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The $^{12}C(\alpha,\gamma)^{16}O$ Cross Section at Low Energies

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Abstract

An *R*-matrix formula for the cross section for radiative capture reactions is developed and applied to fit recently measured $^{12}C(\alpha,\gamma)^{16}O$ data, for both ground-state transitions and cascade transitions through the 6·92 and 7·12 MeV levels. The correct treatment of the channel contributions is significant for the E2 cascade transitions. Consistent fits of the cascade and ground-state data suggest a value of the channel radius larger than those previously used, and consequently a value of the low-energy astrophysical *S*-factor appreciably larger than that adopted recently.

1. Introduction

The low-energy cross section of the $^{12}C(\alpha,\gamma)^{16}O$ reaction is important in astrophysics (Filippone 1986). It is usually considered that the main contributions come from E1 and E2 transitions to the ^{16}O ground state, involving excited 1^- and 2^+ states. Cascade transitions through the particle-bound 6·92 and 7·12 MeV states have also been observed at laboratory energies.

Among the procedures that have been used to fit the measured cross sections and extrapolate them to the low energies of interest in astrophysical calculations have been standard *R*-matrix formulae. In general, however, these are not justified for photon channels, because the basic assumptions that no particles are created or destroyed, and that a channel radius exists, are not satisfied. The electromagnetic interaction is long range, so that contributions to the collision matrix for radiative capture reactions can come from large distances. Thus, in addition to the internal contribution to the collision matrix, which resembles that for particle reactions, there should also be channel contributions. For the E1 components of the $^{12}C(\alpha,\gamma)^{16}O$ cross section, which are nonzero only because of isospin mixing, it can be argued that the channel contributions are negligible. This is not necessarily the case for the E2 components, and it is not clear that previous *R*-matrix fits to E2 data have adequately and accurately included the channel contributions.

In Section 2, an *R*-matrix formula based on perturbation theory is given for the cross section for a radiative capture reaction, for the general case of electric multipole radiation. This is specialised to the case of $^{12}C(\alpha,\gamma)^{16}O$

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in Section 3, which also contains some comments on the formulae used in earlier fits to E2 data. Section 4 gives the results of fits based on the *R*-matrix formula to the recent E1 and E2 data, including the cascade transitions (Redder *et al.* 1987; Kremer *et al.* 1988). The $^{12}C+\alpha$ elastic scattering phase shifts measured recently (Plaga *et al.* 1987) are also fitted. A discussion of the results obtained here and by others is contained in Section 5.

2. *R*-matrix Formulae for Radiative Capture Reactions

Using procedures given in Section XIII·3 of Lane and Thomas (1958), and in Lynn (1968) and Holt *et al.* (1978), we derive a formula for the cross section for the reaction

$$A + a \rightarrow B^* \rightarrow B + \gamma . \tag{1}$$

Since the coupling of the nucleons to the electromagnetic field is weak, first-order perturbation theory is used, and the photon channel is treated in a different way from a normal particle channel. Here we give the formula only for the case of electric multipole radiation, using notation as specified in Fig. 1. The initial state J_i and final state J_f are described by *R*-matrix formulae.

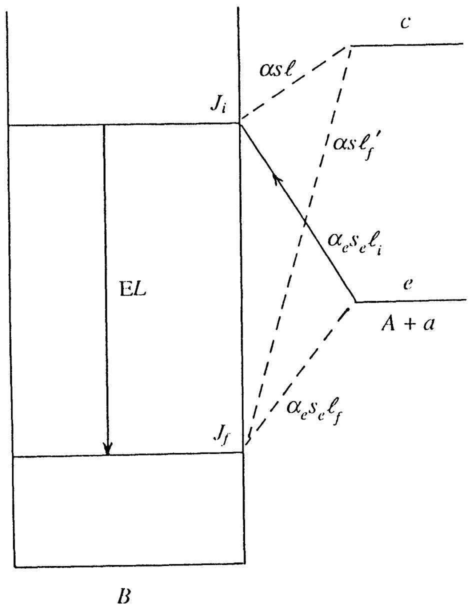

Fig. 1. Diagram showing notation used in *R*-matrix formulae for capture reaction with electric multipole radiation.

For EL radiation to the final state J_f, the total cross section may be written

$$\sigma_{J_f} = \sum_{J_i} \sigma_{J_i J_f} , \tag{2}$$

$$\sigma_{J_i J_f} = \frac{\pi}{k_a^2} \frac{2J_i + 1}{(2I_a + 1)(2I_A + 1)} \sum_{s_e l_i} | U^{J_i}_{s_e l_i, J_f} |^2 , \tag{3}$$

where

$$U^{J_i}_{s_e l_i, J_f} = -ie^{i(\omega_i - \phi_i)} 2P^{1/2}_{l_i} k_\gamma^{L+1/2} \Bigg[\sum_{\lambda\mu} \gamma^{J_i}_{\lambda s_e l_i} \gamma^{J_i}_{\mu\gamma J_f} A^{J_i}_{\lambda\mu}$$

$$+ \frac{2\mu_e^{1/2}\bar{e}_L}{\hbar k_a} \left\{ \frac{(L+1)(2L+1)}{L} \right\}^{1/2} \frac{1}{(2L+1)!!} N_f^{1/2} a_e^L F_{l_i}(a_e) G_{l_i}(a_e)$$

$$\times \sum_{l_f} i^{l_i + L - l_f} \theta^{J_f}_{f\alpha_e s_e l_f} (l_i L 0 0 | l_f 0) U(L l_f J_i s_e; l_i J_f) J'_L(l_i, l_f) \Bigg] . \tag{4}$$

Here the level matrix $\mathbf{A}^{J_i}$ is defined by its inverse

$$\left[(\mathbf{A}^{J_i})^{-1} \right]_{\lambda\mu} = \left(E^{J_i}_\lambda - E \right) \delta_{\lambda\mu} - \sum_c (S_l - B_l + iP_l) \gamma^{J_i}_{\lambda c} \gamma^{J_i}_{\mu c} \quad (c \equiv \alpha s l) . \tag{5}$$

The photon reduced-width amplitude has internal and channel contributions

$$\gamma^{J_i}_{\mu\gamma J_f} = \gamma^{J_i}_{\mu\gamma J_f}(\text{int}) + \gamma^{J_i}_{\mu\gamma J_f}(\text{ch}) , \tag{6}$$

where

$$\gamma^{J_i}_{\mu\gamma J_f}(\text{int}) = \left\{ \frac{4\pi(L+1)}{L} \right\}^{1/2} \frac{1}{(2L+1)!!} \sum_{M_f} (J_f L M_f M_i - M_f | J_i M_i)$$

$$\times \left(X_{\mu J_i M_i} | \mathcal{H}^{(L)}_{\text{EM}_i - M_f} | \Phi_{f J_f M_f} \right)_{\text{int}} , \tag{7}$$

$$\mathcal{H}^{(L)}_{\text{EM}} = \sum_{i=1}^{A} e_i r_i^L i^L Y_{LM}(\theta_i \phi_i) , \tag{8}$$

and

$$\gamma^{J_i}_{\mu\gamma J_f}(\text{ch}) = \frac{2\bar{e}_L}{\hbar} \left\{ \frac{(L+1)(2L+1)}{L} \right\}^{1/2} \frac{1}{(2L+1)!!} N_f^{1/2}$$

$$\times \sum_{c l'_f} \mu_c^{1/2} a_c^{L+1} \gamma^{J_i}_{\mu c} i^{l + L - l'_f} \theta^{J_f}_{f\alpha s l'_f} (l L 0 0 | l'_f 0) U(L l'_f J_i s; l J_f) J_{Lc}(l, l'_f) . \tag{9}$$

The normalisation factor N_f for the final state is defined by

$$N_f^{-1} = 1 + \sum_{\alpha s l'_f} \frac{2(\theta^{J_f}_{f\alpha s l'_f})^2}{a_\alpha} \int_{a_\alpha}^{\infty} dr \left[\frac{W_{\alpha s l'_f}(r)}{W_{\alpha s l'_f}(a_\alpha)} \right]^2 . \tag{10}$$

For energies E at which channel c is open, one has

$$J_{Lc}(l, l'_f) = J''_L(l, l'_f) + i \frac{F_l(a_c) G_l(a_c)}{F_l^2(a_c) + G_l^2(a_c)} J'_L(l, l'_f) , \tag{11}$$

where

$$J'_L(l,l'_f) = \frac{1}{a_c^{L+1}} \int_{a_c}^{\infty} dr r^L \frac{W_{\alpha s l'_f}(r)}{W_{\alpha s l'_f}(a_c)} \left[\frac{F_l(r)}{F_l(a_c)} - \frac{G_l(r)}{G_l(a_c)} \right], \tag{12}$$

$$J''_L(l,l'_f) = \frac{1}{a_c^{L+1}} \int_{a_c}^{\infty} dr r^L \frac{W_{\alpha s l'_f}(r)}{W_{\alpha s l'_f}(a_c)} \frac{F_l(a_c)F_l(r) + G_l(a_c)G_l(r)}{F_l^2(a_c) + G_l^2(a_c)}, \tag{13}$$

and for energies E at which channel c is closed,

$$J_{Lc}(l,l'_f) \equiv J_L^-(l,l'_f) = \frac{1}{a_c^{L+1}} \int_{a_c}^{\infty} dr r^L \frac{W_{\alpha s l'_f}(r)}{W_{\alpha s l'_f}(a_c)} \frac{W_{\alpha s l}(r)}{W_{\alpha s l}(a_c)}. \tag{14}$$

The definitions (12) and (13) of the dimensionless radial integrals J' and J'' are extensions of those given in equation (40c) of Thomas (1952). Additional formulae for the dimensionless reduced width amplitude, reduced mass and effective charge are

$$\theta^J_{\lambda c} = \gamma^J_{\lambda c}(\hbar^2/\mu_c a_c^2)^{-1/2}, \tag{15}$$

$$\mu_e = \frac{M_a M_A}{M_a + M_A}, \tag{16}$$

$$\bar{e}_L = \mu_e^L \left[\frac{Z_a}{M_a^L} + (-)^L \frac{Z_A}{M_A^L} \right] e. \tag{17}$$

Other notation is essentially the same as in Lane and Thomas (1958).

There are three contributions to the collision matrix element (4). The part of the photon reduced-width amplitude arising from integration over the internal region, $\gamma^{J_i}_{\mu\gamma J_f}(\mathrm{int})$, given by equation (7), leads to a resonant contribution of the standard R-matrix form (since $\gamma^{J_i}_{\mu\gamma J_f}(\mathrm{int})$ is real and constant). Another resonant contribution comes from integrations over the various channels, leading to $\gamma^{J_i}_{\mu\gamma J_f}(\mathrm{ch})$ given by equation (9), but in general $\gamma^{J_i}_{\mu\gamma J_f}(\mathrm{ch})$ is neither real nor constant. There is also a nonresonant contribution coming from the entrance channel only, which is often referred to as the hard-sphere capture amplitude.

As usual, the astrophysical S-factor is given in terms of the cross section by

$$S = E \exp(2\pi\eta)\sigma, \tag{18}$$

where E is the c.m. energy in the entrance channel and η is the Sommerfeld parameter $\eta = Z_a Z_A e^2/\hbar v = Z_a Z_A e^2(\mu_e/2\hbar^2 E)^{1/2}$.

3. Specialisation to $^{12}C(\alpha,\gamma)^{16}O$

(a) E1 Radiation

Because isospin is expected to be a good quantum number in the channel region, one can neglect the channel contributions to E1 capture (note that $\bar{e}_1 = 0$ if the ^{4}He mass excess is neglected). Thus standard *R*-matrix formulae can be applied to the E1 component.

(b) E2 Radiation

For low-energy α-particles ($E < 4\cdot 44$ MeV), the only open particle channel is ^{12}C(g.s.)+α, and we neglect contributions from all other channels. Then, omitting indices that have fixed values or are otherwise superfluous, we can write the formulae from Section 2 as

$$\sigma_{J_i J_f} = \frac{\pi}{k^2}(2J_i+1)|U^{J_i}_{J_f}|^2\,, \tag{19}$$

where

$$U^{J_i}_{J_f} = -ie^{i(\omega_{J_i}-\phi_{J_i})}2P^{1/2}_{J_i}k^{5/2}_{\gamma}\Bigg[\sum_{\lambda\mu}\gamma^{J_i}_{\lambda}\gamma^{J_i}_{\mu\gamma J_f}A^{J_i}_{\lambda\mu}$$
$$+\frac{3}{\sqrt{10}}\frac{M_n^{1/2}e}{\hbar k}N_f^{1/2}a^2F_{J_i}(a)G_{J_i}(a)i^{J_i+2-J_f}\theta^{J_f}_f(J_i200|J_f0)J'_2(J_i,J_f)\Bigg] \tag{20}$$

with

$$\left[(\mathbf{A}^{J_i})^{-1}\right]_{\lambda\mu} = (E^{J_i}_{\lambda}-E)\delta_{\lambda\mu}-(S_{J_i}-B_{J_i}+iP_{J_i})\gamma^{J_i}_{\lambda}\gamma^{J_i}_{\mu}\,, \tag{21}$$

$$\gamma^{J_i}_{\mu\gamma J_f}(\mathrm{ch}) = \frac{3}{\sqrt{10}}\frac{M_n^{1/2}e}{\hbar}N_f^{1/2}a^3i^{J_i+2-J_f}\gamma^{J_i}_{\mu}\theta^{J_f}_f(J_i200|J_f0)$$
$$\times\left[J''_2(J_i,J_f)+i\frac{F_{J_i}(a)G_{J_i}(a)}{F^2_{J_i}(a)+G^2_{J_i}(a)}J'_2(J_i,J_f)\right], \tag{22}$$

and

$$N_f^{-1} = 1+\frac{2(\theta^{J_f}_f)^2}{a}\int_a^{\infty}dr\left[\frac{W_{J_f}(r)}{W_{J_f}(a)}\right]^2. \tag{23}$$

Because of the one-channel approximation, the resonant term in equation (20) can be written as

$$\sum_{\lambda\mu}\gamma^{J_i}_{\lambda}\gamma^{J_i}_{\mu\gamma J_f}A^{J_i}_{\lambda\mu} = \frac{\sum_{\lambda}\gamma^{J_i}_{\lambda}\gamma^{J_i}_{\lambda\gamma J_f}/(E^{J_i}_{\lambda}-E)}{1-(S_{J_i}-B_{J_i}+iP_{J_i})\sum_{\lambda}(\gamma^{J_i}_{\lambda})^2/(E^{J_i}_{\lambda}-E)} \tag{24}$$

(*cf.* Lane and Thomas 1958, section IX, 1a).

(c) Relative Phase of E1 and E2 Collision Matrix Elements

A formula was derived in Barker (1987) for the relative phase of the E1 and E2 collision matrix elements, based on the assumption that each component could be expressed in standard many-level *R*-matrix form. The above expression for the E2 component does not have this form, in part because the photon reduced width amplitude $\gamma^{J_i}_{\mu\gamma J_f}$ occurring in (20) and (24) is complex (see equation 22), and in part because of the nonresonant term in (20). One can, however, rewrite equation (20), using (24), as

$$U^{J_i}_{J_f} = -ie^{i(\omega_{J_i}-\phi_{J_i})} 2P^{1/2}_{J_i} k^{5/2}_{\gamma} \left\{1-(S_{J_i}-B_{J_i}+iP_{J_i})\sum_{\lambda}\frac{(\gamma^{J_i}_{\lambda})^2}{E^{J_i}_{\lambda}-E}\right\}^{-1}$$

$$\times\left[\sum_{\lambda}\frac{\gamma^{J_i}_{\lambda}}{E^{J_i}_{\lambda}-E}\left\{\gamma^{J_i}_{\lambda\gamma J_f}(\text{int})+\gamma^{J_i}_{\lambda\gamma J_f}(\text{ch})\right\}\right.$$

$$+\left\{1-(S_{J_i}-B_{J_i}+iP_{J_i})\sum_{\lambda}\frac{(\gamma^{J_i}_{\lambda})^2}{E^{J_i}_{\lambda}-E}\right\}\frac{3}{\sqrt{10}}\frac{M_n^{1/2}e}{\hbar k}N_f^{1/2}a^2$$

$$\left.\times F_{J_i}(a)G_{J_i}(a)i^{J_i+2-J_f}\theta^{J_f}_f(J_i 2 0 0|J_f 0)J'_2(J_i,J_f)\right]. \tag{25}$$

Then, by making use of

$$P_{J_i} = ka/\left[F^2_{J_i}(a)+G^2_{J_i}(a)\right], \tag{26}$$

one finds that the imaginary part of the first term in the square brackets in (25), coming from the $J'_2(J_i,J_f)$ term in $\gamma^{J_i}_{\mu\gamma J_f}(\text{ch})$ given by equation (22), just cancels the imaginary part of the second term in the square brackets, coming from P_{J_i}. Thus the quantity in the square brackets in (25) is real, and the argument given in Barker (1987) still applies, so that equation (8) of that reference is still valid, *i.e.*

$$\phi_{12} = \delta_2-\delta_1+\arctan(\tfrac{1}{2}\eta). \tag{27}$$

(d) Change of B_{J_i} Value

For standard *R*-matrix formulae, the collision matrix is independent of the choice of the boundary condition parameters B_c provided that the level parameters E_λ and $\gamma_{\lambda c}$ satisfy certain relations (Barker 1972). Similarly the collision matrix element given by equation (20) is independent of the choice of B_{J_i} if $E^{J_i}_{\lambda}$ and $\gamma^{J_i}_{\lambda}$ satisfy these relations, and if the $\gamma^{J_i}_{\mu\gamma J_f}(\text{int})$ satisfy the relations for feeding amplitudes (denoted by $g_{\lambda x}$ in Barker 1972); $\theta^{J_f}_f$ must be independent of B_{J_i}.

It is expedient to use these relations because suitable choices of the B_{J_i} values can simplify the imposition of restrictions on some of the parameter values.

(e) Comments on the Formulae used in Earlier Fits to E2 Data

In the fits to E2 data by Kettner *et al.* (1982) and Redder *et al.* (1987), the resonant contributions were taken to have the standard *R*-matrix form, and these were added coherently to the nonresonant hard-sphere capture amplitude, which was taken from the direct-capture calculation of Rolfs (1973). This implies that the photon reduced-width amplitude $\gamma^{J_i}_{\mu\gamma J_f}$ is real and constant, which is the case if the radial integrals J'_2 in equation (22) are negligible and the J''_2 are energy independent (but not necessarily negligible). This is approximately true if the final state is strongly bound, as then $W_{\alpha s l'_f}(\mathrm{r})$ in equations (12) and (13) decreases rapidly as r increases from a_c. For ground-state transitions, the final state is bound by 7·16 MeV, so this approach may be justifiable, but for the cascade transitions through the 6·92 and 7·12 MeV levels (binding energies 245 and 45 keV respectively) it is not.

Barker (1987) fitted E2 data for ground-state transitions using a standard two-level *R*-matrix approximation, saying that the upper (background) level represented 'all high-lying 2^+ levels of ^{16}O as well as direct capture'. It is not clear that such a background level can adequately represent the nonresonant hard-sphere capture amplitude, although this is proportional to J'_2 and so should be small for a strongly-bound final state.

A rather similar fit to the E2 ground-state data was made by Filippone *et al.* (1989), using *K*-matrix formulae rather than *R*-matrix. These formulae had the standard form for particle reactions, and it is not clear if they are justified for radiative capture reactions.

Thus it seems that one can query the justification for each of the previous fits to the E2 capture data, as distinct from the calculations of the E2 capture cross section based on microscopic models (Langanke and Koonin 1983, 1985; Descouvemont *et al.* 1984; Funck *et al.* 1985; Descouvemont and Baye 1987).

4. Fits to Data

The available $^{12}C(\alpha,\gamma)^{16}O$ data consist essentially of the total cross sections (or *S*-factors) for E1 and E2 capture to the ^{16}O ground state, and for cascade transitions through the 6·92 and 7·12 MeV levels. Values of the relative phase of the E1 and E2 ground-state amplitudes were also extracted from the measured angular distributions (Dyer and Barnes 1974; Redder *et al.* 1987) and compared with values from the formula (27) (see Redder *et al.* 1987, Fig. 7, which gives values averaged over 100 keV energy intervals). Angular distributions of the secondary transitions in the cascades have been measured (Redder *et al.* 1987) but not published.

We consider fits to the data in the order *(a)* E1 ground state, *(b)* E2 ground state, *(c)* cascade through 6·92 MeV level, *(d)* cascade through 7·12 MeV level, *(e)* combined E1 ground state and 7·12 MeV cascade, *(f)* combined E2 ground state and 6·92 MeV cascade.

(a) E1 Capture to ^{16}O Ground State

Previously (Barker 1971, 1987), simultaneous *R*-matrix fits were made to the E1 capture cross section to the ^{16}O ground state, the $^{12}C+\alpha$ p- and f-wave phase shifts, and the delayed α spectrum following ^{16}N β decay. Additional data are now available on the E1 cross section (Redder *et al.* 1987; Kremer *et al.* 1988) and on the phase shifts (Plaga *et al.* 1987); we therefore fit these new data using the same method as before.

The ^{16}N delayed α-spectrum data are the same as used previously (Barker 1971, 1987). The p- and f-wave phase shifts and their uncertainties are taken from Table 2 of Plaga *et al.* (1987). Separate fits are made with the E1 capture data taken either from Table 1 of Redder *et al.* (1987) or from Fig. 3 of Kremer *et al.* (1988). As Filippone *et al.* (1989) pointed out, these two sets of data differ significantly. In all fits the radiation width of the $7\cdot12$ MeV 1^- level of ^{16}O is taken as 55 ± 3 MeV (Ajzenberg-Selove 1986).

As in the previous three-level *R*-matrix fits (Barker 1971, 1987), we assume no feeding of the background levels in ^{16}N β decay and no γ-decay of the 1^- background level:

$$A^{(3)}_{3l} = 0 \quad (l = 1,3), \quad \gamma^{(3)}_{3\gamma} = 0 . \tag{28}$$

(The significance of the superscripts in parentheses is explained in Barker 1971.)

Results of best fits to the data for various channel radii are given in Table 1; the notation is the same as in Barker (1987), except that $S_{\rm E1}$ ($0\cdot3$ MeV) is abbreviated to $S(0\cdot3)$. The X values, which give the quality of fit to the various data, are defined by

$$X = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{V_{\rm exp}(E_i) - V_{\rm calc}(E_i)}{\epsilon(E_i)}\right|^2 , \tag{29}$$

where E_i are the N energies at which the experimental values $V_{\rm exp}$ with errors ϵ are fitted. The dimensionless reduced width $\theta^{(1)2}_{11}$, derived from $\gamma^{(1)2}_{11}$ by means of equation (15), is 3/2 times the more familiar quantity $\theta^2_\alpha(7\cdot12)$, which comes from the relation $\gamma^2 = \theta^2(3\hbar^2/2\mu a^2)$; for ease of comparison with earlier work, we give values of $\theta^2_\alpha(7\cdot12)$ in Table 1. Acceptable fits to the data are obtainable for ranges of parameter values around the best-fit values. Fig. 2 shows minimum values of $X_{\rm tot}$ as functions of $\theta^2_\alpha(7\cdot12)$ for the various channel radii [for a given value of the channel radius, $S(0\cdot3)$ is proportional to $\theta^2_\alpha(7\cdot12)$, within about ±5%, over the ranges of the curves shown in Fig. 2]. It is seen that, for a given channel radius, the Kremer fits favour smaller values of $\theta^2_\alpha(7\cdot12)$ and of $S(0\cdot3)$ than do the Redder fits. Also the Kremer fits favour the smaller channel radii, while the Redder fits favour the larger channel radii. Fits to the data are shown later (Figs 6 and 7).

(b) E2 Capture to ^{16}O Ground State

From their measured values of $\sigma_{\rm E2}/\sigma_{\rm E1}$, averaged over 100 keV intervals, and their calculated (best-fit) values of $\sigma_{\rm E1}$, Redder *et al.* (1987) extracted values of $\sigma_{\rm E2}$, the E2 component of the ^{12}C(α,γ_0)^{16}O cross section. The corresponding *S*-factor values are shown in their Fig. 9. Filippone *et al.* (1989) in their Fig. 8 give an alternative set of $S_{\rm E2}$ values, derived from the same $\sigma_{\rm E2}/\sigma_{\rm E1}$ values but with interpolated measured values of $\sigma_{\rm E1}$. The two sets

Table 1. Parameter values from best fits to $^{12}C+\alpha$ phase shifts, α spectrum from ^{16}N β decay and $^{12}C(\alpha,\gamma_0)^{16}O$ E1 cross section for various channel radii

The cross section data are from either Redder *et al.* (1987), R, or Kremer *et al.* (1988), K

a (fm)	Data	l	$\gamma_{1l}^{(1)2}$ (MeV)	$E_{2l}^{(1)}$ (MeV)	$\gamma_{2l}^{(1)2}$ (MeV)	$E_{3l}^{(1)}$ (MeV)	$\gamma_{3l}^{(1)2}$ (MeV)	R_l	X_l	X_β	R_γ	X_γ	X_{tot}	$\theta_\alpha^2(7\cdot12)$	$S(0\cdot3)$ (MeV b)
5·0	R	1	0·140	3·91	0·457	29·4	7·18	0·32	2·73	0·48	−0·47	2·69	7·97	0·167	0·132
		3	0·074	5·94	0·218	109·7	21·6	−0·31	2·07						
	K	1	0·126	3·88	0·459	26·5	6·07	0·32	2·65	0·41	−0·43	2·26	7·33	0·151	0·119
		3	0·056	5·91	0·213	106·6	20·8	−0·30	2·01						
5·5	R	1	0·080	3·37	0·304	17·4	2·37	0·33	2·72	0·71	−0·41	2·08	7·67	0·116	0·164
		3	0·0265	5·49	0·184	32·2	3·67	−0·29	2·16						
	K	1	0·070	3·34	0·299	16·6	2·10	0·32	2·63	0·69	−0·38	2·11	7·60	0·101	0·144
		3	0·0190	5·50	0·187	31·4	3·49	−0·30	2·17						
6·0	R	1	0·048	3·05	0·180	16·1	1·75	0·32	2·77	0·93	−0·38	1·69	7·32	0·083	0·203
		3	0·0086	5·10	0·118	27·8	2·54	−0·30	1·93						
	K	1	0·042	3·03	0·177	15·6	1·60	0·31	2·75	1·02	−0·35	2·30	7·95	0·072	0·180
		3	0·0061	5·08	0·117	28·1	2·58	−0·30	1·88						
6·5	R	1	0·030	2·85	0·110	16·2	1·65	0·31	2·96	1·28	−0·36	1·48	7·27	0·061	0·256
		3	0·0029	4·82	0·072	28·9	2·59	−0·29	1·55						
	K	1	0·026	2·83	0·107	15·9	1·55	0·30	3·12	1·51	−0·33	2·71	8·94	0·053	0·222
		3	0·0017	4·83	0·073	28·5	2·52	−0·30	1·60						

Table 2. Parameter values from best fits to $^{12}C+\alpha$ d-wave phase shift and $^{12}C(\alpha,\gamma_0)^{16}O$ E2 cross section for various channel radii

a (fm)	$\gamma_1^{(1)2}$ (MeV)	$E_2^{(1)}$ (MeV)	$\gamma_2^{(1)2}$ (MeV)	$\gamma_{1\gamma}^{(1)}$(int) ($MeV^{1/2}fm^{5/2}$)	$\gamma_{2\gamma}^{(2)}$(int) ($MeV^{1/2}fm^{5/2}$)	$\theta_\alpha^2(0\cdot0)$	X_2	X_γ	X_{tot}	$\theta_\alpha^2(6\cdot92)$	$S(0\cdot3)$ (MeV b)
5·0	0·361	58·9	11·74	−0·573	−7·02	0·282	0·68	1·45	2·13	0·431	0·028
5·5	0·167	39·7	6·05	−0·731	−4·75	0·131	0·67	1·46	2·13	0·242	0·033
6·0	0·130	40·9	6·07	−0·704	−5·11	0·136	0·70	1·40	2·10	0·224	0·047
6·5	0·116	43·4	6·55	−0·705	−4·72	0·107	0·71	1·32	2·03	0·235	0·090

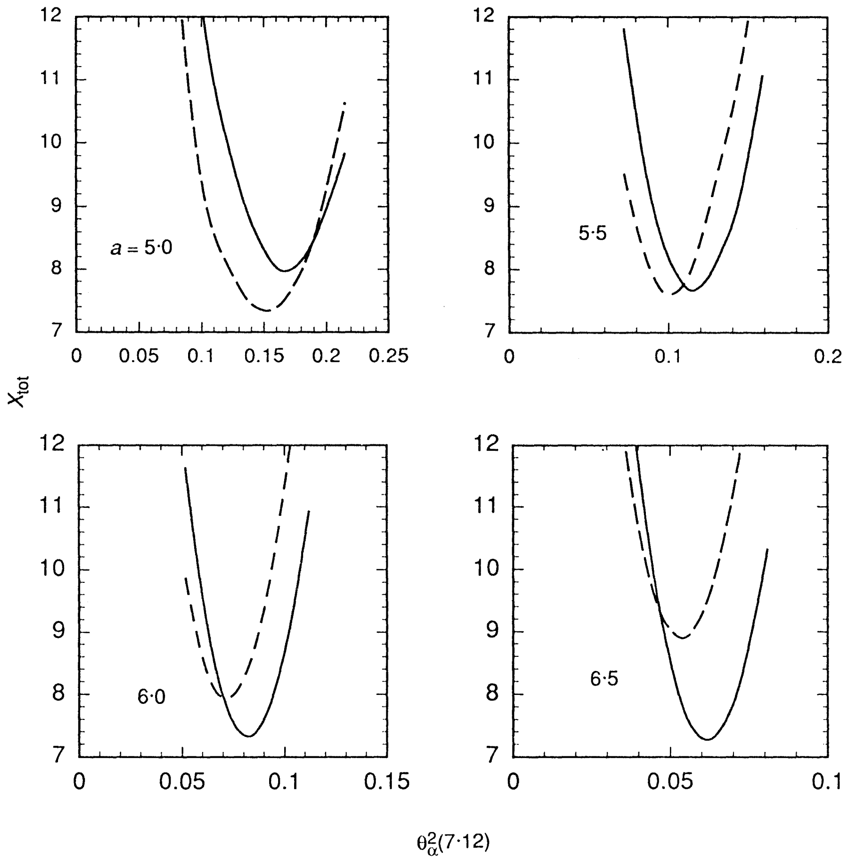

Fig. 2. Minimum values of $X_{\rm tot}$ as functions of $\theta_\alpha^2(7\cdot12)$ for the ^{12}C$(\alpha,\gamma_0)^{16}$O E1 component, for the indicated values of a (in fm) (solid curves—Redder *et al.* data; dashed curves—Kremer *et al.* data).

are not significantly different, in view of the considerable scatter of the points in each set, except that Filippone *et al.* do not have any point corresponding to the $\sigma_{\rm E2}/\sigma_{\rm E1}$ value at the lowest energy of 0·94 MeV.

We fit the $S_{\rm E2}$ values and uncertainties as given in Fig. 9 of Redder *et al.* (1987) for energies E from 1·10 to 2·45 MeV. Their uncertainty for the point at 0·94 MeV is probably much underestimated, because they did not allow for any uncertainty in the value of $\sigma_{\rm E1}$. Points at $E \geq 2\cdot55$ MeV are omitted in order to avoid contributions from the narrow 2^+ levels at $E = 2\cdot68$ MeV ($\gamma = 0\cdot625$ keV) and 4·36 MeV (71 keV). Likewise we fit the ^{12}C+α d-wave phase shift data of Plaga *et al.* (1987) only for $E_\alpha \leq 4\cdot451$ MeV ($E \leq 3\cdot34$ MeV). The radiation width of the 6·92 MeV level is taken as 97±3 meV (Ajzenberg-Selove 1986).

As in Barker (1987), we use a two-level R-matrix approximation, the two levels corresponding to the subthreshold 6·92 MeV level and a background level. Here, however, the background is taken to represent only the higher-lying 2^+ levels and not a direct-capture component, since the resonant and nonresonant

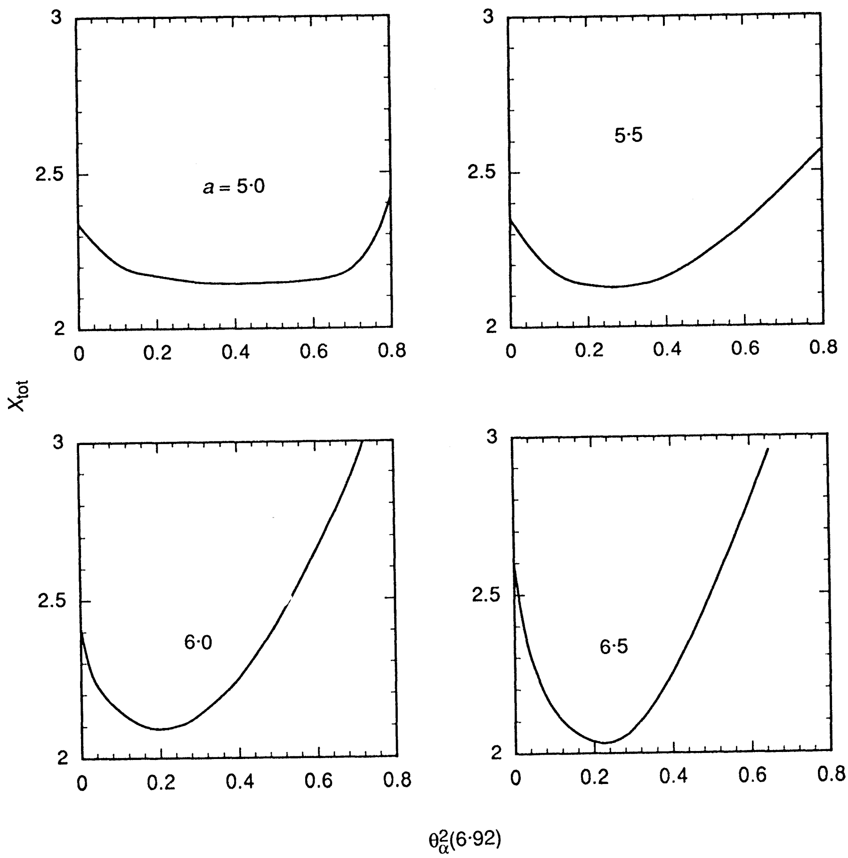

Fig. 3. Minimum values of $X_{\rm tot}$ as functions of $\theta_\alpha^2(6\cdot 92)$ for the $^{12}C(\alpha,\gamma_0)^{16}O$ E2 component, for the indicated values of a (in fm).

channel contributions are included explicitly in the formulae of Section 3*b*. For given channel radius, there are then seven parameters entering the *R*-matrix formula for $\sigma_{\rm E2}$, namely E_γ, γ_λ, $\gamma_{\lambda\gamma}(\rm int)$ $(\lambda = 1,2)$, and θ_f^0 $(\equiv (3/2)^{1/2}\theta_\alpha(0\cdot 0))$. Three constraints come from fitting the energy and radiation width of the $6\cdot 92$ MeV level and the energy of the background level, assumed to be $E_2^{(2)} = 15$ MeV, as in Barker (1987).

Parameter values for the best fits for various values of the channel radius are given in Table 2. The quality of fit is similar for each value of *a*, but the value of $S(0\cdot 3)$ increases rapidly with *a*. For each value of *a*, Fig. 3 gives minimum values of $X_{\rm tot}$ as functions of $\theta_\alpha^2(6\cdot 92)$. Fits to the data are shown later (Figs 10 and 11).

(c) Cascade Transitions through the 6·92 MeV Level

Experimental data as *S*-factors are given in Fig. 8*a* and Table 3 of Redder *et al.* (1987). We fit the data only for $E \leq 2\cdot 61$ MeV, so that resonant contributions due to the $10\cdot 36$ MeV 4^+ level of ^{16}O may be neglected.

We assume, as did Redder *et al.*, that there are four contributions, which add incoherently: a resonant contribution due essentially to the E1 transition between the 9·58 MeV 1^- level and the 6·92 MeV 2^+ level, and three nonresonant contributions due to E2 channel capture from initial s-, d- and g-wave states (taking $A^{0^+}_{\lambda\mu} = A^{2^+}_{\lambda\mu} = A^{4^+}_{\lambda\mu} = 0$).

Values of the 1^- level parameters are taken from Table 1 (data of Redder *et al.*). The E1 transition between the 7·12 and 6·92 MeV level has not been observed; we assume that $\gamma^{1(1)}_{1\gamma 2} = 0$ for this transition, based on the argument that the E1 transition between the 7·12 and 6·05 MeV levels is very weak (Ajzenberg-Selove 1986, Table 2) and that the 6·92 and 6·05 MeV levels are probably generically related. We also assume that $\gamma^{1(3)}_{3\gamma 2} = 0$ for the background level, for the reason given in Section 5*b*. Then, for a given channel radius a, the adjustable parameters are $\gamma^{1}_{2\gamma 2}$ and θ^2_f. Since the final state here (the 6·92 MeV level) is effectively described by a one-level approximation, the value of θ^2_f corresponds to $B_2 = S_2(6\cdot 92)$, *i.e.* $\theta^2_f = \theta^{2(1)}_1 = (3/2)^{1/2}\theta_\alpha(6\cdot 92)$.

Table 3. Parameter values for best fits to S-factor for cascade transition through 6·92 MeV level

a (fm)	$\theta^2_\alpha(6\cdot 92)$	$\gamma^{1(2)}_{2\gamma 2}$ (MeV$^{1/2}$fm$^{3/2}$)	$\Gamma^0_{\rm E1}(9\cdot 58 \rightarrow 6\cdot 92)$ (meV)	X	$S(0\cdot 3)$ (MeV b)
5·0	2·69	0·0298	2·18	0·59	0·008
5·5	1·039	0·0270	2·35	0·65	0·009
6·0	0·540	0·0256	2·44	0·72	0·010
6·5	0·310	0·0252	2·58	0·80	0·012

Parameter values giving best fits to the data for various values of a are given in Table 3; this also contains the corresponding values of the observed radiation width for the E1 transition between the 9·58 and 6·92 MeV levels, given by

$$\Gamma^0_{\rm E1}(9\cdot 58 \rightarrow 6\cdot 92) = 2(E_\gamma/\hbar c)^3(\gamma^{1(2)}_{2\gamma 2})^2 \Big/ \left[1 + (\gamma^{1(2)}_2)^2 (dS_1/dE)_{E^{1(2)}_2}\right]. \qquad (30)$$

Use of 1^- level parameters from Table 1 for the Kremer *et al.* data gives values not significantly different from those in Table 3. Fig. 4 shows minimum values of X as functions of $\theta^2_\alpha(6\cdot 92)$, for each value of a. A best fit to the data is shown later in Fig. 12.

(d) Cascade Transitions through the 7·12 MeV Level

Experimental S-factors are given in Fig. 8*b* and Table 3 of Redder *et al.* (1987). Redder *et al.* argued that the primary transition to the 7·12 MeV level is predominantly E2, comprising a resonant contribution due to the 9·58 MeV 1^- level interfering coherently with a p-wave direct-capture contribution, together with an incoherent f-wave direct-capture component. We also assume that the primary transition is E2, with resonant and nonresonant components. Values of the 1^- and 3^- level parameters are taken from Table 1 (Redder *et al.* data). Because of the limited data, we assume that $\gamma^{1(3)}_{3\gamma 1}(\text{int}) = \gamma^{3(3)}_{3\gamma 1}(\text{int}) = 0$.

We choose $\gamma^{3(1)}_{1\gamma 1}(\text{int})$ to fit the observed width of the strong E2 transition between the 7·12 and 6·13 MeV levels, $\Gamma^0_{\rm E2}(7\cdot 12 \rightarrow 6\cdot 13) = (4 \pm 1) \times 10^{-5}$ eV =

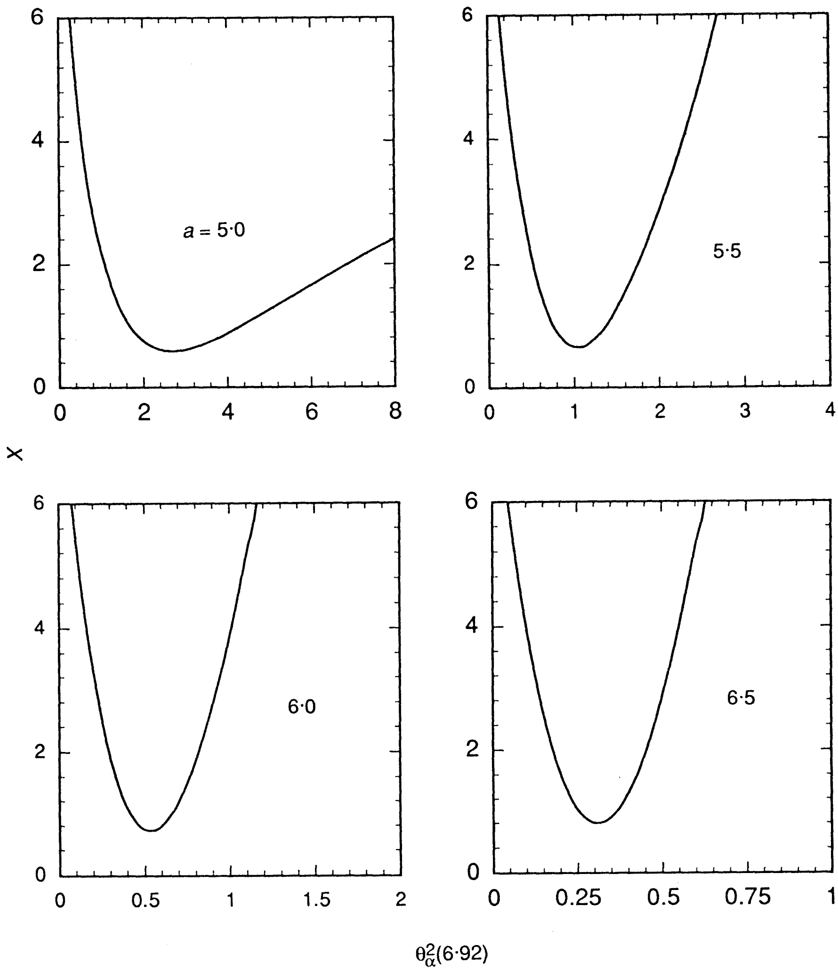

Fig. 4. Minimum values of X as functions of $\theta^2_\alpha(6\cdot92)$ for the $^{12}C(\alpha,\gamma)^{16}O$ cascade transition through the 6·92 MeV level, for the indicated values of a (in fm).

21 ± 5 W.u. (Ajzenberg-Selove 1986), using

$$\Gamma^0_{E2}(7\cdot12\rightarrow6\cdot13)=2(E_\gamma/\hbar c)^5(\gamma^{1(1)}_{1\gamma3})^2/\left[1+(\gamma^{1(1)}_1)^2(dS_1/dE)_{E^{1(1)}_1}\right], \tag{31}$$

with

$$\gamma^{1(1)}_{1\gamma3}=\gamma^{1(1)}_{1\gamma3}(\text{int})+\frac{3}{5\sqrt{2}}e\,a^2N_3^{1/2}\theta^{1(1)}_1\theta^{3(1)}_1\mathcal{J}_2(1,3) \tag{32}$$

$$\gamma^{3(1)}_{1\gamma1}(\text{int})=(3/7)^{1/2}\gamma^{1(1)}_{1\gamma3}(\text{int})\,. \tag{33}$$

The contribution to the cascade of interest here comes from the transition between the extension of the 6·13 MeV level above the $^{12}C+\alpha$ threshold (the ghost of the 6·13 MeV level—see Barker and Treacy 1962) and the subthreshold 7·12 MeV level. Similarly one may expect a contribution from the E2 transition to the subthreshold 7·12 MeV level from its own ghost, proportional to the reduced width amplitude $\gamma_{1\gamma 1}^{1(1)}$, which is related to the quadrupole moment of the 7·12 MeV level by

$$Q(7\cdot 12) = \frac{2\sqrt{3}}{e} N_1^{1/2} \gamma_{1\gamma 1}^{1(1)}, \tag{34}$$

where

$$\gamma_{1\gamma 1}^{1(1)} = \gamma_{1\gamma 1}^{1(1)}(\mathrm{int}) + \frac{\sqrt{3}}{5} e\, a^2 N_1^{1/2} \theta_1^{(1)2} J_2^{-}(1,1). \tag{35}$$

Thus the value of the parameter $\gamma_{1\gamma 1}^{1(1)}(\mathrm{int})$ could be obtained if the value of $Q(7\cdot 12)$ were known. No experimental value is available. We therefore calculate both $Q(7\cdot 12)$ and $\Gamma_{\mathrm{E2}}^{0}(7\cdot 12 \to 6\cdot 13)$ using shell model wavefunctions for the 1^- and 3^- states, and choose the parameter values to fit the experimental value of γ_{E2}^{0}. The Oxford–Buenos Aires–MSU shell model code (Brown *et al.* 1986) is used in the $1\hbar\omega$ approximation with the interaction of van Hees and Glaudemans (1983, 1984) and harmonic oscillator single-particle wavefunctions. Independently of the length parameter or the isoscalar effective charge, we obtain the relation

$$Q(7\cdot 12) = 0\cdot 583[B(\mathrm{E2}; 7\cdot 12 \to 6\cdot 13)]^{1/2}, \tag{36}$$

where

$$\Gamma_{\mathrm{E2}}^{0}(7\cdot 12 \to 6\cdot 13) = \frac{4\pi}{75}(E_\gamma/\hbar c)^5 e^2 B(\mathrm{E2}; 7\cdot 12 \to 6\cdot 13). \tag{37}$$

With the experimental value of Γ_{E2}^{0}, these equations give $Q(7\cdot 12) = 4\cdot 2 \pm 1$ fm^2, and we choose $\gamma_{2\gamma 1}^{1(1)}(\mathrm{int})$ to fit this. Then the adjustable parameters are $\gamma_{2\gamma 1}^{1}(\mathrm{int})$, $\gamma_{2\gamma 1}^{3}(\mathrm{int})$ and $\theta_f^1 \equiv (3/2)^{1/2}\theta_\alpha(7\cdot 12)$.

The best-fit parameter values for various values of a are given in Table 4, case A. Almost identical fits to the data can be obtained with positive values of $\gamma_{1\gamma 1}^{3(1)}(\mathrm{int})$, the values of the other quantities in Table 3 except $\gamma_{2\gamma 1}^{3(2)}$ remaining essentially unchanged. Values obtained when $Q(7\cdot 12)$ and $\Gamma_{\mathrm{E2}}^{0}(7\cdot 12 \to 6\cdot 13)$ are both fixed at zero are also given in Table 3, case B; these show that the values of $\theta_\alpha^2(7\cdot 12)$, $\Gamma_{\mathrm{E2}}^{0}(9\cdot 58 \to 7\cdot 12)$, X and $S(0\cdot 3)$ are not sensitive to the fitted values of $Q(7\cdot 12)$ and $\Gamma_{\mathrm{E2}}^{0}(7\cdot 12 \to 6\cdot 13)$. Fig. 5 shows minimum values of X as functions of $\theta_\alpha^2(7\cdot 12)$, for each value of a. A best fit to the data is given later in Fig. 8.

(e) Combined E1 Ground State and 7·12 MeV Cascade Transitions

The four separate fits to the ^{12}C$(\alpha,\gamma)^{16}$O data given in Sections 4*a*–4*d* are similar to earlier fits, in particular those of Redder *et al.* (1987). There is an inconsistency, however, in our procedure in Section 4*d*, in that $\theta_\alpha^2(7\cdot 12)$ is treated as an adjustable parameter, even though a particular value of $\theta_\alpha^2(7\cdot 12)$ is implied by the fixed value of $\gamma_{11}^{(1)2}$ taken from Table 1. It is seen that the best-fit values of $\theta_\alpha^2(7\cdot 12)$ in Tables 1 and 4 agree only for the largest value of the channel radius a = 6·5 fm. There are, however, ranges of acceptable values of $\theta_\alpha^2(7\cdot 12)$, as indicated in Figs 2 and 5, and it is possible that acceptable consistent fits could be obtained for the smaller channel radii.

Table 4. Parameter values for best fits to *S*-factor for cascade transition through 7·12 MeV level

a (fm)	Case	$\theta_\alpha(7\cdot 12)$	$\gamma^{1(1)}_{1\gamma 1}$(int) (MeV$^{1/2}fm^{5/2}$)	$\gamma^{1(2)}_{2\gamma 1}$(int) (MeV$^{1/2}fm^{5/2}$)	$\gamma^{3(1)}_{1\gamma 1}$(int) (MeV$^{1/2}fm^{5/2}$)	$\gamma^{3(2)}_{2\gamma 1}$(int) (MeV$^{1/2}fm^{5/2}$)	$\Gamma^0_{E2}(9\cdot 58 \to 7\cdot 12)$ (meV)	X	$S(0\cdot 3)$ (MeV b)
5·0	A	0·356	0·498	2·01	−1·344	−2·42	8·4	2·04	0·0019
	B	0·326	−0·986	2·11	0·321	−3·54	8·4	2·05	0·0018
5·5	A	0·194	0·829	2·36	−1·472	−2·71	8·2	2·01	0·0021
	B	0·168	−0·594	2·52	0·179	−3·38	8·2	2·03	0·0018
6·0	A	0·107	1·055	2·67	−1·557	−2·94	8·0	1·98	0·0021
	B	0·098	−0·395	2·74	0·097	−3·14	8·0	2·00	0·0020
6·5	A	0·060	1·200	2·89	−1·603	−2·95	7·8	1·97	0·0021
	B	0·058	−0·261	2·87	0·052	−3·37	7·7	1·99	0·0020

A: $Q(7\cdot 12) = 4\cdot 2\ \mathrm{fm}^2$, $\Gamma^0_{E2}(7\cdot 12 \to 6\cdot 13) = 4\times 10^{-5}$ eV.
B: $Q(7\cdot 12) = 0$, $\Gamma^0_{E2}(7\cdot 12 \to 6\cdot 13) = 0$.

Table 6. Resultant values from simultaneous best fits to $^{12}C+\alpha$ d-wave phase shift and cross sections for $^{12}C(\alpha,\gamma_0)^{16}O$ E2 transition and cascade transition through 6·92 MeV level

a (fm)	$\gamma^{(1)2}_1$ (MeV)	$\gamma^{(1)}_{1\gamma}$(int) (MeV$^{1/2}$fm$^{5/2}$)	$\theta^2_\alpha(0\cdot 0)$	$\theta^2_\alpha(6\cdot 92)$	$\Gamma^0_{E1}(9\cdot 58 \to 6\cdot 92)$ (meV)	X_2	X_γ	X_{casc}	X_{tot}	$S_{E2}(0\cdot 3)$ (MeV b)	$S_{casc}(0\cdot 3)$ (MeV b)
5·0	0·649	−0·338	0·434	0·776	4·56	0·92	1·39	2·93	5·24	0·042	0·004
5·5	0·504	−0·184	0·530	0·730	3·28	1·12	1·37	1·07	3·56	0·061	0·007
6·0	0·293	−0·395	0·309	0·505	2·63	1·10	1·30	0·73	3·13	0·083	0·010
6·5	0·158	−0·623	0·140	0·319	2·42	0·84	1·26	0·81	2·90	0·117	0·012
7·0	0·077	−0·814	0·040	0·181	2·80	0·71	1·78	0·88	3·38	0·142	0·013
7·5	0·039	−0·943	0·002	0·105	3·02	1·54	4·52	1·16	7·22	0·209	0·014

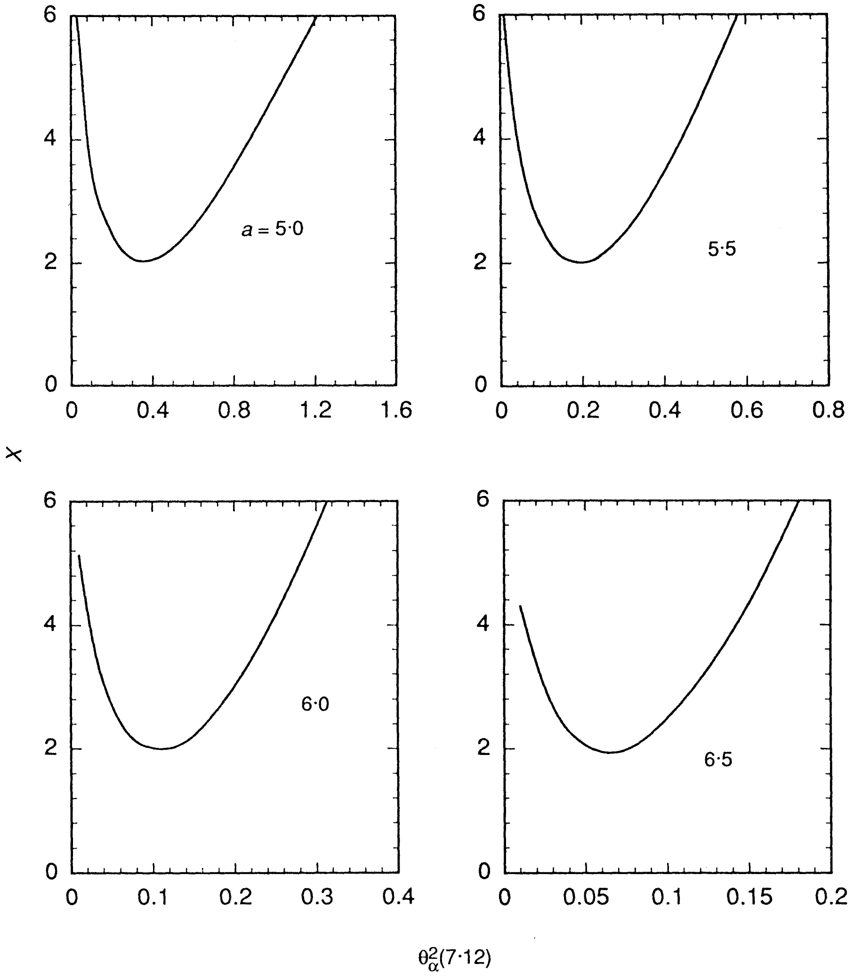

Fig. 5. Minimum values of X as functions of $\theta_\alpha^2(7\cdot 12)$ for the $^{12}C(\alpha,\gamma)^{16}O$ cascade transition through the $7\cdot 12$ MeV level, for the indicated values of a (in fm).

We have therefore carried out simultaneous fits to the E1 ground-state data, including the ^{16}N delayed alpha spectrum and the $^{12}C+\alpha$ phase shifts, and to the cascade data through the $7\cdot 12$ MeV level. Again the E1 ground-state capture data of Redder *et al.* (1987) and Kremer *et al.* (1988) are treated separately.

Resultant values from the best fit for each value of the channel radius are given in Table 5. Values of parameters not included in Table 5 are not significantly different from those in Tables 1 and 4. Because the previous fits favoured large channel radii, we have extended the range of a values to $7\cdot 5$ fm. The smallest value of X_{tot} is obtained for $a = 6\cdot 5$ fm for the fits involving the Redder *et al.* E1 data and for $a = 5\cdot 5$ fm for the Kremer *et al.* data. The corresponding fits to the data are shown in Figs 6–8; Fig. 6 for the E1 capture data of Redder *et al.* and Kremer *et al.*, Fig. 7 for the phase shifts of Plaga *et al.* (1987), and Fig. 8 for the $7\cdot 12$ MeV cascade data from Redder *et al.*

Table 5. Resultant values from simultaneous best fits to $^{12}C+\alpha$ phase shifts, α spectrum from ^{16}N β decay and cross sections for $^{12}C(\alpha,\gamma_0)^{16}O$ E1 transition and cascade transition through 7·12 MeV level

The E1 cross section data are from either Redder *et al.* (1987), R, or Kremer *et al.* (1988), K

a (fm)	Data	l	$\gamma_{1l}^{(1)2}$ (MeV)	$\theta_\alpha^2(7\cdot12)$	$\Gamma_{E2}^0(9\cdot58\to7\cdot12)$ (meV)	X_l	X_β	X_γ	X_{casc}	X_{tot}	$S_{E1}(0\cdot3)$ (MeV b)	$S_{casc}(0\cdot3)$ (MeV b)
5·0	R	1	0·148	0·178	8·7	2·80	0·69	2·43	2·63	10·66	0·138	0·0010
		3	0·083			2·10						
	K	1	0·133	0·160	8·7	2·69	0·42	2·25	2·81	10·22	0·125	0·0009
		3	0·065			2·04						
5·5	R	1	0·084	0·122	8·6	2·80	0·84	1·92	2·31	10·02	0·172	0·0013
		3	0·0283			2·15						
	K	1	0·074	0·107	8·4	2·63	0·65	2·21	2·51	10·17	0·152	0·0012
		3	0·0219			2·17						
6·0	R	1	0·048	0·084	8·2	2·79	0·97	1·64	2·10	9·44	0·207	0·0016
		3	0·0088			1·94						
	K	1	0·043	0·074	8·2	2·74	0·95	2·42	2·26	10·25	0·184	0·0015
		3	0·0066			1·89						
6·5	R	1	0·030	0·061	7·8	2·96	1·28	1·49	1·97	9·26	0·258	0·0021
		3	0·0027			1·56						
	K	1	0·027	0·054	7·8	3·03	1·40	2·91	2·05	10·97	0·228	0·0019
		3	0·0021			1·58						
7·0	R	1	0·0208	0·049	7·4	3·62	1·93	1·55	2·02	10·67	0·340	0·0029
		3	0·00066			1·55						
	K	1	0·0194	0·045	7·2	3·78	1·75	4·72	1·98	13·83	0·312	0·0027
		3	0·00050			1·60						
7·5	R	1	0·0130	0·035	6·9	6·86	2·72	1·98	2·26	15·65	0·401	0·0035
		3	0·00001			1·84						

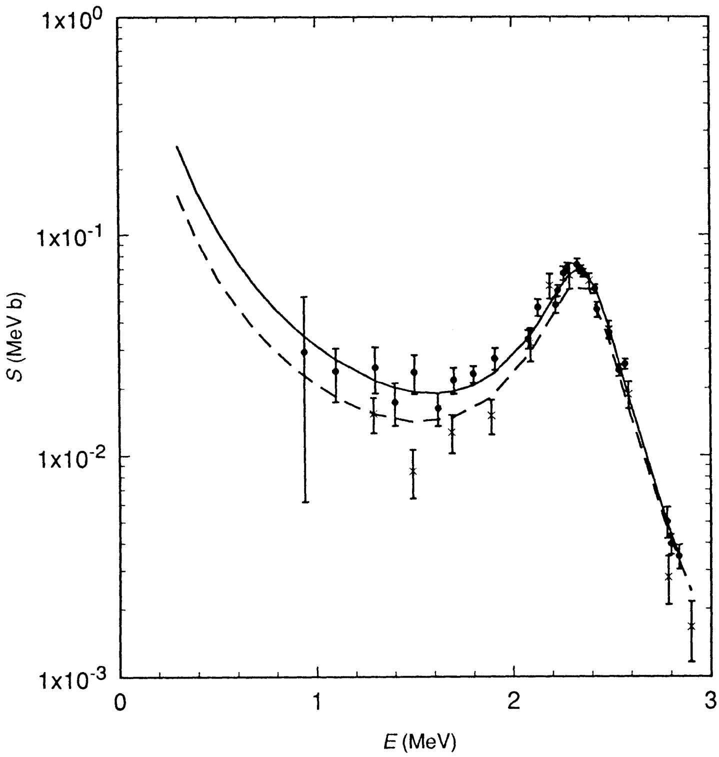

Fig. 6. The $^{12}C(\alpha,\gamma_0)^{16}O$ E1 S-factor as a function of c.m. energy. The experimental points are from Redder *et al.* (1987) (solid circles) and Kremer *et al.* (1988) (crosses). The solid (dashed) curve is the best simultaneous fit to the Redder *et al.* (Kremer *et al.*) E1 data, the $^{12}C+\alpha$ p- and f-wave phase shifts, the α spectrum from ^{16}N β decay and the $^{12}C(\alpha,\gamma)^{16}O(7\cdot12)$ S-factor, for $a = 6\cdot5$ $(5\cdot5)$ fm.

Fig. 9 shows the variation of X_{tot} as a function of $S_{E1}(0\cdot3)$ in the regions of best fit. If we restrict consideration to $a \gtrsim 5$ fm (because reduced widths exceeding the Wigner limit are obtained for smaller channel radii—see Barker 1971), and take as acceptable fits that give $X_{tot} \lesssim 1\cdot5(X_{tot})_{min}$ (as in Barker 1971, 1987), then the best values and acceptable ranges of $S_{E1}(0\cdot3)$ are $0\cdot26(0\cdot10$–$0\cdot40)$ MeV b for the data of Redder *et al.* (1987) and $0\cdot15(0\cdot08$–$0\cdot32)$ MeV b for the data of Kremer *et al.* (1988). These ranges of acceptable values would be considerably smaller if a single value of the channel radius were assumed. The values of $S_{casc}(0\cdot3)$ range from $0\cdot001$ to $0\cdot003$ MeV b, so that their contribution to $S_{tot}(0\cdot3)$ is negligible.

(f) Combined E2 Ground State and 6·92 MeV Cascade Transitions

There are considerable differences between the values of $\theta^2_\alpha(6\cdot92)$ given in Table 2 and in Table 3; these values were obtained respectively from fits to the $^{12}C(\alpha,\gamma_0)^{16}O$ E2 capture data and to the data for cascade transitions through the $6\cdot92$ MeV level. The difference is especially marked for the smaller channel radii. We therefore do simultaneous fits to both sets of data, in order

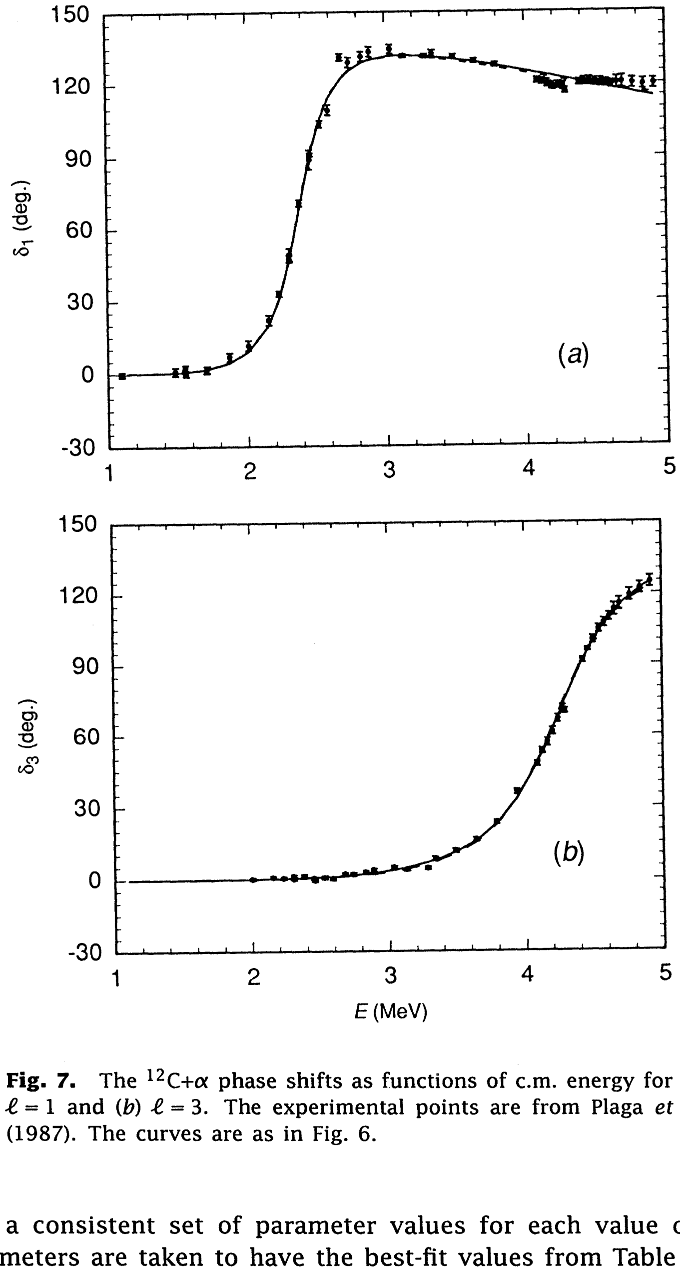

Fig. 7. The $^{12}C+\alpha$ phase shifts as functions of c.m. energy for (*a*) $\ell = 1$ and (*b*) $\ell = 3$. The experimental points are from Plaga *et al.* (1987). The curves are as in Fig. 6.

to obtain a consistent set of parameter values for each value of a. The 1^- level parameters are taken to have the best-fit values from Table 5 (Redder *et al.* data).

The results are given in Table 6. The overall best fit is obtained for $a = 6 \cdot 5$ fm. The corresponding fit to the ground-state E2 capture data is shown in Fig. 10, to the d-wave phase shift in Fig. 11, and to the cascade data in Fig. 12.

Fig. 13 shows the variation of X_{tot} as a function of $S_{E2}(0 \cdot 3)$ in the regions of the best fit. The best value and acceptable range of $S_{E2}(0 \cdot 3)$ is $0 \cdot 12(0 \cdot 05–0 \cdot 18)$ MeV b, the corresponding value of $S_{casc}(0 \cdot 3)$ being about $0 \cdot 01$ MeV b.

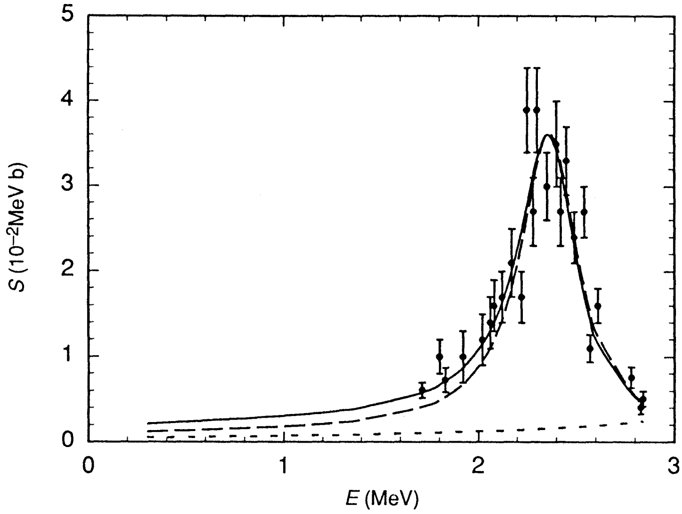

Fig. 8. The $^{12}C(\alpha,\gamma)^{16}O(7\cdot12)$ S-factor as a function of c.m. energy. The experimental points are from Redder *et al.* (1987). The solid and dashed curves are as in Fig. 6. The short-dash curve shows the f-wave E2 contribution (for the fit to the Redder *et al.* E1 data).

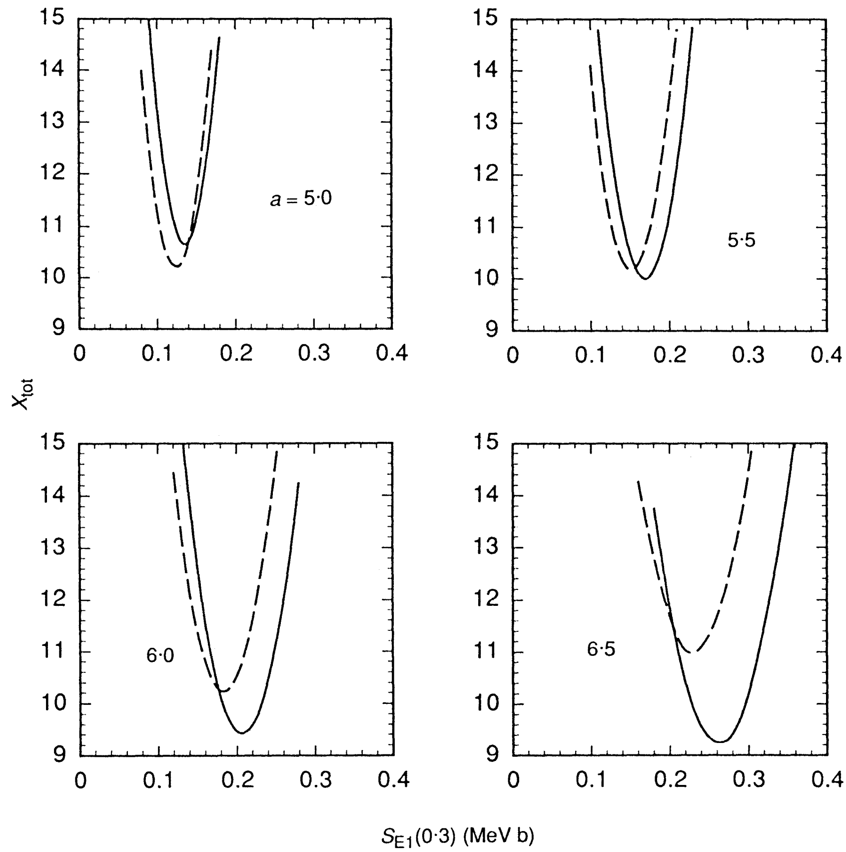

Fig. 9. Minimum values of X_{tot} as functions of $S_{E1}(0\cdot3)$. The solid (dashed) curves are from simultaneous fits to the Redder *et al.* (Kremer *et al.*) E1 data, the $^{12}C+\alpha$ p- and f-wave phase shifts, the α spectrum from ^{16}N β decay and the $^{12}C(\alpha,\gamma)^{16}O(7\cdot12)$ S-factor, for the indicated values of a (in fm).

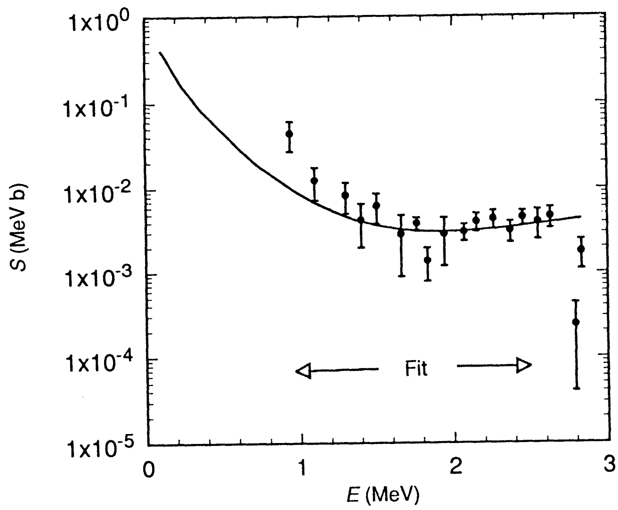

Fig. 10. The $^{12}C(\alpha,\gamma_0)^{16}O$ E2 S-factor as a function of c.m. energy. The experimental points are from Redder *et al.* (1987). The curve is the best simultaneous fit to these data, the $^{12}C+\alpha$ d-wave phase shift and the $^{12}C(\alpha,\gamma)^{16}O(6\cdot92)$ S-factor, for $a = 6\cdot5$ fm. The range of fitted data is indicated.

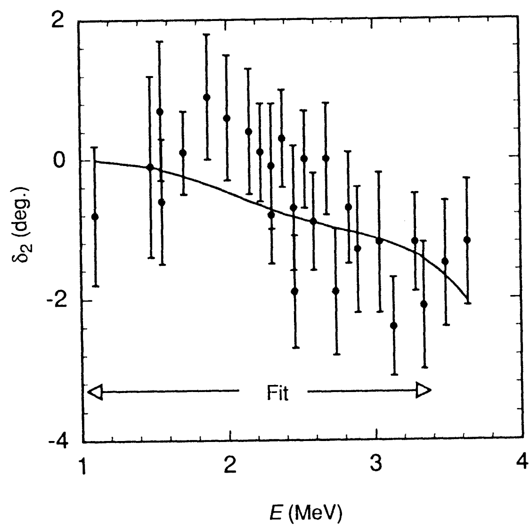

Fig. 11. The $^{12}C+\alpha$ d-wave phase shift as a function of c.m. energy. The experimental points are from Plaga *et al.* (1987). The curve is as in Fig. 10.

5. Discussion

(a) Comparison with Results of Previous Fits

Fits to the same $^{12}C(\alpha,\gamma)^{16}O$ and $^{12}C+\alpha$ elastic scattering data (used *in toto* or in part) have previously been made by Redder *et al.* (1987), Plaga *et al.* (1987)

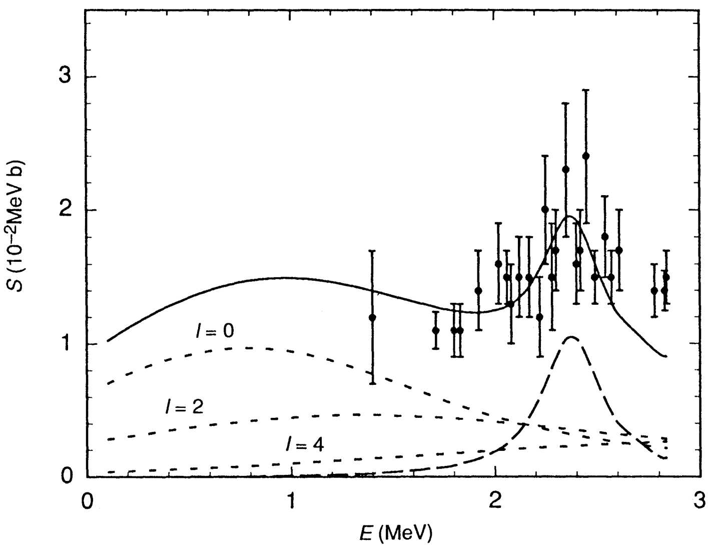

Fig. 12. The $^{12}C(\alpha,\gamma)^{16}O(6\cdot92)$ S-factor as a function of c.m. energy. The experimental points are from Redder *et al.* (1987). The solid curve is as in Fig. 10. The dashed curve shows the resonant E1 contribution, and the short-dash curves the nonresonant s-, d- and g-wave E2 contributions.

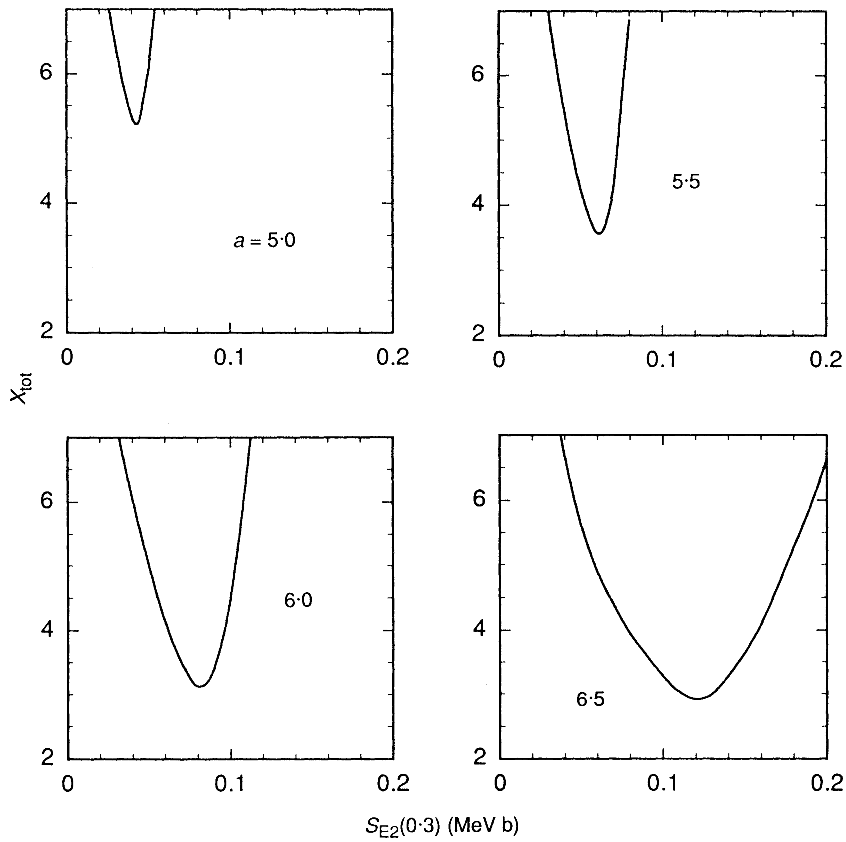

Fig. 13. Minimum values of $X_{\rm tot}$ as functions of $S_{\rm E2}(0\cdot3)$. The curves are from simultaneous fits to the E2 ground state cross section, the $^{12}C+\alpha$ d-wave phase shift and the $^{12}C(\alpha,\gamma)^{16}O(6\cdot92)$ S-factor.

and Kremer *et al.* (1988), all using *R*-matrix formulae with $a \approx 5 \cdot 5$ fm, and by Filippone *et al.* (1989) with *K*-matrix formulae. From the results of these fits Caughlan and Fowler (1988), in their latest compilation of thermonuclear reaction rates, have adopted for $^{12}C(\alpha,\gamma)^{16}O$ the values $S_{E1}(0 \cdot 3) = 0 \cdot 06$ MeV b and $S_{E2}(0 \cdot 3) = 0 \cdot 04$ MeV b, giving a total $S(0 \cdot 3) = 0 \cdot 10$ MeV b, with an uncertainty of a factor of two up or down (Fowler, personal communication, 1989). These values are much smaller than those that we have found, $S_{E1}(0 \cdot 3) = 0 \cdot 15(0 \cdot 08–0 \cdot 32)$ MeV b (data of Kremer *et al.*) and $0 \cdot 26(0 \cdot 10–0 \cdot 40)$ MeV b (data of Redder *et al.*), and $S_{E2}(0 \cdot 3) = 0 \cdot 12(0 \cdot 05–0 \cdot 18)$ MeV b. Here we give the results of the previous fits in some detail, and in the next subsection we comment on the reasons for the differences from our values.

In passing, we note that our qualities of fit to the data are comparable with the earlier values, any differences being attributable largely to the choice of the data fitted.

The adopted value of $S_{E1}(0 \cdot 3)$ was based on fits to the E1 capture data of Kremer *et al.* (1988); *R*-matrix fits using various models gave $S_{E1}(0 \cdot 3) = 0 \cdot 01(0 \cdot 00–0 \cdot 14)$, $0 \cdot 08$ and $0 \cdot 14$ MeV b (Kremer *et al.* 1988), and *K*-matrix fits with different choices of background gave $0 \cdot 028(0 \cdot 00–0 \cdot 15)$ and $0 \cdot 051(0 \cdot 00–0 \cdot 18)$ MeV b (Filippone *et al.* 1989). Various *R*-matrix fits to the E1 data of Redder *et al.* (1987) gave $S_{E1}(0 \cdot 3) = 0 \cdot 20^{+0 \cdot 28}_{-0 \cdot 18}$, $0 \cdot 09^{+0 \cdot 10}_{-0 \cdot 06}$ and $0 \cdot 14^{+0 \cdot 12}_{-0 \cdot 08}$ MeV b (Redder *et al.* 1987), and $0 \cdot 20 \pm 0 \cdot 08$ and $0 \cdot 16 \pm 0 \cdot 10$ MeV b (Plaga *et al.* 1987), while *K*-matrix fits gave $0 \cdot 050(0 \cdot 00–0 \cdot 19)$ and $0 \cdot 079(0 \cdot 00–0 \cdot 29)$ MeV b (Filippone *et al.* 1989).

The *R*-matrix fits to the E2 capture data of Redder *et al.* gave $S_{E2}(0 \cdot 3) = 0 \cdot 096^{+0 \cdot 024}_{-0 \cdot 030}$ MeV b (Redder *et al.* 1987) and $0 \cdot 089 \pm 0 \cdot 030$ MeV b (Plaga *et al.* 1987), while *K*-matrix fits (Filippone *et al.* 1989) gave best values and allowed ranges depending on the choice of background energy dependence $S_{E2}(0 \cdot 3) = 0 \cdot 014(0 \cdot 005–0 \cdot 028)$ MeV b (echo-pole background), $4 \cdot 0 \times 10^{-6}$ $(0 \cdot 000–0 \cdot 034)$ MeV b (linear background), and $(0 \cdot 00–0 \cdot 16)$ MeV b (quadratic background).

The values of $S_{casc}(0 \cdot 3)$ obtained by Redder *et al.* (1987), $0 \cdot 0013^{+0 \cdot 0005}_{-0 \cdot 0010}$ MeV b for the $7 \cdot 12$ MeV cascade and $0 \cdot 007 \pm 0 \cdot 002$ or $0 \cdot 0042 \pm 0 \cdot 0013$ MeV b for the $6 \cdot 92$ MeV cascade, are comparable with or somewhat smaller than ours.

Also the values of $\Gamma_{E2}(9 \cdot 58 \rightarrow 7 \cdot 12)$ and $\Gamma_{E1}(9 \cdot 58 \rightarrow 6 \cdot 92)$ given in Tables 5 and 6 respectively agree reasonably well with the only previous values, $\Gamma_{E2} = 7 \cdot 8 \pm 1 \cdot 6$ meV and $\Gamma_{E1} = 1 \cdot 4 \pm 1 \cdot 4$ or $2 \cdot 2 \pm 1 \cdot 4$ meV, obtained by Redder *et al.* (1987).

(b) Comments on Previous Fits

The previous fits in general gave values of $S_{E1}(0 \cdot 3)$ and $S_{E2}(0 \cdot 3)$ smaller than those that we have found. All the previous *R*-matrix fits used $a \approx 5 \cdot 5$ fm. Our fit to the Redder *et al.* (1987) E1 data with $a = 5 \cdot 5$ fm gives $S_{E1}(0 \cdot 3) = 0 \cdot 17$ MeV b (Table 5), in good agreement with the previous fits to these data. Thus in this case our larger recommended value of $S_{E1}(0 \cdot 3)$ is due to our larger value of the channel radius, $a = 6 \cdot 5$ fm.

For fits to the data of Kremer *et al.* (1988), we favour $a = 5 \cdot 5$ fm, and our value of $S_{E1}(0 \cdot 3)$ is in good agreement with the value $0 \cdot 14$ MeV b found by

Kremer *et al.* when they imposed on their R-matrix fit the restriction $\gamma_{3\gamma}^{(3)}=0$ (see equation 28). Filippone *et al.* (1989) criticised the model-dependent shell model argument on which this assumption for $\gamma_{3\gamma}^{(3)}$ was based (Barker 1971), on the grounds that 'there are experimentally observed states in ^{16}O with higher excitation that could, in principle, contribute'. It is true that if one extrapolates the ^{12}C$(\alpha,\gamma_0)^{16}$O E1 cross section measured in the region of the $12\cdot44$ MeV 1^- level down to the energies of interest here, using a one-level approximation, one finds a large contribution, and the same is true for the $13\cdot09$ MeV 1^- level. The contributions from these two levels should, however, be added coherently. Because the cross section is nonzero only because of isospin mixing, and because the properties of the $12\cdot44$ and $13\cdot09$ MeV levels are well described by a two-state isospin mixing model (Barker 1978), the contributions from the two levels interfere constructively in the energy region between the levels but destructively elsewhere, and far from the levels the total contribution is more or less the same as that for a pure $T=0$ level and a pure $T=1$ level, *i.e.* zero. The same applies to other pairs of $T=0$ and $T=1$ 1^- states that are expected to occur at higher energies (because of the small Coulomb mixing matrix elements, only nearby levels are isospin mixed to an appreciable extent). This argument (in abbreviated form) was given in Barker (1987). It can be extended to any number of mixing $T=0$ and $T=1$ states provided that they can be considered as degenerate. We therefore think that the smaller values of $S_{\rm E1}(0\cdot3)$ obtained by Kremer *et al.*, which were not based on the assumption $\gamma_{3\gamma}^{(3)}=0$, should not be given undue weight.

In their fits to the ground-state E2 data, Redder *et al.* (1987) used formulae from Kettner *et al.* (1982) (with an additional contribution from the narrow $9\cdot85$ MeV level). Kettner *et al.* described the direct capture component by

$$S_{\rm DC}(E)=(0\cdot03618+0\cdot00146E-0\cdot00136E^2)\theta_\alpha^2(0\cdot0)\ \ \text{MeV b} \tag{38}$$

(with E in MeV). Within 5%, we can fit our low-energy ($E<2$ MeV) hard-sphere component for a = $5\cdot5$ fm (Kettner *et al.* assumed a = $5\cdot4$ fm) with the quadratic expression

$$S_{\rm HS}(E)=(0\cdot199–0\cdot0087E-0\cdot0158E^2)\theta_\alpha^2(0\cdot0)N_f\ \ \text{MeV b}, \tag{39}$$

where $N_f=[1+0\cdot229\theta_\alpha^2(0\cdot0)]^{-1}$, which is about $0\cdot95$ for $\theta_\alpha^2(0\cdot0)$ = $0\cdot25$ as used by Kettner *et al.* It is seen that our expression for this component of S is several times that of Kettner *et al.* Also Redder *et al.* fitted their ground-state E2 data using a = $5\cdot5$ fm, obtaining $S(0\cdot3)=0\cdot096$ MeV b, about three times our result for the same value of a. From their Fig. 9, it seems that the interference between their resonant ($6\cdot92$ MeV) and direct-capture amplitudes was destructive in the region of the experimental data, but constructive at $E=0\cdot3$ MeV. Our fits give destructive interference throughout the energy range considered. Our values of $\theta_\alpha^2(0\cdot0)$ in Table 6 may be contrasted with the value $0\cdot012\pm0\cdot012$ given by Redder *et al.* (1987) for a = $5\cdot5$ fm. It appears that the values of $S_{\rm E2}(0\cdot3)$ given by Plaga *et al.* (1987) were obtained from those of Redder *et al.* by scaling according to the value of $\theta_\alpha^2(6\cdot92)$.

A previous fit (Barker 1987) to earlier ground-state E2 data (Dyer and Barnes 1974; Redder *et al.* 1985) favoured $a = 5 \cdot 5$ fm and gave $S_{E2}(0 \cdot 3) = 0 \cdot 03^{+0 \cdot 05}_{-0 \cdot 03}$ MeV b; in this case the small value of $S_{E2}(0 \cdot 3)$ is probably not connected with the (relatively) small value of a, but is due to the limited range of the data fitted, $E \geq 1 \cdot 71$ MeV only.

Filippone *et al.* (1989) fitted the $^{12}C(\alpha,\gamma_0)^{16}O$ and phase-shift data using K-matrix rather than R-matrix formulae. They list the many advantages of the K-matrix approach in the penultimate paragraph of their introduction; there is, however, still a problem in the freedom one has in choosing the background terms. Filippone *et al.* fitted the data, for both E1 and E2 cases, using either a background consisting of a constant plus an echo pole, or a linear background. Although the qualities of fit are similar for the two choices, the best values of $S(0 \cdot 3)$ depend very strongly on the form of background. For the E1 case, Filippone *et al.* preferred the echo pole background, because it 'allows a more physically reasonable parametrisation of δ_1 above 3 MeV', which they say has a downward trend beyond $3 \cdot 34$ MeV. They fitted experimental values of δ_1 only up to $E = 4 \cdot 3$ MeV, as shown in their Fig. 5. Plaga *et al.* (1987) gave values of δ_1 up to $4 \cdot 9$ MeV, and earlier measurements by Morris *et al.* (1968) extend the range up to $6 \cdot 4$ MeV. These values of δ_1 do not continue to drop, but reach a minimum of about 120° and then increase. With Filippone *et al.*'s choice of an echo pole at 7 MeV, their calculated δ_1 would decrease through 90° at 7 MeV. Thus it is not clear that the echo pole background has more justification than the linear background. Similar remarks apply to the E2 case.

In Barker (1987), it was said that the K-matrix fit is essentially identical with an R-matrix fit with zero channel radius. This is incorrect or at least misleading because reasonable R-matrix fits to data are not always possible with zero channel radius, *e.g.* the one-level R-matrix approximation for the observed width

$$\Gamma^0(E_r) = 2\gamma^2 P(E_r)/[1 + \gamma^2 (dS/dE)_{E_r}] \tag{40}$$

gives an upper limit on Γ^0 as $\gamma^2 \to \infty$, and this limit decreases to zero as a decreases to zero. It is still true, however, that the K-matrix penetration factor has the same energy dependence as the R-matrix penetration factor in the limit of zero channel radius. Consequently K-matrix fits tend to give small values of $S(0 \cdot 3)$, because the value of $S(0 \cdot 3)$ from R-matrix fits decreases as a decreases (see Tables 1 and 2).

(c) Comparison with Results from Other Sources

We note that microscopic and semi-microscopic calculations have given values of $S_{E2}(0 \cdot 3)$: $0 \cdot 09$ MeV b (Descouvemont *et al.* 1984), $0 \cdot 07$ MeV b (Langanke and Koonin 1985), $0 \cdot 10$ MeV b (Funck *et al.* 1985), $0 \cdot 07$ MeV b (Descouvemont and Baye 1987) and $0 \cdot 05$ MeV b (Redder *et al.* 1987). Our present recommended value is somewhat larger than these values, but not inconsistent with them.

The values of $\theta^2_\alpha(6 \cdot 92)$ and $\theta^2_\alpha(7 \cdot 12)$ obtained above may be compared with values derived from α-particle transfer reactions, *e.g.* Becchetti *et al.* (1989) studied $^{12}C(^7Li,t)^{16}O$ at high energies and deduced ratios of γ^2_α (or θ^2_α) values for the $6 \cdot 92$, $7 \cdot 12$, $9 \cdot 58$ and $10 \cdot 36$ MeV levels. Values of θ^2_α depend sensitively on the value of the channel radius, but ratios of θ_α^2 values are less sensitive to a (see Tables 5 and 6); nevertheless, it is prudent to compare ratios at the same value of a. The α-transfer values of Becchetti *et al.* are based on a channel radius of 5·4 fm. We therefore compare with our values for $a = 5 \cdot 5$ fm. Values of $\theta_\alpha^2(6 \cdot 92)$ and $\theta_\alpha^2(7 \cdot 12)$ are given in Tables 6 and 5 respectively (for the latter we use the average of the R and K values). Also $\theta_\alpha^2(9 \cdot 58) = (3\hbar^2/2\mu a^2)^{-1}\ [\gamma_{21}^{(2)}]^2$, where $\gamma_{21}^{(2)}$ is taken from the fits of Section 4*e*. These values of θ_α^2 are given in Table 7, where their ratios are compared with those of Becchetti *et al.* (1989). Probably one should not take the agreement between the values of $\theta_\alpha^2(7 \cdot 12)/\theta_\alpha^2(6 \cdot 92)$ too seriously, as Plaga *et al.* (1987) also pointed out the excellent agreement between the value of this ratio deduced from their fits to $^{12}C(\alpha,\gamma)^{16}O$ and phase shift data and that obtained from α-transfer reactions (Becchetti *et al.* 1980); these values were, however, 0·42 and 0·41 respectively.

Table 7. Ratios of θ_α^2 values from radiative capture data and from α-transfer reactions

	$^{12}C(\alpha,\gamma)^{16}O^A$	$^{12}C(^7Li,t)^{16}O^B$
$\theta_\alpha^2(6 \cdot 92)$	0·730	
$\theta_\alpha^2(7 \cdot 12)$	0·114	
$\theta_\alpha^2(9 \cdot 58)$	0·794	
$\theta_\alpha^2(7 \cdot 12)/\theta_\alpha^2(6 \cdot 92)$	0·16	0·17±0·05
$\theta_\alpha^2(7 \cdot 12)/\theta_\alpha^2(9 \cdot 58)$	0·14	0·35±0·07

[A] Present results; $a = 5 \cdot 5$ fm.
[B] Becchetti *et al.* (1989); $a = 5 \cdot 4$ fm.

Table 8. Contributions to $S_{tot}(0 \cdot 3)$ from best fits to data

Final State	Radiation	$S(0 \cdot 3)$ (MeV b)
0	E1	$0 \cdot 15(0 \cdot 08–0 \cdot 32)^A$, $0 \cdot 26(0 \cdot 10–0 \cdot 40)^B$
	E2	$0 \cdot 12(0 \cdot 05–0 \cdot 18)^C$
6·92	E2	$0 \cdot 01^C$
7·12	E2	$0 \cdot 001^A$, $0 \cdot 002^B$
	Total	$0 \cdot 28(0 \cdot 14–0 \cdot 51)^A$, $0 \cdot 39(0 \cdot 16–0 \cdot 59)^B$

[A] Fit to $^{12}C(\alpha,\gamma_0)^{16}O$ E1 data of Kremer *et al.* (1988); $a = 5 \cdot 5$ fm.
[B] Fit to $^{12}C(\alpha,\gamma_0)^{16}O$ E1 data of Redder *et al.* (1987); $a = 6 \cdot 5$ fm.
[C] $a = 6 \cdot 5$ fm.

(d) Recommended Values and Additional Comments

The quantity of prime interest here is the value of the total astrophysical S-factor for the reaction $^{12}C(\alpha,\gamma)^{16}O$ at 0·3 MeV, $S_{tot}(0 \cdot 3)$. We have considered four contributions to $S_{tot}(0 \cdot 3)$: the ground-state E1 and E2 transitions, and the cascade transitions through the 6·92 and 7·12 MeV levels. Best values and acceptable ranges of these contributions obtained from our fits to the available data are given in Table 8. Although the cascade contributions to $S_{tot}(0 \cdot 3)$ are not themselves very significant, the fits to the cascade data are important in giving constraints on the dimensionless reduced widths of the 6·92 and 7·12

MeV levels; these have led to values of the channel radius larger than those that have been used in previous fits, and so indirectly to larger values of $S_{tot}(0\cdot3)$. Additional and improved data for the cascade transitions would be most welcome.

There is an appreciable discrepancy between the ground-state E1 data of Redder *et al.* and of Kremer *et al.*, and new measurements should be made to resolve this. One approach that would not require any new measurements would be to analyse the ground-state angular distributions of Redder *et al.* (1987) assuming the relative E1/E2 phase given by equation (27); the uncertainties in the resultant values of σ_{E2}/σ_{E1} should be considerably reduced.

In this paper, following the earlier work by Barker (1971, 1987), we have used additional information from the delayed α spectrum following ^{16}N β decay. A recent paper (Ji *et al.* 1990) has also explored the usefulness of this approach. A new measurement of the ^{16}N α spectrum is planned at TRIUMF (Azuma, personal communication, 1989). A similar but alternative approach using $^{15}N(p,\gamma\alpha)^{12}C$ through the 0^- level of ^{16}O at $12\cdot80$ MeV, which avoids the problem of contributions from 3^- levels, has also been suggested, and work on this is in progress at Toronto (King, personal communication, 1989).

6. Summary

Recent cross section measurements for the $^{12}C(\alpha,\gamma)^{16}O$ reaction have been fitted using *R*-matrix formulae. The data are for E1 and E2 ground-state transitions, and for cascade transitions through the $6\cdot92$ and $7\cdot12$ MeV levels. The correct treatment of the channel contributions is important for the cascade transitions, because the $6\cdot92$ and $7\cdot12$ MeV levels are weakly bound. Consistent fits of the ground-state and cascade data suggest values of the channel radius larger than those that have been used previously. Consequently the low-energy astrophysical *S*-factor, represented by $S_{tot}(0\cdot3)$, has values (given in Table 8) that are appreciably larger than the value, $S_{tot}(0\cdot3) = 0\cdot10(0\cdot05\text{–}0\cdot20)$ MeV b, adopted in the latest compilation of Thermonuclear Reaction Rates (Caughlan and Fowler 1988).

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