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Generation via Limiting Procedures of New Solutions in a Nonsymmetric Gravitational Theory

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Abstract

New electromagnetic Bianchi type I solutions with nonzero cosmological constant, which satisfy the field equations of a nonsymmetric gravitational theory, are obtained from static solutions. This is achieved by taking a limit of the nonsymmetric static solutions.

1. Introduction

It has been observed by Geroch (1969), in the context of General Relativity (GR), that the procedure of taking limits of a family of solutions is coordinate dependent. As an example, Geroch considered the Schwarzschild solution and showed that both flat and nonflat spacetimes can be obtained as limiting cases. The idea is to redefine an existing parameter and introduce a coordinate transformation which depends on this parameter. The original metric is then transformed and the parameter is allowed to vanish. Such procedures have also been applied by Plebañski and Demianski (1976) to obtain the Taub–NUT, Robinson–Bertotti and other GR solutions from their type D solution.

The above limiting procedures will also work in other theories of gravity. We can take advantage of this to obtain new solutions from old ones without having to solve the field equations. This is very useful in the context of nonsymmetric theories of gravity where the field equations are very complicated even in the case of simple fundamental tensors.

The theory of gravity considered in this paper is a member of the *Algebraically Extended* class of gravitational theories. The simplest of these theories, in which tensors on the spacetime manifold ${}^4\mathcal{M}$ take their values in the algebra $\mathcal{A}$ of real numbers, is GR. The other theories in this class arise when one allows geometrical objects defined on the (real) spacetime, ${}^4\mathcal{M}$, to take their values in an arbitrary algebra $\mathcal{A}$. It has been shown by Mann (1984) that, apart from real numbers, only four algebras are acceptable. These are the complex, hypercomplex, quaternion and hyperquaternion numbers. The complex theory was first proposed as a possible theory of gravity by Moffat (1979). It has the interesting property that its solutions are, unlike those of GR, singularity free. This phenomenon occurs because the fundamental metric tensor has unphysical signature in a region surrounding the singularity. Thus, for example, the perfect fluid cosmological solution derived by Kunstatter *et al.* (1980) displays finite spacetime curvature as well as finite density of matter at the beginning of expansion.

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However, an analysis of the particle spectra by Kelly and Mann (1986) revealed that the complex, quaternion and hyperquaternion versions contain ghost particles and must therefore be rejected on physical grounds. This leaves the hypercomplex theory as the only viable algebraic extension of GR. Consequently we confine ourselves to finding solutions of the hypercomplex nonsymmetric theory which is known in the literature as NGT.

In the following section we shall write down the NGT field equations in the presence of a sourceless electromagnetic field. In Section 3 we shall illustrate the limiting procedure by obtaining a vacuum-plane symmetric NGT Bianchi type I solution from the NGT Schwarzschild solution. In Section 4 we solve the NGT field equations to get electromagnetic solutions with nonzero cosmological constant. These solutions are used in the final section to obtain an electromagnetic cosmological solution with nonzero cosmological constant.

2. Field Equations

The gravitational field is determined, in the hypercomplex theory, by a nonsymmetric pseudo-Hermitian tensor $g_{\mu\nu}$ which can be decomposed into symmetric and antisymmetric parts:

$$g_{\mu\nu} = g_{(\mu\nu)} + g_{[\mu\nu]} \,. \tag{1}$$

There is more than one way of constructing the contravariant tensor $g^{\mu\nu}$. We shall define it by the relation

$$g^{\mu\nu}\, g_{\lambda\nu} = g^{\nu\mu}\, g_{\nu\lambda} = \delta_\lambda{}^\mu \,. \tag{2}$$

In the case of the complex theory $g_{[\mu\nu]}$ is purely imaginary, whereas in the hypercomplex theory considered in this paper it is real.

A nonsymmetric pseudo-Hermitian affine connection $\Gamma_{\mu\nu}{}^\lambda$ is obtained by solving

$$\partial_\omega\, g_{\mu\nu} - \Gamma_{\mu\omega}{}^\alpha\, g_{\alpha\nu} - \Gamma_{\omega\nu}{}^\alpha\, g_{\mu\alpha} = 0 \,. \tag{3}$$

The order of indices is, as shown by Hlavatý (1957), of the utmost importance. This set of linear equations can be solved by elimination or an inversion formula derived by Tonnelat (1954) may be employed. The symmetric part of the connection can be written in terms of the antisymmetric part as

$$\Gamma_{(\mu\nu)}{}^\beta = \{{}^{\,\beta}_{\mu\nu}\} + \gamma^{\beta\rho}\,(\Gamma_{[\mu\rho]}{}^\alpha\, g_{[\alpha\nu]} + \Gamma_{[\nu\rho]}{}^\alpha\, g_{[\alpha\mu]}) \,, \tag{4}$$

where

$$\{{}^{\,\beta}_{\mu\nu}\} = \tfrac{1}{2}\gamma^{\beta\rho}\,(\partial_\nu g_{(\mu\rho)} + \partial_\mu g_{(\rho\nu)} - \partial_\rho g_{(\mu\nu)}) \,, \quad \gamma^{\beta\rho}\, g_{(\alpha\rho)} = \delta_\alpha{}^\beta \,. \tag{5}$$

The antisymmetric part of the connection is given by a very complicated expression (see Tonnelat 1954) which, even for simple metrics, is best evaluated using an algebraic manipulator.

Once the pseudo-Hermitian connection coefficients have been calculated we can construct, following Moffat (1984), a nonsymmetric Ricci tensor

$$R_{\mu\nu}(\Gamma) = \partial_\alpha\, \Gamma_{\mu\nu}{}^\alpha - \partial_\nu\, \Gamma_{\mu\alpha}{}^\alpha - \Gamma_{\alpha\nu}{}^\beta\, \Gamma_{\mu\beta}{}^\alpha + \Gamma_{\alpha\beta}{}^\beta\, \Gamma_{\mu\nu}{}^\alpha \,. \tag{6}$$

The field equations to be solved are given by

$$\Gamma^{\nu}_{[\mu\nu]} = 0, \tag{7a}$$

$$R_{(\mu\nu)}(\Gamma) + \lambda g_{(\mu\nu)} = 8\pi\, S_{(\mu\nu)}\,, \tag{7b}$$

$$R_{[\mu\nu,\rho]}(\Gamma) + \lambda g_{[\mu\nu,\rho]} = 8\pi\, S_{[\mu\nu,\rho]}\,, \tag{7c}$$

where λ is the cosmological constant, $S_{\mu\nu}$ is constructed from the metric tensor and the matter energy–momentum tensor,

$$S_{\mu\nu} = T_{\mu\nu} - \tfrac{1}{2}\, g_{\mu\nu}\, g^{\alpha\beta}\, T_{\alpha\beta}\,, \tag{8}$$

and $R_{[\mu\nu,\rho]}$ is defined by

$$R_{[\mu\nu,\lambda]} = \partial_\lambda\, R_{[\mu\nu]} + \partial_\mu\, R_{[\nu\lambda]} + \partial_\nu\, R_{[\lambda\mu]}\,. \tag{9}$$

It should be noted that the NGT affine connection and the field equations reduce to those of GR whenever $g_{[\mu\nu]} = 0$. Consequently all solutions of the GR field equations are special solutions of the NGT field equations.

3. A Simple Example

As an example Geroch (1969) showed how to obtain the plane-symmetric Kasner and Minkowski spacetimes from the Schwarzschild metric. This limiting procedure can be generalised to the NGT theory in a straightforward manner, as shown by the following calculation.

Consider the NGT generalisation of the Schwarzschild solution

$$ds^2 = -\left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 d\Omega^2 + \left(1 - \frac{2m}{r}\right)\left\{1 + \frac{c_1^2}{r^4\,(1+c_2^2)}\right\} d\tau^2\,,$$

$$g_{[14]} = \frac{c_1}{r^2\,\sqrt{1+c_2^2}}\,, \qquad g_{[23]} = -c_2\, r^2 \sin\theta\,, \tag{10}$$

where m, c_1, c_2 are constants and $d\Omega^2 \equiv d\theta^2 + \sin^2\theta\, d\phi^2$. Note that this solution reduces to the GR Schwarzschild solution if we set the NGT parameters c_1 and c_2 to zero. A parameter ϵ such that

$$2m = \epsilon^{-3}, \qquad c_1 = c\epsilon^{-2}\,, \tag{11}$$

is introduced, we apply the following coordinate transformation

$$r = \epsilon^{-1} T\,, \qquad \tau = \epsilon R\,, \qquad \theta = \epsilon\,\rho\,, \tag{12}$$

to (10) and take the limit $\epsilon \to 0$. The result of these operations is

$$ds^2 = -\frac{1}{T}\left\{1 + \frac{c^2}{T^4\,(1+c_2^2)}\right\} dR^2 - T^2\,(d\rho^2 + \rho^2\, d\phi^2) + T\, dT^2\,,$$

$$g_{[14]} = \frac{c}{T^2\,\sqrt{1+c_2^2}}\,, \qquad g_{[23]} = -c_2\, T^2\, \rho\,. \tag{13}$$

If we apply to this solution another coordinate transformation, defined by

$$\begin{aligned} T &= (3/2)^{2/3}\, t^{2/3}\,, \qquad R = (3/2)^{1/3}\, x\,, \\ \rho &= (3/2)^{-2/3}\sqrt{y^2+z^2}\,, \qquad \phi = \tan^{-1}(z/y)\,, \end{aligned} \tag{14}$$

and introduce a new constant $c_1 = (3/2)^{-4/3}c$ then

$$\begin{aligned} ds^2 &= -t^{-2/3}\left\{1 + \frac{c_1^2}{t^{8/3}\,(1+c_2^2)}\right\} dx^2 - t^{4/3}\,(dy^2 + dz^2) + dt^2\,, \\ g_{[14]} &= \frac{c_1}{\sqrt{1+c_2^2}}\, t^{-5/3}\,, \qquad g_{[23]} = -c_2\, t^{4/3}\,. \end{aligned} \tag{15}$$

It can be checked by direct calculation that the above solution satisfies the vacuum NGT field equations. In fact, it is a generalisation of the plane-symmetric NGT solution derived by Kunstatter *et al.* (1979). Thus we have obtained an NGT generalisation of the plane-symmetric Kasner solution by taking a *limit* of the NGT–Schwarzschild solution.

4. Static Electromagnetic NGT Solutions with Nonzero Cosmological Constant

We can take advantage of this method in order to derive the NGT version of the plane-symmetric Bianchi I solution with sourceless electromagnetic field and nonzero cosmological constant. As a first step we shall derive the appropriate spherically symmetric NGT solution. The static NGT metric can be expressed, in terms of coordinates (r, θ, ϕ, τ) and a parameter k, in the Papapetrou form

$$g_{\mu\nu} = \begin{pmatrix} -a_{11}(r) & 0 & 0 & b_{14}(r) \\ 0 & -a_{22}(r) & b_{23}(x)\, h(\theta,k) & 0 \\ 0 & -b_{23}(r)\, h(\theta,k) & -a_{22}(r)\, h^2(\theta,k) & 0 \\ -b_{14}(r) & 0 & 0 & a_{44}(r) \end{pmatrix}, \tag{16a}$$

and the corresponding electromagnetic field tensor (which inherits the symmetry properties of $g_{(\mu\nu)}$) is given by

$$f_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & e_{14}(r) \\ 0 & 0 & e_{23}(r)\, h(\theta,k) & 0 \\ 0 & -e_{23}(r)\, h(\theta,k) & 0 & 0 \\ -e_{14}(r) & 0 & 0 & 0 \end{pmatrix}. \tag{16b}$$

The field equations to be satisfied are (3) and (7) together with Maxwell's equations. The field equation (3) can be solved by implementing Tonnelat's (1954) inversion formula. An algebraic manipulator such as REDUCE can be used for this purpose. The resulting nonzero connection coefficients are

$$\Gamma_{[12]}{}^3 = \frac{A}{2h}, \qquad \Gamma_{[13]}{}^2 = -\frac{h\,A}{2}, \qquad \Gamma_{[14]}{}^1 = \frac{b_{14}\,B}{2a_{11}},$$

$$\Gamma_{[23]}{}^1 = \frac{h}{2A}\,(b_{23}\,C - a_{22}\,A), \qquad \Gamma_{[24]}{}^2 = \Gamma_{[34]}{}^3 = -\frac{b_{14}\,C}{2a_{11}},$$

$$\Gamma_{11}{}^1 = \frac{a_{11}{}'}{2a_{11}}, \qquad \Gamma_{(12)}{}^2 = \Gamma_{(13)}{}^3 = \frac{C}{2},$$

$$\Gamma_{(14)}{}^4 = \frac{b_{14}^2 B}{2a_{11}\,a_{44}} + \frac{a_{44}{}'}{2a_{44}}, \qquad \Gamma_{22}{}^1 = -\frac{1}{2a_{11}}(a_{22}\,C + b_{23}\,A),$$

$$\Gamma_{(23)}{}^3 = \frac{\dot{h}}{h}, \qquad \Gamma_{(24)}{}^3 = \frac{b_{14}\,A}{2a_{11}\,h}, \qquad \Gamma_{33}{}^1 = -\frac{h^2}{2a_{11}}(a_{22}\,C + b_{23}\,A),$$

$$\Gamma_{33}{}^2 = -h\,\dot{h}, \qquad \Gamma_{(34)}{}^2 = -\frac{h\,b_{14}\,A}{2a_{11}}, \qquad \Gamma_{44}{}^1 = \frac{b_{14}^2 B}{a_{11}^2} + \frac{a_{44}{}'}{2a_{11}}, \tag{17}$$

where

$$A = \frac{a_{22}\,b_{23}{}' - b_{23}\,a_{22}{}'}{a_{22}^2 + b_{23}^2}, \qquad B = \frac{d}{dr}\ln\left(1 - \frac{a_{11}\,a_{44}}{b_{14}^2}\right),$$

$$C = \frac{b_{23}\,b_{23}{}' + a_{22}\,a_{22}{}'}{a_{22}^2 + b_{23}^2}, \qquad a' \equiv \frac{da}{dr}, \qquad \dot{a} \equiv \frac{da}{d\theta}. \tag{18}$$

The field equation (7a) and Maxwell's equations lead to

$$\Gamma_{[\mu\nu]}{}^{\nu} = 0\,, \Rightarrow a_{44} = \frac{b_{14}^2}{a_{11}}\left(1 + \frac{a_{22}^2 + b_{23}^2}{\alpha_1^2}\right), \tag{19a}$$

$$\partial_\nu\,\mathbf{f}^{\mu\nu} = 0\,, \Rightarrow e_{14} \propto b_{14}\,, \tag{19b}$$

$$\partial_{[\sigma}\,f_{\mu\nu]} = 0\,, \Rightarrow e_{23} = \alpha_3\,. \tag{19c}$$

We expect, in order to get back to the Schwarzschild solution, that $a_{11} \to (a_{44})^{-1}$ as $b_{14} \to 0$. This implies that

$$b_{14} = \frac{\alpha_1}{a_{22}\,\sqrt{1 + c_2^2}} \qquad \text{and} \qquad a_{44} = \frac{1}{a_{11}}\,(1 + b_{14}^2)\,. \tag{20}$$

Assume, as in the vacuum static (spherically symmetric) case, that $A = 0$. This assumption together with (19a) leads to

$$b_{23} = -c_2 a_{22}\,, \qquad B = \frac{2a_{22}{}'}{a_{22}}, \qquad C = \frac{a_{22}{}'}{a_{22}}. \tag{21}$$

Finally, we shall assume, again by analogy with the vacuum case, that $a_{22} = r^2$. As a result of the above assumptions we are left with only one unknown, a_{11},

$$
\begin{aligned}
a_{44} &= \frac{1}{a_{11}}\left(1+\frac{\alpha_1^2}{r^4\,(1+c_2^2)}\right), \qquad a_{22}=r^2\,,\\
b_{14} &= \frac{\alpha_1}{r^2\sqrt{1+c_2^2}}\,, \qquad b_{23}=-c_2r^2\,,\\
e_{14} &= \frac{\alpha_2}{r^2}\,, \qquad e_{23}=\alpha_3\,.
\end{aligned} \tag{22}
$$

The construction of the generalised Ricci tensor, from the connection coefficients (subject to the above simplifying assumptions), followed by substitution into the field equation (7b) leads to two ordinary differential equations for $a_{11}(r)$ and one for $h(\theta,k)$:

$$\frac{a_{11}''}{a_{11}^2}-\frac{(a_{11}')^2}{a_{11}^3}+\frac{2a_{11}'}{ra_{11}^2}+\frac{8\pi}{r^4}\left(\alpha_2^2+\frac{\alpha_3^2}{1+c_2^2}\right)-2\lambda=0\,, \tag{23a}$$

$$\frac{a_{11}'}{a_{11}^2}-\frac{1}{ra_{11}}-\frac{4\pi}{r^3}\left(\alpha_2^2+\frac{\alpha_3^2}{1+c_2^2}\right)-\lambda r+\frac{k}{r}=0\,, \tag{23b}$$

$$\ddot{h}-kh=0\,. \tag{23c}$$

In the last equation we can, without loss of generality, take $k=0,\pm1$ and hence obtain the following solutions:

$$h(\theta,-1)=\sinh\theta\,,\qquad h(\theta,0)=\theta\,,\qquad h(\theta,+1)=\sin\theta\,. \tag{24}$$

We note that solving (23b) for a_{11}' and substituting into (23a) reduces this equation identically to zero. Thus any solution a_{11} of (23b) is automatically a solution of (23a). The substitution $a_{11}=u^{-1}$ reduces the equation (23b) to the following first-order linear ordinary differential equation

$$u'+\frac{u}{r}-\frac{k}{r}+\lambda r+\frac{4\pi}{r^3}\left(\alpha_2^2+\frac{\alpha_3^2}{1+c_2^2}\right)=0\,, \tag{25}$$

which can be integrated to give

$$u=k-\frac{2m}{r}+\frac{4\pi}{r^2}\left(\alpha_2^2+\frac{\alpha_3^2}{1+c_2^2}\right)-\frac{\lambda r^2}{3}\,, \tag{26}$$

where $2m$ is the constant of integration. Finally, we have to show that the remaining field equation (7c) is also satisfied. Now

$$\widetilde{R}_{[\mu\nu]}\equiv R_{[\mu\nu]}+\lambda g_{[\mu\nu]}-8\pi T_{[\mu\nu]} \tag{27}$$

is not identically zero only for

$$
\begin{aligned}
\widetilde{R}_{[14]} &= \widetilde{R}_{[14]}(r)\,, \qquad \text{the functional form is irrelevant,}\\
\widetilde{R}_{[23]} &= c_2rh(\theta,k)\left\{\frac{a_{11}'}{a_{11}}-\frac{1}{r\,a_{11}}-\frac{4\pi}{r^3}\left(\alpha_2^2+\frac{\alpha_3^2}{1+c_2^2}\right)-\lambda r\right\}.
\end{aligned} \tag{28a}
$$

Substituting from (23b) into the last equation we obtain

$$\widetilde{R}_{[23]} = -c_2 k\, h(\theta, k)\,. \tag{28b}$$

It is now clear, since $\widetilde{R}_{[14]}$ and $\widetilde{R}_{[23]}$ are only functions of r and θ respectively, that the field equation (7c) is also satisfied.

To summarise, we have derived three electrovac static NGT solutions with a nonzero cosmological constant:

$$\begin{aligned} ds^2 = &-\left\{k - \frac{2m}{r} + \frac{4\pi}{r^2}\left(\alpha_2^2 + \frac{\alpha_3^2}{1+c_2^2}\right) - \frac{\lambda r^2}{3}\right\}^{-1} dr^2 - r^2 d\Omega_k^2 \\ &+ \left\{k - \frac{2m}{r} + \frac{4\pi}{r^2}\left(\alpha_2^2 + \frac{\alpha_3^2}{1+c_2^2}\right) - \frac{\lambda r^2}{3}\right\}\left\{1 + \frac{\alpha_1^2}{r^4\,(1+c_2^2)}\right\} d\tau^2\,, \end{aligned} \tag{29a}$$

$$\begin{aligned} g_{[14]} &= \frac{\alpha_1}{r^2\sqrt{1+c_2^2}}\,, \qquad & g_{[23]} &= -c_2 r^2 \sin\theta\,, \\ f_{14} &= \frac{\alpha_2}{r^2}\,, & f_{23} &= \alpha_3 \sin\theta\,, \end{aligned} \tag{29b}$$

where

$$d\Omega_{-1}^2 = d\theta^2 + \sinh^2\theta\, d\phi^2,\ \ d\Omega_0^2 = d\theta^2 + \theta^2\, d\phi^2,\ \ d\Omega_{+1}^2 = d\theta^2 + \sin^2\theta\, d\phi^2\,. \tag{30}$$

In the case $k = +1$ this solution corresponds to an NGT generalisation (containing two independent NGT parameters α_1 and c_2) of the exterior Reissner–Nordström GR solution (m, α_2 and α_3 are the mass, electric charge and magnetic charge respectively) with cosmological constant λ. In the limit α_2, $\alpha_3 \to 0$ we obtain Tiwari's (1970) solution to the Einstein and Straus (1946) *weak field equations*. Note that Tiwari's field equations differ from those that we are using in the sign of λ because his definition of the generalised Ricci tensor has opposite sign to (6).

If we set the cosmological constant and the parameters α_1, α_2, α_3 and c_2 equal to zero then we recover class A degenerate static vacuum solutions of Einstein's equations (GR). These solutions were listed by Kramer *et al.* (1980). It turns out that there exist three class B degenerate static vacuum solutions which are connected to class A by the complex substitution $\tau \to i\phi$, $\phi \to i\tau$. We can use this substitution, together with $\alpha_1 \to -i\alpha_1$, $\alpha_2 \to -i\alpha_2$, $\alpha_3 \to -i\alpha_3$ and $c_2 \to -ic_2$, to obtain the NGT generalisation of the class B Einstein solutions:

$$\begin{aligned} ds^2 = &-\left\{k - \frac{2m}{r} - \frac{4\pi}{r^2}\left(\alpha_2^2 + \frac{\alpha_3^2}{1-c_2^2}\right) - \frac{\lambda r^2}{3}\right\}^{-1} dr^2 - r^2 d\Psi_k^2 \\ &- \left\{k - \frac{2m}{r} - \frac{4\pi}{r^2}\left(\alpha_2^2 + \frac{\alpha_3^2}{1-c_2^2}\right) - \frac{\lambda r^2}{3}\right\}\left\{1 - \frac{\alpha_1^2}{r^4\,(1-c_2^2)}\right\} d\phi^2\,, \end{aligned} \tag{31a}$$

$$\begin{aligned} g_{[13]} &= \frac{\alpha_1}{r^2\sqrt{1-c_2^2}}\,, \qquad & g_{[24]} &= -c_2 r^2 \sin\theta\,, \\ f_{13} &= \frac{\alpha_2}{r^2}\,, & f_{24} &= \alpha_3 \sin\theta\,, \end{aligned} \tag{31b}$$

where

$$d\Psi_{-1}^2 = d\theta^2 - \sinh^2\theta\, d\tau^2, \;\; d\Psi_0^2 = d\theta^2 - \theta^2\, d\tau^2, \;\; d\Psi_{+1}^2 = d\theta^2 - \sin^2\theta\, d\tau^2 . \tag{32}$$

Note that for these NGT class B solutions, as opposed to the NGT class A solutions above, the two NGT parameters α_1 and c_2 are no longer arbitrary. In fact, $-1 < c_2 < 1$ and α_1 must satisfy the inequality $r^4(1-c_2^2) - \alpha_1^2 \geq 0$ in order to preserve signature.

5. NGT Plane-Symmetric Bianchi Type I Solution with EM Field

In order to obtain the desired solution we start from the NGT generalisation of the class AI (Kramer *et al.* 1980) GR solution, i.e. (29) with $k = +1$. As in Section 3, we introduce a new parameter ϵ such that

$$2m = m_1\epsilon^{-3}, \quad \alpha_1 = -c_1\epsilon^{-2}, \quad \alpha_2 = -q_e\epsilon^{-2}, \quad \alpha_3 = -q_m\epsilon^{-2}, \tag{33}$$

apply the coordinate transformation

$$r = \epsilon^{-1}T, \quad \tau = \epsilon x, \quad \theta = \epsilon\sqrt{y^2+z^2}, \quad \phi = \tan^{-1}\left(\frac{z}{y}\right), \tag{34}$$

and take the limit $\epsilon \to 0$. The result of these operations is

$$ds^2 = -A(T)\left\{1 + \frac{c_1^2}{T^4(1+c_2^2)}\right\} dx^2 - T^2\,(dy^2 + dz^2) + A(T)\,dT^2, \tag{35a}$$

$$g_{[14]} = \frac{c_1}{T^2\sqrt{1+c_2^2}}, \qquad g_{[23]} = -c_2T^2,$$
$$f_{14} = \frac{c_e}{T^2}, \qquad f_{23} = -c_m, \tag{35b}$$

where

$$A(T) \equiv \frac{m_1}{T} - \frac{4\pi}{T^2}\left(c_e^2 + \frac{c_m^2}{1+c_2^2}\right) + \frac{\lambda T^2}{3}. \tag{36}$$

In order to interpret this as a plane-symmetric cosmological solution we require T to be the time coordinate. This implies that T can take only those values for which $A > 0$.

In terms of the comoving coordinate system (x, y, z, t) we can write our six-parameter plane-symmetric Bianchi type I solution as follows:

$$ds^2 = -A(T)\left\{1 + \frac{c_1^2}{T^4(1+c_2^2)}\right\} dx^2 - T^2\,(dy^2 + dz^2) + dt^2, \tag{37a}$$

$$g_{[14]} = \frac{c_1}{T^2\sqrt{A(T)(1+c_2^2)}}, \qquad g_{[23]} = -c_2T^2,$$
$$f_{14} = \frac{c_e}{T^2\sqrt{A(T)}}, \qquad f_{23} = -c_m, \tag{37b}$$

with $T(t)$ given, implicitly, by

$$t = \int^{T} \sqrt{A(u)}\, du \,, \tag{38}$$

where A is defined in (36). Among the parameters λ is the cosmological constant and c_e and c_m are the electric and magnetic charges, respectively. There are two NGT parameters, c_1 and c_2, which distinguish this solution from the corresponding Einstein–Maxwell solution. If we set, for example, $c_1 = c_2 = c_e = m_1 = \lambda = 0$, in (35a,b) and (36), then we obtain a general relativistic Bianchi type I cosmological model with pure magnetic field discussed by De (1975) (case D, solution 1).

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