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Collisionless Damping of Fast and Ion–Cyclotron Surface Waves

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Abstract

A recently developed general kinetic theory of surface waves is used to calculate the collisionless damping of low frequency fast and ion–cyclotron surface waves on a magnetised plasma–vacuum interface. In particular, the possibility of Cherenkov (Landau and transit-time magnetic) absorption by electrons is accounted for, assuming a bi-Maxwellian distribution of electrons in velocity space. It is shown that in general the surface waves are damped via mode conversion to a short-wavelength mode, such as the kinetic Alfvén wave, which is subsequently Landau absorbed within the plasma. For high temperatures this short-wavelength mode can also be radiated into the plasma without being completely absorbed. It is also shown that the related ion–sound surface wave mode and instability identified by Alexandrov *et al.* (1984) are unphysical, and are the result of neglecting the gas pressure in the first-order magnetic field boundary condition.

1. Introduction

The three low-frequency ($\omega \ll \Omega_e,\ \omega_{pe}$) magnetohydrodynamic (MHD) bulk wave modes which can propagate in an unbounded homogeneous and magnetised plasma are well known. They are the fast magneto–sound (FMS) and Alfvén modes, referred to as the compressional and shear Alfvén modes respectively in some of the older literature, and the ion–sound or slow magneto–sound mode. In a bounded plasma there are, in general, two related surface wave modes which can also propagate. They are the fast and ion–cyclotron surface waves considered by Cramer and Donnelly (1983). These MHD surface modes propagate along the bounding surface and decay exponentially in amplitude into both the plasma and the bounding medium. A third surface wave mode, the ion–sound surface mode identified by Alexandrov *et al.* (1984), can be shown to be unphysical in that it arises due to the neglect of the gas pressure in the boundary condition for the wave magnetic field. This point is discussed in detail in Section 7.

A substantial amount of work has already been done on fast and ion–cyclotron surface waves, particularly in the ideal MHD limit, that is, for wave frequencies well below the ion–cyclotron frequency ($\omega \ll \Omega_i$). In this limit these surface waves are degenerate (same dispersion relation) and are commonly referred to as magneto–sound or Alfvén surface waves. Of particular interest in many applications are the various mechanisms via which these surface waves are damped, as it is through wave absorption that plasma heating occurs. Collisional (resistive

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and viscous) damping, for instance, has been studied in the ideal MHD limit by Gordon and Hollweg (1983) and Steinolfson *et al.* (1986) in connection with solar coronal heating. A form of collisionless damping which has also been studied in some detail is Alfvén resonance absorption. In the ideal MHD limit this has been considered by Chen and Hasegawa (1974), Ott *et al.* (1978), Ionson (1978) and Kuperus *et al.* (1981). More recently, Cramer and Donnelly (1983) have extended the analysis to frequencies up to and above the ion–cyclotron frequency. This mechanism, however, operates only when the surface layer is smooth.

In the case of a sharp plasma–vacuum interface an alternative collisionless damping mechanism has been identified for ideal MHD Alfvén surface waves in high temperature plasmas [$V_{e\|} \gg v_A$, where $V_{e\|}$ is the thermal (r.m.s.) electron speed along the ambient magnetic field and v_A is the Alfvén speed] by Cramer and Donnelly (1992). This mechanism is similar to Alfvén resonance absorption in that it involves mode conversion to a short-wavelength mode, in this case the kinetic Alfvén wave (KAW), which propagates energy away from the surface. We note that the calculation of Cramer and Donnelly involved a fluid theory approach which is incapable of accounting for Cherenkov (Landau and transit-time magnetic) and cyclotron absorption due to resonant wave–particle interactions. As in the case of Alfvén resonance absorption, the surface wave damping is due to the mode conversion itself and is independent of the subsequent fate of the KAW which, in the absence of Cherenkov and cyclotron absorption, is radiated into the plasma without being absorbed.

In general one is justified in neglecting electron–cyclotron absorption of fast and ion–cyclotron surface waves given that they satisfy the condition $\omega \ll \Omega_e$, that is, they have frequencies well below the electron–cyclotron frequency. Ion–cyclotron absorption can be significant for wave frequencies close to the ion–cyclotron frequency ($\omega \approx \Omega_i$), however, in the present paper we restrict our attention to Cherenkov absorption. It is important to consider Cherenkov absorption for two reasons. Firstly, it enables us to consider the absorption of the radiating KAW and thereby the possibility of plasma heating. Secondly, the inclusion of Cherenkov absorption is important when $V_{e\|} \lesssim v_{\mathrm{A}}$, as it can lead to significant damping of the surface waves even though, in that case, the KAW does not radiate.

We note that Cherenkov absorption of ideal MHD Alfvén surface waves has been considered recently by Assis and Busnardo-Neto (1987) and De Assis and Tsui (1991) as a possible mechanism for the heating of the solar corona, and it was concluded that the surface waves are absorbed mainly via transit-time magnetic damping. In these papers, however, the plasma was effectively assumed to be infinitely conducting through the neglect of the short-wavelength mode in the analysis, and as discussed by Rowe (1992) this is a valid simplification only when one is free to assume that the electron mass $m_e = 0$. Given that the damping rate found in these papers is proportional to m_e the theory appears to be inherently inconsistent, and it is thus important to reconsider Cherenkov absorption of these surface waves.

In the present paper we account for the possibility of Cherenkov absorption of fast and ion–cyclotron surface waves using a general kinetic theory of surface waves developed by Rowe (1991), and generalised slightly to account for the gas pressure in the first-order magnetic field boundary condition (this extension of the theory is detailed in Appendix 1). This theory correctly accounts for the short-wavelength mode and is a relatively straightforward way of calculating surface wave damping. The results thus obtained correctly reproduce the ideal MHD results of Cramer and Donnelly (1992) in the cold plasma ($V_{e\|} \ll v_A$) and hot plasma ($V_{e\|} \gg v_A$) limits, for which Cherenkov absorption of the waves can be neglected. In Section 2 the method of calculation is summarised and we define a general surface wave absorption coefficient in analogy with the familiar bulk wave absorption coefficient. In Section 3 we summarise the properties of the fast and ion–cyclotron surface waves in a cold plasma. This serves two useful purposes. Firstly, it allows some notation to be introduced, and secondly, it is a natural point from which we can consider the thermal effects as a perturbation.

Section 4 then details the appropriate dielectric tensor for these waves in a thermal plasma. Note that Cramer and Donnelly (1992) only considered electron motion along the magnetic field lines [$V_{e\perp} = 0$, where $V_{e\perp}$ is the thermal (r.m.s.) electron speed around the ambient magnetic field lines] through their choice of the dielectric tensor (although they do not explicitly state this), while Assis and Busnardo-Neto (1987) and De Assis and Tsui (1991) considered an isotropic ($V_{e\perp} = V_{e\|}$) velocity distribution of electrons. In this paper we use a more general distribution with an arbitrary electron velocity anisotropy. Section 5 deals with the calculation of the surface wave growth rate (which is negative for wave damping) and frequency shift, and the results are discussed in detail in Section 6. In Section 7 we consider the ion–sound surface wave identified by Alexandrov *et al.* (1984), and show that it is unphysical. We note that an important consequence of this result is that the ion–sound surface wave instability discussed by these authors does not exist.

2. Method of Calculation

A general method for calculating surface wave dispersion relations was first given by Rowe (1991) and has subsequently been discussed in detail by Rowe (1992), in the context of low frequency surface waves in a cold magnetised plasma. In this section we give only a general outline, concentrating on some definitions which we will need for the following sections. Any remaining definitions can be found in the aforementioned papers. We do, however, extend the above treatments by including a discussion of the surface wave absorption coefficient.

(a) Surface Wave Dispersion Relation

We assume that a homogeneous plasma occupies the $x < 0$ half space and a vacuum occupies the $x > 0$ half space, the interface between the two media being sharp. The dispersion equation for surface waves propagating along the interface with frequency ω and surface wavevector $\mathbf{k}_s = (k_y, k_z)$ then takes the general form

$$Z_s(\omega, \mathbf{k}_s) = 0\,, \tag{1}$$

where $Z_s(\omega, \mathbf{k}_s)$ is the determinant of the 2×2 matrix

$$\mathbf{Z}_s(\omega, \mathbf{k}_s) = \int \frac{dr_x}{2\pi} \boldsymbol{\Delta}_s(\omega, \mathbf{k}) \mathbf{Q}_s(\omega, \mathbf{k})\,, \tag{2}$$

with (:= denotes a definition)

$$\int \frac{dr_x}{2\pi} := \lim_{x\to 0^-} \int \frac{dr_x}{2\pi} e^{ik_x x}, \tag{3}$$

$r_x := k_x/k_s$ and $k_s := |\mathbf{k}_s| = (k_y^2 + k_z^2)^{\frac{1}{2}}$. The surface wave dispersion relations are then identified as the solutions $\omega = \omega_s(\mathbf{k}_s)$ of equation (1).

The matrix $\mathbf{Z}_s(\omega, \mathbf{k}_s)$ describes the response of the plasma–vacuum system to the surface wave fields and is thus called the response tensor of the system. The other two matrices involved in the integrand are the surface field propagator $\mathbf{Q}_s(\omega, \mathbf{k})$, which determines the surface components of the electric fields associated with the surface wave, and the kernel matrix $\mathbf{\Delta}_s(\omega, \mathbf{k})$, which encompases the electromagnetic boundary conditions. General forms of these matrices have been given in Rowe (1991, 1992) subject to the assumption that the first-order wave magnetic fields are continuous across the interface. This assumption is, however, not valid in general for a thermal magnetised plasma, and one requires a slightly more general form of the kernel matrix than that given in the above papers. This extension of the theory is not relevant to the following discussion and is detailed in Appendix 1. Specific forms of $\mathbf{Q}_s(\omega, \mathbf{k})$ and $\mathbf{\Delta}_s(\omega, \mathbf{k})$ appropriate for the problem of interest in this paper are provided in Section 5, and these generalise the specific forms given in Rowe (1992) for a cold magnetised plasma to the case of low frequency waves in a thermal plasma.

In general the response tensor (2) can be evaluated via contour integration, the poles in the integrand being determined by the zeros of the denominators in $\mathbf{Q}_s(\omega, \mathbf{k})$ and given by the familiar bulk wave dispersion equation

$$\Lambda(\omega, \mathbf{k}) = 0, \tag{4}$$

where $\Lambda(\omega, \mathbf{k})$ is the determinant of the 3×3 bulk plasma response tensor (Melrose 1986)

$$\mathbf{\Lambda}(\omega, \mathbf{k}) = n^2(\boldsymbol{\kappa}\boldsymbol{\kappa} - \boldsymbol{\delta}) + \mathbf{K}(\omega, \mathbf{k}), \tag{5}$$

and $\mathbf{K}(\omega, \mathbf{k})$ is the dielectric tensor of the plasma. Here $n = ck/\omega$ is the refractive index, $\boldsymbol{\kappa} = \mathbf{k}/k$ is the unit vector in the direction of propagation and $\boldsymbol{\delta}$ is the unit matrix. The response can thus be interpreted as a sum

$$\mathbf{Z}_s(\omega, \mathbf{k}_s) = \sum_M \mathbf{Z}_{sM}(\omega, \mathbf{k}_s) \tag{6}$$

over bulk mode contributions (labelled by M)

$$\mathbf{Z}_{sM}(\omega, \mathbf{k}_s) := \mathbf{\Delta}_{sM}(\omega, \mathbf{k}_s)\mathbf{Q}_{sM}(\omega, \mathbf{k}_s) \tag{7}$$

with

$$\begin{aligned} \mathbf{\Delta}_{sM}(\omega, \mathbf{k}_s) &:= \mathbf{\Delta}_s(\omega, k_M, \mathbf{k}_s), \\ \mathbf{Q}_{sM}(\omega, \mathbf{k}_s) &:= -i \lim_{r_x \to r_M} [(r_x - r_M)\mathbf{Q}_s(\omega, \mathbf{k})], \end{aligned} \tag{8}$$

corresponding to a given pole $r_x = r_{xM} \equiv r_M$ (or equivalently, $k_x = k_{xM} \equiv k_M$). Note that in general there are contributions only from the poles located in the lower half of the complex r_x plane in which the contour of integration must be closed, that is, $\mathrm{Im}\, r_M \leq 0$. As noted in Rowe (1991, 1992) this corresponds to wave fields (for each bulk mode M) which decay exponentially into the plasma, given that the wave fields vary as $\exp(ik_M x)$ for each mode, and is in accordance with the definition of a surface wave. (In Section 6 it is shown that poles with $\mathrm{Im}\, r_M > 0$ can contribute when the surface wave is a radiating mode or *leaky* surface wave.)

It was noted in Rowe (1992) that for a magnetised plasma, as we are dealing with in this paper, the determinants of the matrices $\mathbf{Z}_{sM}(\omega, \mathbf{k}_s)$ are identically zero and that as a result the surface wave dispersion equation (1) may be written in the form of a trace. In the case considered in this paper there are only two contributing bulk modes ($M = \pm$, say) and we obtain specifically the result (Tr[ ] denotes the trace)

$$Z_s(\omega, \mathbf{k}_s) = \mathrm{Tr}[\mathbf{Z}_{s+}(\omega, \mathbf{k}_s)\boldsymbol{\zeta}_{s-}(\omega, \mathbf{k}_s)] = 0\,, \tag{9}$$

where $\boldsymbol{\zeta}_{s-}(\omega, \mathbf{k}_s)$ is the matrix of cofactors of $\mathbf{Z}_{s-}(\omega, \mathbf{k}_s)$. This result will be used in Section 5 when we consider the fast and ion–cyclotron surface waves in a thermal plasma.

(b) Surface Wave Absorption Coefficient

Surface wave damping is treated by including the antihermitian part of the plasma–vacuum response tensor $\mathbf{Z}_s(\omega, \mathbf{k}_s)$ and allowing the frequency and wavenumber (here ω and $\mathbf{k}_s$) to have small imaginary parts, assuming that the dissipative effects are of first order (weak damping assumption). Including dissipative effects arising from the antihermitian part of the response as a first-order correction $\mathrm{Im}\, \Delta Z_s(\omega, \mathbf{k}_s)$ to the dispersion equation (1) we write

$$Z_s^0(\omega, \mathbf{k}_s) + \mathrm{Im}\ \Delta Z_s(\omega, \mathbf{k}_s) = 0\,, \tag{10}$$

where the solutions $\omega = \omega_s(\mathbf{k}_s)$ of $Z_s^0(\omega, \mathbf{k}_s) = 0$ determine the frequencies (dispersion relations) of the undamped surface waves in the absence of dissipation. Equation (10) is solved by balancing the small correction $\mathrm{Im}\, \Delta Z_s(\omega, \mathbf{k}_s)$ with the small corrections to the frequency and surface wavenumber by making the replacements $\omega \to \omega_s(\mathbf{k}_s) + i\omega_i(\mathbf{k}_s)$ and $\mathbf{k}_s \to \mathbf{k}_s + i\mathbf{k}_{si}$. Here, $\omega_i(\mathbf{k}_s)$ is the *growth rate* of the surface waves, and $|\mathbf{k}_{si}|$ is inversely proportional to the *absorption* (or *attenuation*) *length* of the surface waves in the surface plane. We then obtain an equation which relates all first-order quantities, and defining the *surface wave absorption coefficient*

$$\gamma_s(\mathbf{k}_s) := -2\{\omega_i(\mathbf{k}_s) - \mathbf{k}_{si} \cdot \mathbf{v}_{gs}(\mathbf{k}_s)\}\,, \tag{11}$$

where

$$\mathbf{v}_{gs}(\mathbf{k}_s) := \frac{\partial \omega_s(\mathbf{k}_s)}{\partial \mathbf{k}_s} \tag{12}$$

is the *surface wave group velocity*, we obtain in analogy with the bulk wave result (e.g. Melrose 1986)

$$\gamma_s(\mathbf{k}_s) = \left.\frac{2\mathrm{Im}\ \Delta Z_s(\omega, \mathbf{k}_s)}{\partial Z_s^0(\omega, \mathbf{k}_s)/\partial\omega}\right|_{\omega_s(\mathbf{k}_s)} . \tag{13}$$

In the context of surface waves we note that the term 'absorption' as used above refers to the decay of the surface wave energy within the vicinity of the surface at which it is localised. Thus, surface wave 'absorption' can occur either via resonant wave–particle interactions which transfer energy from the surface waves to the plasma particles, resulting in plasma heating, or via radiation of surface wave energy away from the surface due to mode coupling to a freely propagating wave such as in the case of a leaky surface mode. In the latter case, absorption of the surface waves does not immediately imply that plasma heating occurs, and one needs to consider separately the various absorption mechanisms applicable to the propagating mode.

Finally, let us note that if we assume $\mathbf{k}_{si} = 0$ (corresponding to a surface wave which is uniformly excited along the surface plane at some initial time), and we allow for a real frequency shift $\Delta\omega(\mathbf{k}_s)$ in addition to the imaginary shift $i\omega_i(\mathbf{k}_s)$, we can write instead of (13)

$$\Delta\omega(\mathbf{k}_s) + i\omega_i(\mathbf{k}_s) = -\left.\frac{\Delta Z_s(\omega, \mathbf{k}_s)}{\partial Z_s^0(\omega, \mathbf{k}_s)/\partial\omega}\right|_{\omega_s(\mathbf{k}_s)} . \tag{14}$$

In this paper we are mostly concerned with the $\mathbf{k}_{si} = 0$ case, and we include the real frequency shift for completeness. This provides an additional point of comparison between the results to be derived in this paper and those of Cramer and Donnelly (1992) who obtained the frequency shift for the ideal MHD surface waves in a cold plasma (using a different approach to that of this paper).

3. Review of the Cold Plasma Results

We begin our investigation of fast and ion–cyclotron surface waves in a thermal magnetised plasma by summarising the results for a cold plasma. In Section 5 thermal effects will be considered by treating them as small corrections to these cold plasma results. The cold plasma dispersion relation has previously been obtained by Cramer and Donnelly (1983) and more recently by Rowe (1992), the latter using the method which we have briefly described in the preceding section of this paper. In the interests of continuity, we shall follow the paper of Rowe here.

Assuming that the ambient magnetic field $\mathbf{B}_0$ is directed along the z-axis (and is thus parallel to the bounding surface), the dielectric tensor for a cold plasma is of the form (Stix 1962; Melrose 1986)

$$\mathbf{K}(\omega) = \begin{pmatrix} S(\omega) & -iD(\omega) & 0 \\ iD(\omega) & S(\omega) & 0 \\ 0 & 0 & P(\omega) \end{pmatrix} , \tag{15}$$

where $S(\omega)$, $D(\omega)$ and $P(\omega)$ are functions of the frequency only. For the description of low frequency ($\omega \ll \Omega_e$, ω_{pe}) fast and ion–cyclotron surface waves the relevant approximations of these functions can be written as

$$S \approx \frac{n_A^2}{(1-f^2)}, \quad D \approx -\frac{n_A^2 f}{(1-f^2)}, \quad P \approx -\frac{n_A^2}{\mu f^2}, \tag{16}$$

where $f := \omega/\Omega_i$ is defined to be the normalised frequency,

$$n_A := c/v_A \gg 1 \tag{17}$$

is the Alfvén refractive index, with $v_A = B_0/(\mu_0 \rho_i)^{\frac{1}{2}}$ the Alfvén speed, and

$$\mu := \frac{\Omega_i}{\Omega_e} \ll 1 \,. \tag{18}$$

The specific forms of the surface field propagator $\mathbf{Q}_s(\omega, \mathbf{k})$ and the kernel matrix $\mathbf{\Delta}_s(\omega, \mathbf{k})$ corresponding to the dielectric tensor above are given in Rowe (1992) and we will not repeat them here. We note, however, that these may be obtained from the more general results of Section 5 by considering the cold plasma limit.

The first step in the determination of the cold plasma dispersion relation is the identification of the poles which appear in the integrand of the response tensor (2). As stated in Section 2, these are determined by solving (4) for the wavenumber k_x. In general, only two poles contribute to the response in the cold plasma approximation (as there are only two poles in the lower half of the complex $r_x := k_x/k_s$ plane in which our contour of integration must be closed) and in the low frequency regime the squares of these poles (which shall be denoted by $r_\pm \equiv r_{x\pm}$) take the approximate forms

$$\begin{aligned} r_+^2 &\approx \frac{P(S - n_s^2 r_z^2)}{S n_s^2}, \\ r_-^2 &\approx \left(\frac{S - n_s^2}{n_s^2}\right) - \frac{D^2}{n_s^2(S - n_s^2 r_z^2)}, \end{aligned} \tag{19}$$

to dominant terms in μ, provided $(S - n_s^2 r_z^2) \not\approx 0$. Here, $n_s := ck_s/\omega$ is the surface refractive index and $r_z := k_z/k_s$. These poles correspond to the two bulk modes of the plasma which contribute to the surface wave fields. The two modes here labelled simply $+$ and $-$ have been given the generic names short-wavelength or quasi-electrostatic wave (QEW) and magnetohydrodynamic (MHD) modes respectively by Cramer and Donnelly (1992).

The next step in the determination of the dispersion relation is the calculation of the contributions $\mathbf{Z}_{s\pm}(\omega, \mathbf{k}_s)$ to the response [see equation (6) with $M = \pm$]. In view of the inequality (18) these contributions may be expanded in powers of $\sqrt{\mu}$, and the approximate forms (19) for the two poles are valid to $O(\sqrt{\mu})$, as are the approximate forms (16) for the dielectric tensor elements. The zeroth-order

$[O(1)]$ contributions lead to the zeroth-order dispersion equation which, neglecting displacement current effects, may be written in the form

$$r_- = i\left[\frac{(S - n_s^2)r_z^2 + Dr_y}{(S - n_s^2 r_z^2)}\right], \tag{20}$$

with $r_y := k_y/k_s = \pm(1 - r_z^2)^{\frac{1}{2}}$. This dispersion equation can be solved for the surface refractive index n_s by squaring both sides and using (19) to eliminate r_-^2. The result is

$$n_s^2 = \frac{n_A^2}{r_z^2[(f + r_y)^2 + (1 - f^2)]}, \tag{21}$$

and after further manipulation the positive frequency solutions are found to be

$$f = \alpha|r_z|[(\alpha^2 r_y^2 r_z^2 + 2 - r_z^2)^{\frac{1}{2}} + \alpha r_y|r_z|], \tag{22}$$

where we have defined the dimensionless parameter $\alpha := v_A k_s/\Omega_i$. As noted in Rowe (1992), equation (22) is equivalent to the solution of Cramer and Donnelly (1983), apart from notational differences.

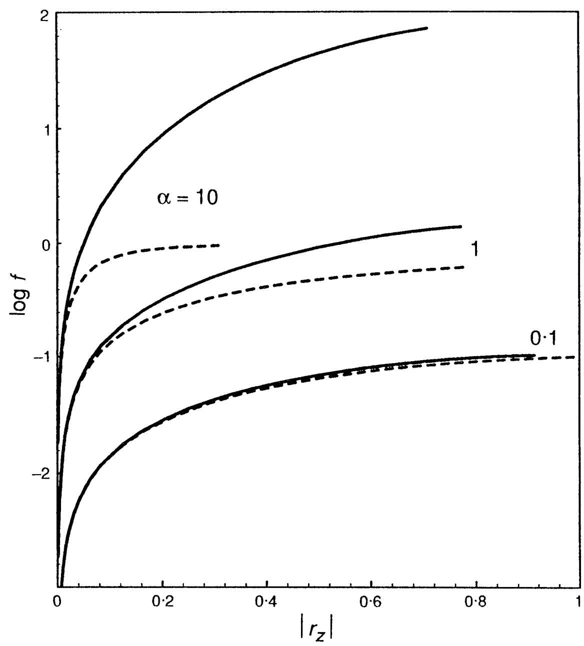

Fig. 1. Surface wave frequency f as a function of $|r_z|$ for various values of α. The solid curves are for the *fast* surface mode ($r_y > 0$) and the broken curves are for the *ion–cyclotron* surface mode ($r_y < 0$).

Plots of the cold plasma surface wave frequency f are shown in Fig. 1 for various values of α. As indicated, the solutions are valid only for restricted ranges of $|r_z| = |\cos\theta|$ (here θ is the angle between $\mathbf{r}_s := \mathbf{k}_s/k_s$ and the ambient

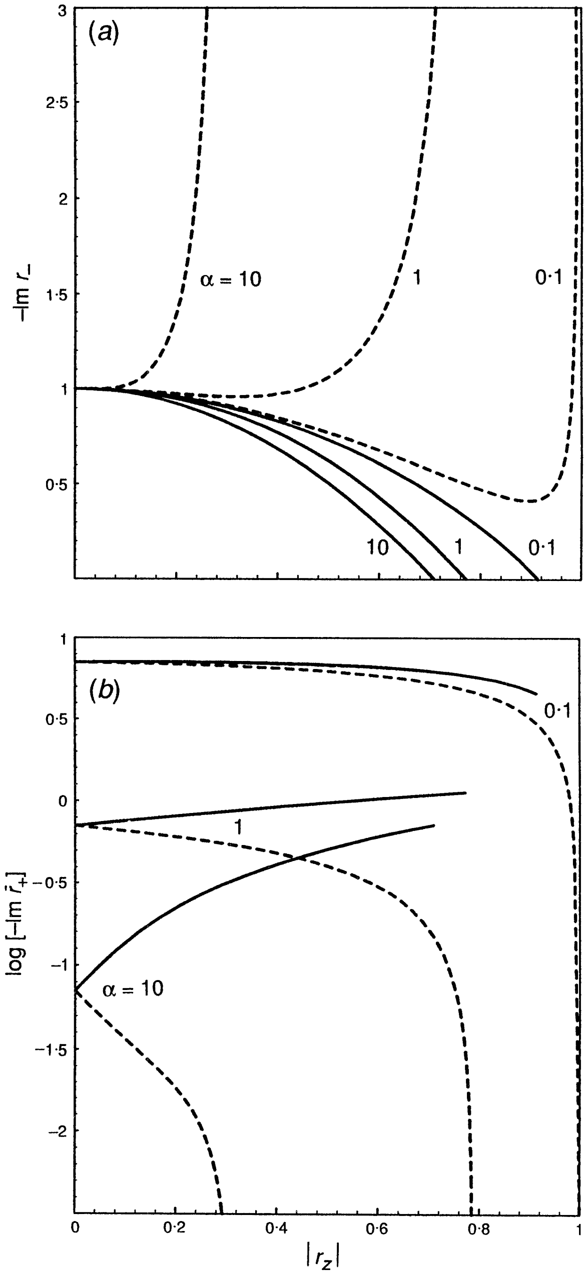

Fig. 2. Plots of (*a*) $-\mathrm{Im}\ r_-$ and (*b*) $-\mathrm{Im}\ \bar{r}_+$ as functions of $|r_z|$ for various values of α. The solid curves are for the *fast* surface mode ($r_y > 0$) and the broken curves are for the *ion–cyclotron* surface mode ($r_y < 0$).

magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$) beyond which (22) corresponds to spurious solutions introduced by squaring (20). Specifically, note that (21) and (22) are valid surface wave solutions of (20) only when the right-hand side of (20) is negative imaginary (we have already established that we need Im $r_- < 0$ for surface waves). Substituting the solution (21) into the right-hand side of (20) we obtain the $-$ mode wavenumber as a function of frequency:

$$r_- = i\left[\frac{f(1-2r_y^2) - r_y^3}{(f+r_y)0}\right] \tag{23}$$

with f as given in (22). Plots of $-$Im r_- as given by (23) are shown in Fig. 2*a*. For completeness note also that evaluating r_+^2 as given in (19), and taking the relevant root so that Im $r_+ < 0$, we have the $+$ mode wavenumber

$$\bar{r}_+ := \sqrt{\mu}r_+ = -i\frac{|(f+r_y)r_z|}{f}\,. \tag{24}$$

Plots of $-$Im $\bar{r}_+$ are given in Fig. 2*b*.

We now consider some of the features of these surface wave solutions. For frequencies close to and above the ion–cyclotron frequency Ω_i there are two distinct surface wave modes corresponding to $r_y > 0$ and $r_y < 0$. In the ideal MHD regime $f \ll 1$ (corresponding to $\alpha|r_z| \ll 1$) these two wave modes are degenerate with frequency given approximately by

$$f = \alpha|r_z|(1+r_y^2)^{\frac{1}{2}}\,. \tag{25}$$

This is the familiar result for Alfvén or magneto–sound surface waves and is the dispersion relation derived by Assis and Busnardo-Neto (1987).

In general, the cut-off frequency of the $r_y > 0$ mode corresponds to the point at which $r_- = 0$ (see Figs 1 and 2*a*). The cut-off frequency for this mode may thus be determined by setting $r_-^2 = 0$ in (19), which yields the familiar dispersion relation for the compressional and shear Alfvén bulk waves (e.g. Akhiezer *et al.*). Specifically, the cut-off frequency may be identified as the frequency of the fast magneto–sound (FMS) or compressional Alfvén branch of the bulk mode dispersion relation and at the cut-off the surface wave solution is said to merge with the FMS bulk wave spectrum. For this reason the $r_y > 0$ surface wave mode has been named the *fast* surface wave by Cramer and Donnelly (1983). Beyond the cut-off point only bulk waves can propagate with $r_y > 0$.

In the case of the $r_y < 0$ surface mode the cut-off frequency corresponds to the resonance $r_- = -i\infty$. In this case the cut-off frequency is given by the generalised Alfvén resonance frequency (Cramer and Donnelly 1992)

$$\omega_r = v_A|k_z|/(1+v_A^2k_z^2/\Omega_i^2)^{\frac{1}{2}}\,, \tag{26}$$

corresponding to $(S - n_s^2r_z^2) = 0$. It is easily shown that $\omega_r < \Omega_i$, and the $r_y < 0$ mode is thus closely related to the shear Alfvén branch of the bulk mode dispersion relation. Near $f \approx 1$ the shear Alfvén bulk mode is often referred to as the ion–cyclotron mode, and in analogy the surface wave mode has been referred to as the *ion–cyclotron* surface wave by Cramer and Donnelly (1983). Beyond this cut-off point only bulk waves can propagate with $r_y < 0$.

Note that at the ion–cyclotron surface mode resonance $r_- = -i\infty$ the surface wave dispersion relation is not strictly valid, as it was assumed in its derivation that $(S - n_s^2 r_z^2) \not\approx 0$ which implied, given $P \sim 1/\mu \gg 1$, that $|r_+| \gg |r_-|$. That this condition is violated near the resonance may been seen from Fig. 2 where the plots for r_- and $\bar{r}_+ := \sqrt{\mu} r_+$ are shown. In general the assumption $|r_+| \gg |r_-|$ is valid but near the resonance $r_- \to -i\infty$ while $r_+ \to 0$. It has been shown by Cramer and Donnelly (1992) that a proper expansion of the wavenumbers around the resonance point leads to the result $|r_+| \approx |r_-|$ at the ion–cyclotron surface wave cut-off. A numerical solution of the exact cold plasma dispersion equation (i.e. retaining all orders of $\sqrt{\mu}$) carried out by the author indicates that the sole effect of this is that the actual cut-off frequency for the ion–cyclotron surface wave is reduced slightly from that given by the approximate results of this section. For frequencies $f \approx 1$ we note that the resonance can also be removed by thermal effects associated with ion–cyclotron absorption.

4. Dielectric Tensor for a Thermal Plasma

In order to account for Cherenkov damping of fast and ion–cyclotron surface waves in a thermal plasma we must first identify the appropriate dielectric tensor $\mathbf{K}(\omega, \mathbf{k})$ using plasma kinetic theory. The most general result for a magnetised plasma is that of Melrose (1986) [equation (10·21)] which involves a distribution of particles $f(\mathbf{p})$ over particle momenta $\mathbf{p}$. In this paper we shall consider the non-streaming bi-Maxwellian particle distribution (normalised to number density n)

$$f(\mathbf{p}) = \frac{n}{(2\pi)^{3/2} m^3 V_\parallel V_\perp^2} \exp\left[-\frac{v_\parallel^2}{2V_\parallel^2} - \frac{v_\perp^2}{2V_\perp^2}\right], \tag{27}$$

which is appropriate in the nonrelativistic approximation. Here, $v_\parallel$ and $v_\perp$ are the particle speeds parallel and perpendicular to the ambient magnetic field, $V_\parallel$ and $V_\perp$ are the corresponding thermal (r.m.s.) particle speeds, and m is the particle mass. The exact dielectric tensor for this distribution is given by Melrose (1986) [equation (11·27) with $U = 0$] for the $k_y = 0$ reference frame. This form is too complicated, however, to be of any practical use and a number of approximations and simplifications, in addition to those which were made for the cold plasma dielectric tensor of the previous section, must be made. These are now outlined below.

(a) Simplifying Assumptions

We assume that the radius of gyration (Larmor radius) $R = V_\perp/\Omega$ of all species of particle is small compared with the wavelength $\lambda_\perp := 2\pi/k_\perp$ [where $k_\perp := (k_x^2 + k_y^2)^{\frac{1}{2}}$] so that the inequality

$$\frac{k_\perp^2 V_\perp^2}{\Omega^2} \ll 1 \tag{28}$$

is satisfied. In this limit (known as the *small gyroradius* limit) the only significant contributions to the time-irreversible (antihermitian) part of the dielectric tensor are those corresponding to the particle–wave resonances $\omega - k_z v_\| = 0$ ($s = 0$, where s is the harmonic number) and $\omega - k_z v_\| \mp \Omega = 0$ ($s = \pm 1$), which relate to Cherenkov and cyclotron absorption respectively. This condition also ensures that we are justified in our assumption that the surface layer can be regarded as arbitrarily sharp [recall that this is the assumption which was made in the derivation of the general surface wave dispersion equation (1)]. As shown in Appendix 2, the thickness of the surface layer is equal to the Larmor radius of the particles.

A further simplification is to assume that the ions (denoted by subscript i) are sufficiently cold that the conditions

$$|\omega \pm \Omega_i| \gg \sqrt{2}|k_z|V_{i\|}, \qquad \omega \gg \sqrt{2}|k_z|V_{i\|} \tag{29}$$

are satisfied and all thermal effects due to the ions can be ignored (effectively, $V_{i\|} \equiv 0$). Clearly, the first of these approximations breaks down for frequencies close to the ion–cyclotron frequency $|\omega| \approx \Omega_i$ where cyclotron absorption by ions becomes significant. In the present paper, however, we are only interested in Cherenkov absorption by electrons. In a similar manner we assume that the electrons satisfy (noting that $\omega \ll \Omega_e$ from Section 3)

$$|\omega \pm \Omega_e| \approx \Omega_e \gg \sqrt{2}|k_z|V_{e\|} \tag{30}$$

so that electron–cyclotron damping can also be neglected. Unlike the corresponding ion inequality, this condition is always easily satisfied in practice.

(b) Dielectric Tensor for $k_y = 0$

After making the above approximations the exact dielectric tensor reduces to (in the $k_y = 0$ frame)

$$\mathbf{K}(\omega, \mathbf{k}) = \begin{pmatrix} S & -iD & 0 \\ iD & S + \mu G k_x^2/k_z^2 & -iRk_x/k_z \\ 0 & iRk_x/k_z & F/\mu \end{pmatrix}, \tag{31}$$

where

$$S = \frac{n_A^2}{(1-f^2)}, \quad D = -\frac{n_A^2 f}{(1-f^2)},$$

$$G = -n_A^2(1+A_e)^2\frac{\bar{\phi}(z)}{z^2}, \quad R = \frac{n_A^2}{f}(1+A_e)[1-\bar{\phi}(z)], \tag{32}$$

$$F = 2z^2\frac{n_A^2}{f^2}[1-\bar{\phi}(z)],$$

and the plasma dispersion function $\bar{\phi}(z)$ is given by

$$\bar{\phi}(z) = ze^{-z^2}\left[2\int_0^z dte^{t^2} - i\sqrt{\pi}\frac{k_z}{|k_z|}\right] \tag{33}$$
$$= -i\sqrt{\pi}ze^{-z^2}\left[\frac{k_z}{|k_z|} - \mathrm{Erf}(-iz)\right]$$

with $\mathrm{Erf}(z)$ denoting the familiar error function (Abramowitz and Stegun 1965) and

$$z := \frac{\omega}{\sqrt{2}k_z V_{e\|}} \,. \tag{34}$$

The other quantity introduced above is the dimensionless parameter

$$A_e := \frac{V_{e\perp}^2}{V_{e\|}^2} - 1 = \frac{T_{e\perp}}{T_{e\|}} - 1\,, \tag{35}$$

which is a measure of the electron velocity, or temperature anisotropy. Note that the parallel and perpendicular plasma temperatures are defined by $T_{e\|} := m_e V_{e\|}^2$ and $T_{e\perp} := m_e V_{e\perp}^2$ respectively, in units where Boltzmann's constant is set to unity.

In the isotropic temperature case ($A_e = 0$) the dielectric tensor given above is consistent with that which appears in Stepanov (1958), apart from notational differences. It is also the dielectric tensor used by Assis and Busnardo-Neto (1987), De Assis and Tsui (1991) and Akhiezer *et al.* (1975), although in these papers and text numerous typographical errors have been made. We stress here that the dielectric tensor accounts only for Cherenkov damping, which includes Landau damping and transit-time magnetic damping, through Im $\bar{\phi}(z)$ which leads to the antihermitian part of the bulk plasma response.

It should be noted that in writing down the form of F as above we have also made the implicit assumption that $|z| \gg \sqrt{\mu}$. It is instructive to note that in the ideal MHD regime, where the fast and ion–cyclotron surface modes are degenerate with frequency given by (25), this condition is satisfied for these waves provided

$$\beta_{p\|} := \frac{P_{e\|}}{B_0^2/2\mu_0} = 2\left(\frac{v_s^2}{v_A^2}\right) \ll 1\,, \tag{36}$$

where $\beta_{p\|}$ is the parallel plasma beta (ratio of parallel gas pressure $P_{e\|} = n_e T_{e\|}$ to magnetic pressure $B_0^2/2\mu_0$) due to thermal electrons, and $v_s := V_{e\|}\sqrt{\mu}$ is the ion–sound speed [this definition is consistent with Melrose (1986), equation (1·6)]. Thus, in general the form of F given above is suitable for a low parallel beta plasma only. Also note that, provided the quantity $(1 + A_e)$ is not too large, (36) implies that the perpendicular plasma beta ($P_{e\perp} = n_e T_{e\perp}$)

$$\beta_{p\perp} := \frac{P_{e\perp}}{B_0^2/2\mu_0} = \beta_{p\|}(1 + A_e) \ll 1\,. \tag{37}$$

As shown in Appendix 2, this is a necessary condition for the ambient magnetic field to be assumed continuous across the interface as was done in the derivation of the general surface wave dispersion relation (Rowe 1991). The more general form of F is

$$F = \frac{n_A^2}{f^2}\left(2z^2[1-\bar{\phi}(z)] - \mu\right), \tag{38}$$

where the additional term is a contribution from the cold ions.

(c) Dielectric Tensor for $k_y \neq 0$

In the case of bulk waves the above result for the $k_y = 0$ frame of reference is sufficient as the bulk plasma exhibits cylindrical symmetry about the ambient magnetic field and consequently one may arbitrarily set $k_y = 0$ (or $k_x = 0$) to simplify the analysis. This symmetry is lost, however, in the presence of the surface. In particular, note that the low frequency surface waves in which we are interested exist only for $k_y \neq 0$, the surface wave modes being cutoff in general well before $|r_z| = 1$ (or equivalently $r_y = 0$) as was illustrated in Fig. 1. As a result we need the more general dielectric tensor with $k_y \neq 0$. This can be easily obtained from the $k_y = 0$ result by a unitary rotation about the z-axis given by

$$\mathbf{K}(k_y \neq 0) = \mathbf{R}\mathbf{K}(k_x = k_\perp, k_y = 0)\mathbf{R}^T, \tag{39}$$

where

$$\mathbf{R} = \frac{1}{k_\perp}\begin{pmatrix} k_x & -k_y & 0 \\ k_y & k_x & 0 \\ 0 & 0 & k_\perp \end{pmatrix} \tag{40}$$

is the unitary rotation matrix, $\mathbf{R}^T$ denotes the transpose of $\mathbf{R}$ and $k_\perp := (k_x^2+k_y^2)^{\frac{1}{2}}$. The dielectric tensor in this more general frame of reference is

$$\mathbf{K}(\omega,\mathbf{k}) = \begin{pmatrix} S+\mu G k_y^2/k_z^2 & -iD-\mu G k_x k_y/k_z^2 & iRk_y/k_z \\ iD-\mu G k_x k_y/k_z^2 & S+\mu G k_x^2/k_z^2 & -iRk_x/k_z \\ -iRk_y/k_z & iRk_x/k_z & F/\mu \end{pmatrix}. \tag{41}$$

We note that this reduces to the cold plasma dielectric tensor (15) with (16) in the cold plasma limit $V_{e\parallel} \to 0$ (or $|z| \gg 1$), as then $\bar{\phi} \approx 1 + 1/2z^2$ and both G and $R \to 0$, while $F/\mu \to P$. Conversely, in the high temperature limit $|z| \ll 1$ with an isotropic velocity distribution ($A_e = 0$) and $k_y = 0$, this result reproduces that of Melrose (1986) [equation (10·74)] used to derive the thermal corrections for ideal MHD bulk FMS and Alfvén waves [provided we use the more general form of F as given in (38)]. We are now in a position to calculate the response tensor for the plasma–vacuum system, and the growth rate of the fast and ion–cyclotron surface waves.

5. Calculation of the Growth Rate

(a) Propagator and Kernel Matrix

The elements of the surface field propagator $\mathbf{Q}_s(\omega,\mathbf{k})$ corresponding to the dielectric tensor (41) of the previous section can be written down with respect to the standard basis in the form

$$
\begin{aligned}
Q_{yy}(\omega,\mathbf{k}) &= \frac{[(S-n_s^2)+\mu G r_y^2/r_z^2][F/\mu - n_s^2 r_\perp^2] - [n_s^4 r_x^2 r_z^2 + R^2 r_y^2/r_z^2]}{\Lambda(\omega,\mathbf{k})}\,,\\
Q_{yz}(\omega,\mathbf{k}) &= -\frac{[(S-n_s^2)-n_s^2 r_x^2]n_s^2 r_y r_z - iHr_x/r_z + [DR+\mu G n_s^2 r_\perp^2]r_y/r_z}{\Lambda(\omega,\mathbf{k})}\,,\\
Q_{zy}(\omega,\mathbf{k}) &= -\frac{[(S-n_s^2)-n_s^2 r_x^2]n_s^2 r_y r_z + iHr_x/r_z + [DR+\mu G n_s^2 r_\perp^2]r_y/r_z}{\Lambda(\omega,\mathbf{k})}\,,\\
Q_{zz}(\omega,\mathbf{k}) &= \frac{(S-n_s^2 r_z^2)[(S-n_s^2 r_z^2)-(n_s^2 r_z^2-\mu G)r_\perp^2/r_z^2]-D^2}{\Lambda(\omega,\mathbf{k})}\,,
\end{aligned}
\tag{42}
$$

where $r_\perp^2 := r_x^2 + r_y^2$ and we have defined

$$
H := D n_s^2 r_z^2 + R(S - n_s^2 r_z^2)\,. \tag{43}
$$

The corresponding elements of the kernel matrix $\mathbf{\Delta}_s(\omega,\mathbf{k})$ are, quite generally (using the form given in Appendix 1),

$$
\begin{aligned}
\Delta_{yy}(\omega,\mathbf{k}) &= \frac{(1-n_s^2 r_z^2)}{n_s^2} - \frac{r_x r_x^v (S-n_s^2 r_z^2) - iDr_x^v r_y}{[(S-n_s^2)+\mu G r_y^2/r_z^2]}\,,\\
\Delta_{yz}(\omega,\mathbf{k}) &= \frac{(n_s^2 r_y r_z^2 - i r_x^v R)}{n_s^2 r_z} - \frac{r_x^v r_y [n_s^2 r_x r_z + iRr_y/r_z]}{[(S-n_s^2)+\mu G r_y^2/r_z^2]}\,,\\
\Delta_{zy}(\omega,\mathbf{k}) &= \frac{[(S-n_s^2)-n_s^2 r_x r_x^v]r_y r_z + iDr_x^v r_z + \mu G(r_x r_x^v + r_y^2)r_y/r_z}{[(S-n_s^2)+\mu G r_y^2/r_z^2]}\,,\\
\Delta_{zz}(\omega,\mathbf{k}) &= \frac{(1-n_s^2 r_y^2)}{n_s^2} - \frac{r_x r_x^v[(S-n_s^2 r_y^2)+\mu G r_y^2/r_z^2] + iRr_y r_x^v}{[(S-n_s^2)+\mu G r_y^2/r_z^2]}\,,
\end{aligned}
\tag{44}
$$

where

$$
r_x^v := \frac{k_x^v}{k_s} = \left(\frac{1-n_s^2}{n_s^2}\right)^{\frac{1}{2}}, \tag{45}
$$

with Im $r_x^v \geq 0$, is the normalised vacuum wavenumber. These are generalisations of the cold plasma results quoted in Rowe (1992) to allow for the thermal corrections to the dielectric tensor as given in the previous section.

As in the case of a cold plasma we can write the determinant $\Lambda(\omega,\mathbf{k})$ in the form of a quadratic

$$
\begin{aligned}
\Lambda(\omega,\mathbf{k}) &= A(\omega,\mathbf{k}_s)r_\perp^4 - B(\omega,\mathbf{k}_s)r_\perp^2 + C(\omega,\mathbf{k}_s)\\
&= A(\omega,\mathbf{k}_s)(r_x^2 - r_+^2)(r_x^2 - r_-^2)\,,
\end{aligned}
\tag{46}
$$

where $r_\perp^2 := r_x^2 + r_y^2$ as above and the zeros are given by

$$
r_\pm^2 = \frac{B \pm (B^2 - 4AC)^{\frac{1}{2}}}{2A} - r_y^2\,. \tag{47}
$$

The coefficients in the thermal plasma case are

$$
\begin{aligned}
A(\omega,\mathbf{k}_s) &= Sn_s^4\left(1-\frac{\mu G}{n_s^2 r_z^2}\right),\\
B(\omega,\mathbf{k}_s) &= n_s^2\left[\left(\frac{F}{\mu}-\frac{(FG-R^2)}{n_s^2 r_z^2}+S\right)(S-n_s^2 r_z^2)-D^2+2DR\right],\\
C(\omega,\mathbf{k}_s) &= \frac{F}{\mu}\left[(S-n_s^2 r_z^2)^2-D^2\right],
\end{aligned}
\tag{48}
$$

which we note reduce to the correct cold plasma coefficients in the cold plasma limit discussed in the previous section. While we have not outlined the steps involved in the derivation of these coefficients it is worthwhile noting that they are most simply derived by appealing to the cylindrical symmetry (about the ambient magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$) of the bulk plasma response, that is, by using the $(k_y = 0)$ frame dielectric tensor (31) rather than the more complicated general frame result (41) which was required for the calculation of $\mathbf{Q}_s(\omega,\mathbf{k})$ and $\mathbf{\Delta}_s(\omega,\mathbf{k})$. The determinant (46) for the general frame is then retained by simply making the replacement $k_x^2 \to k_\perp^2$.

(b) Response Tensor

We can now evaluate the contributions $\mathbf{Z}_{s\pm}(\omega,\mathbf{k}_s)$ to the plasma–vacuum response corresponding to the two contributing bulk modes, according to (7) and (8). As in the cold plasma case, these contributions can be expanded in powers of $\sqrt{\mu}$. We note that the poles corresponding to (47) can be approximated to $O(\sqrt{\mu})$ by

$$
\begin{aligned}
r_+^2 &\approx \frac{F(S-n_s^2 r_z^2)}{\mu S n_s^2},\\
r_-^2 &\approx \left(\frac{S-n_s^2}{n_s^2}\right)-\frac{D^2}{n_s^2(S-n_s^2 r_z^2)},
\end{aligned}
\tag{49}
$$

which are the same as the cold plasma results (19) except that now r_+^2 involves thermal effects through F. The approximations (32) for the dielectric tensor elements are also valid to $O(\sqrt{\mu})$, and in order to obtain the thermal corrections to the dispersion equation we must now include the $O(\sqrt{\mu})$ terms in the expansions of $\mathbf{Z}_{s\pm}(\omega,\mathbf{k}_s)$. Again, we write $\bar{r}_+ := \sqrt{\mu} r_+$ so that all μ-dependence is explicit.

For the $-$ mode the surface field propagator, kernel matrix and response tensor all take the same forms to $O(\sqrt{\mu})$ as in the cold plasma case of Rowe (1992) (except that $\bar{r}_+$ and F are as defined in the present paper), and we can thus immediately write down the response tensor

$$
\mathbf{Z}_{s-}(\omega,\mathbf{k}_s) = \frac{iF(S-n_s^2)}{2Sn_s^4 r_- \bar{r}_+^2}\begin{pmatrix}\Delta_{yy-} & 0\\ \Delta_{zy-} & 0\end{pmatrix}+O(\mu)\,,
\tag{50}
$$

where

$$\begin{aligned}
\Delta_{yy-} &= \frac{(1-n_s^2r_z^2)(S-n_s^2) - r_-r_x^vn_s^2(S-n_s^2r_z^2) + iDr_x^vr_yn_s^2}{n_s^2(S-n_s^2)}\,, \\
\Delta_{zy-} &= \frac{[(S-n_s^2) - n_s^2r_-r_x^v]r_yr_z + iDr_x^vr_z}{(S-n_s^2)}\,. \qquad (51)
\end{aligned}$$

For the + mode the surface field propagator is given by the cold plasma result (again with $\bar{r}_+$ and F defined as in this paper) plus an additional $O(\sqrt{\mu})$ correction which, for convenience, we write in the form

$$\frac{\Delta\mathbf{Q}_{s+}(\omega,\mathbf{k}_s)}{\sqrt{\mu}} = \frac{iF}{2S^2n_s^4\bar{r}_+^3}\begin{pmatrix} 0 & -\dfrac{iSH\bar{r}_+}{Fr_z}\sqrt{\mu} \\ \dfrac{iSH\bar{r}_+}{Fr_z}\sqrt{\mu} & 0 \end{pmatrix} + O(\mu)\,. \qquad (52)$$

The kernel matrix for the + mode is, in full,

$$\sqrt{\mu}\boldsymbol{\Delta}_{s+}(\omega,\mathbf{k}_s) = \begin{pmatrix} \bar{\Delta}_{yy+} & \bar{\Delta}_{yz+} \\ \bar{\Delta}_{zy+} & \bar{\Delta}_{zz+} \end{pmatrix} + O(\mu)\,, \qquad (53)$$

where

$$\begin{aligned}
\bar{\Delta}_{yy+} &= \frac{-\bar{r}_+r_x^vn_s^2(S-n_s^2r_z^2) + [(1-n_s^2r_z^2)(S-n_s^2) + iDr_x^vr_yn_s^2]\sqrt{\mu}}{n_s^2(S-n_s^2)}\,, \\
\bar{\Delta}_{yz+} &= \frac{-n_s^4\bar{r}_+r_x^vr_yr_z^2 + \{(S-n_s^2)n_s^2r_yr_z^2 - ir_x^vR(S-n_s^2r_z^2)\}\sqrt{\mu}}{(S-n_s^2)n_s^2r_z}\,, \\
\bar{\Delta}_{zy+} &= \frac{-n_s^2\bar{r}_+r_x^vr_yr_z + [(S-n_s^2)r_yr_z + iDr_x^vr_z]\sqrt{\mu}}{(S-n_s^2)}\,, \qquad (54) \\
\bar{\Delta}_{zz+} &= \frac{-\bar{r}_+r_x^vn_s^2(S-n_s^2r_y^2) + [(1-n_s^2r_y^2)(S-n_s^2) - iRr_yr_x^vn_s^2]\sqrt{\mu}}{n_s^2(S-n_s^2)}\,.
\end{aligned}$$

The response tensor for this mode is then

$$\mathbf{Z}_{s+}(\omega,\mathbf{k}_s) = \begin{pmatrix} Z_{yy+} & Z_{yz+} \\ Z_{zy+} & Z_{zz+} \end{pmatrix} + O(\mu)\,, \qquad (55)$$

where we need only write down the two elements

$$\begin{aligned}
Z_{yz+} &= \frac{iF(S-n_s^2r_z^2)}{2S^2n_s^4\bar{r}_+^3}[(S-1)r_y + ir_x^vD]r_z\sqrt{\mu}\,, \\
Z_{zz+} &= -\frac{iF(S-n_s^2r_z^2)}{2S^2n_s^6\bar{r}_+^3}[Sn_s^2\bar{r}_+r_x^v - \{S(1-n_s^2r_y^2) - n_s^2r_z^2\}\sqrt{\mu}]\,, \qquad (56)
\end{aligned}$$

in view of the fact that the determinant $Z_s(\omega,\mathbf{k}_s)$ (in which we are interested) can be written as a trace as was outlined in Section 2 [see also equation (57) below]. We note that the determinants $Q_{s\pm}(\omega, \mathbf{k}_s) \equiv 0$ and $Z_{s\pm}(\omega, \mathbf{k}_s) \equiv 0$ to $O(\sqrt{\mu})$, as they must in accordance with the discussion in Rowe (1992).

(c) Growth Rate

Using (9) and the above results for $\mathbf{Z}_{s\pm}(\omega, \mathbf{k}_s)$, the determinant of the plasma–vacuum response evaluates to

$$\begin{aligned} Z_s(\omega, \mathbf{k}_s) &= Z_{zz+}Z_{yy-} - Z_{yz+}Z_{zy-} + O(\mu) \\ &= Z_s^0(\omega, \mathbf{k}_s) + \Delta Z_s(\omega, \mathbf{k}_s) + O(\mu)\,, \end{aligned} \tag{57}$$

where we can now identify the zeroth-order $[O(1)]$ and first-order $[O(\sqrt{\mu})]$ terms as

$$\begin{aligned} Z_s^0(\omega, \mathbf{k}_s) &= \left[\frac{(S-n_s^2)r_x^v}{4n_s^4(S-n_s^2r_z^2)r_-}\right]\Delta_{yy-} \\ &= \frac{r_x^v[(S-n_s^2)(1-n_s^2r_z^2) - r_x^v r_- n_s^2(S-n_s^2r_z^2) + iDn_s^2r_x^v r_y]}{4n_s^6(S-n_s^2r_z^2)r_-}\,, \end{aligned} \tag{58}$$

and

$$\begin{aligned} \Delta Z_s(\omega, \mathbf{k}_s) &= \frac{(S-n_s^2)}{4Sn_s^6(S-n_s^2r_z^2)r_-\bar{r}_+}[\{(S-1)r_y + ir_x^v D\}n_s^2 r_z \Delta_{zy-} \\ &\quad - \{S(1-n_s^2r_y^2) - n_s^2r_z^2\}\Delta_{yy-}]\sqrt{\mu} \end{aligned} \tag{59}$$

respectively. The zeroth-order dispersion equation $Z_s^0(\omega, \mathbf{k}_s) = 0$ with the identification (58) is the cold plasma dispersion equation given by Rowe (1992) and reduces to (20) when the displacement current effects are ignored. There are no thermal corrections in the zeroth order; these appear only in the first-order correction $\Delta Z_s(\omega, \mathbf{k}_s)$.

In order to evaluate the surface wave absorption coefficient as given by (13), or the frequency shift and growth rate as in (14), we must evaluate the derivative $\partial Z_s^0(\omega, \mathbf{k}_s)/\partial\omega$. The calculation is simplified somewhat by writing $Z_s^0(\omega, \mathbf{k}_s)$ in the form (discarding displacement current corrections)

$$Z_s^0(\omega, \mathbf{k}_s) = \frac{iS(S-n_s^2)[\{2fr_y + (1+r_y^2)\}n_s^2r_z^2 - n_A^2]}{4n_s^6(S-n_s^2r_z^2)r_-[(S-n_s^2)r_z^2 + Dr_y - ir_-(S-n_s^2r_z^2)]}\,, \tag{60}$$

and the required derivative evaluated at the zeroth-order surface wave frequency (22) is then found to be

$$\left.\frac{\partial Z_s^0(\omega, \mathbf{k}_s)}{\partial\omega}\right|_{\omega_s(\mathbf{k}_s)} = \left[\left(\frac{-iS(S-n_s^2)n_s^2r_z^2}{4n_s^6(S-n_s^2r_z^2)r_-}\right)\frac{\{fr_y + (1+r_y^2)\}}{\omega\{(S-n_s^2)r_z^2 + Dr_y\}}\right]_{\omega_s(\mathbf{k}_s)}. \tag{61}$$

We also have

$$\Delta Z_s(\omega_s(\mathbf{k}_s), \mathbf{k}_s) = \frac{(S-n_s^2)(Sr_y - D)r_z\Delta_{zy-}\sqrt{\mu}}{4Sn_s^4(S-n_s^2r_z^2)r_-\bar{r}_+}\,, \tag{62}$$

where Δ_{zy-} evaluates to $\Delta_{zy-} = (1 + ir_-)r_z/r_y$ and where the zeroth-order wavenumber $\bar{r}_+$ evaluates to (using equation 21)

$$\bar{r}_+ = |f + r_y|\xi/\alpha\beta_e\,, \tag{63}$$

with $\beta_e := V_{e\|}/v_A$ and $\xi := [1 - \bar{\phi}(z)]^{\frac{1}{2}}$. Here, the branch of the square root is chosen such that Im $\xi < 0$ and Re $\xi < 0$, corresponding to + mode fields which spatially decay into the plasma (Im $\bar{r}_+ < 0$) and a + mode energy flux away from the surface (Re $\bar{r}_+ < 0$). After further algebra the frequency shift and the growth rate are found to be (taking $\mathbf{k}_{si} = 0$ and omitting the argument $\mathbf{k}_s$ for brevity)

$$\begin{aligned}\frac{\Delta\omega}{\omega_s} &= \alpha\beta_e\left|\frac{f(1-2r_y^2)-r_y^3}{1+(f+r_y)r_y}\right|\frac{\mathrm{Im}\,\xi}{|\xi|^2}\sqrt{\mu}\,, \\ \frac{\omega_i}{\omega_s} &= \alpha\beta_e\left|\frac{f(1-2r_y^2)-r_y^3}{1+(f+r_y)r_y}\right|\frac{\mathrm{Re}\,\xi}{|\xi|^2}\sqrt{\mu}\,,\end{aligned} \tag{64}$$

where we stress that f is the normalised zeroth-order frequency of (22). We note for completeness that the absorption coefficient in general (i.e. for ω_i, $\mathbf{k}_{si} \neq 0$) is given by $\gamma_s = -2\omega_i$ with ω_i as given in (64). This may be used to calculate the absorption length of the surface waves along the surface when $\mathbf{k}_{si} \neq 0$.

Given that Im $\xi < 0$ and Re $\xi < 0$ we immediately note that the surface waves are frequency-shifted downwards ($\Delta\omega < 0$) and temporally damped ($\omega_i < 0$). We also note that the frequency shift and growth rate depend only upon the temperature of the electrons parallel to the ambient magnetic field via $V_{e\|}$. This dependence comes from the K_{zz} element of the dielectric tensor (41) which determines the Landau damping of the surface waves, which we note is absent in the fluid model of the plasma used by Cramer and Donnelly (1992). The results given in (64) also generalise those of Cramer and Donnelly (1992) in that they allow for frequencies close to and above the ion–cyclotron frequency (i.e. $f \not\ll 1$) and arbitrary $|r_z|$. In the following section we discuss the above results in detail, and show how those of Cramer and Donnelly (1992) are retained upon taking the limit $|r_z| \to 0$ (and therefore $f \ll 1$) and either $|z| \gg 1$ (*cold plasma limit*) or $|z| \ll 1$ (*hot plasma limit*).

6. Discussion of Results

(a) General Discussion

The damping of the surface waves can be attributed in general to the mode coupling of the − (MHD) mode in the plasma and the vacuum mode to the + (QEW) mode due to the finite conductivity of the plasma. [As discussed in Rowe (1992) this mode coupling is always present except in the infinite conductivity case, $\mu \equiv 0$.] Specifically, the damping rate arises from the $O(\sqrt{\mu})$ thermal corrections to the + mode response tensor $\mathbf{Z}_{s+}(\omega, \mathbf{k}_s)$, recalling that there are no thermal corrections to the − mode response, and $\omega_i \sim$ Re $\bar{r}_+ < 0$ [see equation (63)] corresponds to a + mode energy flux away from the surface and into the plasma.

Given that the antihermitian part of the dielectric tensor $\mathbf{K}(\omega, \mathbf{k})$ appears only in the + mode corrections to the response, it is clear that the − mode is not directly Landau absorbed. That the − mode cannot be Landau absorbed can be understood on a physical basis, given that Landau absorption of waves in magnetised plasmas is due to the acceleration of particles (in this case electrons) along the background magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$ via the z component E_z of the wave electric field. As shown in Rowe (1992), only the + mode has a nonzero E_z. Thus, taking into account that both the − mode and the vacuum mode are evanescent (and therefore cannot radiate energy away from the surface), it is clear that the temporal decay of both of these modes is due entirely to mode conversion to the + mode. This damping mechanism is similar to Alfvén

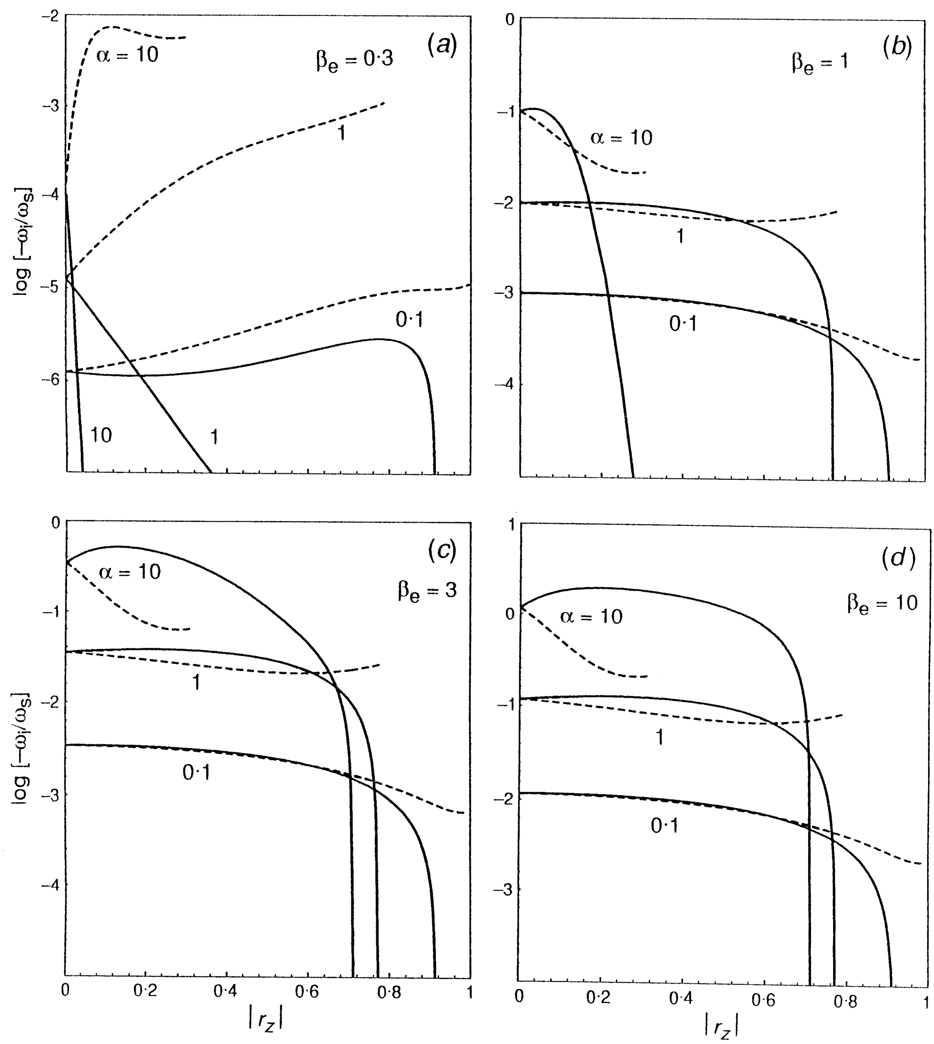

Fig. 3. Surface wave damping rate $-\omega_i/\omega_s$ as a function of $|r_z|$ for $Z_i/N_i = 1$ and various values of α and β_e. The solid curves are for the *fast* surface mode ($r_y > 0$) and the broken curves are for the *ion–cyclotron* surface mode ($r_y < 0$).

resonance absorption which occurs at smooth interfaces, except in this case the interface is sharp and the mode conversion takes place upon the reflection and transmission of the $-$ and vacuum mode energy fluxes at the surface. This conversion process may be interpreted as a *scattering* process in which the $-$ and vacuum modes are scattered into the $+$ mode by the sudden density variation which defines the plasma–vacuum interface.

Note that both the frequency shift and the growth rate vanish at the fast surface wave cut-off where the fast surface wave merges with the bulk FMS mode. This is consistent with the fact that the FMS bulk mode frequency shift and damping rate ($=$ $-$growth rate) are of second order $[O(\mu)]$ rather than first order $[O(\sqrt{\mu})]$, and we need to include the next order in our expansions of the previous section to obtain a nonzero result in that case. Also note that neither the frequency shift nor the growth rate obtained above vanish at the ion–cyclotron cut-off as we would also expect, but as stated in Section 3 the theory breaks down close to this cut-off.

Plots of the damping rate (divided by the surface wave frequency) $-\omega_i/\omega_s$ are given in Fig. 3 for plasma temperatures corresponding to $\beta_e = 0{\cdot}3$, 1, 3, and 10. In obtaining these plots we have used (noting that the neutron mass $\approx$ the proton mass)

$$\mu \approx \frac{m_e}{m_p}\frac{Z_i}{N_i}\,, \tag{65}$$

where $Z_i := q_i/q_e$ and N_i are the ionisation and nucleon numbers of the ions respectively, and $m_e/m_p = 5{\cdot}446 \times 10^{-4}$ is the ratio of the electron mass to proton mass. We have also taken $Z_i/N_i = 1$ which corresponds to a fully ionised hydrogen plasma. Given that $Z_i/N_i \leq 1$ these plots effectively give the maximum possible damping rates for these waves in an arbitrary electron–(single) ion plasma. Note also that we are limited to temperatures corresponding to $\beta_e \lesssim O(10)$ by the low parallel plasma beta assumption (36) used in the derivation of the dielectric tensor. This is not a great limitation, however, with $\beta_e = 10$ being suitable even for solar coronal conditions. The other relevant simplifying assumption used to approximate the dielectric tensor is (30), which is easily satisfied for the range of values of α and β_e as given.

The plots for $\beta_e = 0{\cdot}3$ and $\beta_e = 1$ are representative of a warm plasma $[|z| \sim O(1)]$ for which, as already pointed out, Landau absorption by electrons plays a crucial role. In this case the $+$ mode [or surface electrostatic wave (SEW)] is evanescent (Im $\bar{r}_+ < 0$) and can only be dissipated via Landau absorption, that is, it cannot be radiated to $x = -\infty$ as in the case of a leaky surface wave. In the absence of Landau absorption [Im $\bar{\phi}(z) = 0$] and thus of a dissipation mechanism for the $+$ mode, Re $\bar{r}_+ \sim$ Re $\xi \equiv 0$ [as Re $\bar{\phi}(z) > 1$ for $z \sim O(1)$] and mode conversion to the $+$ mode is inhibited. There is then no surface wave damping as in the fluid model used by Cramer and Donnelly (1992). The inclusion of Landau absorption shows that it can be an effective dissipation mechanism for the surface waves, as the $\beta_e = 1$ curves illustrate, particularly for shorter wavelengths (larger α). The absorption of the wave energy results in plasma electron heating within the vicinity of the surface as determined by the penetration depth $d_{s+} := 1/|k_{+i}|$ (where $k_{+i} = \text{Im}\ k_+$) of the $+$ mode into the plasma.

The plots given for $\beta_e = 3$ and in particular $\beta_e = 10$ are typical of a hot plasma ($|z| \ll 1$). In this case Landau absorption of the $+$ mode [or the kinetic Alfvén wave (KAW)] is not relevant to the calculation of the surface wave damping and can be neglected in the first instance by putting Im $\bar{\phi}(z) = 0$. Then $\bar{r}_+ \sim \xi$ is negative real for $\omega_i < 0$, by virtue of the fact that $|z| \ll 1$ implies Re $\bar{\phi}(z) \ll 1$, and the $+$ mode is not bound to the surface, that is, it is not evanescent but is propagating. The $+$ mode energy in this case is dissipated (removed from the vicinity of the surface) via radiation to $x = -\infty$ and the surface wave is damped due to mode conversion of the $-$ and vacuum modes to this radiating mode. This is an example of a leaky surface wave and is consistent with the result obtained by Cramer and Donnelly (1992) using fluid theory. From the plots we note that the wave damping can be large, and the weak damping assumption of Section 2 is not strictly valid for $\alpha \sim O(10)$. In that case $\omega_i/\omega_s \sim O(1)$ and the surface modes (particularly the fast surface mode) are strongly damped, that is, they damp out in about one wave period.

It is now instructive to consider the analytic results (64) in the cold and hot plasma limits, and show how the results of Cramer and Donnelly (1992) are retained explicitly. In the above discussion of the hot plasma case we have also omitted a discussion of the possibility of Landau absorption of the radiating KAW. In practice it is important to consider this as it is through absorption of the KAW by the plasma particles that the plasma can be heated. This is also considered in the remainder of this section.

(b) Cold Plasma Limit

In the cold plasma limit $|z| \gg 1$ the plasma dispersion function $\bar{\phi}(z)$ can be approximated by

$$\bar{\phi}(z) \approx 1 + \frac{1}{2z^2}\left[1 + \frac{3}{2z^2} - 2i\sqrt{\pi}|z|^3 e^{-z^2}\right]. \tag{66}$$

Then, upon approximating ξ (taking Im $\xi < 0$ as before), the frequency shift of (64) evaluates to

$$\frac{\Delta\omega}{\omega_s} = -\frac{f}{|r_z|}\left|\frac{f(1-2r_y^2) - r_y^3}{1+(f+r_y)r_y}\right|\left(1 - \frac{3\alpha^2\beta_e^2 r_z^2}{2f^2}\right)\sqrt{\mu}\,, \tag{67}$$

while the damping rate is exponentially small $[\omega_i \sim \exp(-z^2)]$ and can be neglected. Note that we have retained the first-order temperature correction to the frequency shift.

In the ideal MHD limit ($f \ll |r_y|$), corresponding to the approximate zeroth-order frequency solution (25), the frequency shift reduces further to

$$\frac{\Delta\omega}{\omega_s} = -\frac{f}{|r_z|}\frac{|r_y|^3}{(1+r_y^2)}\left(1 - \frac{3\beta_e^2}{2(1+r_y^2)}\right)\sqrt{\mu}\,, \tag{68}$$

which, given $|z| \gg 1$, is valid for electron temperatures corresponding to

$$\beta_e \ll \left(\frac{1+r_y^2}{2}\right)^{\frac{1}{2}} \leq 1\,. \tag{69}$$

Upon making the additional approximation $|r_z| \to 0$ ($|r_y| \to 1$) the frequency shift given in (68) reproduces that of Cramer and Donnelly (1992) [equation (51)] which we note is not valid for all values of k_z but only for $k_z \to 0$, due to their assumption on the low frequency dispersion relation, that is, they assume $f = \sqrt{2}\alpha|r_z|$. This result verifies the thermal correction of their work, provided we reinterpret the quantity β_e as $\beta_e/\sqrt{3}$, noting that their definition of V_e includes an additional factor of $\sqrt{3}$. This may be seen by noting that in the low temperature limit, K_{zz} of Section 4 can be written in the form

$$K_{zz} \approx -\frac{\omega_{pe}^2}{\omega^2}\left(\frac{\omega^2 + 3k_z^2 V_{e\|}^2}{\omega^2}\right) \approx -\frac{\omega_{pe}^2}{(\omega^2 - 3k_z^2 V_{e\|}^2)}\,, \tag{70}$$

which can then be directly compared with equations (3) and (8) of their work.

(c) Hot Plasma Limit

In the hot plasma limit $|z| \ll 1$, the appropriate approximation for the plasma dispersion function is

$$\bar{\phi}(z) \approx -i\sqrt{\pi}|z|\,, \tag{71}$$

and evaluating ξ (with Im $\xi < 0$) gives

$$\begin{aligned} \frac{\Delta\omega}{\omega_s} &= -\left(\frac{\pi}{8}\right)^{\frac{1}{2}} \frac{f}{|r_z|}\left|\frac{f(1-2r_y^2) - r_y^3}{1 + (f + r_y)r_y}\right|\sqrt{\mu}\,, \\ \frac{\omega_i}{\omega_s} &= -\alpha\beta_e\left|\frac{f(1-2r_y^2) - r_y^3}{1 + (f + r_y)r_y}\right|\sqrt{\mu}\,. \end{aligned} \tag{72}$$

As noted previously the surface wave damping in this case is independent of the antihermitian part of the dielectric tensor, that is, ω_i is independent of Im $\bar{\phi}(z)$. The frequency shift is, however, proportional to Im $\bar{\phi}(z)$ and remains finite in the limit $|z| \to 0$. Comparison of the frequency shift above with that of the cold plasma limit indicates that the temperature dependence of the frequency shift is weak.

In the ideal MHD limit the hot plasma results reduce to

$$\begin{aligned} \frac{\Delta\omega}{\omega_s} &= -\left(\frac{\pi}{8}\right)^{\frac{1}{2}} \frac{f}{|r_z|}\frac{|r_y|^3}{(1 + r_y^2)}\sqrt{\mu}\,, \\ \frac{\omega_i}{\omega_s} &= -\alpha\beta_e\frac{|r_y|^3}{(1 + r_y^2)}\sqrt{\mu}\,, \end{aligned} \tag{73}$$

which, given $|z| \ll 1$, are valid for

$$\beta_e \gg \left(\frac{1 + r_y^2}{2}\right)^{\frac{1}{2}} \geq (\tfrac{1}{2})^{\frac{1}{2}}\,. \tag{74}$$

The damping rate in (73) reproduces that of Cramer and Donnelly (1992), again upon taking the limit $|r_z| \to 0$, except their result was too large by a factor of $\sqrt{3}$, taking into account their different definition of the electron velocity V_e as previously discussed. The frequency shift was not found by Cramer and Donnelly as they used a fluid theory approach which does not include the antihermitian part of the dielectric tensor as we have done here. We stress here that the surface wave damping in the hot plasma case is independent of whether or not the radiated KAW is Landau absorbed by the plasma electrons.

(d) Landau Absorption of the KAW

Landau absorption of the KAW in a hot plasma can be accounted for by considering the KAW wavenumber r_+ of (49), which we noted previously is valid to $O(\sqrt{\mu})$. Expanding the wavenumber to first order in small quantities we obtain the zeroth-order real part

$$\bar{r}_+ = -\frac{|f + r_y|}{\alpha\beta_e}, \tag{75}$$

which is equivalent to (63) with $\xi = -1$ [$\bar{\phi}(z) = 0$], and the first-order (in small corrections) imaginary part (assuming $\mathbf{k}_{si} = 0$)

$$\bar{r}_{+i} = \bar{r}_+ \left[\frac{\sqrt{\pi}}{2}|z| + \left\{ \frac{1 + (f + r_y)^2}{(f + r_y)^2} \right\} \frac{\omega_i}{\omega_s} \right], \tag{76}$$

where it is understood that f is as given by (22). The first term arises from the $\bar{\phi}(z)$ correction to ξ, with $\bar{\phi}(z)$ given as in (71), while the second term arises from the $O(\sqrt{\mu})$ imaginary correction $i\omega_i$ to the surface wave frequency. In general these two terms are of the same order of magnitude for $\beta_e \sim O(10)$, and of opposite sign, and the imaginary contribution $\bar{r}_{+i}$ to the wavenumber may be either negative or positive.

The above result can be written in the alternative form

$$\gamma_s(\mathbf{k}_s) = \gamma_+(\mathbf{k}_s) - 2k_{+i}(\mathbf{k}_s)v_{gx+}(\mathbf{k}_s), \tag{77}$$

where we now write $\gamma_s = -2\omega_i$, and where

$$\frac{\gamma_+}{\omega_s} = \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \frac{f}{\alpha\beta_e|r_z|} \left[\frac{(f + r_y)^2}{1 + (f + r_y)^2} \right], \tag{78}$$

and

$$\frac{v_{gx+}}{v_A} = \frac{f}{\alpha\bar{r}_+} \left[\frac{(f + r_y)^2}{1 + (f + r_y)^2} \right] \sqrt{\mu}. \tag{79}$$

We can identify γ_+ as the bulk wave absorption coefficient for the KAW, that is, $\gamma_+(\mathbf{k}_s) := \gamma_+(k_+(\mathbf{k}_s), \mathbf{k}_s)$, where $\gamma_+(\mathbf{k})$ is given by the familiar result of Melrose (1986) [equation (2·67)]. We can also identify v_{gx+} as the x-component of the KAW group velocity, that is, $v_{gx+}(\mathbf{k}_s) := v_{gx+}(k_+(\mathbf{k}_s), \mathbf{k}_s)$, where

$$v_{gx+}(\mathbf{k}) := \frac{\partial\omega_+(\mathbf{k})}{\partial k_x}. \tag{80}$$

In making these identifications we note that the KAW frequency $\omega_+(\mathbf{k})$ as a function of $\mathbf{k}$ is obtained by rewriting equation (49) for r_+^2 as an equation for ω. Putting $\bar{\phi}(z) = 0$ we get the zeroth-order result $(k_+ \to k_x)$

$$\omega_+(\mathbf{k}) = v_A|k_z|\left(\frac{\Omega_i^2 + v_s^2k_x^2}{\Omega_i^2 + v_A^2k_z^2}\right)^{\frac{1}{2}}, \tag{81}$$

and it is then easily shown that $\omega_+(\mathbf{k}_s) := \omega_+(k_+(\mathbf{k}_s), \mathbf{k}_s) = \omega_s(\mathbf{k}_s)$, that is, the KAW frequency evaluated at the KAW wavenumber $k_+(\mathbf{k}_s)$ matches that of the surface wave. This is a necessary condition for the surface wave to couple to the KAW. Given these identifications, (77) is clearly a transfer equation for the KAW which states that the rate at which the surface wave energy density decays due to mode conversion to the KAW is equal to the rate at which the KAW energy density decays (at any fixed point in the plasma) due to Landau absorption and radiation into the plasma. Note that as v_{gx+} is of the same sign as $\bar{r}_+$ the KAW is a forward propagating mode, and given that $\bar{r}_+ < 0$, we also have $v_{gx+} < 0$, which corresponds to a flow of energy away from the surface and is consistent with the damping of the surface wave due to mode coupling to the KAW.

It is instructive to note that in the ideal MHD limit, $f \ll 1$, the KAW absorption coefficient can be written in the form

$$\frac{\gamma_+}{\Omega_i} = \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \alpha^3\beta_e|r_z|r_+^2\mu\,, \tag{82}$$

which agrees with the result of Melrose (1986) [equation (10·81a)] for the absorption coefficient of bulk Alfvén waves in an isotropic temperature plasma, provided we take into account that there $\omega = v_A|k_z|$ and $\tan^2\theta \approx r_+^2/r_z^2 \sim O(1\sqrt{\mu}) \gg \cot^2\theta$. This result is also consistent with the result for the damping rate of the KAW given by De Assis and Tsui (1991), again noting that they have $\omega = v_A|k_z|$.

Plots of $\gamma_+/2\omega_s$ as given by (78) are shown in Fig. 4 for $\beta_e = 10$. These plots effectively represent the rate at which the KAW is Landau absorbed by the plasma electrons. That the absorption coefficient γ_+ corresponds to Landau absorption due to the acceleration of electrons via E_z is clear, as γ_+ vanishes at the ion–cyclotron surface wave cut-off, for which $E_z = 0$ (Rowe 1992). The plots indicate that Landau absorption via mode conversion to the KAW is more efficient for fast surface waves than for ion–cyclotron surface waves, and also that the former are strongly absorbed for $\alpha \gtrsim O(10)$. The magnitude of the KAW group speed v_{gx+} normalised with the Alfvén speed is shown in Fig. 5 for $Z_i/N_i = 1$, $\beta_e = 10$ and two values of α. As indicated, the fast surface wave propagates energy into the plasma via the KAW at a greater speed than does the ion–cyclotron surface mode.

The imaginary part $r_{+i}/|r_+|$ of the KAW wavenumber is shown in Fig. 6 for $Z_i/N_i = 1$, $\beta_e = 10$ and two values of α. Note that the expansion used to obtain r_{+i} is only valid when $r_{+i}/|r_+| \ll 1$, and thus the approximation fails near the ion–cyclotron surface mode cut-off. Also, the expansion is not valid for $\alpha \gtrsim O(10)$, given that the weak damping assumption $\omega_i \ll \omega_s$ is then violated and the ω_i contribution to r_{+i} is no longer small (first order). Thus, we have presented plots only for $\alpha = 0{\cdot}1$ and $\alpha = 1$.

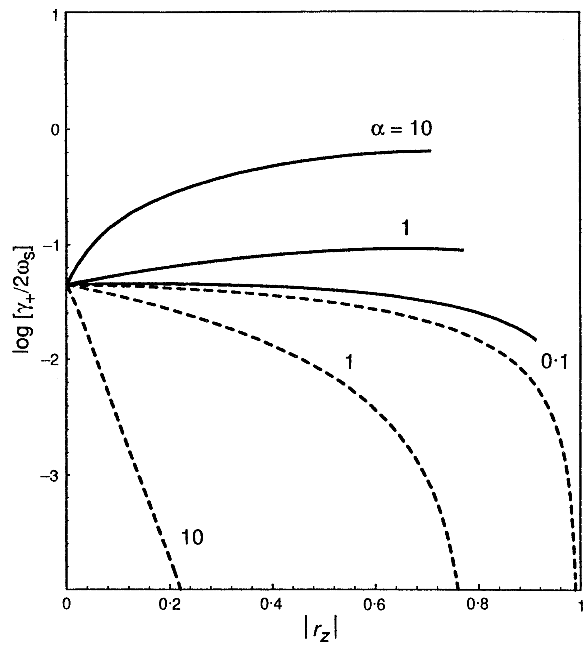

Fig. 4. Kinetic Alfvén wave absorption coefficient $\gamma_{+}/2\omega_s$ as a function of $|r_z|$ for various values of α and $\beta_e = 10$. The solid curves are for the *fast* surface mode ($r_y > 0$) and the broken curves are for the *ion-cyclotron* surface mode ($r_y < 0$).

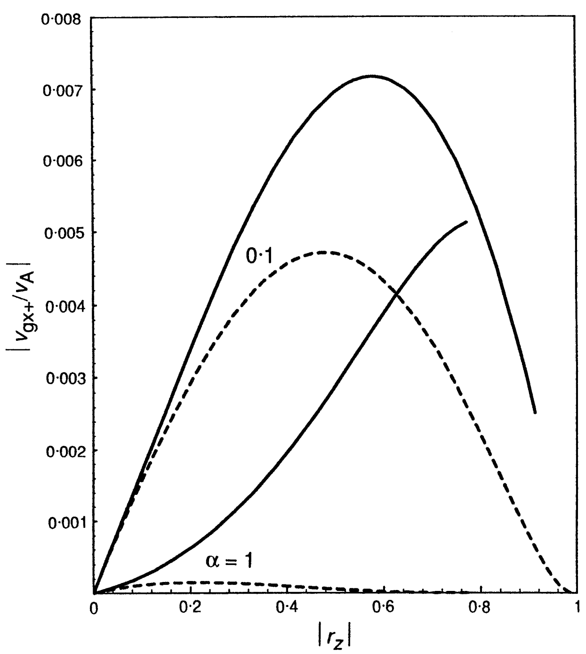

Fig. 5. Kinetic Alfvén wave group speed $|v_{gx+}/v_A|$ as a function of $|r_z|$ for $Z_i/N_i = 1$, $\beta_e = 10$ and two values of α. The solid curves are for the *fast* surface mode ($r_y > 0$) and the broken curves are for the *ion-cyclotron* surface mode ($r_y < 0$).

For $\alpha = 0 \cdot 1$, $r_{+i} < 0$ (except near the ion–cyclotron surface wave cut-off), and the plasma is opaque to the KAW. This corresponds to $\gamma_s < \gamma_+$, which indicates that the surface wave is absorbed into the bulk plasma via mode conversion to the KAW at a *slower* rate than that at which the radiated KAW is Landau absorbed by electrons. In this case all the surface wave energy is absorbed by the plasma electrons within the penetration depth $d_{s+} = 1/|k_{+i}|$ of the KAW, and the KAW does not radiate to $x = -\infty$. This is similar to the warm plasma case except that in the present case the penetration depth of the + mode is greater and as a result more of the plasma is heated.

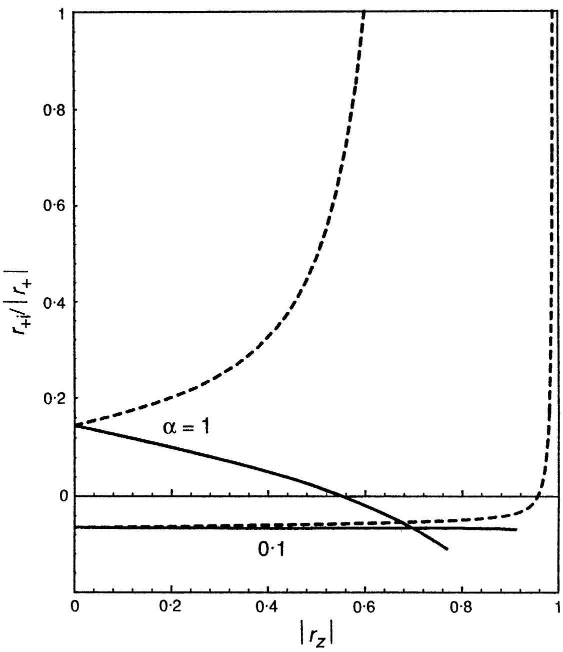

Fig. 6. Imaginary part of the kinetic Alfvén wave wavenumber $r_{+i}/|r_+|$ as a function of $|r_z|$ for $Z_i/N_i = 1$, $\beta_e = 10$ and two values of α. The solid curves are for the *fast* surface mode ($r_y > 0$) and the broken curves are for the *ion–cyclotron* surface mode ($r_y < 0$).

For $\alpha = 1$, we have $r_{+i} > 0$ (except close to the fast surface wave cut-off), and the KAW is no longer bound to the surface but grows in amplitude exponentially from the bounding surface. In this case $\gamma_s > \gamma_+$, which indicates that the surface wave is absorbed into the bulk plasma via mode conversion to the KAW at a *greater* rate than that at which the radiated KAW energy is dissipated by Landau absorption, and the remaining energy is radiated deeper into the plasma. In the idealised case of no Landau absorption ($\gamma_+ = 0$), as in the fluid model of Cramer and Donnelly (1992), all the surface wave energy is radiated away via the KAW without being absorbed by the plasma particles.

That the radiated KAW grows exponentially in amplitude away from the surface can be understood as follows. As the surface wave radiates, its energy is depleted and all the wave fields (including those of the KAW) decrease in amplitude at the surface in such a way that the boundary conditions remain satisfied. As the radiated KAW propagates through the plasma, however, its energy density remains constant in the absence of absorption. Thus, the energy density of the KAW is necessarily greater further away from the surface. When absorption is included the KAW will still grow exponentially into the plasma unless the rate of absorption exceeds the rate at which the KAW is radiated from the surface, as was the case for $\alpha = 0{\cdot}1$. A similar description has been given by Cally (1986) who considered the case of a cold plasma flux tube embedded in a vacuum. In that case it was found that surface waves can leak into the vacuum via mode conversion into a propagating vacuum wave which grows exponentially in amplitude into the vacuum.

The above discussion indicates that there is a subtlety in the interpretation of the contour involved in the determination of the plasma–vacuum response tensor. In Section 2 it was stated that in general only poles with negative imaginary parts contribute to the response of the plasma–vacuum system. However, in the above we can clearly have a contribution from the + mode corresponding to a pole $r_x = r_+$ in the upper half of the complex r_x plane, that is, Im $r_+ > 0$. From the above we see that the pole is initially in the lower half of the complex plane for small α, but crosses the real r_x axis as α is increased. In order that the pole does not cross the contour of integration it is thus implicit that we deviate the contour so that it always remains above the pole in question. At the same time the contour is deviated below the corresponding pole $r_x = -r_+$ which starts in the upper half of the complex plane and crosses into the lower half of the plane as α is increased. This ensures that this pole never contributes to the plasma–vacuum response. That neither pole crosses the contour of integration ensures that the flow of energy associated with the KAW remains directed away from the plasma–vacuum interface for all α. This prescription is not unlike the Landau prescription for evaluating singular integrals relating to causal functions (e.g. Melrose 1986).

7. Non-existence of the Ion–Sound Surface Mode

It was noted in Section 1 that in addition to the fast and ion–cyclotron surface modes a third surface wave mode, referred to as the ion–sound surface mode, has been identified by Alexandrov *et al.* (1984). In that work it was found that ion–sound surface waves are unstable, and this instability was attributed to the presence of diamagnetic surface currents which arise as a result of the cyclotron motion of electrons (for $V_{e\perp} \neq 0$) within the plasma and the sharp inhomogeneity across the plasma–vacuum interface. An immediate objection to this explanation is that the authors assumed that both the ambient and first-order magnetic fields are continuous across the interface, so that the diamagnetic surface currents are not taken into account in their boundary conditions. We note that the ion–sound surface mode solution obtained by these authors is not found to be a solution of the dispersion equation (57) presented herein. In the following we show that the ion–sound mode is in fact unphysical and arises (incorrectly) due to the neglect of the gas pressure in their first-order magnetic field boundary condition, that is, their assumption that the wave magnetic fields are continuous across the interface is invalid.

Neglecting the gas pressure contribution to the kernel matrix (Appendix 1) one can show that the following result for the $O(\sqrt{\mu})$ correction to the determinant of the response arises:

$$\Delta Z_s(\omega, \mathbf{k}_s) = \frac{(S - n_s^2)}{4Sn_s^6(S - n_s^2 r_z^2) r_z r_- \bar{r}_+} [\{(S-1) n_s^2 r_y r_z^2 + i r_x^v H\} \Delta_{zy-}$$
$$- \{S(1 - n_s^2 r_y^2) - n_s^2 r_z^2\} r_z \Delta_{yy-}] \sqrt{\mu} . \tag{83}$$

We immediately note that this result differs from the correct result of (59) only in that it involves the function R through H. The ion–sound surface mode of Alexandrov *et al.* (1984) satisfies the inequality $f \sim O(\sqrt{\mu}) \ll \alpha |r_z| < \alpha$, corresponding to $S \ll n_s^2 r_z^2 < n_s^2$, in which case the determinant of the response tensor simplifies to [to $O(1)$]

$$Z_s(\omega, \mathbf{k}_s) = \frac{S\bar{r}_+ + iRr_y\sqrt{\mu}}{2Sn_s^4 \bar{r}_+} . \tag{84}$$

We note specifically that this result is obtained from the zeroth- and first-order contributions to $Z_s(\omega, \mathbf{k}_s)$ as given in (58) and (83) by neglecting displacement current corrections, taking $D = 0$ and $r_- = -i$, and retaining only terms which are proportional to the function R in the first order contribution (83).

Neglecting dissipative effects [i.e. taking $\bar{\phi}(z) \equiv 0$], the dispersion equation $Z_s(\omega, \mathbf{k}_s) = 0$ leads to

$$f = \alpha\beta_e [r_z^2 + (1 + A_e)^2 r_y^2]^{\frac{1}{2}} \sqrt{\mu} . \tag{85}$$

Including the dissipative effects through the retention of Im $\bar{\phi}(z)$ as given in (71), we obtain the growth rate

$$\frac{\omega_i}{\omega_s} = -\left(\frac{\pi}{8}\right)^{\frac{1}{2}} \alpha\beta_e \frac{[r_z^2 - (1 + A_e)^2 r_y^2]}{f |r_z|} \mu . \tag{86}$$

Note that we have used the general form of the function F [equation (38)], given that we have $|z| \sim O(\sqrt{\mu})$. Upon making the assumptions $A_e = 0$ and $|r_z| \ll |r_y|$ these results simplify, and can be written in the form ($k_s \approx |k_y|$)

$$\omega_s = v_s |k_y|, \qquad \omega_i = \left(\frac{\pi}{8}\right)^{\frac{1}{2}} \frac{\omega^2}{|k_z| V_{e\|}} , \tag{87}$$

where $v_s := V_{e\|}\sqrt{\mu}$ is the ion–sound speed. These are precisely the dispersion relation and growth rate obtained by Alexandrov *et al.* (1984). Note that here the growth rate is positive ($\omega_i > 0$) which indicates that the surface waves grow. We stress here that the above results are unphysical in that they are derived without the pressure balance correctly accounted for in the magnetic field boundary condition.

It is instructive to consider the effect on the fast and ion–cyclotron surface waves of neglecting the gas pressure in the first-order magnetic field boundary condition. For these surface waves one then has

$$\Delta Z_s(\omega_s(\mathbf{k}_s), \mathbf{k}_s) = \frac{(S - n_s^2)[Sn_s^2 r_y r_z^2 - H] \Delta_{zy-} \sqrt{\mu}}{4Sn_s^6 (S - n_s^2 r_z^2) r_z r_- \bar{r}_+} , \tag{88}$$

in place of the correct result (62). This leads incorrectly to the following frequency shift and growth rate contributions:

$$\frac{\Delta\omega_\perp}{\omega_s} = \frac{\alpha\beta_e}{f}\left|\frac{f(1-2r_y^2)-r_y^3}{1+(f+r_y)r_y}\right|(1+A_e)(f+r_y)\mathrm{Im}\,\xi\sqrt{\mu}\,,$$

$$\frac{\omega_{i\perp}}{\omega_s} = -\frac{\alpha\beta_e}{f}\left|\frac{f(1-2r_y^2)-r_y^3}{1+(f+r_y)r_y}\right|(1+A_e)(f+r_y)\mathrm{Re}\,\xi\sqrt{\mu}\,, \tag{89}$$

in addition to those of (64). The subscript $\perp$ is used to indicate that these contributions vanish when $V_{e\perp}=0$, corresponding to zero gas pressure perpendicular to the interface.

Given that Re $\xi<0$ and Im $\xi<0$ as before, the signs of these frequency shift and growth rate contributions are determined by the sign of $(f+r_y)$. For the fast surface mode $(r_y>0)$ it is immediately obvious that $(f+r_y)\geq 0$, while for the ion–cyclotron surface mode $(r_y<0)$ we find that $(f+r_y)\leq 0$, noting that as discussed in Section 3 the ion–cyclotron surface mode is cut-off for frequencies $f>-r_y=|r_y|$. Thus, for the fast surface mode, $\omega_{i\perp}>0$, while for the ion–cyclotron waves, $\omega_{i\perp}<0$. For sufficiently low frequencies it follows that the incorrect neglect of the gas pressure implies that the fast surface waves are unstable in analogy with the unphysical ion–sound surface wave results.

8. Concluding Remarks

In this paper we have investigated the collisionless damping of fast and ion–cyclotron surface waves propagating on a sharp, thermal magnetised plasma–vacuum interface. The analysis has shown that (i) the dominant damping process for these surface waves is mode conversion to a short-wavelength mode such as the kinetic Alfvén mode; and (ii) the mode conversion requires the presence of a dissipation mechanism for the short-wavelength mode, such as Landau damping in a warm plasma or radiation in a hot plasma. These results differ from those of Assis and Busnardo-Neto (1987) and De Assis and Tsui (1991) who did not include the short-wavelength mode in their analysis. In particular we note that they concluded that the surface waves are damped predominantly due to transit-time magnetic damping. The results obtained in this paper are, however, in agreement with those of Cramer and Donnelly (1992) in the relevant limits. We have also shown that the ion–sound surface mode and instability identified by Alexandrov *et al.* (1984) are unphysical and are the result of an unphysical first-order pressure discontinuity across the interface.

The results obtained have application to the heating of the solar corona and to the magnetosphere. We note that one limitation of the analysis presented herein is the assumption of a low beta plasma, an assumption which is clearly not always valid, particularly in the astrophysical context. Thus, an important extension of the present work would be the inclusion of finite zeroth-order gas pressure in the plasma and the associated ambient magnetic field discontinuity across the interface.

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Appendix 1. Extension of the General Dispersion Relation

In the derivation of the general dispersion relation of Rowe (1991) it was assumed that both the ambient (zeroth-order) and the wave (first-order) magnetic fields are continuous across the plasma–vacuum interface. The assumption that the ambient field is continuous may be readily justified when the plasma beta is low (see Appendix 2) and is implicitly made in the following. The boundary condition on the first order magnetic field is, however, more generally

$$\mathbf{J}_s(\omega, \mathbf{k}_s) = -\frac{\hat{\mathbf{x}}_n \times \Delta\mathbf{B}(\omega, \mathbf{k}_s)}{\mu_0}\,, \tag{A1}$$

where $\Delta\mathbf{B}(\omega, \mathbf{k}_s) := \mathbf{B}(\omega, x = 0^-, \mathbf{k}_s) - \mathbf{B}(\omega, x = 0^+, \mathbf{k}_s)$ denotes the discontinuity in the magnetic field and $\mathbf{J}_s(\omega, \mathbf{k}_s)$ is a surface current. In the absence of extraneous surface currents (that is, in a medium of finite conductivity) one identifies $\mathbf{J}_s(\omega, \mathbf{k}_s)$ as the induced surface current which, in the case of a collisionless magnetised plasma, is most generally derived by considering the boundary terms which arise when the Vlasov equation is Fourier transformed according to the rules of Rowe (1991). The result for a low beta plasma–vacuum interface is

$$\mathbf{P}(\omega, \mathbf{k}_s) \cdot \hat{\mathbf{x}}_n = -\mathbf{J}_{s\mathrm{ind}}(\omega, \mathbf{k}_s) \times \mathbf{B}_0\,, \tag{A2}$$

with $\mathbf{P}(\omega, \mathbf{k}_s) := \mathbf{P}(\omega, x = 0^-, \mathbf{k}_s)$ and where

$$\mathbf{P}(\omega, \mathbf{k}) := m \int d^3\mathbf{p}\mathbf{v}\mathbf{v}f(\mathbf{p}, \omega, \mathbf{k}) \tag{A3}$$

is the first-order kinetic pressure tensor for a given species of particle with $f(\mathbf{p},\omega,\mathbf{k})$ the first-order perturbation to the distribution function of that species, as determined by solving the linearised Vlasov equation. The boundary condition above is consistent with the boundary condition of Stix (1962), and in physical terms states that the kinetic pressure exerted perpendicular to the boundary by a given species of particle is balanced by the pressure associated with the surface current due to that species and the background magnetic field.

A general result for the pressure tensor has been derived by Rowe (unpublished notes) and for the equilibrium particle distribution (27), using the simplifying assumptions as used in the derivation of the dielectric tensor in Section 4, one obtains the following result for arbitrary k_y:

$$\mathbf{P}(\omega,\mathbf{k}) = \frac{ien_e}{k_z}E_z(\omega,\mathbf{k})\begin{pmatrix} fR/n_A^2 & 0 & 0 \\ 0 & fR/n_A^2 & 0 \\ 0 & 0 & 1+f^2F/n_A^2 \end{pmatrix}, \tag{A4}$$

where the functions R and F are defined as in (32). We note that in the cold plasma limit this reduces to $\mathbf{P}(\omega,\mathbf{k}) = 0$ as expected, whilst in the high temperature limit of Section 6 one has, for an isotropic distribution ($A_e = 0$),

$$\mathbf{P}(\omega,\mathbf{k}) = \frac{ien_e}{k_z}E_z(\omega,\mathbf{k})\boldsymbol{\delta}\,. \tag{A5}$$

This is in fact the result obtained if one assumes an ideal gas law with isotropic pressure and uses the simpler fluid theory approach. We note that for a plasma with $V_{e\perp} = 0$ ($A_e = -1$) only the zz element of the pressure tensor is nonzero, in which case one is free to assume continuity of the first-order magnetic fields as in the calculation of Cramer and Donnelly (1992). In general, however, one must use the more general result (A 4), in which case the invalid assumption that the wave magnetic fields are continuous would lead to unphysical instabilities as discussed in Section 7.

Given the pressure tensor as above, one immediately finds that the induced surface current implied by the boundary condition (A 2) is

$$\mathbf{J}_{s\mathrm{ind}}(\omega,\mathbf{k}_s) = -\frac{i\omega\varepsilon_0 R}{k_z}E_z(\omega,\mathbf{k}_s)\hat{\mathbf{y}}\,, \tag{A6}$$

where $E_z(\omega,\mathbf{k}_s) := E_z(\omega, x = 0^-, \mathbf{k}_s)$ denotes the plasma field evaluated at the boundary and $\hat{\mathbf{y}}$ is the unit vector in the positive y direction. Using the magnetic field boundary condition (A1) it is then easy to show that the kernel matrix of Rowe (1991) and Rowe (1992) is generalised in this particular case to

$$\boldsymbol{\Delta}_s(\omega,\mathbf{k}) = \mathbf{r}_s\mathbf{r}_s + r_x^v(r_x^v - r_x)\boldsymbol{\delta} - r_x^v\mathbf{r}_s\frac{[\hat{\mathbf{x}}_n\cdot\boldsymbol{\Lambda}(\omega,\mathbf{k})]_s}{\Lambda_{xx}(\omega,\mathbf{k})} - \frac{ir_x^vR}{n_s^2r_z}\hat{\mathbf{y}}\hat{\mathbf{z}}\,, \tag{A7}$$

where the only change is the addition of the final gas pressure contribution.

It is instructive to note that the induced surface current given above may be regarded as being due to a magnetisation

$$\mathbf{M}(\omega, \mathbf{k}) = -\frac{i\omega\varepsilon_0 R}{k_z} E_z(\omega, \mathbf{k})\hat{\mathbf{z}} \tag{A8}$$

directed along the ambient magnetic field $B_0\hat{\mathbf{z}}$. Physically, this states that the magnetic dipoles in the plasma are aligned along the background magnetic field. In an unmagnetised plasma there is no such alignment and the net magnetisation $\mathbf{M}(\omega, \mathbf{k}) = 0$ by assumption. In that case one thus generally assumes continuity of the wave magnetic fields. The induced bulk current arising from this magnetisation is

$$\mathbf{J}_{s\mathrm{ind}}(\omega, \mathbf{k}) = i\mathbf{k} \times \mathbf{M}(\omega, \mathbf{k}) = \frac{\omega\varepsilon_0 R}{k_z} E_z(\omega, \mathbf{k})\mathbf{k} \times \hat{\mathbf{z}}\,, \tag{A9}$$

from which it immediately follows that the K_{xz} and K_{yz} elements of the dielectric tensor (41) correspond to this magnetisation, the remaining elements arising due to polarisation effects.

Appendix 2. Validity of the General Dispersion Relation

In the derivation of the general dispersion relation (Rowe 1991) and the extension of Appendix 1 two important assumptions were made which are not valid in a thermal magnetised plasma in general. We now wish to consider under what conditions these assumptions are valid, and in particular whether they are valid for a plasma described by the dielectric tensor given in Section 4 of this paper.

The first assumption is that the ambient magnetic field tangential to the surface is continuous across the plasma–vacuum interface. Consider the equation for pressure (gas and magnetic) balance across the equilibrium plasma–vacuum interface:

$$P_\perp + \frac{B_{01}^2}{2\mu_0} = \frac{B_{02}^2}{2\mu_0}\,, \tag{A10}$$

where B_{01} and B_{02} are the magnitudes of the ambient magnetic fields in the plasma and vacuum respectively. This equation must be satisfied in order for a stable bounding surface to exist in the absence of wave perturbations. Assuming that $B_{01} \approx B_{02} \approx B_0$, so that $\Delta B_0 := B_{02} - B_{01} \ll B_0$, this can be written in the form

$$P_\perp = \frac{(B_{02} - B_{01})(B_{02} + B_{01})}{2\mu_0} \approx 2\left(\frac{B_0^2}{2\mu_0}\right)\frac{\Delta B_0}{B_0}\,. \tag{A11}$$

We then find that for pressure balance, the perpendicular plasma beta [as defined in equation (37)] and the discontinuity in the ambient magnetic field are related by

$$\beta_{p\perp} = 2\left(\frac{\Delta B_0}{B_0}\right), \tag{A12}$$

provided they are both sufficiently small. Thus, we are justified in neglecting the discontinuity in the ambient magnetic field only when the perpendicular plasma beta is small, that is, $\beta_{p\perp} \ll 1$. This is one of the conditions which the dielectric tensor of this paper satisfies.

The second important assumption made in the derivation of the plasma–vacuum response was that the surface layer could be treated as being arbitrarily sharp. Specifically, it was assumed that the surface layer is thin compared with the wavelength $\lambda_\perp = 2\pi/k_\perp$ of the surface waves in the direction perpendicular to the surface, that is,

$$k_\perp \delta_s \ll 1\,, \tag{A13}$$

where $k_\perp$ is the wavenumber perpendicular to the surface and δ_s is the thickness of the surface layer. In order to justify this assumption we need to identify the surface thickness. This may be done by considering the following.

When $V_{e\perp}$ is nonzero, zeroth-order bulk currents of magnitude

$$J_0 = q_e n_e V_{e\perp} \tag{A14}$$

flow throughout the plasma and in all directions. Within the plasma the bulk currents flowing in opposite directions cancel so that the total zeroth-order bulk current in the plasma is zero and the background magnetic field is homogeneous. Within the surface layer, however, there are no such cancellations, and a zeroth-order surface current

$$J_{0s} = J_0 \delta_s \tag{A15}$$

flows throughout the surface layer in the negative $\hat{\mathbf{y}}$ direction, corresponding to an apparent electron drift within the surface layer in the positive $\hat{\mathbf{y}}$ direction. The surface current is related to the discontinuity in the ambient magnetic field of (A12) by the electromagnetic boundary condition

$$J_{0s} = \frac{\Delta B_0}{\mu_0}\,. \tag{A16}$$

Then, using (A12) to relate the surface current to the perpendicular plasma beta, we end up with the identification

$$\delta_s = \frac{V_{e\perp}}{\Omega_e} = R_e\,. \tag{A17}$$

Hence the thickness of the surface layer is equal to the Larmor radius of the electrons (assuming the ion temperature $T_i = 0$). The assumed condition (A13) then is clearly equivalent to the small gyroradius approximation which was used in the approximation of the dielectric tensor. Thus we are justified in assuming that the surface layer may be regarded as sharp for a plasma with the dielectric tensor as given in Section 4.

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