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Quantum Measurements in Atomic Optics*

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Abstract

We consider atoms traversing a cavity filled with an optical field. When the atoms are well detuned from the optical resonance the output momentum distribution of the atoms is found to be a sensitive probe of the photon statistics of the light field. Near resonance spontaneous emission smears the diffractive peaks. We obtain a good fit to the experimental data of Gould *et al.* (1991). As the atoms pass through the optical field they impart a position-dependent phase shift to the field. By making a quadrature phase measurement on the optical field a position measurement of the atom is achieved. We show that it is possible to prepare the atom in a 'contractive state' which beats the standard quantum limit for position measurements.

1. Introduction

Recent advances in the cooling and trapping of atoms have led to a number of possible quantum measurements involving the interaction of atoms with the electromagnetic field. We shall consider the scattering of atoms from a standing wave light field. As an atom passes through the field, momentum exchanges occur between the light field and the atom in multiples of the photon momentum. If the atoms are sufficiently detuned so that spontaneous emission is negligible, the situation is analogous to the lossless diffraction grating in optics. The momentum distribution in the far field consists of several sharply defined diffractive peaks. For smaller atomic detunings, spontaneous emission becomes important and the diffractive peaks become smeared out. In a recent paper by Gould *et al.* (1991), the transition from the diffractive to the diffusive regime has been observed. We present the results of an analysis (Tan and Walls 1991) which provide an accurate fit to the results of Gould *et al.* (1991).

In the far from resonance situation, when spontaneous emission can be neglected, the momentum distribution of the atoms in the far field depends on the photon statistics of the light from which they are scattered. We show that the atomic deflection may be used to give a quantum non-demolition (QND) measurement of the photon number of the field (Holland *et al.* 1991). An alternative scheme has been proposed whereby the photon number is determined via the phase shift imparted to the atom by the interaction with the light field (Brune *et al.* 1990). Using atoms as a probe to determine the statistical properties of a light field is

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a new direction in the rapidly developing field of the mechanical effects of light on atoms.

Recently several experiments demonstrating atomic interferometry have been reported (Carnal and Mlynek 1991; Keith *et al.* 1991; Kasevich and Chu 1991). A question of current interest is how to best localise the position of an atom. If the position of an atom is determined by observing light scattered from the atom (the Heisenberg microscope) the position cannot be localised to better than a wavelength. Several methods to make atomic position measurements using an optical field have been proposed. These include channelling in an off-resonant optical standing wave (Salomon *et al.* 1991) and using spatially varying level shifts which enable one to correlate the position of the atom with its resonant frequency (Thomas 1990). Micron resolution in experiments using the latter method have been reported (Stokes *et al.* 1991).

We wish to propose a new scheme to localise the atom's position using an optical field. In a similar arrangement to the QND measurement of photon number the two level atom is passed through an off-resonant standing wave light field. The interaction with the atom imparts a phase shift to the light field which depends on the atom's position in the field. This phase shift may be measured by making a homodyne measurement of the quadrature phase of the light field.

It has been demonstrated by Yuen (1983) and Ozawa (1988) that the standard quantum limit (SQL) on position measurements of a free mass may be surpassed if the free mass is initially prepared in a contractive state. We shall show that for certain measurements of the quadrature phase of the field, the atom is prepared in a contractive state. We show that it is possible to localise the position of the atom very precisely within a wavelength of the light in the cavity. Depending on the initial position distribution of the atom, a field measurement may localise the atom into one or more virtual slits. We present situations exhibiting diffraction from a single virtual slit and interference from two such slits.

2. Quantum Non-demolition Measurement of Photon Number

QND measurements are designed to evade the back action noise inherent in a quantum measurement (Braginsky *et al.* 1977; Unruh 1978; Caves *et al.* 1980). The goal of a QND measurement is to determine the value of an observable of a system without in any way disturbing it. That is, we require that the act of measurement itself does not degrade the predictability of subsequent measurements. This requirement is satisfied if, for an observable $A^{\mathrm{I}}(t)$ (in the interaction picture),

$$[A^{\mathrm{I}}(t),\, A^{\mathrm{I}}(t')] = 0\,. \tag{1}$$

This condition ensures that if the system is in an eigenstate of $A^{\mathrm{I}}(t_0)$ it remains in this eigenstate for all subsequent times although the eigenvalues may change. Such observables are called QND observables. For example, for a free particle the position is not a QND observable, but the momentum is.

In order to measure the observable of a system it is necessary to couple the system to a meter or probe. It is essential that the coupling to the meter does not feed back fluctuations into the QND variable of the detector. In order to avoid this it is sufficient if the QND variable A commutes with the Hamiltonian

coupling the system and the meter, $H_{\rm sm}$, that is

$$[A, H_{\rm sm}] = 0\,. \tag{2}$$

This is known as the back action evasion criterion.

A number of QND schemes to measure the photon number of a signal beam have involved a four-wave interaction with a probe beam (Milburn and Walls 1983; Imoto *et al.* 1985; Levenson *et al.* 1986; Imoto *et al.* 1987). In the case of a Kerr medium this interaction may be written in the form

$$H = \chi a^\dagger a b^\dagger b\,, \tag{3}$$

where a (b) is the annihilation operator for the signal (probe) mode. Clearly the photon number in the signal mode $a^\dagger a$ is a QND observable. The coupling with the probe also satisfies the back action evasion criterion. The photon number in the signal mode is determined by making a measurement on a quadrature phase $X_\theta = b\mathrm{e}^{-\mathrm{i}\theta} + b^\dagger \mathrm{e}^{\mathrm{i}\theta}$ of the probe.

We wish to discuss a new type of QND coupling generated by the interaction of atoms with a standing wave light field. We consider a standing wave electromagnetic field containing a small number of photons in a high quality cavity. The measurement probe is a beam of two-level atoms crossing perpendicularly through the cavity. The radiation field is detuned far from the atomic resonance so that the photon number is not changed due to absorption or emission. The photon number of the field is therefore a QND observable. Using the atoms as a probe the photon number of the field may be obtained in two ways. In one proposal for Rydberg atoms, the phase shift of the atomic wavefunction due to the interaction with the field is measured by the Ramsey separated oscillatory fields technique (Brune *et al.* 1990). This phase shift is directly proportional to the field intensity. The other method makes use of the exchange of momentum between atoms and photons. The transverse momentum transfer of the atom–photon interaction is such that in the far field the position of the atom carries a significant amount of information about the intensity of the field. The atomic deflection profile carries information on the photon statistics of the light field.

We shall present an analysis which can describe both the above methods. We assume that a two-level atom interacts with a single cavity field mode. The interaction is assumed to be sufficiently non-resonant that we may neglect spontaneous emission. The cavity lifetime is assumed to be long compared with the interval between atom arrivals, so that a measure of the photon number of the field may be obtained before it decays.

We denote the boson annihilation operator for the field by a and the atomic inversion and atomic coherences by the Pauli spin operators σ_z, σ_+ and σ_-. The operators π and x describe the momentum and position of the centre of mass of the atom. The Hamiltonian for the confined atom and radiation field system in the rotating wave approximation is

$$\begin{aligned} H \;=\; & \hbar\omega_a\, a^\dagger a + \hbar\omega_0\, \sigma_z + \frac{p^2}{2m} \\ & + \hbar(g\sigma_-\, a^\dagger + g^*\sigma_+\, a)\cos(kx + \phi)\,, \end{aligned} \tag{4}$$

where m is the atomic mass, g is the single-photon Rabi frequency and k is the wave number of the cavity mode. We consider the Raman–Nath regime in which the transverse kinetic energy absorbed by the atom during the interaction can be neglected. In the large detuning limit

$$g\sqrt{n}/\Delta \ll 1\,, \tag{5}$$

where $\Delta = \omega_0 - \omega_a$, the effective interaction Hamiltonian is

$$V_{\mathrm{i}} = 2\hbar \frac{|g|^2}{\Delta} \sigma_z a^{\dagger} a \cos^2(kx + \phi)\,. \tag{6}$$

It is immediately apparent that the photon number $a^{\dagger}a$ satisfies the back action evasion criterion. We may consider this interaction as describing an atomic phase shift proportional to the field intensity, in which case the photon number is determined by measuring the atomic coherence σ_+. Alternatively we can measure the momentum distribution of the scattered atoms. We shall give a more complete analysis of the latter scheme.

Let the initial state of the system be represented by $\mathcal{E}$. The probability of the atom exiting with momentum p after interaction time t is given by

$$P(p, t) = \langle p \,|\, \mathrm{Tr}\{U(t) \,|\, \mathcal{E}\rangle\langle \mathcal{E} \,|\, U^{\dagger}(t)\} \,|\, p\rangle\,, \tag{7}$$

where the trace is taken over the field and atomic internal variables, and U is the time evolution operator. Since the possible momentum shifts are discrete multiples of $2\hbar k$, the final output momentum probability is composed of a comb of images of the initial momentum distribution. In order to resolve these peaks it is necessary that the initial momentum spread is $\Delta p < 2\hbar k$. When the momentum peaks are distinguishable the output momentum distribution is given by

$$\begin{aligned} P(p, t) \;&=\; P(p, 0) \mathbin{\text{☆}} \sum_r Q_r(t)\, \delta(p - 2r\hbar k)\,, \\ Q_r(t) &= \sum_n J_r^2\!\left(\frac{|g|^2 nt}{\Delta}\right) P(n)\,, \end{aligned} \tag{8}$$

where ☆ denotes momentum convolution, δ and J are the Dirac delta and Bessel functions respectively, and $P(n)$ describes the photon statistics of the cavity. The amplitudes Q_r represent the probability that a momentum of $2r\hbar k$ will be transferred to the atom in the cavity. The probability amplitudes are strongly dependent on n so that fields with different photon statistics exhibit very different output momentum distributions.

In Fig. 1*a* we plot the output momentum distribution for an atom traversing a cavity composed of a number state field containing 50 photons. In Fig. 1*b* we consider a thermal field where the maximum $P(n)$ corresponds to no photons in the cavity, so the most likely momentum shift is zero. In Fig. 1*c* we consider a coherent field where the photon statistics are Poissonian. In this case the cavity is well described as an atomic beam splitter in which the greatest probability lies in regions of nonzero atomic deflection (Meystre *et al.* 1989).

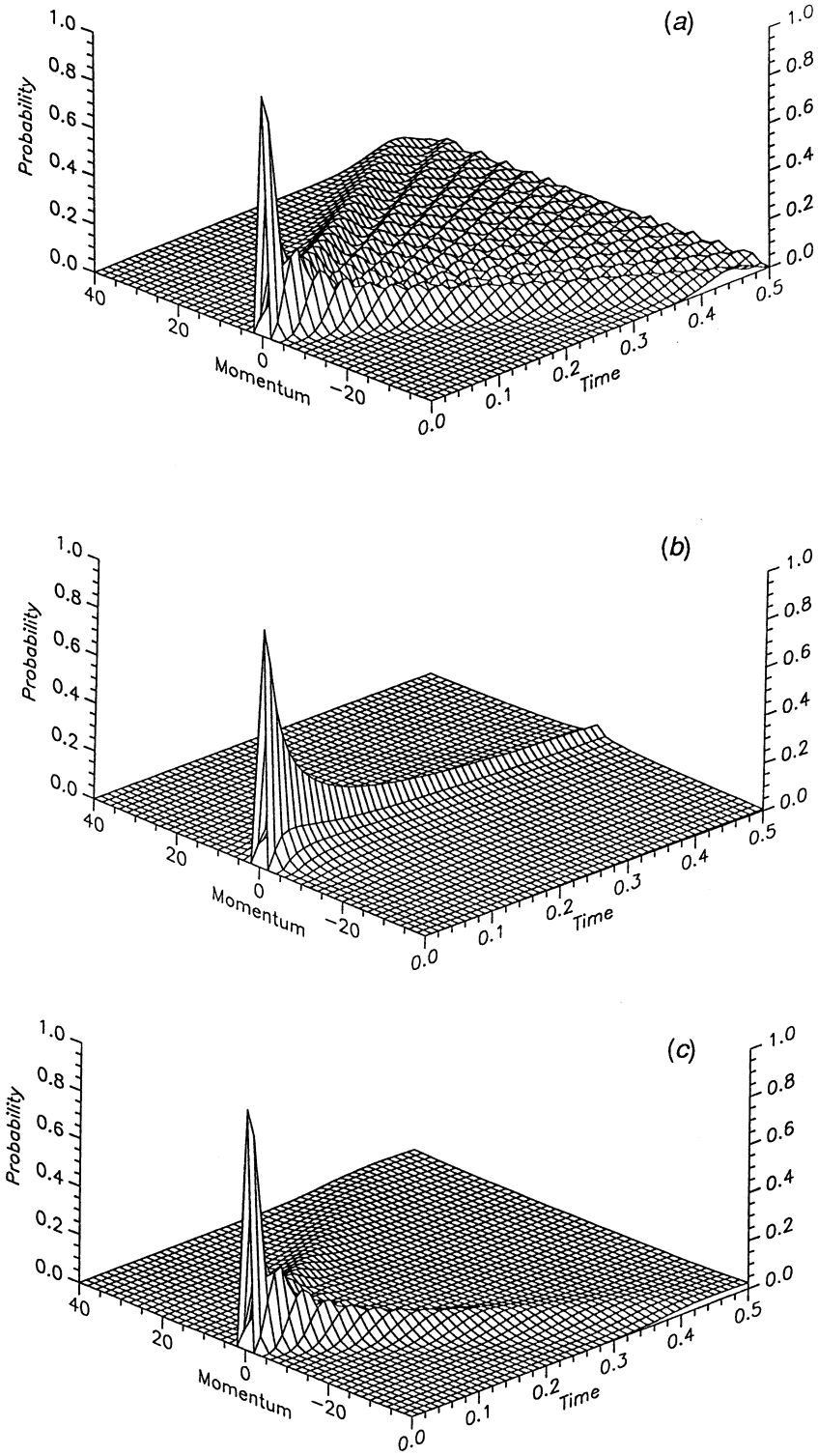

Fig. 1. Output momentum distribution for an atom traversing a cavity composed of (*a*) a number state field with 50 photons; (*b*) a thermal field with mean photon number 50; and (*c*) a coherent field with mean photon number 50.

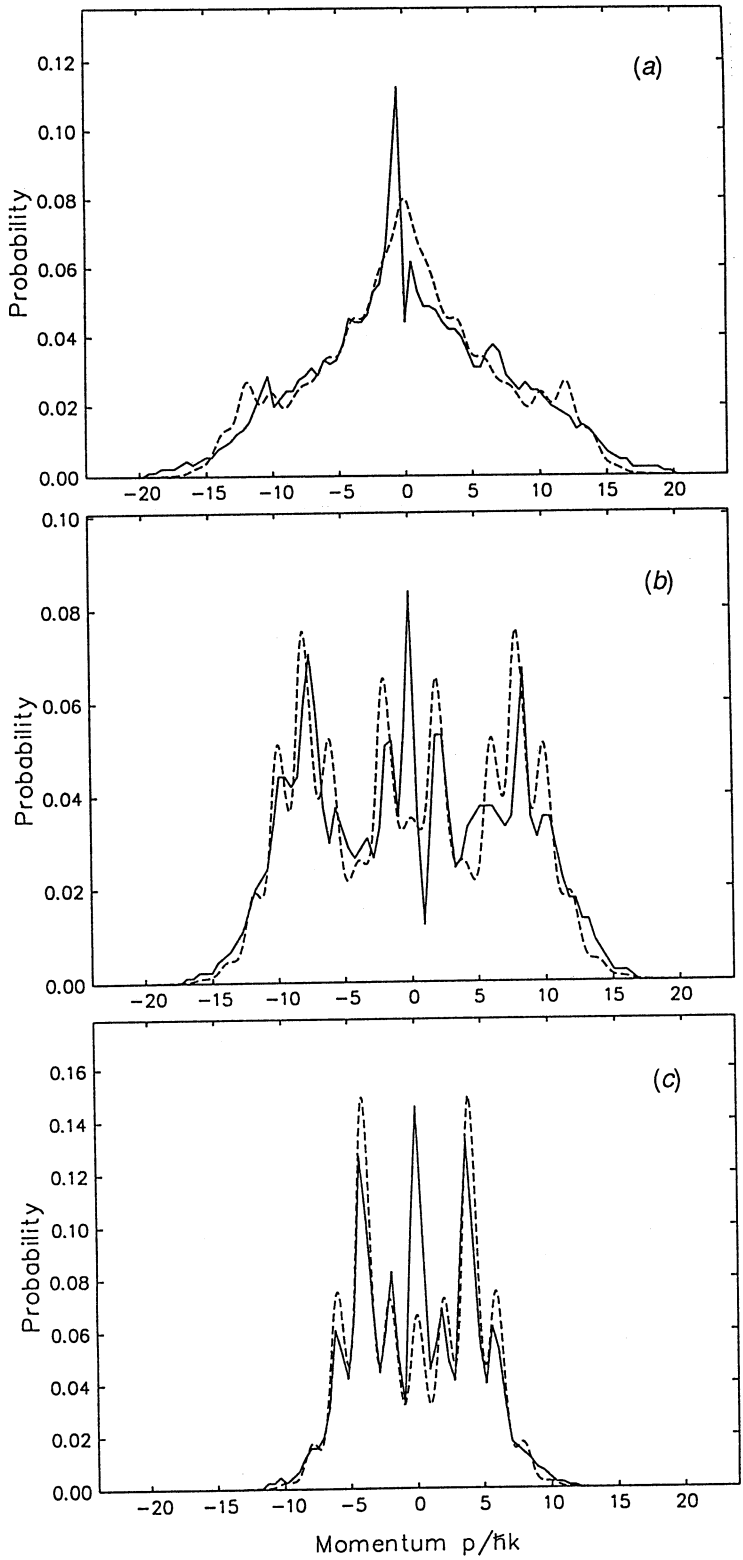

Fig. 2. Output momentum distribution for atomic scattering from a standing light wave. The experimental data (solid curve) are compared with the simulation results (dashed curve) for the detunings: (*a*) $\delta\omega = 0$; (*b*) $\delta\omega = 4\gamma$; and (*c*) $\delta\omega = 8\gamma$.

If one tunes closer to the atomic resonance, spontaneous emission becomes important. The recoil imparted to an atom by a spontaneously emitted photon occurs in a random direction so that its momentum component in the direction of the standing wave can range from $-\hbar k$ to $+\hbar k$. This spontaneous emission causes the diffractive peaks to be smeared out. In Fig. 2 we show some experimental momentum distributions obtained by Gould *et al.* (1991), compared with theoretical fits by Tan and Walls (1991). Fig. 2*a* is on-resonance ($\delta\omega = 0$), where the spontaneous emission completely smears out the diffractive structure. Figs 2*b* ($\delta\omega = 4\gamma$) and 2*c* ($\delta\omega = 8\gamma$) show how the diffraction peaks appear as the spontaneous emission is reduced.

We now return to the case well off-resonance where we can neglect spontaneous emission. We shall show that by measuring the deflection of a sequence of atoms through the cavity, the cavity field will be reduced to a near number state. We simulate the effect of repeated atomic position measurements and calculate the resulting field statistics.

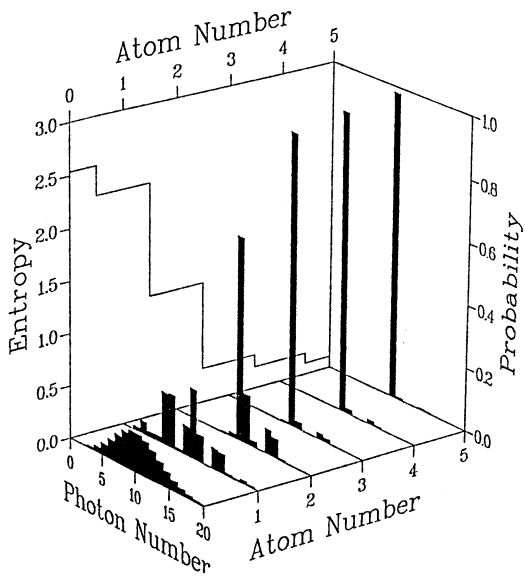

Fig. 3. Simulation of the collapse of the field density of states to a single photon number. The nonrelaxing cavity was initially described by a coherent state (Poissonian photostatistics with mean 10). Projected onto the back wall is the entropy of the field state with scale denoted by the entropy vertical axis on the left.

Based on an initial choice of field statistics, a particular output momentum p_0 for an atom exiting the cavity is chosen. The diagonal elements of the field density matrix $P(n)$ are then altered by the back action of the measurement

$$P(n \,|\, p_0) = MP(p_0 \,|\, n)P(n)\,, \tag{9}$$

where M is a normalisation constant. The next momentum p_1 is then selected with this probability weighting for the statistics of the field, and the process is

repeated. Fig. 3 illustrates a simulation of five probe atoms with a field initially described by a coherent state with a mean of ten photons. Each such simulation collapses the field to a single photon number. The proportion of times that each number is selected is completely determined by the initial photon statistics. The entropy of the field $\Sigma_n P_n \ln P(n)$ can be used as an indicator of the quality of the measurement.

3. Measurement of Atomic Position with Optical Fields

The measurement of the position of an atom runs into the difficulty that the position x is not a QND observable for a free particle. Uncertainty in the atom's momentum created by a position measurement feeds back into subsequent measurements of position. The standard quantum limit (SQL) for the measurement of the position of a free mass relates to the uncertainty $\Delta x(\tau)$ in a measurement of the position at time τ, given that a measurement had been initially made with uncertainty $\Delta x(0)$. The SQL is given by

$$\Delta x(\tau) \geq \left(\frac{\hbar\tau}{m}\right)^{\frac{1}{2}}, \tag{10}$$

which is obtained by the following argument. After the first measurement with accuracy $\Delta x(0)$, the uncertainty in momentum $\Delta p(0)$ leads to an uncertainty in position at time τ given by

$$\begin{aligned} \Delta x^2(\tau) &= \Delta x^2(0) + \frac{\Delta p^2(0)\tau^2}{m^2} \\ &\geq 2\Delta x(0)\,\Delta p(0)\,\frac{\tau}{m} \geq \frac{\hbar\tau}{m}, \end{aligned} \tag{11}$$

where we have used the Heisenberg uncertainty relation

$$\Delta x(0)\,\Delta p(0) \geq \hbar/2\,. \tag{12}$$

This argument has the following flaw as pointed out by Yuen (1983). Since the evolution of a free mass is given by

$$x(\tau) = x(0) + p(0)\,\frac{\tau}{m}, \tag{13}$$

the variance of x at time τ is given by

$$\Delta x^2(\tau) = \Delta x^2(0) + \frac{\Delta p^2(0)\tau^2}{m^2} + \langle \Delta x(0)\,\Delta p(0) + \Delta p(0)\,\Delta x(0)\rangle \frac{\tau}{m}. \tag{14}$$

In the standard argument the correlation term is assumed to be positive. However, if the initial measurement leaves the free mass in a state with negative correlation, then $\Delta x^2(\tau) < \hbar\tau/m$.

Squeezed states may exhibit this property. These may be defined, following Yuen (1976), as the eigenstates of the operator $\mu a + \nu a^\dagger$:

$$\mu a + \nu a^\dagger \,|\, \mu,\, \nu,\, \alpha\rangle = (\mu,\, \nu,\, \alpha\rangle\,, \tag{15}$$

where

$$|\mu|^2 - |\nu|^2 = 1\,.$$

The wavefunction $\langle x\,|\,\mu,\,\nu,\,\alpha\rangle$, where $\hat{x}\,|\,x\rangle = x\,|\,x\rangle$, is given by

$$\langle x\,|\,\mu,\,\nu,\,\alpha\rangle = \left(\frac{m\omega}{\pi\hbar\,|\,\mu-\nu\,|^2}\right)^{\frac{1}{4}} \times \exp\left(-\frac{m\omega}{2\hbar}\frac{1+2\mathrm{i}\xi}{|\,\mu-\nu\,|^2}(x-x_0)^2 + \frac{\mathrm{i}p_0}{\hbar}(x-x_0)\right), \tag{16}$$

where

$$\xi = \mathrm{Im}(\mu^*\nu)\,,$$

$$\alpha = \left(\frac{m\omega}{2\hbar}\right)^{\frac{1}{2}} x_0 + \frac{\mathrm{i}}{(2\hbar m\omega)^{1/2}}\,p_0\,. \tag{17}$$

When $\xi = 0$ the wavefunction (16) is the usual minimum uncertainty state. The first two moments of $|\,\mu,\,\nu,\,\alpha\rangle$ are

$$\langle x\rangle = x_0, \qquad \langle p\rangle = p_0\,,$$

$$\Delta x^2 = \frac{\hbar\,|\,\mu-\nu\,|^2}{2m\omega}, \qquad \Delta p^2 = \hbar m\omega\frac{|\mu+\nu|^2}{2}\,,$$

$$\langle \Delta x\,\Delta p + \Delta p\,\Delta x\rangle = -2\hbar\xi\,. \tag{18}$$

When $\xi > 0$ the x-dependent phase in (16) leads to a narrowing of $\Delta x(t)$ from $\Delta x(0)$ during free evolution.

The position fluctuation for a free mass starting in an arbitrary squeezed state is

$$\frac{m\Delta x^2(t)}{2\hbar} = \zeta\omega - \xi t + \eta\omega t^2\,, \tag{19}$$

where

$$\zeta = \frac{|\,\mu-\nu\,|^2}{4}, \qquad \eta = \frac{|\,\mu+\nu\,|^2}{4}\,,$$

$$\zeta\eta = (1+4\xi^2)/16\,.$$

The minimum fluctuation is $1/16\omega\eta$. This occurs at $t_\mathrm{m} = \xi/2\eta\omega$ and is $1/2(2\xi)^{1/2}$ times the SQL. Thus the minimum fluctuation $\Delta x(\tau)$ is related only to the momentum uncertainty as follows:

$$\Delta x(\tau) = \frac{\hbar}{2\Delta p(0)} = \frac{\Delta x(0)}{(1+4\xi^2)^{1/2}}\,. \tag{20}$$

This shows that $\Delta x(\tau)$ can be made arbitrarily small with sufficiently large ζ. Note that the minimum uncertainty product is realised between the momentum

uncertainty at $t = 0$ and the position uncertainty at $t = \tau$, since the momentum is a constant of the motion.

Thus the SQL can be surpassed if there is a measurement which places the free mass in a contractive state. Ozawa (1988) has proposed a scheme where the system is coupled to a probe which is initially prepared in a contractive state. In the following we shall describe a measurement of an atom's position which puts it in a contractive state, without requiring special preparation of the probe (Storey *et al.* 1992).

When a two-level atom is passed through a standing wave mode in an optical cavity the interaction with the field depends on the position of the atom. By making a quadrature phase measurement on the field, it is possible to localise the position of the atom very precisely within a wavelength of the light in the cavity. Certain of these measurements may prepare the atom in a contractive state.

The experimental arrangement for this measurement is the same as for the QND measurement of photon number described in Section 2. A two-level atom is passed through a standing light wave in an optical cavity. The interaction of the atom with the light field imparts a phase shift to the field which is dependent on the atomic position. This phase shift is determined by measuring the quadrature phase of the light by a homodyne detection scheme. The Hamiltonian is given by (4).

The cavity mode is assumed to be initially in a coherent state $|\alpha\rangle$. The atom is assumed to be initially in the ground state with a transverse spread in position given by $\phi(x)$. This implies phase coherence across the width of the distribution, so the atom must be cooled before entering the cavity. We can write the initial state of the system as

$$|\psi(0)\rangle = \int \mathrm{d}x\, \phi(x) |\alpha, x, g\rangle\,, \tag{21}$$

where the state is labelled by the field amplitude, atomic position and internal atomic state respectively. After an interaction time t in the field, the state of the system is

$$\begin{aligned} |\psi(0)\rangle &= \int \mathrm{d}x\, \phi(x) \exp\left(\frac{-\mathrm{i}(V_\mathrm{i} + \hbar\Delta\sigma_z)t}{\hbar}\right) |\alpha, x, g\rangle \\ &= \mathrm{e}^{\mathrm{i}\Delta t/2} \int \mathrm{d}x\, \phi(x) \,|\, \alpha \exp\left(\frac{\mathrm{i}|g|^2 t}{\Delta} \cos^2(kx + \phi)\right), x, g\rangle\,. \end{aligned} \tag{22}$$

That is, the interaction has changed the phase of the coherent state. This field phase shift depends on the vacuum light shift of the atomic state. [Vacuum field-induced level shifts of barium atoms have been measured recently by Heinzen and Feld (1987).] Because the phase shift depends on the position of the atom, a measurement of the quadrature phase X_θ of the field will localise the atom.

We define

$$X_\theta = a\mathrm{e}^{-\mathrm{i}\theta} + a^\dagger \mathrm{e}^{\mathrm{i}\theta}\,. \tag{23}$$

To find the atomic state after the field measurement we project the field state onto an eigenstate $|X_\theta\rangle$ of the quadrature phase:

$$|\psi(t)\rangle_{\text{atom}} = N\int \mathrm{d}x\,\phi(x)\langle X_\theta|\,\alpha\exp\left(\frac{\mathrm{i}|g|^2t}{\Delta}\cos^2(kx+\phi)\right)\rangle|x,g\rangle$$

$$= N\int \mathrm{d}x\,\phi(x)\,\frac{1}{4\sqrt{2\pi}}\exp\left[-\left(\alpha_1-\frac{X_\theta}{2}\right)^2-\mathrm{i}\alpha_2(\alpha_1-X_\theta)\right]|x,g\rangle\,, \qquad (24)$$

where

$$\alpha_1+\mathrm{i}\,\alpha_2 \equiv \alpha\exp\left[\mathrm{i}\left(\frac{|g|^2t}{\Delta}\cos^2(kx+\phi)-\theta\right)\right].$$

In order to observe the correlation between the atomic position and the value of the quadrature phase, the transit time of each atom through the cavity must be much shorter than the lifetime of the cavity, which in turn must be shorter than the interval between the times that successive atoms enter the cavity.

The best localisation is obtained with $|g|^2t/\Delta=\pi$ and a high field intensity. However, if the scheme is implemented at optical wavelengths the Raman–Nath condition imposes a severe restriction on the interaction time and the field strength. The additional requirement of a low transition probability between internal atomic states leads to the following condition on the atom–field coupling constant:

$$|g| \gg 2\langle n\rangle^{\frac{3}{2}}\left(\frac{\pi^2\eta\hbar}{\lambda^2 m}\right), \qquad (25)$$

where η is a proportionality factor characterising the momentum uncertainty of the atom after the interaction, and is independent of the cavity frequency and approximately independent of the field strength. Assuming $|g|^2t/\Delta=\pi$, we can obtain significant localisation if the mean number of photons in the field is greater than about 8. For optical transitions the required value for $|g|$ is extremely high (of the order of 10^8 Hz). Such high values have recently been obtained by Rempe *et al.* (1991) using a very short cavity of high finesse.

Fig. 4*a* shows the initial position distribution of the atom (iii) and the near-field distributions resulting from the field measurement $X_0=0$ (i) and $X_{\pi/2}=2\alpha$ (ii). In this graph, and in all subsequent graphs, we have used $|g|^2t/\Delta=\pi$ with $\alpha=\sqrt{8}$. By varying the phase of the field quadrature measured, we have varied the degree of localisation and effectively created an atomic slit of adjustable width. Fig. 4*b* shows the far-field distribution of the atom after the measurement of X_0 (i) and after the measurement of $X_{\pi/2}$ (ii). It is clear that the wide virtual slit has produced a narrower diffraction pattern than the narrow virtual slit. This is exactly what we would expect if the atom passed through a real physical slit. In fact, as the width of the virtual slit is reduced, by varying the phase of the field quadrature from $\pi/2$ to 0, the width of the diffraction pattern increases smoothly. The product $\Delta x\,\Delta p$ is close to the uncertainty limit, but there is some excess noise due to a small degree of nonlinearity in the phase change across the atomic wavefront. A linear component in the phase change across the virtual slit produces a deflection of the atomic beam, which is responsible for the asymmetry of the far-field distribution.

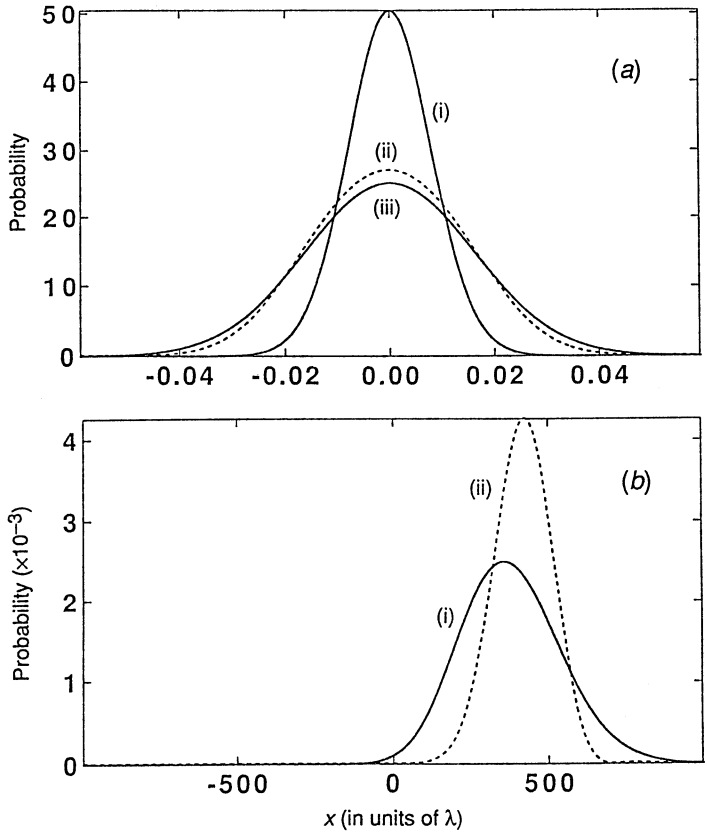

Fig. 4. (*a*) Near-field position distribution of the atom before the field measurement (iii), after a field measurement yielding the value $X_0 = 0$ (i), and after a field measurement yielding the value $X_{\pi/2} = 2\alpha$ (ii). The position distribution of the atom before it enters the cavity is taken to be gaussian with a standard deviation $\sigma = 0{\cdot}1\lambda/2\pi$ centred midway between a node and an antinode of the field ($\phi = -\pi/4$). (*b*) Far-field position distribution for the same parameters after the measurement of X_0 (i) and after the measurement of $X_{\pi/2}$ (ii).

Of course this 'slit' is not a real physical slit which the atom passes through, but a virtual slit created by our knowledge from the field measurement of where the atom is. Whether diffraction can be observed from such a virtual slit was suggested by Popper (1982) as a crucial test of the Copenhagen interpretation of quantum mechanics. Popper proposed a scheme which he claimed would create such a virtual slit of adjustable width, and suggested that such a scheme would test whether knowledge alone is sufficient to create uncertainty (as is contended under the Copenhagen interpretation) or whether scattering of a particle depends on the physical presence of a slit. Popper's proposed experiment does not, however, provide such a test, due to other uncertainties inherent in the scheme which Popper had not included in his analysis. The experiment we propose should resolve Popper's question in favour of the Copenhagen interpretation.

If the measurement is such that the atom is localised at an antinode of the field, then the phase change across the atomic wavefront is approximately

parabolic, and the atom is focused. That a measurement can produce focusing is contrary to the intuitive notion that momentum uncertainty introduced by a position measurement should cause the position distribution to spread out with time. The requirement for focusing is that the measurement leave the system in a state with a negative correlation between its position and momentum. States whose position distribution contracts under free evolution (termed 'contractive states') have been described by Yuen (1983) and Ozawa (1988). Fig. 5 compares the focusing of a contractive state produced by our position localisation scheme with the theoretical maximum focusing, achieved by Yuen's 'twisted coherent state' or squeezed state.

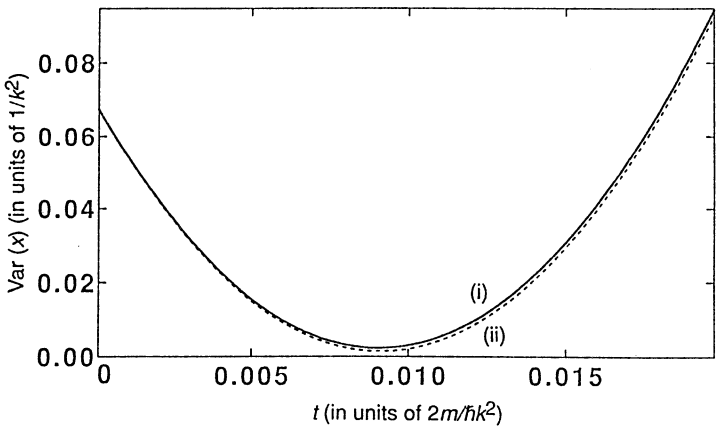

Fig. 5. Position variance (i) as a function of time for the contractive state produced by our position localisation scheme, compared with the ideal focusing achieved by a 'twisted coherent state' with the same momentum variance and initial position variance (ii). Our contractive state (i) is produced by making a field measurement $X_0 = -2\alpha$ after the atom has crossed the cavity. The position distribution of the atom before it enters the cavity is taken to be gaussian with standard deviation $\sigma = 0{\cdot}5\lambda/2\pi$ centred at an antinode of the field ($\phi = 0$).

If the initial position distribution of the atom is wider than that used in Fig. 4 then a measurement of $X_0 = 0$ will localise the atom as indicated by curve (i) in Fig. 6*a*. The position distribution now has two smaller peaks located at $x = \lambda/4$ and $-\lambda/4$ on either side of the central peak. The initial gaussian distribution of the atom is indicated by curve (iii), and the phase of the atomic wavefront after the field measurement by curve (ii). The phase shift across each of the three peaks is approximately linear. However, the phase change across the central peak is positive and that across each of the two side peaks is negative. Hence the central atomic beam is deflected to the right and the two side beams are deflected to the left. Here the term 'atomic beam' is used loosely because the three 'beams' constitute the wavepacket of a single atom. Fig. 6*b* shows the far-field position distribution of the atom. The left half of the distribution shows complete interference between the atomic beams from the side peaks. The right half of the distribution is the diffraction pattern of the central peak. The partial interference in the right half of the distribution is due to the presence of very small peaks located at $x = \lambda/2$ and $-\lambda/2$ in curve (i) of Fig. 6*a*.

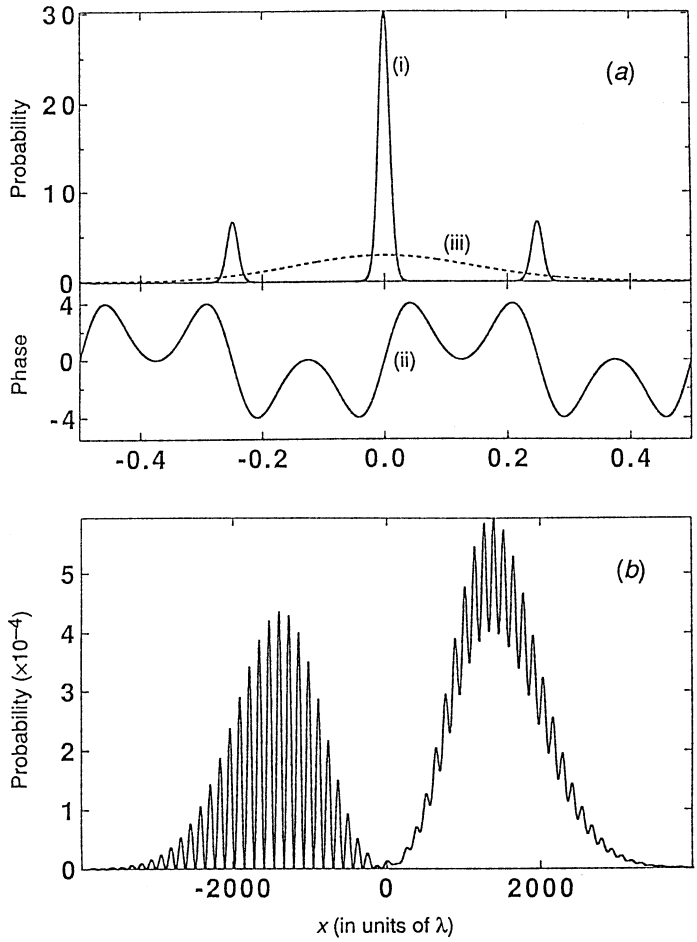

Fig. 6. (*a*) Near-field position distribution of the atom with a broader initial distribution, before the field measurement (iii), and after the value $X_0 = 0$ has been measured for the field (i). Curve (ii) shows the phase (in radians) of the atomic wavefront after the measurement. The position distribution of the atom before it enters the cavity is assumed to be gaussian with standard deviation $\sigma = 0{\cdot}9\lambda/2\pi$, centred midway between a node and antinode of the field ($\phi = -\pi/4$). (*b*) Far-field position distribution for the same parameters after the field measurement.

If the initial position distribution of the atom is centred at an antinode of the field and the value $X_0 = 0$ is measured after the interaction, then the near-field position distribution of the atom will exhibit two slits, with opposite phase change across each slit. The two atomic beams converge, and interference can be observed in the near field where they cross. Alternatively, a second cavity in antiphase with the first can be placed immediately after the first cavity to act as an atomic lens. If the value $X_{0(\text{second cavity})} = -X_{0(\text{first cavity})}$ is measured, the phase change across the atomic wavefront is eliminated and interference from the two adjacent slits can be observed in the far field.

We have shown that a quadrature phase measurement on the field can localise the position of the atom very precisely within a wavelength of the cavity field.

However, a single field measurement cannot determine 'which wave' the atom went through, so if the initial position distribution is spread over many wavelengths the distribution after a field measurement will contain correspondingly many peaks. But if the atom then passes through a second cavity tuned to a slightly different frequency immediately after exiting the first cavity, one or more of these peaks can be selected out by a quadrature phase measurement of the field in the second cavity.

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