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Numerical Modelling of Star Formation in Giant Molecular Clouds*

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Abstract

We review some of the difficulties associated with understanding star formation, and discuss some numerical approaches. In particular we describe a method which seems to hold promise for future studies.

1. Introduction

Although stars can be approximated very satisfactorily by spherical models, it is clear from observations of star forming regions that the break-up of molecular gas to form proto-stars is a highly complex process that cannot be approximated simply. As a consequence, the theoretical picture of fragmentation, which is based on assuming fragmentation occurs in uniform, static, isothermal clouds, is too crude to provide a guide to the star formation process. The alternative is to simulate the fragmentation process numerically with the ultimate aim of evolving the gas to proto-stars. The difficulties involved with this procedure are discussed in this paper. We also discuss our proposed solution to this problem, together with a very preliminary calculation.

2. Theory and Simulation

The classical picture of fragmentation and star formation is summarised by Fowler and Hoyle (1963). Fragmentation is supposed to begin in a uniform spherical cloud when the radius is larger than the Jeans length. For an infinite medium with plane-wave density perturbations of the form $e^{iwt}\cos(kx)$ the resulting dispersion relation is

$$\omega^2 = \gamma^2 c^2 k^2 - 4\pi G\rho\,, \tag{1}$$

where c is the isothermal sound speed and γ is the adiabatic index. The same dispersion relation is obtained for a spherically symmetric disturbance of the

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form $r^{-1}\sin(kr)$ (Larson 1985). Clearly there is a critical wavenumber k_c and wavelength $\lambda_c = 2\pi/k_c$ which separates real and complex ω:

$$\lambda_c = \left(\frac{\pi\gamma c^2}{G\rho}\right)^{1/2}. \tag{2}$$

Taking the critical mass M_c to be that interior to the first zero of $r^{-1}\sin(kr)$ we obtain:

$$M_c = 8{\cdot}53\left(\frac{\gamma c^2}{G\rho}\right)^{3/2}\rho. \tag{3}$$

During isothermal collapse both λ_c and M_c decrease, allowing perturbations to grow independently. This may be expected to produce a cascade of less massive clouds, ceasing when the opacity of the gas prevents radiation escaping (Hoyle 1953). Estimates of the cloud mass at this final stage are roughly comparable to a solar mass. Direct numerical simulations, however, show that this picture is naive. In particular, if a spherical cloud is followed as it collapses isothermally, or even with cooling, it produces a dense core with no sign of fragments forming. The reason for this disagreement with the theoretical prediction is that the Jeans model is concerned with perturbations growing in a static background. In a collapsing cloud the background is never static and the perturbations do not have time to grow. To find the wavenumber with the fastest growth rate we take $\mathrm{d}\omega/\mathrm{d}k = 0$, i.e.

$$\frac{\mathrm{d}\omega}{\mathrm{d}k} = \frac{\gamma c^2 k}{\omega} = 0 \Rightarrow k = 0. \tag{4}$$

But $k = 0$ means that $\lambda \to \infty$. This means that the entire cloud (i.e. the largest length-scale in the problem) collapses on a time-scale which is shorter than that required for the fragments to separate out of the background. It appears that to get fragments the cloud of gas needs to be supported against overall collapse (a counter example to this may be the case of thermal instability, as studied by Murray and Lin 1989).

If the initial cloud has some angular momentum it can form a disk, and there is then ample time for the fragments to form within this rotating disc. Hence it is appropriate to look at the stability of discs, even in the non-rotating limit. For an infinite non-rotating, self-gravitating disc with density perturbations of the form $\cos(kx)$ the dispersion relation shows that the maximum growth rate occurs for wavenumber

$$k_{\max} = \frac{\pi G\sigma}{c^2},$$

where σ is the surface density of the disk. Thus there is a preferred length-scale for the fragmentation (e.g. Larson 1985).

Toomre (1964) showed that the stability of a rotating disc is determined primarily by

$$Q = \frac{\kappa c}{\pi G\sigma}, \tag{5}$$

where κ is the epicyclic frequency:

$$\kappa = \left[2\Omega\left(\tilde{\omega}\frac{d\Omega}{d\tilde{\omega}} + 2\Omega\right)\right]^{1/2}.$$

Here $\tilde{\omega}$ and Ω are the radial coordinate in the disc and the angular frequency, respectively. For $Q < Q_c \simeq 0 \cdot 55$ we expect fragmentation. Numerical calculations of Maclaurin discs were performed by Monaghan and Lattanzio (1991; hereafter ML). For a Maclaurin disc, where

$$\sigma = \frac{3M}{2\pi R^2}\left[1 - \left(\frac{r}{R}\right)^2\right]^{1/2}$$

and

$$\Omega = \left(\frac{3G\pi M}{4R^3}\right)^{1/2},$$

we find

$$Q \simeq \frac{2\zeta^{1/2}}{[1-(r/R)^2]^{1/2}},$$

where

$$\zeta = \frac{\mathcal{R}TR}{GM\mu}.$$

We chose $T = 70\,\mathrm{K}$, $M = 10^4 M_{\odot}$ and $R = 12 \cdot 6\,\mathrm{pc}$, so that $\zeta = 0 \cdot 073$ and $Q \simeq 0 \cdot 54\ [1-(r/R)^2]^{1/2}$. We thus expect fragmentation near the centre of an isothermal disc where $Q < Q_c \simeq 0 \cdot 55$. On the other hand, if the disc is allowed to cool then ζ decreases linearly with temperature, and the entire disc should fragment. These predictions are verified, rather dramatically, in Fig. 1. But these initial states are not realistic: they are near-equilibrium discs, with constant Ω. Although we seem to understand their behaviour, their relation to star formation is dubious.

The evolution of clouds which are initially spherical and have constant density and rotation has been studied by Miyama *et al.* (1984) and others. The results here depend on the product of the two parameters:

$$\alpha = \frac{E_{\mathrm{thermal}}}{|E_{\mathrm{grav}}|}, \qquad \beta = \frac{E_{\mathrm{rot}}}{|E_{\mathrm{grav}}|}.$$

For $\alpha\beta \gtrsim 0 \cdot 20$ the cloud becomes an oblate spheroid, in approximate hydrostatic equilibrium (details depend on the interaction with the external medium). For $0 \cdot 13 \lesssim \alpha\beta \lesssim 0 \cdot 20$ the result is a flattened disc, which fragments if $\alpha\beta \lesssim 0 \cdot 12$. Although the details of these calculations have been questioned (e.g. Lattanzio and Henriksen 1988), there is general agreement with the overall picture (see also Stahler 1983; Hachisu and Eriguchi 1985; ML).

A major restriction in these studies was the use of an isothermal equation of state. To study the effects of removing this, we have added cooling due to various molecules, as given by Hollenbach and McKee (1979). This algorithm,

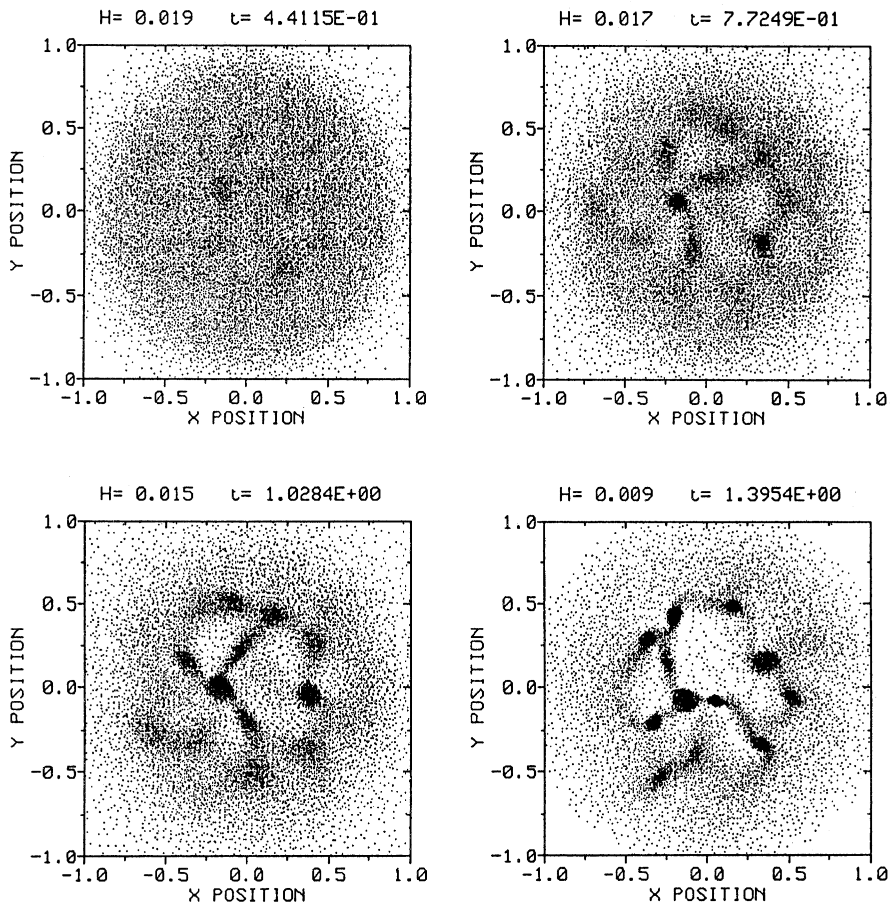

Fig. 1. Particle positions for the evolution of an isothermal Maclaurin disc (*above*) and a cooling Maclaurin disc (*opposite page*).

described in ML, allows for cooling by CO, CH, H_2O, HCl and H_2. Because of the disparate time-scales, the energy equation was solved implicitly (see ML for details).

A case was run with $\alpha\beta = 0{\cdot}141$, both with an isothermal equation of state and with molecular cooling. In the former no fragmentation was seen, as expected (see ML). But in the latter a ring-mode instability developed, as shown in Fig. 2, which we associate with the instability of Goldreich and Lynden-Bell (1965). Although the calculations were terminated at this stage, we would expect the ring to fragment. Thus we see clearly that cooling can substantially alter the evolution of the cloud.

But all of these models began with highly idealised states: spheres of constant density with solid-body rotation. Observations indicate that the interstellar medium is a highly disordered place, and we should not expect reality to be constrained by our simple models.

A possible exception to this may be W49A, which exhibits a rotating ring of HII regions. This object is thus unusual (indeed, it is presently unique) in that it does display a high degree of symmetry. It is interesting to note that

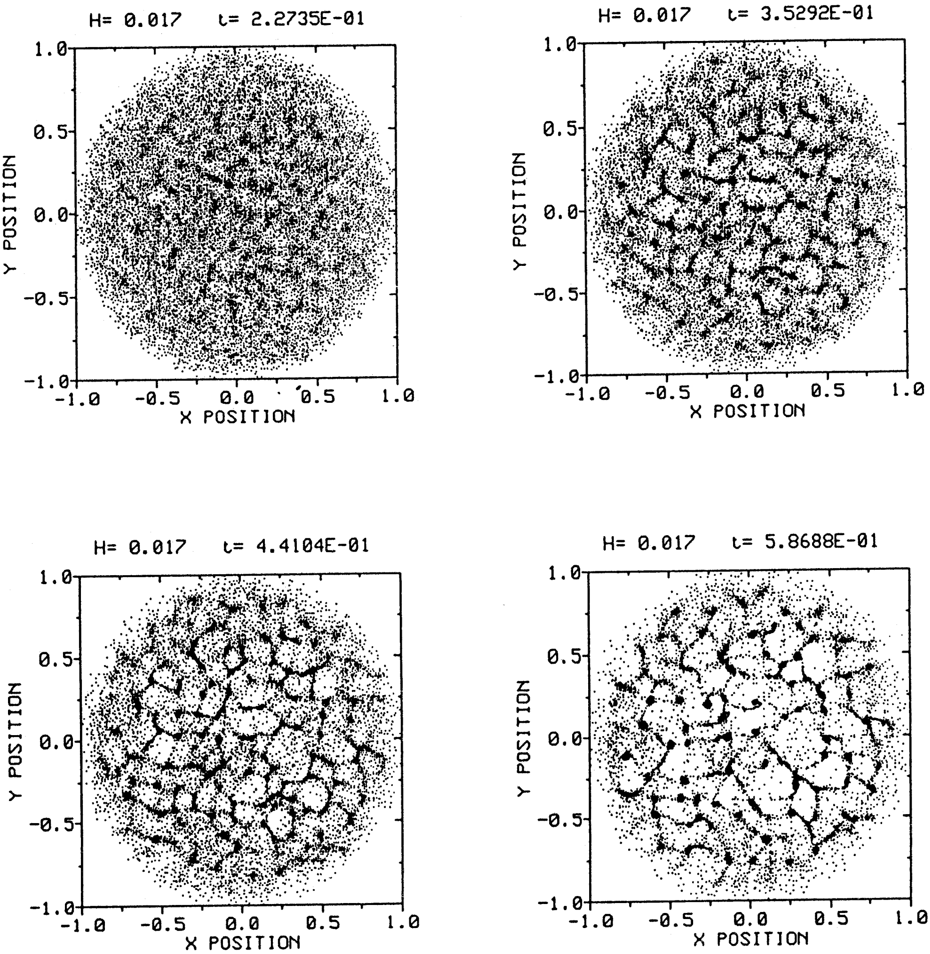

Fig. 1. (*Continued*)

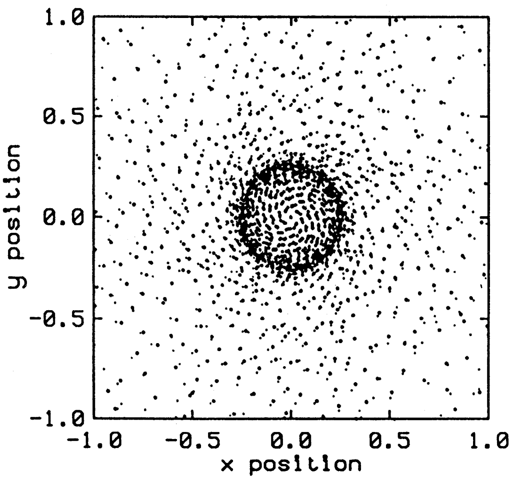

Fig. 2. Projected particle positions for the evolution of a smooth initial cloud (see ML for details).

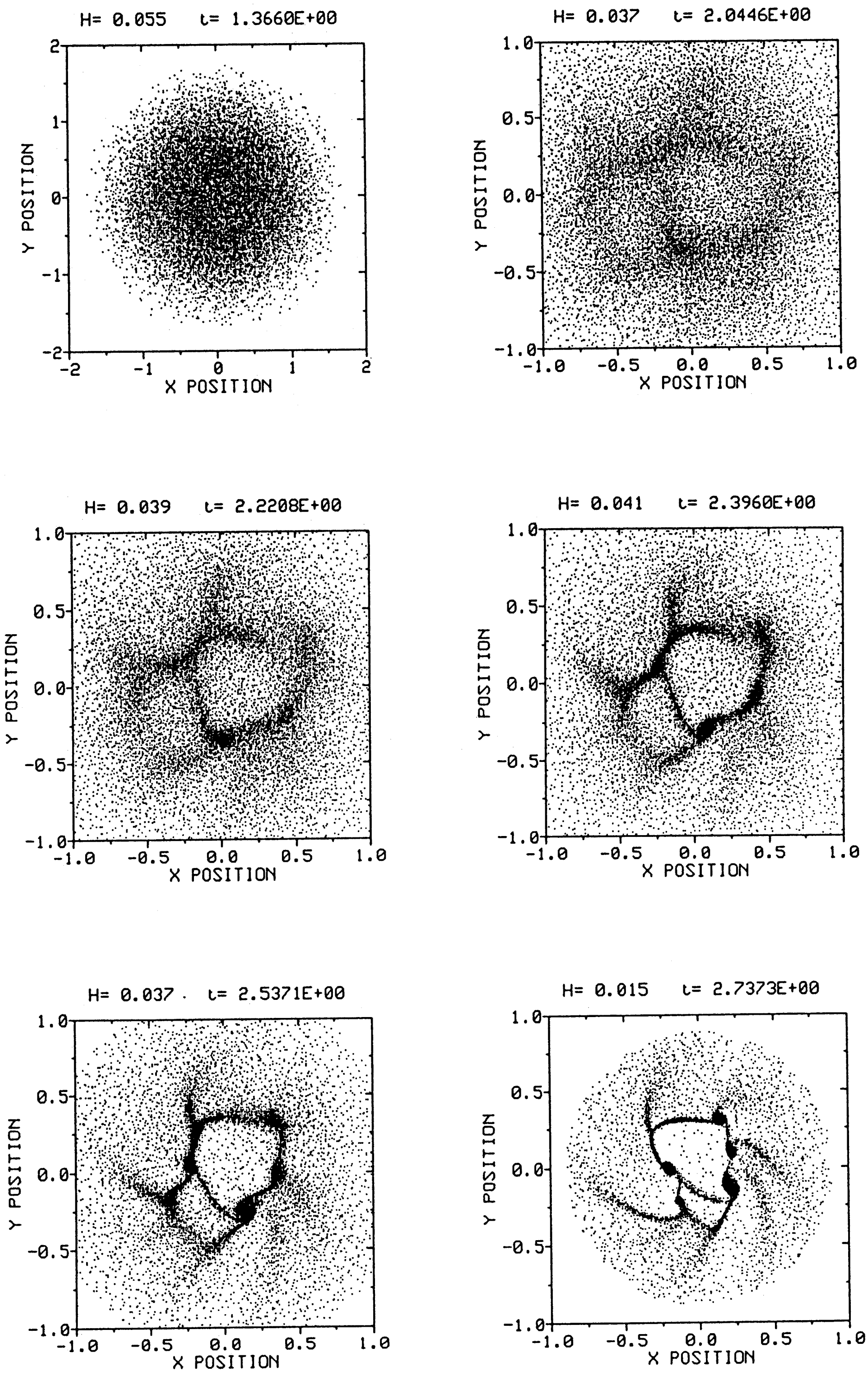

Fig. 3. Projected particle positions for the evolution of a perturbed initial cloud (see ML for details).

relatively symmetric initial conditions can indeed produce such a system. ML showed that an initially spherical cloud, rotating with constant Ω and a field of random density perturbations (of standard deviation 14%) resulted in the configuration shown in Fig. 3. This does indeed look much like W49A, and was considered in further detail in Keto *et al.* (1991). But this object appears to be unique, and cannot be taken as evidence for the universal applicability of such simple initial conditions. Indeed, the current challenge is to determine reliable (and accurate) initial states for studies of star formation.

3. Initial States

A substantial problem to be overcome is the specification of the original velocity field. Most previous studies have assumed solid-body rotation. Hunter (1970*a*, 1970*b*) has considered the restricted case where the velocity field is a linear function of the coordinates. (Solid-body rotation is only one example of this.) He found that the ratio of vorticity to angular momentum is of crucial importance. These quantities may appear in *any* ratio, although this is perhaps not obvious at first sight. The tendency has been, wherever possible, to interpret (projected) velocity fields as rotation, and this can give a false impression of the actual velocity field. The problem of inverting observations to a 3D velocity field is very difficult. To the extent that it is possible, it requires a full mathematical solution (e.g. Keto and Lattanzio 1989; Keto *et al.* 1991), and not an 'eyeball' plausibility argument.

In view of these facts, we are forced to accept that the 'initial' velocity field is a crucial and largely unknown factor in any simulation. For this reason we have adopted the following approach.

4. Simulating a GMC

The only reliable way to study star formation is to look at the evolution of an 'entire' giant molecular cloud (GMC). Thus we may expect agglomeration and fragmentation to occur, and if the calculation is accurate, we should see the resulting star formation through *whichever mechanism(s) nature chooses.* We expect to see discs, and these should follow naturally from the simulation.

We have the numerical tools to handle 3D hydrodynamics, self-gravity, heating and cooling. Magnetic fields are expected to play an important role, and we are currently working on an accurate way of including these in the simulation, but feel sure that much can be learnt even in their absence.

A preliminary study of this kind was performed by Monaghan and Varnas (1988). They took an ensemble of $\simeq 50$ spherical clouds, with different velocity dispersions. These calculations resulted in both fragmentaion and agglomeration, with entities of preferred masses being seen. However, the simulation did not include enough clouds to be reliable, and the cooling used was not appropriate for molecular clouds. In unpublished calculations, Monaghan then chose interesting mass concentrations from these simulations and removed them from the calculation. Preserving their density and velocity fields, their evolution was continued at higher resolution. This resulted in some fragmentation, the formation of some discs, and dispersal of some of the matter into an intercloud medium. This process was repeated, with another clump extracted for further study at even

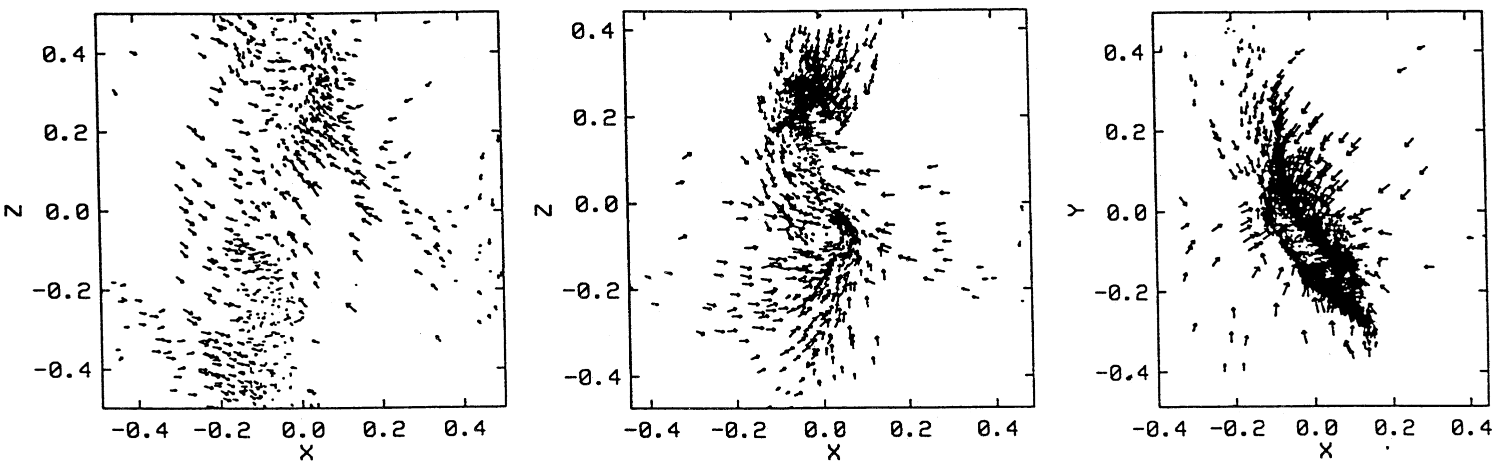

Fig. 4. Close-up of some substructure in a simulated GMC. The plots are two-dimensional projections of a small part of the calculational volume, shown at different times (increasing to the right).

higher resolution. In this way different 'generations' can be followed over many different length- and time-scales.

Our aim is to improve this approach. We begin by giving our computational domain approximately periodic boundary conditions. Perfect periodicity is not required, but we do require that our box of gas knows that it is part of a larger structure—a GMC—and therefore does not immediately collapse to the centre. By imposing known amounts of vorticity and angular momentum we can study their effects on the resulting structures, and hence the subsequent star formation. Again, interesting clumps can be extracted for studies at higher resolution.

A very preliminary simulation has been run, with a simple Kolmogorov velocity spectrum. The initial temperature was 70 K, and we included molecular cooling but no heating. Fig. 4 shows 2D projections of the region near the centre of our box, where one interesting clump forms. We also see the development of a dense knot, which collides with the clump. This object is a prime candidate for further study at higher resolution.

5. Conclusion

Appropriate initial conditions are the most crucial unknown in numerical studies of star formation. Yet we have all the tools necessary for a *direct* simulation of star-forming environments. By varying the velocity field in such a region we hope to see the development of the overall structure of a GMC. By extracting distinct self-gravitating regions we can follow their evolution at higher resolution and on time-scales smaller than that of the entire GMC. Hence we can deduce the resulting initial mass function. Our expectation is that this will be largely independent of the initial velocity field, provided that the latter is consistent with the orbit of the GMC in the galactic potential. Time will tell.

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