{f^4} \tag{9}$$

and the total energy of the soliton is

$$E = N_q\epsilon_0 + \tfrac{8}{3}f^2(1-\epsilon_0^2)^{\frac{3}{2}}\,. \tag{10}$$

For values of f and N_q such that $f^2 > N_q/4$ there are two roots of (6) satisfying $0 < \epsilon_0 < 1$. For the case of the 'nucleon' with $N_q = 3$ and f set arbitrarily to unity, the solution with the least energy is plotted in Fig. 1. The quarks are localised in a shallow depression in the scalar field which does not depart much from its vacuum value. The other solution which has smaller ϵ_0 but larger field energy and total energy is shown in Fig. 2.

An example of the kink solution,

$$\sigma = tanh(x), \tag{11}$$

$$u_0 = [cosh(x)]^{-1},$$

$$v_0 = u_0, \tag{12}$$

is shown in Fig. 3. These are solutions where arbitrarily many quarks are trapped in a kink in the σ field which, since it interpolates between the degenerate vacuum states $\sigma = \pm 1$, carries topological charge. We note that the quark source term in (5) vanishes and the energy $E = \frac{4}{3}f^2$ is independent of N_q. We will not consider these solutions further here, as we are more interested in the non-topological soliton solutions as being representative of the soliton bag models of nuclear physics. In the next section we describe a model of nuclear matter formed by arranging the soliton bags on a one-dimensional lattice.

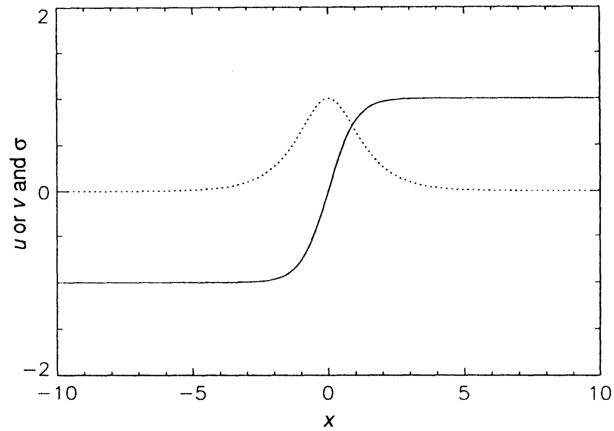

Fig. 3. The kink solution. The σ field interpolates between the degenerate vacuum values. In this case the upper and lower spinor components are equal (dotted curve).

3. Multi-soliton Solutions

Since our interest is in many bag configurations, we seek semiclassical solutions of the system described by (1) on a line of length $2NR$ with periodic boundary conditions. We assume the σ field repeats in each cell,

$$\sigma(x + 2R) = \sigma(x), \tag{13}$$

and is symmetric about any cell centre. There are three quarks per cell, so the total number of quarks is $3N$. The quark basis states must now exhibit discrete translational symmetry and their form is prescribed by Bloch's theorem, familiar from solid state physics (see e.g. Jones and March 1973),

$$\psi_j = e^{ik_j x}\begin{pmatrix} u_j\,(x) \\ v_j\,(x) \end{pmatrix} e^{-i\epsilon_j t}, \tag{14}$$

where $k_j = \pi j/(RN)$ is the crystal momentum, the integer j taking the values $-N/2 < j \leq N/2$ for the first Brillouin zone, and u_j and v_j, the upper and lower components of the quark spinors, are periodic on the lattice like $\sigma(x)$.

If n_j is the number of fermions occupying the single particle state ψ_j, the probability of occupancy of the state is $p_j = n_j/(3N)$, and the mean field equations of interest become (Dodd *et al.* 1987)

$$\begin{aligned} u_j' + ik_j u_j &= -(\epsilon_j + \sigma)v_j\,, \\ v_j' + ik_j v_j &= (\epsilon_j - \sigma)u_j\,, \end{aligned} \tag{15}$$

$$\sigma'' = \sum_j p_j(|u_j|^2 - |v_j|^2) + 2\sigma(\sigma^2 - 1), \tag{16}$$

$$\int_0^R (|u_j|^2 + |v_j|^2)dx = \frac{N_q}{2f^2}. \tag{17}$$

From the assumed symmetry of the solutions it follows that $\psi_j^* = \psi_{-j}$, and it is sufficient to solve this nonlinear system for nonnegative j on the interval $[0, R]$. The boundary conditions are

$$\begin{aligned} \sigma'(0) &= \sigma'(R) = 0, \\ v_0(0) &= v_0(R) = 0 \end{aligned} \tag{18}$$

for the real-valued quark components with $j = 0$,

$$v_{N/2}(0) = u_{N/2}(R) = 0 \tag{19}$$

for the complex-valued quark components with $j = N/2$, and

$$\begin{aligned} \mathrm{Im}\, u_j(0) &= \mathrm{Im}\, u_j(R) = 0 \\ \mathrm{Re}\, v_j(0) &= \mathrm{Re}\, v_j(R) = 0 \end{aligned} \tag{20}$$

for the levels between the bottom and the top of the band.

The property of the system of principal interest is the dependence of the soliton energy on the density, which is found by calculating the total energy per cell, given by

$$\frac{E}{N} = N_q \sum_j p_j \epsilon_j + 2\int_0^R \left[\tfrac{1}{2}\left(\frac{d\sigma}{dx}\right)^2 + U(\sigma) \right] dx \tag{21}$$

for the cell length $2R$ corresponding to a density $N_q/(2R)$.

Because the quarks move in a periodic scalar potential, the eigenvalues ϵ_j exhibit the usual crystal band structure. The standard computational problem of solid state physics is complicated by the additional requirement that the potential in the Dirac equation is to be found self-consistently by solving the nonlinear equation (16). The equations (15), (16) and (17) together with the boundary conditions (18), (19) and (20) can be converted to a two-point boundary value system of order $3N+6$ for $N > 1$ (Dodd and Lohe 1985). This system can be solved using standard numerical techniques provided N is small (Dodd *et al.* 1987) but clearly this approach is prohibitive for large N.

The numerical calculations of Dodd *et al.* (1987) were restricted to four or fewer cells but remarkably we have recently found explicit solutions for arbitrarily large numbers of cells. These exact solutions, which involve Jacobi elliptic functions, generalise the isolated soliton solutions of the previous section. The integration of the field equations (15) and (16) is based on the key assumption due to Lohe, that the quark wavefunctions depend quadratically on σ,

$$|u_j|^2 + |v_j|^2 = \beta_j(\alpha_j - \sigma^2), \tag{22}$$

where α_j and β_j are constants. The mathematical details of the solution (Dodd and Lohe 1991) are too lengthy to be repeated here so we will limit ourselves to stating the main results.

It turns out that the multi-soliton solution can be completely characterised by two parameters η and Δ. The σ field in any cell has the form

$$\frac{\sigma}{\epsilon_0} = 1 + \frac{\Delta(t+\eta)}{1+\eta t}, \tag{23}$$

where the position x is related to the new variable t by the elliptic integral

$$x = \frac{1}{\epsilon_0 c}\int_{-1}^{t} \frac{dt}{[(1-k^2t^2)(1-t^2)]^{1/2}} \tag{24}$$

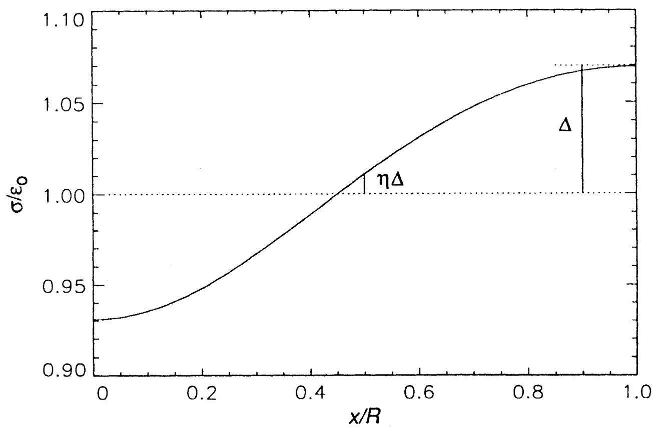

Fig. 4. The σ field in a half-cell for a multi-soliton solution, showing the parameters Δ and η. The field takes a universal form, given by equations (23) and (24) of the text.

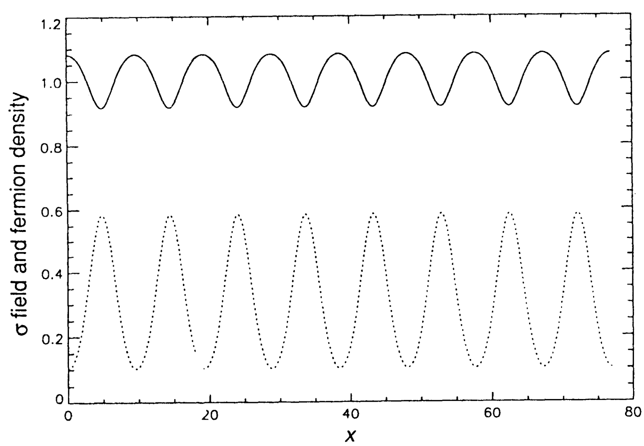

Fig. 5. A typical crystal solution for 24 quarks. The dotted curve shows the quark density concentrated in the periodic potential wells formed by the σ field.

and

$$k^2 = \frac{\eta(2\eta + \Delta)}{2 + \eta\Delta}, \quad c^2 = \Delta\left(\Delta + \frac{2}{\eta}\right). \tag{25}$$

The cell size is obtained when $t = 1$ in the integral (24). Thus we get

$$R = \frac{2}{c\epsilon_0} K(k^2), \tag{26}$$

where $K(k^2)$ is the complete elliptic integral of the first kind.

From (23) we see that $\sigma(0)$ corresponds to the value $t = -1$ and $\sigma(R)$ corresponds to the value $t = 1$, so that the lowest single particle energy ϵ_0 is just the mean of the field at the centre and the field at the edge of the cell. The geometrical interpretation of the parameters Δ and η is shown in Fig. 4.

Explicit expressions for the quark eigenvalues and spinor wavefunctions have also been derived but will not be repeated here. However, we mention in passing that the eigenvalue for the top level of the ground state band is given by

$$\frac{\epsilon_{N/2}}{\epsilon_0} = \tfrac{1}{2}\left[-\Delta + \left\{\Delta^2 + 2\Delta\left(\frac{1}{\eta} + \eta\right) + 4\right\}^{\frac{1}{2}}\right], \tag{27}$$

and the energy gap at the top of the band is $\Delta\epsilon_0$. Fig. 5 shows the quark density and σ field for a typical 24 quark solution with 8 cells. Calculations with up to 128 cells have been made.

The energy density (21) has also been evaluated in terms of complete elliptic integrals. In the next section we show how the energy varies with density using these exact solutions.

4. The Soliton Crystal

If the isolated soliton of Section 2 is taken to represent a single nucleon, we can interpret the multi-soliton solutions of the preceding section as a crystal of solitons representing nuclear matter. Of course with static solitons the relative motion of the nucleons, which is an important contribution to the saturation mechanism in nuclear matter, is ignored. However, the dynamics of moving solitons would involve the very difficult problem of constructing time-dependent mean field solutions and to date calculations for nuclear matter have assumed static solitons (see e.g. Achtzehnter *et al.* 1985; Birse *et al.* 1988; Glendenning and Banerjee 1986).

A necessary ingredient in the model is the choice of p_j in (16) which fix the quark numbers in the ground state band of the soliton crystal. Since we have ignored colour forces between the quarks, the state of the system with the least energy is obtained by filling the Bloch states from the lowest level up with the maximum number of quarks consistent with the exclusion principle. There is no spin in one dimension but we have assumed a six-fold degeneracy of quark states associated with two flavours and three colours. Thus the ground state band is half-filled, there being $3N$ quarks in the system and $6N$ Bloch states in the band. We call this mode of band filling 'tight filling'.

An alternative way of distributing the quarks through the ground state band, which we call 'uniform filling', simulates the effects of the colour forces between the quarks. This is best described by using a basis of Wannier states, which describe localised quarks. These are related to the Bloch states by a unitary transformation,

$$\psi_j^W(x) = N^{-1/2} \sum_k e^{2iRk(j-1)} \psi_k(x) . \tag{28}$$

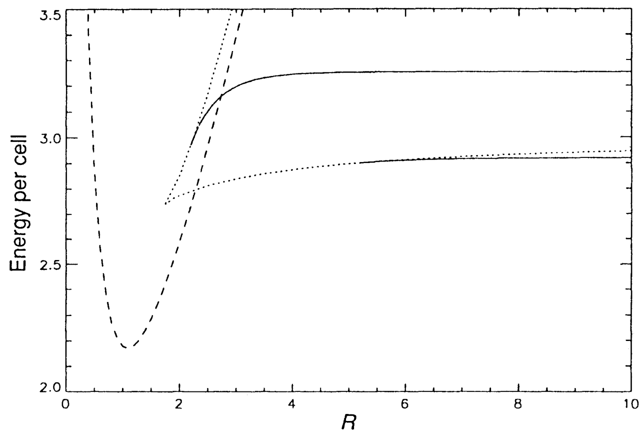

Fig. 6. The energy per cell as a function of the lattice spacing for tight filling of the ground state band. The branches for the shallow bag and deep bag crystal solutions (solid curves) bifurcate from the gas-like solutions with constant non-vanishing values of the σ field (dotted curves). The dashed curve is the solution where σ vanishes everywhere.

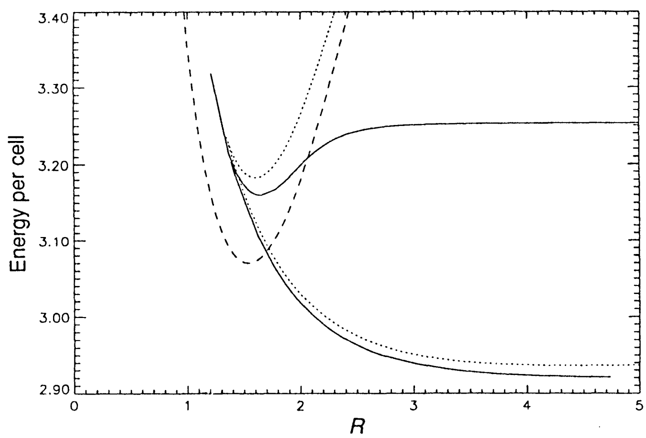

Fig. 7. The energy per cell as a function of the lattice spacing for uniform filling of the ground state band. In contrast to tight filling (Fig. 6), the shallow bag solution (lower solid curve) has increasing energy as the lattice spacing is decreased, corresponding to positive pressure. The periodic crystal solutions always have lower energy than the solutions with constant, non-vanishing σ.

The wavefunction $\psi_j^W(x)$ describes a quark localised in the jth cell. With a suitable superposition of Wannier states we can place three quarks in each cell in a colour singlet combination to form an array of colourless nucleons. After transforming back to the Bloch basis we find that all states in the ground state band have equal weight, i.e. $p_j = 1/N$.

Let us now show some typical results for $N = 8$, i.e. a system with 24 quarks and $f^2 = 1$. Fig. 6 shows the energy per cell as a function of the lattice spacing for tight filling of the band, i.e. when colour correlations are neglected. In addition to the branches for the periodic solutions there are also three branches for solutions where the σ field takes a constant value everywhere on the lattice. A striking feature of these results is that the deep and shallow bag solutions bifurcate from the solutions with constant, non-vanishing values of the σ field. The detailed analysis (Dodd and Lohe 1991) gives an explicit formula for the densities at which bifurcation occurs. At low densities we see that multi-soliton solution is energetically favoured, but that the pressure (negative slope of the energy versus lattice spacing curve) is negative and the system collapses. The stable ground state of the system is a rather uninteresting configuration where the σ field vanishes everywhere and the quarks form a massless Fermi gas.

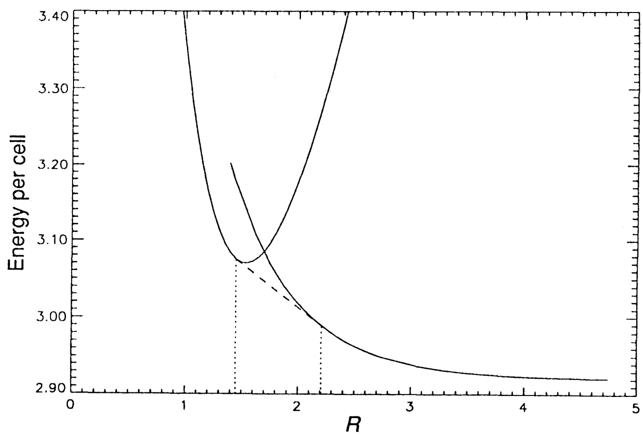

Fig. 8. The Maxwell construction (dashed curve) interpolating between the massless quark gas at high densities and the soliton crystal at low densities. The two phases can co-exist for a range of lattice spacings.

These results may be contrasted with those for uniform band filling, which simulates colour forces, shown in Fig. 7. Here there is no bifurcation and the periodic shallow bag solution has lower energy per nucleon than the solutions with constant, non-vanishing σ field at all densities where both exist. The exact analysis of the multi-soliton solutions referred to in the last section shows that if there are any quarks present in the top level of the ground state band then the soliton solutions cannot bifurcate from the constant solutions without violating normalisation constraints. With uniform band filling the system has positive pressure and there are two distinct phases, at low densities or large lattice spacings, a crystal of nucleons, and at high densities or small lattice spacings, a massless Fermi gas. Fig. 8 shows the Maxwell construction which interpolates between the two phases. For $1 \cdot 4 \lesssim R \lesssim 2 \cdot 2$ the pressure is constant and the two phases co-exist in equilibrium.

We have seen in this very simple example that the soliton model is capable of describing a transition from nuclear matter to quark plasma. There are several obstacles to making the description more realistic. Nuclear matter is generally thought of as a Fermi liquid rather than a crystalline solid. The problem of analysing the dynamics of moving soliton bags seems at present intractable. It would be desirable to include colour forces in the model in a self-consistent manner. Gluon exchange is responsible for the mixing of Bloch states and is ultimately responsible for colour confinement. A more detailed calculation should show that uniform band filling is a consequence of residual colour forces. Three-dimensional calculations which include gluon exchange between neighbouring Wigner–Seitz cells are being undertaken (Wilets, personal communication 1989).

Notwithstanding its limitations, an advantage of the present one-dimensional model is that a rather complete analysis of the multi-soliton solutions can be made. There appears to have been relatively little work done on spatially periodic solitons but we believe their study is of general mathematical interest, as well as being relevant to the investigation of different phases of hadronic matter.

Acknowledgments

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