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Chiral Corrections for Lattice QCD*

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Abstract

We show that in the quenched approximation, lattice QCD calculations miss important corrections to the mass of hadrons. For example, we argue that the nucleon mass is overestimated by up to 30% while the nucleon–delta mass splitting is underestimated by a similar amount. As the quenched approximation is relaxed, the quark mass decreased and the lattice size increased, these errors should go away. A systematic method of monitoring the size of the residual error is suggested.

1. Introduction

It is a pleasure to offer this contribution as part of the 60th birthday celebration for Ian McCarthy. My own research in physics began under Ian's guidance at Flinders University more than twenty years ago. As a second-year student I learnt with considerable excitement about the (p, 2p) reaction, off-shell effects and nuclear structure (McCarthy and Thomas 1969). At that time these were very difficult experiments and before long Ian, together with Eric Weigold, saw the much riper opportunities to exploit knock-out reactions in atomic physics [the (e, 2e) process; Weigold 1991, present issue p. 277].

Only much later did it become possible, at intermediate energy laboratories like TRIUMF and IUCF, to readily explore the (p, 2p) process. In the light of that recent experience it is clear that many of Ian's ideas were a decade or more ahead of their time. A similar observation could be made about his much less formal comments (in the late 60s) concerning the possibility of making Σ-hypernuclei. These were only found at the beginning of the 1980s (at CERN), and the excitement and mystery surrounding their narrowness is by now well documented (Bertini 1989).

My own particular contribution to this Festschrift will deal with the necessity to take chiral symmetry seriously when judging the results of lattice QCD calculations against data. This is a topic fairly far removed from Ian's own areas of research. Nevertheless, there is a very real sense in which my contribution on this topic has its roots in the excitement and enthusiasm for physics that he was able to communicate over twenty years ago. Undoubtedly that enthusiasm is still being shared with each new generation of students.

* Dedicated to Professor Ian McCarthy on the occasion of his sixtieth birthday.

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The plan of the article is as follows. We first remind the reader of the ultimate aims and present limitations of lattice QCD. This is followed by a reminder of the significance of chiral symmetry and a brief review of how it is commonly implemented. This leads us naturally to the conclusion that lattice calculations in the quenched approximation should not be compared directly with experimental masses, but rather with 'bare masses' from which certain pion self-energy corrections have been removed. Finally we suggest a systematic scheme for undoing these corrections as the lattice calculations improve in future years.

2. Lattice QCD in the Quenched Approximation

There is no serious theoretical model of the strong interaction other than QCD. Furthermore most of the identified mass of the universe is tied up in nucleons. It is therefore very important to see if one can derive the mass of the nucleon from QCD. At present the only practical suggestion for doing so without making (physically motivated) simplifying assumptions is lattice QCD (Wilson 1975, 1977). As there are many excellent reviews of the lattice approach we need say very little here. The major problem is one of a lack of computing power. Indeed it may yet be a decade or two before it is possible to accurately compute the mass of the nucleon (Petronzio 1987; Martinelli 1990).

At the present time there seems to be a consensus that calculations of hadronic masses, form factors and even low moments of structure functions are fairly reliable in the so-called 'quenched approximation'. In this approximation, it is possible to work with a spatial lattice of order 16^3 and a lattice spacing as small as $0 \cdot 1$ fm. That is, the hadron is necessarily contained in a cubic box about 3 fm on a side. Modulo the long-range pion tail, which will be discussed at length below, this could be expected to be sufficient. However, we shall suggest that the pion tail cannot be ignored.

It is perhaps worth while, in view of what follows, to explain a little of the physical meaning of quenching. Mathematically it is defined by setting the fermion determinant to unity, or excluding all fermion loops. (Gluon exchange, including self-coupling and loops is nevertheless treated to all orders.) This means that once three quarks (in the case of a baryon) are initially placed on the lattice we can follow their unbroken propagator lines until they are removed in the last time slice. It would be tempting to equate this with a valence quark picture in familiar hadronic models, but this would be misleading. Fig. 1 illustrates schematically a typical path on the lattice (in quenched approximation) which goes beyond the valence approximation. (In Fig. 1 one should understand that all possible gluon exchanges are included.)

It is particularly relevant to what follows that if one were to calculate the axial form factor of the nucleon it would have an induced pseudoscalar piece, $g_p(q^2)$. Furthermore, to the extent that an interacting $q-\overline{q}$ pair can represent a pion, through processes like that shown in Fig. 1, $g_p(q^2)$ may have a pion pole. But, and this is crucial, processes such as that shown in Fig. 2, which would include a pion loop, are definitely excluded by quenching.

To conclude this section we quote some recent results for hadron masses in state-of-the-art, quenched lattice QCD (Petronzio 1987; Negele 1990). By

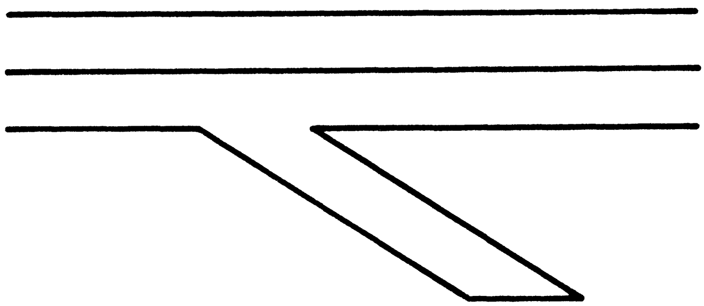

Fig. 1. One possible path allowed even in the quenched approximation. (All possible gluon exchanges are included.) The extension downwards would give rise to an induced pseudoscalar form factor.

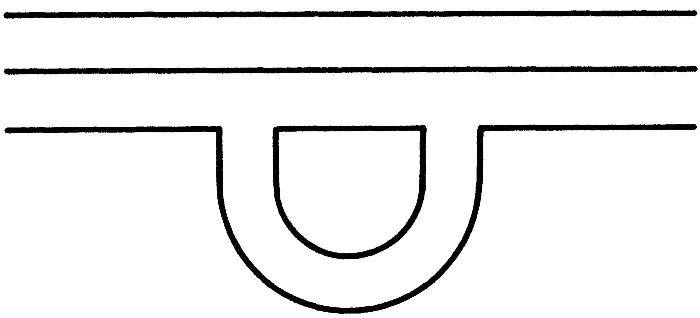

Fig. 2. A loop correction to the nucleon mass which is forbidden in the quenched approximation. It is expected to be well approximated by the processes shown in Figs 3*a* and 3*c* below.

using m_π and m_ρ to set the lattice spacing and the light quark mass, one can predict m_N and m_Δ. The result is that m_N always comes out too high (of order 1·2 GeV) while the N–Δ splitting is always too small (of order 0·2 GeV). For a first principles calculation agreement with data at the level of 30% is already very impressive. On the other hand, one would like to do better.

3. Chiral Symmetry—the Role of the Pion

The success of the PCAC hypothesis in low energy physics is very well known (Pagels 1975). From the point of view of QCD the reason for this success is to be found in the small values of the u and d running quark masses (at a typical hadronic scale). In the limit where $\overline{m}$ $[\overline{m} = (m_u + m_d)/2]$ vanishes the axial current would be exactly conserved ($\partial_\mu A_i^\mu = 0$). Then it is straightforward to show that the induced pseudoscalar piece of A_i^μ must have a zero mass pole corresponding to the exchange of a massless, Goldstone pion. As we discussed earlier, even in the quenched approximation $g_p(q^2)$ should have a pion pole, and even though one cannot realistically hope to calculate at exactly $\overline{m} = 0$, for $\overline{m}$ of order 10–20 MeV one could expect to see its effect. One very impressive method for extracting rigorous results from QCD has been developed by Gasser and Leutwyler (1982). The idea is to use the small values of m_u and m_d (and to a lesser extent m_s) to make rigorous expansions for physical hadron masses about the chiral-SU(3) limit. (In this limit, for example, the whole nucleon octet would be degenerate.) It is found that the coupling of the baryons to the pseudo-Goldstone bosons gives rise to non-analytic corrections to properties like the mass of the nucleon and the nucleon σ-term. Relying on the mathematics for guidance, Gasser and Leutwyler argued that it makes sense to keep only the leading non-analytic corrections (LNAC) to the chiral limit—that is terms involving $\overline{m}^{1/2}$ and $\ell n \overline{m}$.

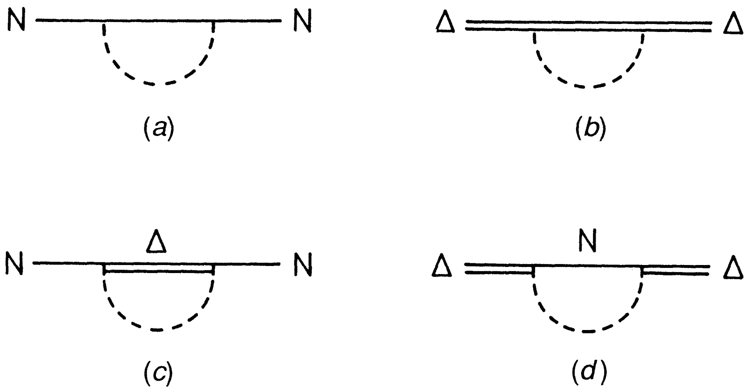

Fig. 3. (*a*) and (*b*). Contributions to the N and Δ self-energy allowed under the LNAC philosophy; (*c*) and (*d*) additional contributions which need to be included in realistic calculations.

For the nucleon and delta the LNAC are shown in Figs 3*a* and 3*b*. At the hadronic level they involve pionic loops, but they clearly correspond to precisely the sort of fermion loops that we saw earlier are excluded from a quenched lattice calculation. In order to calculate these loops one needs only the NNπ and $\Delta\Delta\pi$ coupling constants and form factors (i.e. high momentum cut-offs). The NNπ coupling constant is of course known very accurately, while the $\Delta\Delta\pi$ coupling can reasonably accurately be inferred from it using SU(6) symmetry. Symmetry considerations also suggest that the two form factors should be the same. Gasser and Leutwyler took a dipole of mass 0·4 to 0·7 GeV, in order that the pion loop correction should not be too large. In fact, there is now very good evidence that this cut-off mass should be more like 1 GeV (or 0·73 GeV in a monopole form factor) with an error of perhaps 10% (Thomas and Holinde 1989; Holinde and Thomas 1990; Coon and Scadron 1981). Using this preferred value would dramatically increase the pionic correction to the chiral limit.

Our major concern is not with the correction to the chiral limit but with regard to lattice calculations the effect of ignoring pion loops altogether. As we have seen this is the situation in the quenched approximation. Using the same form factors for the NNπ and $\Delta\Delta\pi$ vertices and the SU(6) ratio of coupling constants, one easily finds that the pion loops shown in Fig. 3 lower the N and Δ masses by the *same* amount—between 100 and 200 MeV depending on the form factor used. In view of the earlier discussion of the results of quenched lattice QCD, it is clear that this is already a step in the right direction!

4. Beyond LNAC

Our discussion thus far has emphasised that chiral symmetry is a vital property of QCD. No quark model can be considered correct if the pion loop corrections demanded by chiral symmetry are omitted. This is also true of the quenched approximation to lattice QCD. We have already seen that the loop corrections to m_N and m_Δ which give rise to the leading non-analytic behaviour in $\overline{m}$ are numerically significant. In this section we ask whether it is sufficient to stop at LNAC.

Over the last decade a great deal of experience has been accumulated with so-called chiral quark models, in which chiral symmetry is restored to a quark model by surrounding it with a pion cloud (Jaffe 1982; Brown and Rho 1979; Théberge *et al.* 1980). For example, in the cloudy bag model (CBM) a perturbative pion could restore the chiral symmetry lost when quarks are confined in the MIT bag. Such a model sacrifices some of the mathematical rigor of the LNAC approach. In compensation it allows one to make predictions for a wide range of phenomena without additional parameters (Thomas 1984; Miller 1984). It also provides a sensible physical picture of hadron structure.

Within the CBM pion loop corrections arise very naturally. However, unlike the LNAC approach, some additional contributions are expected to be important. For the N and Δ these are shown in Figs 3*c* and 3*d*. They are the processes $N\rightarrow\Delta\pi\rightarrow N$ and $\Delta\rightarrow N\pi\rightarrow\Delta$ respectively. They enter on the same footing as $N\rightarrow N\pi\rightarrow N$ and $\Delta\rightarrow\Delta\pi\rightarrow\Delta$ because none of the valence quarks is excited in the intermediate states (Théberge *et al.* 1980). Simply on the basis of the uncertainty principle we expect these processes to dominate the long-range structure of the N and Δ. If further evidence were needed we recall the crucial role of the Δ in the Adler–Weisberger relation for g_A.

The importance of the pion tail in understanding the neutron charge form factor (through the virtual process $n\rightarrow p\pi^-\rightarrow n$) has been stressed many times (e.g. Thomas 1984), although the lesson is occasionally forgotten. The important role of the Δ in calculations of the nucleon magnetic moments (Théberge and Thomas 1983) and the renormalised πN coupling constant has also been detailed elsewhere (Dodd *et al.* 1981; Thomas 1984). With respect to the mass problem the effect of including the Δ for the nucleon, and N for the Δ is dramatic. First the process $N\rightarrow\Delta\pi\rightarrow N$ gives as big a contribution as $N\rightarrow N\pi\rightarrow N$. Thus the attractive nucleon self-energy is doubled. Even more dramatic, the process $\Delta\rightarrow N\pi\rightarrow\Delta$ involves an open channel and hence, for the real part of the self-energy, a principal value integral. This is much smaller than the additional contribution to the nucleon self-energy, and as a consequence the mass of the N moves down with respect to the Δ (Théberge *et al.* 1980, 1982).

Depending on the form factors used the total N self-energy correction should be between −300 MeV and −400 MeV, while the N–Δ splitting from this source is of order 100 MeV. Since, as we have seen, these corrections are entirely absent for quenched lattice QCD, such calculations should be compared not with the experimental masses but rather with masses corrected for the pion loops. It is therefore (to say the least) encouraging that quenched lattice QCD tends to give m_N of order $1 \cdot 2$ to $1 \cdot 3$ GeV and to underestimate the N–Δ splitting.

5. Beyond the Quenched Approximation

It would be tempting to conclude from the success of the pionic corrections in reconciling quenched lattice results for m_N and m_Δ with data that the problem is solved, QCD is correct and the nucleon is essentially three interacting quarks with a perturbative pion cloud. On the other hand, no QCD purist is going to be satisfied by this argument.

Eventually, with a powerful enough computer, no approximations like quenching will need to be made. Unfortunately that day seems a long way off. We would like to suggest a procedure for systematically improving the lattice results as quenching is relaxed. It has the additional advantage that it offers an independent check on the convergence to the limit where chiral symmetry is being accurately incorporated.

The suggestion is simply that for any calculation of m_N and m_Δ in which the quenched approximation is not used, one should also calculate m_π (this would usually be done) *and* $g_p(q^2)$ (this would *not* normally be done). Given m_π and g_p one could estimate the pion loop correction included in the lattice calculation. One would then compare the results with physical masses corrected, not by the full pion loop corrections given above, but by the difference between the full correction and that estimated as we just indicated. As the lattice calculation becomes more reliable this residual correction should vanish (within the errors of the calculation).

6. Concluding Remarks

We believe that the arguments presented here are quite compelling. Quenched lattice QCD for m_N and m_Δ, augmented by pionic self-energy corrections, agree rather well with the experimental values. Furthermore it should be possible to systematically monitor the decreasing need to supplement the lattice results with such a correction as those calculations move beyond quenching.

Rather than end on a note that might be considered indecently optimistic, we mention two problems which stem from our discussion. The first is a caution on just how big a spatial lattice must be to contain a nucleon. As m_π^{-1} is of order $1 \cdot 4$ fm, it is clear that a 3 fm cube can only be considered barely adequate—4 or 5 fm would be better. Since one would also like a spatial resolution not worse than $0 \cdot 05$ fm this means that one should aim for a spatial lattice of order 80^3 or even 100^3.

The second open problem involves an unresolved question of principle. If one is able to work with a quark mass light enough that m_π has its physical value, the Δ will be unstable to decay into Nπ. (The ρ will also be unstable to $\pi\pi$, which could complicate the use of m_ρ to set the lattice scale.) This will lead to complications for the usual lattice procedure whereby the $3q$ correlation function behaves like $\exp[-M\tau]$ in Euclidean time τ. When the Δ acquires a width one might naively expect this to become $\exp[-(M - i\Gamma/2)\tau]$ which would complicate the extraction of M. On the other hand, in this case the Δ is not an eigenstate of the Hamiltonian and the naive expectation is no more than that. A lot more serious thought needs to be put into this problem and one may need to formulate the lattice calculation to yield a (real) bare mass as in the cloudy bag model.

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