

Wave Function Collapse in Atomic Physics*

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Abstract

Wave function collapse has been a contentious concept in quantum mechanics for a considerable time. Here we show examples of how the concept can be used to advantage in predicting the statistical results of three experiments in atomic physics and quantum optics: photon antibunching, single-photon phase difference states and interrupted single-atom fluorescence. We examine the question of whether or not collapse is ‘really’ a physical process, and discuss the consequences of simply omitting it but including the observer as a part of the overall system governed by the laws of quantum mechanics. The resulting entangled world does not appear to be inconsistent with experience.

1. Introduction

The concept of wave function collapse, or state vector reduction, has been a contentious subject of debate since the very early days of quantum mechanics. It is not the purpose of this paper to review the various points of view, but we refer to a recent, and in parts acrimonious, debate initiated by Bell (Bell 1990; van Kampen 1990, 1991; Gottfried 1991; Peierls 1991; Squires 1992). In this paper we describe briefly the essential nature of the collapse and give three examples of ways in which the concept can be used to advantage in atomic physics to predict the statistical results of experiments. This leads on to the question—is the collapse postulate merely a convenient mathematical recipe for obtaining the results we seem to observe, or does it relate to a physical process, that is, is it an essential part of the description of the physical world? We attempt to answer this question in the simplest way possible by examining the effect of removing the collapse entirely, that is by allowing only unitary time evolution, but including the observer and everything else of relevance as part of the total physical system.

2. Reduction as Disentanglement

In the standard interpretation of quantum mechanics (von Neumann 1955) the state vector of a system evolves in two ways: causally and continuously in accord with the time-dependent Schrödinger equation and discontinuously if a measurement is carried out on the system. The latter change in state is the

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reduction of the state vector. We usually assume that, at the initial time t_0 , the system comprising the object A and measuring apparatus B with which A is to interact can be considered as two separate identifiable subsystems, whose possible states are independent of each other, and represent the state of the system as $|A_i\rangle|B_k\rangle$. (For a discussion of the validity of representing the state of a very complicated quantum mechanical system, such as a 'macroscopic' measuring apparatus, by a state vector, see Leggett 1984.) To illustrate the nature of the process in the simplest possible way, let us assume that i takes two values 1, 2 corresponding to two orthogonal states. For B to be a useful measuring device, the system should be set up such that if A is in state $|A_1\rangle$ then B will evolve at time t to some state $|B_1\rangle$, and if A is in state $|A_2\rangle$, B will evolve to a different, distinguishable state $|B_2\rangle$. For example, $|B_1\rangle$ and $|B_2\rangle$ might be orthogonal states of the macroscopic apparatus with very different readings of a pointer. An ideal measurement would also not change the state of A , though this is not essential for our discussion. We have then

$$|A_1\rangle|B_1\rangle = U(t, t_0)|A_1\rangle|B_k\rangle, \quad (1)$$

$$|A_2\rangle|B_2\rangle = U(t, t_0)|A_2\rangle|B_k\rangle, \quad (2)$$

where $U(t, t_0)$ is the time displacement operator. If A is initially in a superposition state $c_1|A_1\rangle + c_2|A_2\rangle$, then from (1) and (2) it follows that the final state will be the superposition

$$c_1|A_1\rangle|B_1\rangle + c_2|A_2\rangle|B_2\rangle. \quad (3)$$

This is a direct consequence of the linearity of quantum mechanics.

The state (3) is entangled, or correlated, that is, it is not expressible as a product of A and B states and so does not allow a description of the object A independent of the measuring apparatus. Such entangled states are the basis of the well-known Einstein–Podolsky–Rosen paradox. State (3) is interpreted in the conditional sense that if, for example, the apparatus is found in $|B_1\rangle$, then A is in the state $|A_1\rangle$. Often this interpretation is all that is needed to predict the statistics of experimental results. Expression (3) presents us with two outcomes of the measurement: the apparatus B being in state $|B_1\rangle$, for example a pointer reading 17, and the apparatus being in state $|B_2\rangle$, for example the pointer reading 10. Reduction of the state is necessary if I, the observer, insist that I can experience only one of these. We shall call such insistence assumption (a). After reduction, the state is $|A_1\rangle|B_1\rangle$ or $|A_2\rangle|B_2\rangle$. In either case the states of A and B have been disentangled, allowing once again a description of the object independent of the apparatus. If for example, $|c_1|^2 = |c_2|^2$, then the standard interpretation is that reduction will produce one or other of the outcomes with a probability of $\frac{1}{2}$. We shall refer to the probabilities linked to c_1 and c_2 as quantum probabilities to distinguish them from 'classical', or statistical, probabilities. An example of the latter is the case of a coin on the table which we have not observed. Before observation, the probability of a head is $\frac{1}{2}$. Observation causes this to change to either 0 or 1. In (3), the values of c_1 and c_2 describe the state of the system. On the other hand, the value of $\frac{1}{2}$ for

the coin refers to our ignorance of the state of the system. Observation reduces this ignorance, not the state of the coin. It is not unusual for both these types of probabilities to be part of a particular problem. Both state reduction and ignorance reduction can contribute to the statistics of the experimental results.

3. Photon Antibunching

The simplest object-detector system is two interacting two-level systems. Let us consider a two-level atom interacting with a two-level detector with the same energy gap. The excited and ground states of the atom are $|e\rangle$ and $|g\rangle$, and of the detector $|d_e\rangle$ and $|d_g\rangle$. To remove unnecessary complications, let us assume that the detector surrounds the atom and is completely absorbing, that is, any photon emitted by the atom must be absorbed by the detector. This allows us to eliminate the field variables and consider the atom as acting directly on the detector. If initially the system is in the state $|e\rangle|d_g\rangle$, it evolves at a later time t to the entangled state

$$a(t)|e\rangle|d_g\rangle + b(t)|g\rangle|d_e\rangle. \quad (4)$$

Suppose the detector is such that if it is excited to $|d_e\rangle$, then a photoelectron is released which, after amplification, produces a signal which tells us that a photon has been detected. If we now say that we must observe either a photon detection event or no photon detection event, that is, we either detect a photon or we do not, then, if a photon has been detected, we reduce (4) to $|g\rangle|d_e\rangle$. This tells us that immediately after a photon has been detected, the atom is in the ground state. If the atom is being weakly and continuously excited this leads to the phenomenon of photon antibunching (Carmichael and Walls 1976; Cohen-Tannoudji 1977; Dagenais and Mandel 1978). In an antibunching experiment a photon can be detected at any time, but if a photon is detected at time t , then there is a waiting time necessary before there is a sizeable probability of detecting the next photon, because the atom has to be re-excited from the ground state.

4. Phase-difference States by Collapse

We consider now the field produced by two separated sources A and B and a two-level photon detector D_1 at a distance r_{A1} and r_{B1} from A and B respectively. If A and B correspond to the signal and idler modes in a degenerate parametric down-conversion process, we can write the initial state of the field and detector as $|1_A\rangle|1_B\rangle|d_g\rangle$, that is, one photon in each mode (Ghosh and Mandel 1987) with D_1 in the ground state. By use of the Hermitian phase operator formalism (Pegg and Barnett 1988, 1989; Barnett and Pegg 1989, 1990) it is not difficult to show that for such a field state the phase difference between the modes is random, that is, there is a uniform probability distribution of phase difference. Thus no classical type of optical interference is expected, that is, no periodic variation of detection probability as the position of D_1 is varied along a line parallel to the line joining A and B . Let us now suppose the field interacts with

D_1 for a time t . The essential term in the interaction Hamiltonian, in terms of photon annihilation operators, is proportional to

$$[\hat{a}_A \exp(i\mathbf{k}_A \cdot \mathbf{r}_{A1}) + \hat{a}_B \exp(i\mathbf{k}_B \cdot \mathbf{r}_{B1})] |d_e\rangle \langle d_g|, \quad (5)$$

with the former factor representing the field at D_1 , where $\mathbf{k}_A$ and $\mathbf{k}_B$ are wave vectors. From perturbation theory we can show reasonably straightforwardly that this interaction transforms the initial state to a superposition

$$\begin{aligned} a(t) |1_A\rangle |1_B\rangle |d_g\rangle + b(t) [|0_A\rangle |1_B\rangle \exp(i\mathbf{k}_A \cdot \mathbf{r}_{A1}) \\ + |1_A\rangle |0_B\rangle \exp(i\mathbf{k}_B \cdot \mathbf{r}_{B1})] |d_e\rangle. \end{aligned} \quad (6)$$

A similar argument to that pertaining to expression (4) shows that a detection of a photon by D_1 reduces the state vector (6) to

$$[|0_A\rangle |1_B\rangle \exp(i\mathbf{k}_A \cdot \mathbf{r}_{A1}) + |1_A\rangle |0_B\rangle \exp(i\mathbf{k}_B \cdot \mathbf{r}_{B1})] |d_e\rangle, \quad (7)$$

which disentangles the field and detector states. The field state has the interesting property of being the simplest field with a reasonably well-defined phase difference. The mean value of this phase difference is $\mathbf{k}_A \cdot \mathbf{r}_{A1} - \mathbf{k}_B \cdot \mathbf{r}_{B1}$, as can be found by using the approach to phase difference calculations of Barnett and Pegg (1990). The variance of the phase difference can also be calculated as $\pi^2/3 - 2$ in a 2π range, compared with $\pi^2/3$ for a uniform, that is, random distribution. Such a two-mode state can give an interference pattern, that is a periodic variation in the probability of detecting a photon as a second detector D_2 is moved away from D_1 along a line parallel to that joining A and B . Indeed, the predicted pattern is similar to that which would be formed by two classical sources at A and B emitting light whose phase difference is such as to produce constructive interference at D_1 . This experiment has been performed by Ghosh and Mandel (1987). It is interesting to note that they simply calculated directly the joint probability of a photon being detected by each detector without explicitly invoking the collapse concept. The results are the same as discussed above.

5. Collapse by Detecting Nothing

The above examples involve the detection of a single photon, with subsequent amplification. We now examine a case where the wave function collapses following a negative result of an observation. Such a concept has been known for some time (see, for example, Dicke 1981, 1986), and has found application recently in interpreting single-atom interrupted fluorescence experiments proposed by Dehmelt (1974) and Cook and Kimble (1985) and investigated experimentally by Nagourney *et al.* (1986), Sauter *et al.* (1986), Bergquist *et al.* (1986) and others subsequently (see the review by Cook 1990).

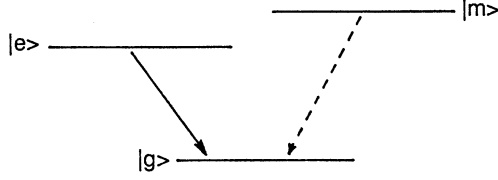


Fig. 1. Energy levels of a three-level atom. State $|e\rangle$ decays quickly to $|g\rangle$ and the metastable state $|m\rangle$ decays slowly to $|g\rangle$. In the interrupted fluorescence experiment, the transition between $|g\rangle$ and $|e\rangle$ is driven strongly and the transition between $|m\rangle$ and $|g\rangle$ is driven weakly.

Consider a three-level atom, as shown in Fig. 1, whose excited state $|e\rangle$ decays rapidly to the ground state $|g\rangle$ with a characteristic time t_e , and whose excited metastable state $|m\rangle$ decays very slowly to $|g\rangle$ with a characteristic time t_m . Let us prepare the atom and field in the superposition state $(\sqrt{\frac{2}{3}}|e\rangle + \sqrt{\frac{1}{3}}|m\rangle)|0\rangle$, where the latter factor is the field state with no photons. Suppose that after a time t such that $t_e \ll t \ll t_m$ no photons have been detected by a detector designed to detect any photon which is emitted. What then is the state of the atom? At first sight, it is tempting to say that, because no decay has occurred, the atom must still be in its original state. A more detailed analysis, however, reveals a different result. In time t the state will evolve to a superposition

$$a(t)|e\rangle|0\rangle + b(t)|m\rangle|0\rangle + \sum c_k(t)|g\rangle|1_k\rangle, \quad (8)$$

where $|1_k\rangle$ is a one-photon field state with a particular wave vector and polarisation and the summation is over all wave vectors and polarisations. Because $t \gg t_e$, the modulus of $a(t)$ will be very small, so the first term in the superposition can be neglected in comparison with both of the other terms. The detection of no photons then reduces the state to $|m\rangle|0\rangle$. Thus the atom has made a transition from a state which was predominantly $|e\rangle$ to the metastable state $|m\rangle$, which may be of higher energy, without the state of the field changing! This transition has been induced by the detection of no photons, and has been referred to as a ‘knowledge-induced transition’ (Cook 1990). It is an actual transition in the sense that if we accept that the atom is in $|m\rangle$, we then are led to the correct prediction of the statistics of subsequent experimental results.

In the single-atom interrupted fluorescence experiments (Cook 1990) the transition between $|e\rangle$ and $|g\rangle$ is driven strongly by external radiation and the $|m\rangle$ to $|g\rangle$ transition is driven very weakly. The fluorescence from the strongly-driven transition is observed continuously, which can be done with the eye, without amplification. Although the fluorescence appears to be continuous, it is actually a succession of antibunched photons with an average time interval between each of t_d which is of the order of t_e . The atom state vector evolves to a superposition of all three states. The observation of no fluorescence for a period

significantly greater than t_e induces a transition to the metastable state $|m\rangle$. The atom must then remain in this state for a period of order t_m before it returns to the ground state to be excited to $|e\rangle$ and begin fluorescing again. The result is a series of long bright fluorescing periods interrupted by dark periods of the order of the same length as the bright periods. Use of this concept allows the precise statistics of the bright and dark periods to be simply calculated, as well as the time taken for the wave function to collapse via a knowledge-induced transition. This time is approximately $t_d \ln(T_B/t_d)$, where T_B is the average length of the bright periods (Pegg and Knight 1988*a*, 1988*b*).

Interrupted fluorescence, often referred to as ‘quantum jumps’, in a single atom is very important as an example of a genuinely random process. At a predetermined time there will be roughly equal probabilities that an observer will see light or darkness. This probability is of the quantum type discussed in Section 2, rather than the classical, coin-type, probability describing our lack of knowledge.

6. Is Wave Function Collapse ‘Real’?

We have illustrated how the concept of wave function collapse, or state reduction, is a useful and efficient means of calculating and understanding the statistics of the results of some experiments in atomic physics. Further, we have given an expression for the time taken for a particular type of collapse. Possibly we should therefore not even ask the question—is wave function collapse really a physical process? If the purpose of physics is merely to predict the results of experiments, then we may as well accept collapse and not worry any further, at least until some experiment gives results which deviate from prediction. On the other hand, there is still something unsatisfying about a process seemingly brought about by observation by an external, and presumably of necessity conscious, observer. We cannot review the extensive literature for and against the concept, but here we wish to explore briefly the consequences of including the observer as part of the quantum mechanical system and of not assuming that any collapse occurs. That is, the only evolution we allow is that governed by the Schrödinger equation to which the observer is also subject.

Let us return to the argument leading to expression (3), but now make the total system as complete as possible by including the observer C and everything else of relevance, which we call D . We assume that these are also separate identifiable subsystems, initially disentangled from the object A and the apparatus B . Again, let the experiment be set up such that the state $|A_1\rangle$ leads to a state $|B_1\rangle$, but now also include the interaction of B with C and D such that state $|B_1\rangle$ leads to a state $|C_1\rangle|D_1\rangle$. Similarly $|A_2\rangle$ now leads not just to $|B_2\rangle$, but to $|B_2\rangle|C_2\rangle|D_2\rangle$. It then follows, in the same way in which we obtained (3), that the superposition $c_1|A_1\rangle + c_2|A_2\rangle$ leads to

$$c_1|A_1\rangle|B_1\rangle|C_1\rangle|D_1\rangle + c_2|A_2\rangle|B_2\rangle|C_2\rangle|D_2\rangle. \quad (9)$$

If, as before, $|B_1\rangle$ and $|B_2\rangle$ are states of a pointer reading 17 and 10, then $|C_1\rangle$ will be the state of the observer upon seeing the pointer read 17, that is a state

of being conscious of, or remembering, the pointer reading 17, with a similar interpretation for $|C_2\rangle$ as remembering a reading of 10. Again expression (9) follows from the linearity of quantum mechanics. We can now introduce state vector reduction at this later stage by postulating that the observer cannot exist in an entangled state such as (9), a postulate which we call assumption (b). Disentanglement means reducing (9) to either its first or last term, which gives effectively the same result as previously, which followed from assumption (a) of Section 2 that the observer could only experience $|B_1\rangle$ or $|B_2\rangle$.

We thus have two assumptions which appear to give the right answer. Assumption (b) appears more fundamental and satisfying, in that the observer is treated as part of the total quantum mechanical system, whereas in the approach involving (a), the observer is treated as somehow external to the quantum mechanical system. Unfortunately, even with assumption (b), there is still something special about the observer, in that it is a quantum mechanical subsystem which cannot be entangled with other such subsystems. This requirement of observer disentanglement always allows a description of the observer independent of the rest of the physical world—which is not greatly different from the notion of an external observer.

Let us return to expression (9) and explore its consequences *without* introducing an extra assumption such as (b). The first question to examine is as follows. Because assumption (b) appears to be consistent with assumption (a), which seems to give the right answers, does removing (b) lead to a contradiction with (a)? Assumption (a) prevents, for example, the observer seeing Schrödinger's cat as a superposition of being alive and dead, a situation often depicted in textbooks by a picture of a cat standing up superimposed on a cat lying dead. In our previous example, it prevents the observer seeing a pointer reading 10 and at the same time reading 17. How do we prevent this without assumption (a)? The crucial point is that such a situation is *not predicted* by (9) in the first place. Indeed, for the observer to see, or be conscious of, the 10–17 superposition state he or she would have to be disentangled, that is, the state would be

$$(c_1|B_1\rangle + c_2|B_2\rangle)|C_{12}\rangle, \quad (10)$$

where $|C_{12}\rangle$ is a state of the observer remembering both $|B_1\rangle$ and $|B_2\rangle$. Clearly this is not the same as (9), indeed it is the very entanglement of the observer with the pointer in (9) which *prevents* this from occurring. We can re-express (9) in a different basis, for example by using $(|B_1\rangle + |B_2\rangle)/\sqrt{2}$ and $(|B_1\rangle - |B_2\rangle)/\sqrt{2}$ as a new basis for the pointer states, but we still cannot obtain a form such as (10). Thus, rather than predicting that an observer will be in a state of being conscious of a superposition of pointer readings 10 and 17, the linear nature of quantum mechanics, without reduction, predicts that the observer will be in a superposition of a state of being conscious that the pointer reads 10 and a state of being conscious that the pointer reads 17, correlating precisely with the states of the pointer. Being in a superposition of states of consciousness is not the same as being conscious of a superposition state. If we were to represent the superposition pictorially, the picture would not show an observer seeing a superposition of pointer positions. Rather there would be two pictures—one showing the pointer reading 17 and the observer thinking it is reading 17, and the other showing 10 and the observer thinking it is 10.

Since the quantum mechanical result (9) does not lead to a cat paradox of the type usually presented, perhaps it is sufficient to describe the actual situation. Before adding further assumptions such as reduction, we should study this possibility. The essential question is—can we devise an experiment to show that the world, including the observer, does not exist in an entangled state such as (9)? How do I put myself into an entangled state? Firstly, a way of *not* doing this is as follows: I decide in advance to look at a coin, if it is heads I shall go to the right side of the room, if tails, then I shall go to the left. Although there is equal probability of my being on one side of the room as the other, this is only ignorance-based probability and I shall be only on one side. To achieve my purpose of entangling myself, I require a *quantum decider*, based on a quantum mechanical superposition. Thus I could use the pointer reading device as described above where A is a spin-half particle, go left if I see the pointer read 10 and go right if I see 17. Alternatively, I could look at the fluorescing three-level atom at a predetermined time and if I see brightness I go right, if darkness then I go left. Would my state be detectable by either myself, or by another person, as being distinguishable from the state I would have been in had I used a coin decider? That is, will I appear strange to the second person, E , who is part of the subsystem D ? If the initial state were $|A_1\rangle$ this would lead to the pointer reading 17, myself being on the right, and E seeing me on the right. An initial state of $|A_2\rangle$ would lead to a reading of 10, myself left and E seeing me on the left. A superposition of $|A_1\rangle$ and $|A_2\rangle$ therefore leads to a superposition of a reading of 10, me on the left, E seeing me on the left and of a reading of 17, me on the right, E seeing me on the right. It does not lead to E seeing me in a superposition of left and right. I would thus not appear strange to E , because of E 's entanglement with me. Am I conscious of having a component on the left and another on the right? The answer is again no. If I look to see where I am, or ask myself what the pointer reading was, that is question my memory, I will not find that I become conscious of being in a superposition of position. I will be in a superposition of states of consciousness, each of which is self-consistent and indistinguishable to me from the situation in which I would be had I used a coin decider. The quantum decider allows me to have my cake and eat it, but unfortunately not to be conscious of doing so! We might ask—what does it mean for me to be in a superposition of states of consciousness if I am not aware of this and if, further, someone watching me is also not aware of this because he or she, and other observers in the chain, are similarly affected? It simply means that the actual entangled state of the world, with its macroscopic superpositions, is not readily perceived by observers who are part of it.

How do we now interpret the antibunching experiment, involving a continuously excited atom, without invoking state reduction as before but, instead, including the observer as part of the system? After the experiment has been running for some time, the observer begins observation. After a time interval Δt , which is very much shorter than the characteristic excitation time, the state of the observer will have two components, one corresponding to observing a photon (1) and one corresponding to not observing a photon (0). Both of these components evolve according to the Schrödinger equation. After the next interval Δt , the component (0) will have become two components, one corresponding to having observed no

photon in the first interval or second interval $(0,0)$ and the other corresponding to having observed no photon and then one photon $(0,1)$. At this time the component (1) will still only be one component $(1,0)$, because $(1,1)$ cannot evolve from (1) in this short time which, of course, is the antibunching effect. After the third time interval, there will be a total of four components to the state of the observer: $(1,0,0)$; $(0,1,0)$; $(0,0,1)$; $(0,0,0)$. All of these are consistent with the observation of antibunching. It is tempting at this stage to apply reduction and say that in the 'real' world, where the observer can have only one component, only one of the four possibilities is realised. In view of our previous discussion, however, there is no need to do this—at least until some experiment tells us that an observer cannot ever have more than one component. The consequences of the multiple components may flow on, depending on the circumstances. For example, if the observer intended to publish a single individual sequence, the journal would have four slightly different components, and so on.

This 'splitting' of the observer's history, and thus the histories of things with which he or she interacts, into separate self-consistent components, each in agreement with our everyday experiences is very similar to the 'many-worlds' interpretation of quantum mechanics (Everett 1957). We wish to stress, however, that there are no additional postulates involved, and there is always only one world, which is essentially quantum mechanical in nature, and with which the observer becomes entangled by being part of it. Rather than many worlds, we have one single, but entangled universe.

7. Conclusions

The similarity of the entangled universe to the 'many-worlds' picture would most likely exclude it from consideration by any of the participants in the debate referred to in the Introduction, if only because it includes the observer as part of the physical system (see, for example, Peierls 1991). Another approach not included in the debate, which we should perhaps mention because of the recent attention it has received, is decoherence (see, for example, Zurek 1981, 1982). The decoherence method avoids macroscopic superpositions by a rapid diagonalisation of a reduced density matrix representing a statistical mixture of states of a selected part of the total system. A step in the procedure involves taking the partial trace of the total density matrix, a mathematical operation which can create an uncorrelated, or partially uncorrelated, statistical mixture from a pure entangled state. If the change in the density matrix caused by the partial tracing operation is not regarded to be a stage in the physical time evolution, then it is difficult to regard decoherence as a physical process. If the change is regarded as a physical change, then the evolution is not unitary. Either way, there is no relation between decoherence and the approach of this paper.

To sum up, wave function collapse, or state reduction, seems to be a useful concept for calculating the statistics of results for some experiments in atomic physics. It is, however, an additional postulate to the laws of quantum mechanics. In its most basic form it is simply an instruction to deny the possibility of a 'macroscopic' object such as a pointer being in an entangled state, that is a statement that macroscopic objects, including observers, should only be described by classical physics (Leggett 1984). When we question whether the collapse is a physical process, that is, a description of the physical world, then it seems

reasonable to include the observer as part of this world. In this case, without any reduction, we are left with macroscopic entanglements which, because they *include* the observer, are not recognisable by the observer as such. This entanglement of the observer prevents the usual paradoxes. If physics is merely a means of calculating results of laboratory experiments then the collapse concept is a useful part of quantum mechanics. If, on the other hand, physics is meant to be a description of the physical world, or at least of a part of the world much larger than, and including, the individual observer, then the collapse concept would seem to preclude an accurate description and so should not be part of physics.

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