

## Energy Dissipation Rates of Turbulence with an Airborne Microwave Refractometer

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### Abstract

Estimates of turbulence energy dissipation rates have been obtained in the free atmosphere of the planetary boundary layer. These are derived in terms of the variance of radio refractive index fluctuations ( $\Delta n^2$ ), the Brunt-Väisälä frequency  $N$ , the mean vertical gradient of generalised potential refractive index  $M$ , and the turbulence structure parameter  $C_n^2$ , using an airborne microwave refractometer. The energy dissipation rates thus derived are comparable with the results from other experiments. The inferred energy dissipation rates (on a near real-time basis) from the airborne microwave refractometer are useful for detection and warning of wind shears in the atmosphere.

### 1. Introduction

Airborne microwave refractometers have proved to be valuable tools for investigating the structure and dynamics of the troposphere on a near real-time basis, especially the microscale dynamics. Small scale motions, in particular turbulence, can be studied extensively over large areas provided that they contain significant three-dimensional radio refractive index structure. When the gradients of wind, humidity and temperature are strong enough, turbulence arises due to instabilities. The so-called clear air turbulence (CAT) in the free atmosphere above the planetary boundary layer often gives rise to severe structural damage to airplanes and is also responsible for aircraft accidents. The turbulence disturbances superimposed on the mean wind have a statistical structure that is determined by wind spread, stability and terrain characteristics. Extensive investigations have to be carried out in the Indian sub-continent to determine its statistical properties. Hence, detection and warning of this hazardous phenomenon of low level wind shear at airports is of considerable importance. Fujita and Caracena (1977) have studied this aspect and concluded that several aircraft accidents, attributed to 'pilot error', might have been caused by unexpected wind conditions. Hazardous wind shear could be detected by the use of electromagnetic waves since these regions are characterised by variations in temperature  $T$  and moisture  $e$ , and hence by varying radio refractive indices  $\Delta n^2$  which often occur in inversions. These turbulent fluctuations of  $T$ ,  $P$  and  $e$  are linked in a complicated manner to the turbulent velocity fluctuations, i.e. the turbulent intensity. The pressure fluctuations can be neglected (Eckart 1960) as compared with temperature

fluctuations. However, in the troposphere, the contributions to  $\Delta n^2$  from humidity fluctuations dominate those from temperature fluctuations. Just as the radar returns give information about hazardous wind shear conditions over a given region, the same information could also be derived on a near real-time basis by using an airborne microwave refractometer.

Airborne microwave refractometers work on the following principle: Comparison of the resonance frequency of an exposed (to the atmosphere) sampling cavity (open ended for free flow of air) with that of a sealed reference cavity gives a measure of the variation of radio refractive index at microwave frequencies (frequency difference). A linear inverse relationship between the cavity resonance frequency and the radio refractive index is a valid assumption (by a proper choice of frequency) in developing a microwave refractometer to read directly the radio refractive index variations.

Quantitative measurements of atmospheric turbulence can be deduced, using an airborne microwave refractometer, from the mean refractivity turbulence structure constant  $C_n^2$ . Since small scale atmospheric turbulence is usually parametrised in terms of  $\epsilon$ , the turbulent kinetic energy (KE) dissipation rate (eddy dissipation rate), its measurement is equivalent to a measurement of the magnitude of KE (or intensity of turbulence). The relationship between the eddy dissipation rate  $\epsilon$  and  $C_n^2$  can be utilised to develop a technique for this estimation of mean  $\epsilon$  from the airborne microwave refractometer measured mean  $C_n^2$ . It should be noted that the derived  $\epsilon$  is the average over about 1 km in the present case.

## 2. Method of Approach

For turbulence generated by shear instability in a hydrostatically stable region of the free atmosphere, Gage and Balsley (1978) derived a relationship between  $C_n^2$  and  $\epsilon$ :

$$C_n^2 = a_n^2 \alpha' R_i N^{-2} \epsilon^{\frac{2}{3}} M^2, \quad (1)$$

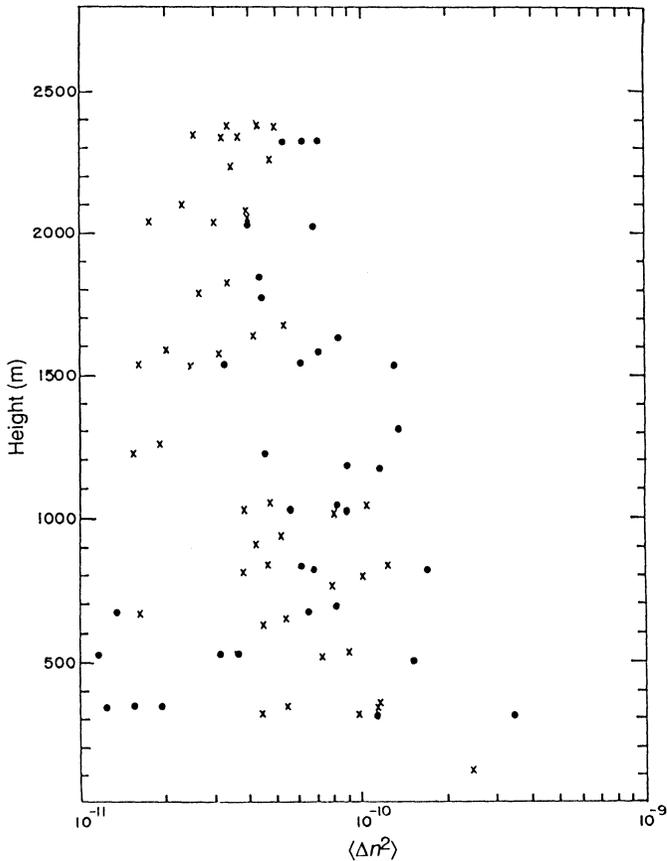
where  $a_n^2$  is a constant which is taken to be 2.8,  $\alpha'$  is the ratio of eddy diffusivities which is taken to be 1,  $R_i$  is a gradient Richardson number which is equal to 0.25,  $N [ \equiv (g \partial \ln \theta / \partial z)^{1/2}$ , where  $\theta$  is the potential temperature,  $g$  the acceleration of gravity and  $z$  the altitude] is the Brunt-Väisälä frequency, and  $M$  is the appropriate gradient of the radio refractive index (Ottersten 1969). The mean vertical gradient  $M$  of the generalised potential radio refractive index can be directly measured at any particular spacing, since it can be expressed in  $N$  units by

$$M \approx -77.6 \times 10^{-6} P_0 T^{-2} (d\theta/dz). \quad (2)$$

The structure parameter  $C_n^2$  can also be written as

$$C_n^2 = \langle [n(r_1) - n(r_2)]^2 \rangle / r_{12}^3 = \langle \Delta n^2 \rangle / r_{12}^3, \quad (3)$$

where  $\langle \Delta n^2 \rangle$  is the variance of radio refractive index fluctuations and  $r_{12}$  the scale size of the inhomogeneities within the inertial subrange of the spatial

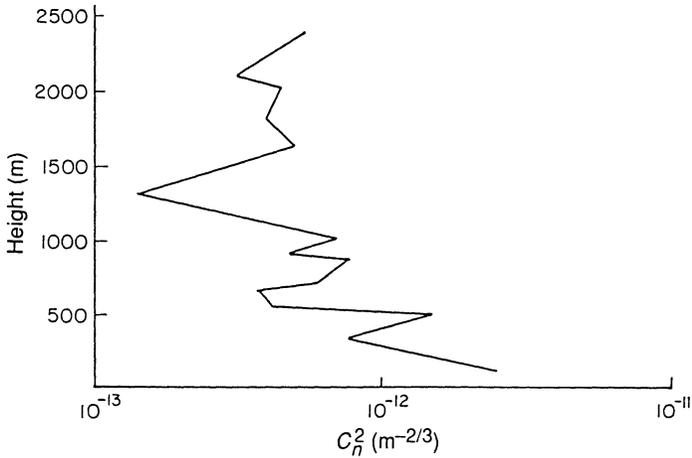


**Fig. 1.** Variance  $\langle \Delta n^2 \rangle$  of radio refractive index fluctuations with height measured with an airborne microwave refractometer on 8 January 1985. Crosses and circles are morning and afternoon data respectively.

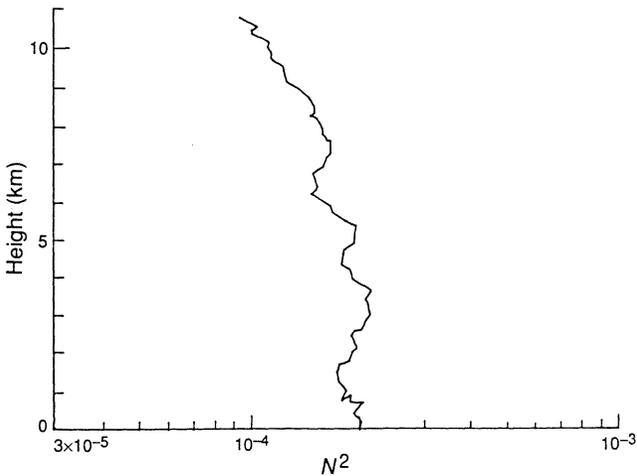
turbulence spectrum. An airborne microwave refractometer makes an in-situ measurement of the variance  $\langle \Delta n^2 \rangle$  in the transverse direction. From the observed temperature profiles of the aircraft's inertial navigation system one can determine the mean profile of  $N^2$  for any particular period. Thus, equation (1) can give the value of  $\epsilon$  from the other known parameters  $C_n^2$ ,  $N^2$ ,  $M$  and  $\langle \Delta n^2 \rangle$  evaluated from airborne microwave refractometer data.

### 3. Results and Discussion

Several authors (Chen 1974; Heck and Panofsky 1975) have demonstrated a strong correlation between the intensity of atmospheric turbulence and the energy dissipation rate  $\epsilon$ . Case studies indicated that the echo power and shear are well correlated. According to Kropfli *et al.* (1968), there exists an excellent agreement between radar reflectivity (from clear air echoes) and the simultaneous in-situ aircraft refractivity turbulence structure constant  $C_n^2$ , provided that the radar half-wavelength lies within the inertial subrange.



**Fig. 2.** Height profile of the mean atmospheric turbulence structure constant  $C_n^2$  measured with an airborne microwave refractometer on 8 January 1985.



**Fig. 3.** Height profile of the mean  $N^2$  for January 1985 derived from the inertial navigation system data of the aircraft.

Further, Lane (1969) has studied radar echoes from clear air in relation to radio refractive index variations from controlled experiments with aircraft supporting microwave refractometer in the radar beam. The measured and calculated values of reflectivity (a quantitative comparison) show striking agreement.

In the present case the author has collected fine structure information on radio refractive index with height using an airborne microwave refractometer, and from the temporal variations in radio refractive index, due to the aircraft motion, the structure parameter  $C_n^2$  has been computed. Fig. 1 gives the

variance of radio refractive index fluctuations  $\langle \Delta n^2 \rangle$  and Fig. 2 depicts the variation of  $C_n^2$  (mean) with height. From the inertial navigation system of the aircraft, the mean height variation of  $N^2$  (where  $N$  is the Brunt-Väisälä frequency) has been evaluated and is shown in Fig. 3. Within the boundary layer its value turns out to be  $2 \times 10^{-4}$  for the period of the present observations. By making use of expression (1), the turbulent energy dissipation rate is  $5 \text{ cm}^2 \text{ s}^{-3}$  from the observed values of  $C_n^2$ ,  $M$ ,  $N^{-2}$  and  $\langle \Delta n^2 \rangle$ . This value seems to be well within the range of values reported during the quiet periods from other experiments.

Since the derived  $N^2$  information has a resolution of about 1 km, we cannot report here the vertical distribution of  $\epsilon$ . Thus, the observed  $\epsilon$  as inferred from the airborne microwave refractometer could be utilised for detection and warning of wind shears in the atmosphere and CAT encounters can be reduced or even eliminated.

#### 4. Conclusions

Airborne microwave refractometer measurements can be used to derive  $C_n^2$ , the radio refractivity turbulence structure constant and, in turn, present a method to determine  $\bar{\epsilon}$ , the mean energy dissipation rate of turbulence. This energy dissipation rate is comparable with the results reported from other experiments. This derivation of  $\epsilon$  is valid under the assumption that the turbulence is locally homogeneous and isotropic. This technique could thus be utilised for detection and warning of wind shears in the atmosphere to reduce or even eliminate the CAT encounters.

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