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Nonlinear Shifts of Refractive Indices from Obliquely Propagating Waves in a Magnetised Plasma in the Presence of Coriolis Rotation

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Abstract

This paper provides an expression for the nonlinear shifts of refractive indices from obliquely propagating waves in a cold, homogeneous, magnetised rotating plasma in the presence of a Coriolis force. The effects of gyrofrequency, rotational frequency, and the amplitude of the waves on the nonlinear shifts of refractive indices are discussed. It is found that these parameters play a dominant role in the nonlinear shifts of refractive indices and the results are more general than those of earlier workers.

1. Introduction

In plasmas nonlinear effects give rise to various phenomena which are important in the investigation of both laboratory and space plasmas (Tsytovich 1970; Kaplan and Tsytovich 1973; Akhiezer *et al.* 1975; Lonngren 1983). Nonlinear mode conversion is found to be responsible for many experimental observations regarding anomalous absorption and scattering of electromagnetic waves and evaluation of various plasma instabilities. Nonlinear interactions of waves in plasmas also give rise to several self-action effects, i.e. shifts of wave parameters, wave precession, filamentation, modulational instabilities (Sodha *et al.* 1976; Chakraborty *et al.* 1983; Shukla *et al.* 1986). In fact, shifts of refractive indices (i.e. shifts of frequency and wave number) are very important as these lead to modulational instabilities which are relevant in understanding various physical processes in the solar and terrestrial atmospheres. For the past few decades many authors have investigated wave parameter shifts of electromagnetic waves due to nonlinear interactions in plasmas. The earliest works on effects causing wave parameter shifts in plasmas were by Sturrock (1957), and others such as Jackson (1960) and Dawson (1969). They neglected the relativistic mass effect for electron motion and evaluated the frequency shift for both travelling and standing waves in plasmas. Sluijter and Montgomery (1965) improved these results for a plane polarised transverse wave in a plasma by including the relativistic mass correction effect for electron motion in a cold, collisionless unmagnetised plasma. They found that the relativistic part of the frequency shift is comparable with the nonrelativistic part and that it does not vanish at the infinite wavelength limit.

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Tidman and Stainer (1965) found the intensity-dependent shifts in frequency and wave number as the nonlinear effect of a wave in a finite temperature plasma, considering both electron motion and electron plasma oscillation. The amplitude-dependent frequency shift for extraordinary standing waves having a propagation vector perpendicular to the direction of an ambient magnetic field has been evaluated by Boyd (1967). Later, several authors (Das 1968; Goldstein and Salu 1973; Schindler and Janick 1973; Chandra 1980; Chakraborty 1991) derived an expression for the frequency shift in different conditions in plasmas and obtained some interesting results. The nonlinear frequency shift of whistlers in the ionosphere has been investigated by Karpman *et al.* (1974*a*, 1974*b*), Vomvoridis and Denavit (1980), Das and Singh (1982), Das (1983) and Paul *et al.* (1987, 1989). Earlier work on the shift of wave parameters has been mainly restricted to a non-rotating plasma medium, but Chandrasekhar (1953*a*, 1953*b*, 1953*c*) showed that the nature of wave propagation in rotating plasmas including the Coriolis force is important in cosmic phenomena. The theory of wave propagation in rotating plasmas has been developed by several authors (Lehnert 1962; Tandon and Bajaj 1966; Bhatia 1967; Uberoi and Das 1970; Bandyopadhyaya 1972; Engels and Verheest 1975; Das *et al.* 1984; Horton and Lin 1984). Recently Paul *et al.* (1992) studied wave propagation in a magnetised rotating plasma and showed the characteristic variation of the refractive indices with wave amplitude and rotational frequency. They also obtained the cut-off and resonance frequencies for both the left circularly polarised (LCP) and right circularly polarised (RCP) waves which are dependent on the rotation of the plasma. In a sequel to that work we are motivated here to investigate the shifts of refractive indices of waves due to nonlinear interactions in a rotating medium in the presence of a Coriolis force. The wave is assumed here to be oblique with respect to the uniform ambient magnetic field. We also derive the nonlinear dispersion relation for the first harmonic part, correct up to third-order field variables, and then discuss the effect of the field amplitude, propagating angle, rotational frequency and gyrofrequency on the shift of the refractive indices.

2. Formulations

We consider a cold, homogeneous and collisionless plasma. The electromagnetic wave propagates at an angle θ to the ambient uniform static magnetic field $\boldsymbol{H}_0$ $[=(0,0,H_0)]$, so the propagation vector is $\boldsymbol{k}$ $[=(k\sin\theta, 0, k\cos\theta)]$. The plasma rotates around the direction of the static magnetic field with a constant angular velocity $\boldsymbol{\Omega}$ $[= 0, 0, \Omega)]$. For simplicity the centrifugal force is neglected and only the effects of the Coriolis force are taken into consideration. In astrophysical plasmas the magnitude of the rotational frequency is small and so neglecting the centrifugal force is justified. In the presence of a Coriolis force, with respect to a rotating frame of reference, the ions and electrons have different equivalent magnetic fields, i.e. $\boldsymbol{H} + 2\boldsymbol{\Omega} m_{\rm i}\, c/e$ and $\boldsymbol{H} - 2\boldsymbol{\Omega} m_{\rm e}\, c/e$ respectively. This generates an additional charge separation effect and is responsible for the stability of the wave (Lehnert 1962; Uberoi and Das 1970).

The electrons are assumed to be mobile, but ions are at rest. The presence of ions is only considered to maintain charge neutrality of the plasma in the equilibrium state.

The basic equations for the plasma particles are

$$\frac{\partial \boldsymbol{v}_{\mathrm{e}}}{\partial t} + (\boldsymbol{v}_{\mathrm{e}} \cdot \nabla)\boldsymbol{v}_{\mathrm{e}} = \frac{q_{\mathrm{e}}}{m_{\mathrm{e}}}\left(\boldsymbol{E} + \frac{\boldsymbol{v}_{\mathrm{e}} \times \boldsymbol{H}}{c}\right) + 2(\boldsymbol{v}_{\mathrm{e}} \times \boldsymbol{\Omega}), \tag{1}$$

$$\frac{\partial n_{\mathrm{e}}}{\partial t} + \nabla \cdot (n_{\mathrm{e}} \boldsymbol{v}_{\mathrm{e}}) = 0, \tag{2}$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{H}}{\partial t}, \tag{3}$$

$$\nabla \times \boldsymbol{H} = \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} + \frac{4\pi q_{\mathrm{e}}}{c}(n_{\mathrm{e}} \boldsymbol{v}_{\mathrm{e}} - n_{\mathrm{i}}\boldsymbol{v}_{\mathrm{i}}), \tag{4}$$

$$\nabla \cdot \boldsymbol{E} = 4\pi q_{\mathrm{e}}(n_{\mathrm{e}} - n_{\mathrm{i}}), \tag{5}$$

$$\nabla \cdot \boldsymbol{H} = 0, \tag{6}$$

where $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric and magnetic fields, m_{e}, n_{e} and v_{e} are the mass, number density and velocity of electrons, and q_{e} $(=-e)$ is the electron charge.

The series expansions of the field variables are

$$\boldsymbol{v}_{\mathrm{e}} = \epsilon^1 \boldsymbol{v}_{\mathrm{e}}^{(1)} + \epsilon^2 \boldsymbol{v}_{\mathrm{e}}^{(2)} + \epsilon^3 \boldsymbol{v}_{\mathrm{e}}^{(3)} + \dots, \tag{7a}$$

$$n_{\mathrm{e}} = n_{\mathrm{e}}^{(0)} + \epsilon^1 n_{\mathrm{e}}^{(1)} + \epsilon^2 n_{\mathrm{e}}^{(2)} + \epsilon^3 n_{\mathrm{e}}^{(3)} + \dots, \tag{7b}$$

$$\boldsymbol{E} = 0 + \epsilon^1 \boldsymbol{E}^{(1)} + \epsilon^2 \boldsymbol{E}^{(2)} + \epsilon^3 \boldsymbol{E}^{(3)} + \dots, \tag{7c}$$

$$\boldsymbol{H} = \boldsymbol{H}_0 + \epsilon^1 \boldsymbol{H}^{(1)} + \epsilon^2 \boldsymbol{H}^{(2)} + \epsilon^3 \boldsymbol{H}^{(3)} + \dots, \tag{7d}$$

where the terms on the right-hand side with a zero represent the equilibrium values and $1, 2, 3, \dots$ represent first-order, second-order, third-order, etc., perturbed values of their respective quantities. Further, ϵ is an arbitrary expansion parameter.

Let the electromagnetic wave be quasi-circular, and so represented as

$$E_{\pm}^{(1)} = a\exp(\pm \mathrm{i}\theta_{\pm}) + \bar{a}\exp(\mp \mathrm{i}\theta_{\mp}), \tag{8}$$

where $\theta_{\pm} = (\boldsymbol{k}_{\pm} \cdot \boldsymbol{r} - \omega t)$, a is the constant complex amplitude of the wave, $\bar{a}$ the complex conjugate of a, $k_{\pm}$ are the wave numbers, ω is the wave frequency, while the plus and minus signs represent the LCP and RCP parts of the wave respectively.

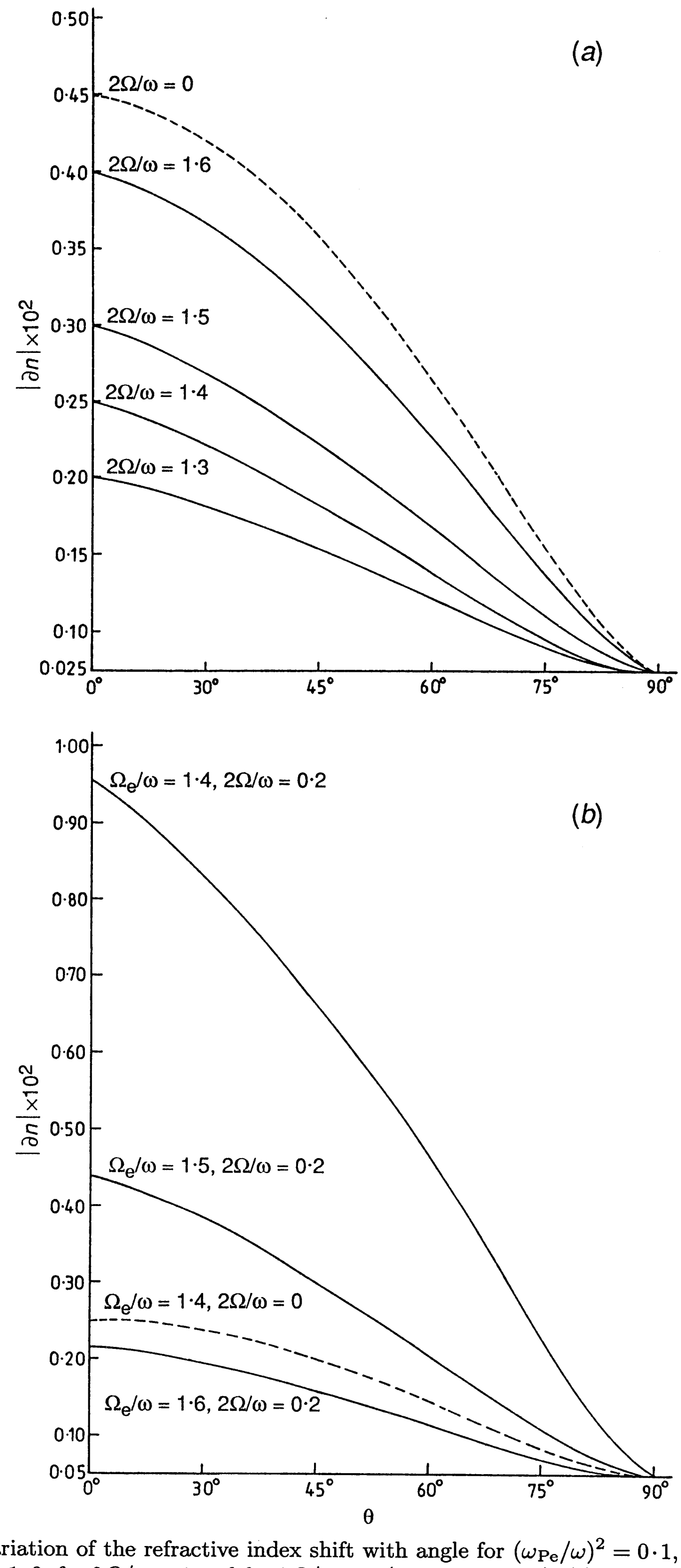

Fig. 1. Variation of the refractive index shift with angle for $(\omega_{\mathrm{Pe}}/\omega)^2 = 0\cdot1$, $\alpha_e^2 = 0\cdot1$ when (a) $\Omega_e/\omega = 1\cdot2$, for $2\Omega/\omega > 1$ and for $2\Omega/\omega = 0$ (i.e. no rotation); (b) $\Omega_e/\omega > 1$ for $2\Omega/\omega = 0$ and $0\cdot2$; (c) $\Omega_e/\omega = 0\cdot2$ for $2\Omega/\omega = 0$ and > 1; and (d) $\Omega_e/\omega = 0\cdot2$ for $2\Omega/\omega < 1$ and 0.

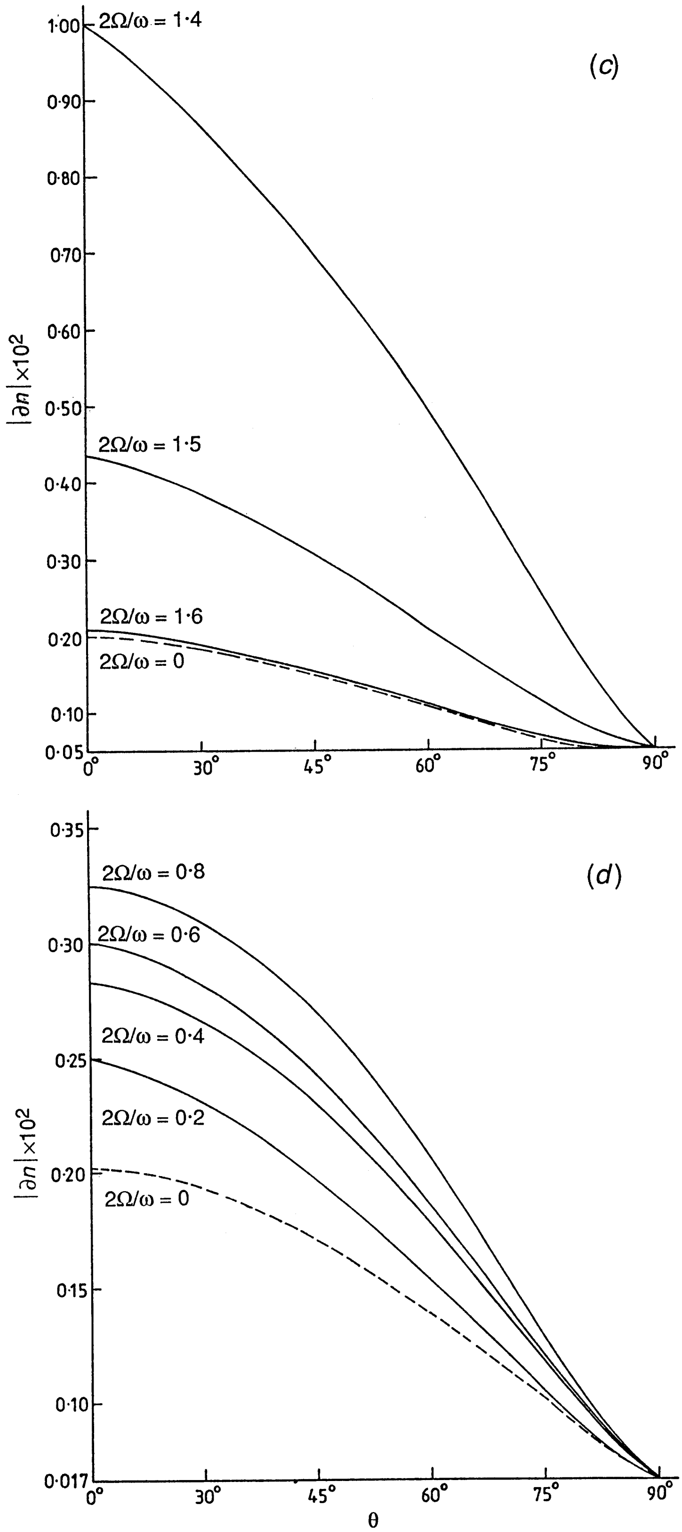

Fig. 1 *(continued)*

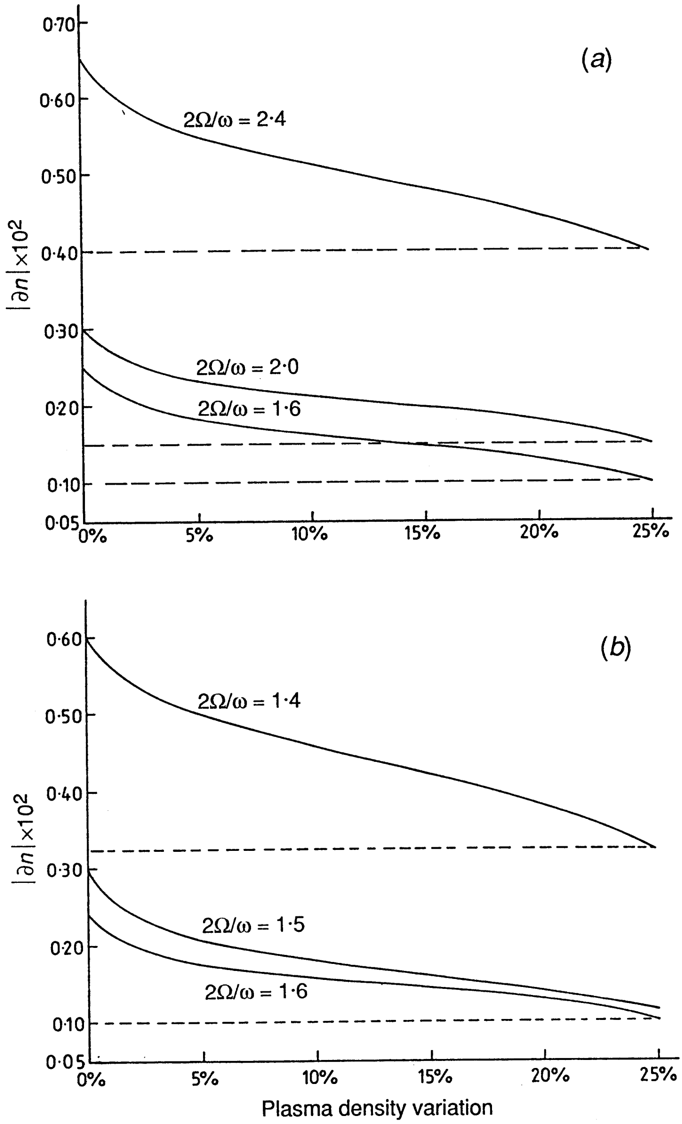

Fig. 2. Variation of the refractive index shift with plasma density for $(\omega_{\rm Pe}/\omega)^2 = 0\cdot 1$, $\alpha_{\rm e}^2 = 0\cdot 1$, $\theta = 30^\circ$ when: (*a*) $\Omega_{\rm e}/\omega = 1\cdot 2$ and $2\Omega/\omega > 1$; (*b*) $\Omega_{\rm e}/\omega = 0\cdot 2$ and $2\Omega/\omega > 1$; (*c*) $\Omega/\omega < 1$ and $2\Omega/\omega = 0$; and (*d*) $\Omega_{\rm e}/\omega = 0\cdot 2$ and $2\Omega/\omega < 1$.

Now, we take the first harmonic part to be correct up to the third-order electric field and obtain the following nonlinear dispersion relations for the LCP and RCP waves:

$$n_+^2 = 1 - \frac{\omega_{\rm Pe}^2}{\omega(\omega+\pi_{\rm e})} - \frac{1}{\omega_{\rm P}^2}\left[\frac{\alpha_{\rm e}^2\cos^2\theta\,\omega_{\rm Pe}^2\,\pi_{\rm e}(ck_-)}{\omega^2-\pi_{\rm e}^2}\left(\frac{ck_+}{\omega-\pi_{\rm e}}+\frac{ck_-}{\omega+\pi_{\rm e}}\right)\right]$$

$$+\frac{\alpha_{\rm e}^2\cos^2\theta\,\omega_{\rm Pe}^2}{2\omega_{\rm P}^2}\,\frac{ck_+ + ck_-}{\omega-\pi_{\rm e}}\left(\frac{ck_+}{\omega-\pi_{\rm e}}+\frac{ck_-}{\omega+\pi_{\rm e}}\right), \qquad (9)$$

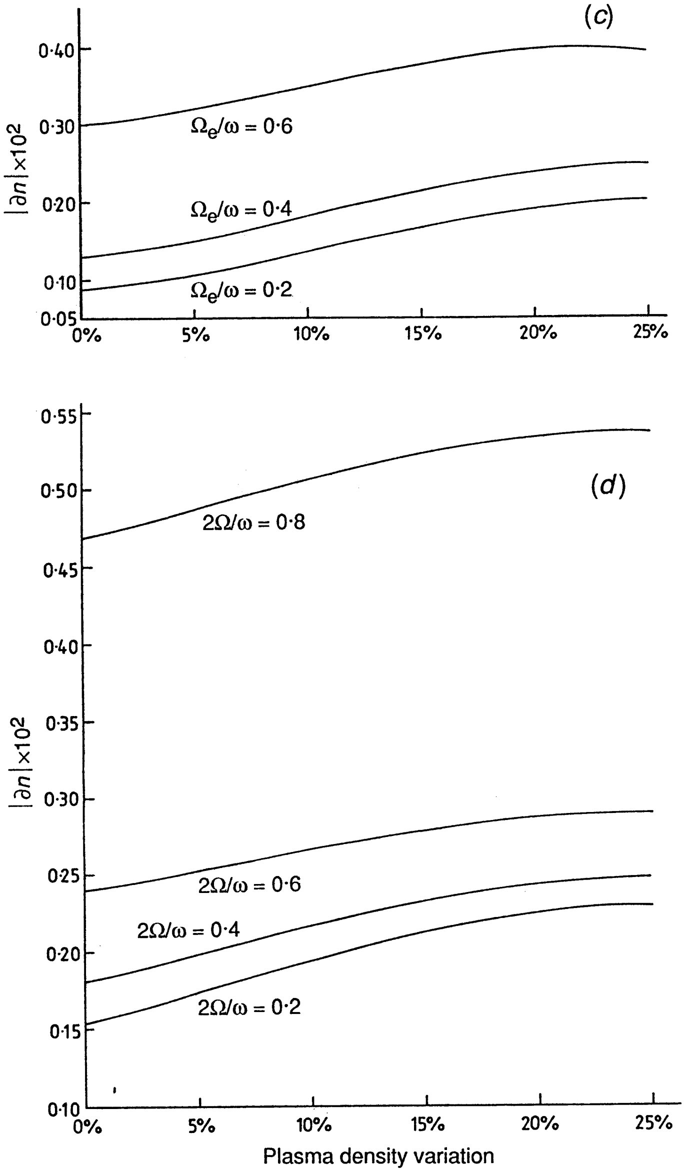

Fig. 2 *(continued)*

$$n_-^2 = 1 - \frac{\omega_{\rm Pe}^2}{\omega(\omega - \pi_{\rm e})} + \frac{1}{\omega_{\rm P}^2}\left[\frac{\alpha_{\rm e}^2 \cos^2\theta\, \omega_{\rm Pe}^2\, \pi_{\rm e}(ck_+)}{\omega^2 - \pi_{\rm e}^2}\left(\frac{ck_+}{\omega - \pi_{\rm e}} + \frac{ck_-}{\omega + \pi_{\rm e}}\right)\right]$$

$$+ \frac{\alpha_{\rm e}^2 \cos^2\theta\, \omega_{\rm Pe}^2}{2\omega_{\rm P}^2}\, \frac{ck_+ + ck_-}{\omega + \pi_{\rm e}}\left(\frac{ck_+}{\omega - \pi_{\rm e}} + \frac{ck_-}{\omega + \pi_{\rm e}}\right), \tag{10}$$

where $\omega_{\rm P}^2 = \omega_{\rm Pe}^2 - 4\omega^2, \alpha_{\rm e} = ea/m_{\rm e}\,\omega c$ is the dimensionless amplitude of the wave, $\pi_{\rm e} = \Omega_{\rm e} - 2\Omega$, $\Omega_{\rm e} = |eH_0/m_{\rm e}\,c|$ is the electron cyclotron frequency, $\omega_{\rm Pe} = [4\pi n_{\rm e}^{(0)} e^2/m_{\rm e}]^{\frac{1}{2}}$ is the electron plasma frequency, while $n_\pm = k_\pm c/\omega$ are the refractive indices for the LCP and RCP waves respectively.

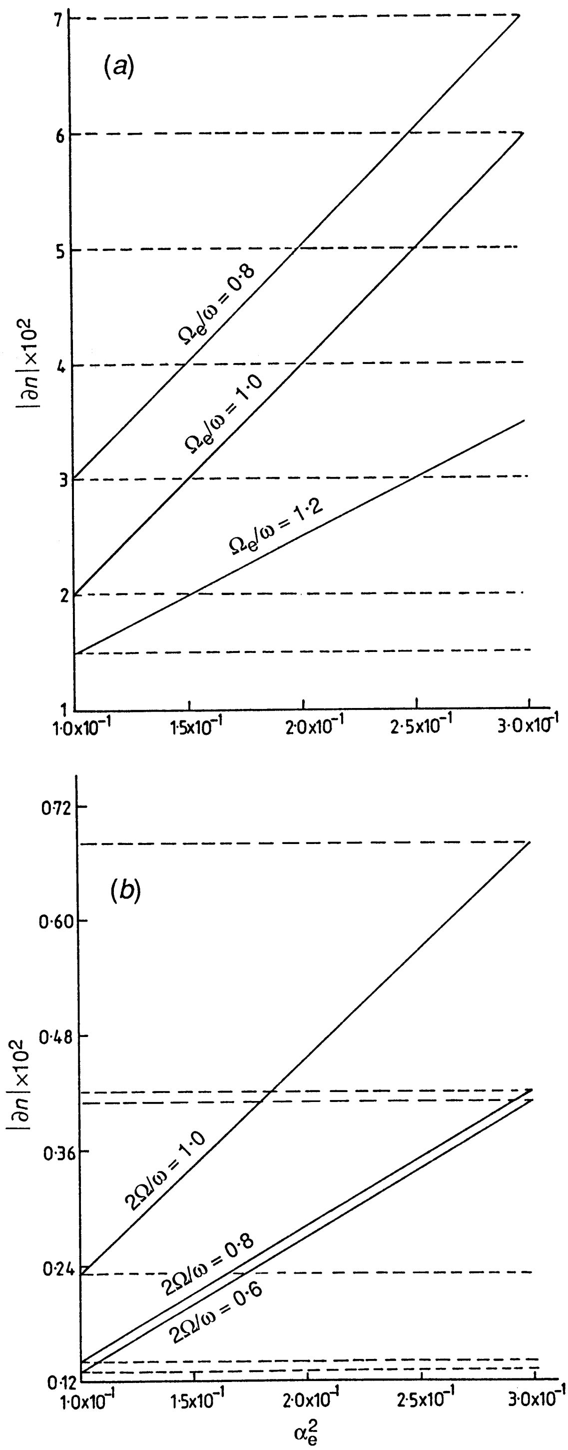

Fig. 3. Variation of the refractive index shift with α_e^2 for $(\omega_{Pe}/\omega)^2 = 0{\cdot}1$, $\theta = 30°$ when: (*a*) $2\Omega/\omega = 0{\cdot}2$ and $\Omega_e/\omega = 0{\cdot}8$, $1{\cdot}0$, $1{\cdot}2$ and (*b*) $\Omega_e/\omega = 0{\cdot}2$ and $2\Omega/\omega = 0{\cdot}6$, $0{\cdot}8$, $1{\cdot}0$.

To obtain the nonlinear refractive index shift for the LCP and RCP waves, we substitute $n_\pm = n_{0\pm} + \partial n_\pm$ in (9) and (10) and find that

$$\partial n_+ = \frac{\alpha_e^2 \cos^2\theta\, \omega_{Pe}^2}{4n_{0+}\,\omega_P^2}\left(\frac{ck_+}{\omega - \pi_e} + \frac{ck_-}{\omega + \pi_e}\right)^2, \tag{11}$$

$$\partial n_- = \frac{\alpha_e^2 \cos^2\theta\, \omega_{Pe}^2}{4n_{0-}\,\omega_P^2}\left(\frac{ck_+}{\omega - \pi_e} + \frac{ck_-}{\omega + \pi_e}\right)^2. \tag{12}$$

Thus, the average nonlinearly induced shift of the refractive indices given by $\partial n = \frac{1}{2}(\partial n_+ + \partial n_-)$ is obtained as

$$\partial n = \frac{\alpha_e^2 \cos^2\theta\, \omega_{Pe}^2}{4\omega_P^2}\left(\frac{\omega^2(\omega^2 + \omega\pi_e - \omega_{Pe}^2)}{(\omega^2 - \pi_e^2)^2} + (\omega^3 + \omega^2\pi_e - \omega\omega_{Pe}^2)^{\frac{1}{2}}\right.$$
$$\left.\times\,\frac{(\omega^3 - \omega^2\pi_e - \omega\omega_{Pe}^2)^{\frac{1}{2}}}{(\omega^2 - \pi_e^2)^{\frac{3}{2}}}\right)\left(\frac{\omega^2(\omega^2 - \pi_e^2 - \omega_{Pe}^2)}{(\omega^2 - \omega_{Pe}^2)^2 - \pi_e^2\omega^2}\right). \tag{13}$$

3. Results and Discussion

From the expression (13) we see that the nonlinear variation of the refractive index shift due to electron motion in the presence of an electromagnetic wave depends on the wave amplitude, the static magnetic field intensity, the rotational frequency, and the angle of wave propagation with respect to the direction of the ambient magnetic field. To get an insight into the role the parameters involved play in the shift of refractive index, we critically consider graphical representations, taking physically attainable values of the rotational frequency, gyrofrequency, plasma frequency, wave amplitude and angle of propagation. Fig. 1 shows the dependence of the refractive index shift on the angle of propagation. It is observed that with an increase in θ, the shift of refractive index ∂n decreases and that it goes to zero for perpendicular propagation ($\theta = 90°$) for all the following four cases: $\Omega_e/\omega > 1$, $2\Omega/\omega > 1$; $\Omega_e/\omega > 1$, $2\Omega/\omega < 1$; $\Omega_e/\omega < 1$, $2\Omega/\omega > 1$; and $\Omega_e/\omega < 1$, $2\Omega/\omega < 1$. But the value of $|\partial n|$ is larger when $\Omega_e/\omega < 1$ and $2\Omega/\omega > 1$. In this case $|\partial n|$ decreases rapidly with an increase in θ. The dashed curves in Fig. 1 represent $|\partial n|$ in the absence of rotation. It is seen that rotational velocity considerably influences the shift in refractive index. The density variations of a plasma also influence $|\partial n|$. In Figs 2*a* and 2*b* we see that $|\partial n|$ decreases with an increase in plasma density variation but the variation is slow. In Figs 2*c* and 2*d*, $|\partial n|$ is found to increase with an increase in plasma density. From (13) it is seen that as the intensity of the wave increases, the value of $|\partial n|$ increases.

Fig. 3 shows the variation of $|\partial n|$ with wave intensity for different values of the rotational frequency and gyrofrequency, taking the value $(\omega_{Pe}/\omega)^2 = 0{\cdot}1$. From Fig. 3*a* it is seen that, for $\Omega_e/\omega < 1$, $|\partial n|$ increases rapidly with the wave intensity when $2\Omega/\omega$ is small, but from Fig. 3*b* it is seen that when $\Omega_e/\omega = 0{\cdot}2$, the shift increases rapidly with intensity for $2\Omega/\omega$ large.

4. Some Concluding Remarks

We have discussed the shift of the refractive index in a magnetised rotating plasma in the presence of a Coriolis force. The corresponding shifts of the wave number as well as wave frequency are important nonlinear effects in plasmas.

The values obtained here could be verified but unfortunately we have not found any published experimental results for a magnetised rotating plasma. In astrophysical media, such as the solar atmosphere or magnetic stars, the effect of both rotation and the magnetic field on the shift of the refractive index or the frequency or wave number should be studied with the help of numerical estimations, which we propose to do in the near future. In astrophysical bodies the static magnetic field and rotational velocity are sometimes taken as depending on position. So, the gradient of the magnetic field and the rotation must be taken into account in the derivation of $|\partial n|$.

We have also assumed a uniform plasma medium, but in most astrophysical bodies the density has a gradient and these variations must be taken into account. Then the mathematical procedure used here cannot be followed and the WKB method for solution of the wave equations is necessary. Moreover, the plasma in this paper is assumed to be cold, but in real situations the temperature effect has an influence on the propagation of waves. So, to get a better idea of the shift of refractive index for a wave in an astrophysical medium, one should consider a finite temperature plasma, as well as the effects of rotation and magnetic field.

It should be mentioned here that our present work has other limitations, as we have considered only the effect of the Coriolis force part of the rotation on the propagating wave. In rotating frames of reference (non-inertial frames) not only the Coriolis force and centrifugal force terms appear in the equation of motion, but also all the other consequences of the tensorial representation in Riemannian non-Euclidean 4-space of the Maxwell equations of electrodynamics, the constitutive relations and the equation of motion. This means a confrontation with a system of equations which cannot be solved with the help of the present tools of mathematics. So, in addition to the balancing of the centrifugal force by a radial electrostatic field, other problems exist. One interesting problem is that the constitutive relations, even for very small rotational velocities, cannot be derived uniquely because several alternative covariant formulations exist for these relations. Volkov and Kisilev (1970) and Chakraborty (1987) have dealt with the non-uniqueness of the constitutive relations in a rotating medium. Chakraborty (1990) has recently discussed several interesting and promising issues of electrodynamics in rotating frames and other related physics. It is also to be noted that for a real rotating plasma one must include confinement in the equilibrium state. This confinement problem need not be studied when, as here, the interior of very large plasma structures like those of the ionosphere, the magnetosphere of pulsars or the sun are considered. Anyway, our present work, with some limitations, gives an insight into developing a theoretical procedure for understanding nonlinear wave processes in a rotating plasma and its characteristics under certain physical situations.

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