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An Analysis of Cosmic Ray Air Showers for the Determination of Shower Age

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Abstract

A sample of 8651 air showers in the size range $10^{4\cdot3}$–$10^{6\cdot2}$ has been analysed to determine the distribution of the measured age in terms of (i) the number of showers in a specified size range, and (ii) the radial distances in individual showers. It is shown that the radial age distribution in an individual shower leads to an average shower age approximately the same as the prediction of the electron–photon cascade theory. The other results include a study of the variation of (i) shower age, as measured by the χ^2-minimisation technique, with shower size of vertically incident showers, and (ii) the measured electron density at any point with its radial distance from the shower axis, as a function of the age of a large shower group with very small spread in size. A comparison of similar measurements with relevant theory is also included.

1. Introduction

The development in the longitudinal direction of electron–photon cascades in cosmic ray extensive air showers is described by a parameter called the shower age s. The cascade grows to a maximum ($s = 1$) and then rapidly decays. In the lateral direction from the axis of the shower, the electron density distribution in the shower is measured in terms of the radial age $S(r)$ as one of the parameters. In most earlier experiments (Idenden 1990; Hara *et al.* 1981, 1983; Abdullah *et al.* 1981, 1983), the shower age determined by the standard least-squares fitting technique differs from the theoretical value at all atmospheric depths. This was taken to be an indication that a shower must be described by two age parameters, one for its longitudinal development and the other for its lateral development (Hara *et al.* 1983; Sasaki 1971; Capdevielle and Gawin 1982, 1985; Dai *et al.* 1990). This aspect of extensive air showers has been under investigation in recent years at various centres (Idenden 1990; Dai *et al.* 1990; Cheung and MacKeown 1987; Samorski and Stamm 1983). In the present work a critical experimental examination is made of the techniques used to determine the shower age from new measurements on smaller air showers in the size range $10^{4\cdot3}$–$10^{6\cdot2}$. An analysis of shower age has also been made to show its dependence on various shower parameters.

2. Experiment

The air shower array at the North Bengal University campus has been developed in stages since 1980 (Basak *et al.* 1984). At present it consists of 21 electron-

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density-sampling plastic scintillation detectors, eight fast timing detectors and two magnet spectrographs. The total area covered by the array is 1176 m^2. The shower size threshold for the array is $N_e = 10^{4\cdot2}$. The radial electron density distribution and muon density distribution are measured simultaneously over a radial distance from the array centre to about 30 m and the muon energy in the range 2·5–220 GeV.

To determine the size of a shower, the electron densities at radial distance intervals of 8 m ($\sim r_0/10$, where r_0 is the Molière radius in air at sea level) were measured by a cluster of 21 scintillation detectors installed at sea level. The dynamic range of the detectors is 1–250 particles/detector and each detector was operated at a threshold of one particle. The shower direction was determined by measuring relative arrival times, while shower size N_e, age parameters s and core location (x_0, y_0) determination was carried out by fitting the radial electron density data of a shower event to an interpolating lateral structure function as given by Hillas and Lapikens (1977):

$$f(r) = c(s)\,(r/r_1)^{a_1+a_2(s-1)}(1+r/r_1)^{b_1+b_2(s-1)}, \tag{1}$$

where $c(s)$ is the normalisation constant and $a_1 = -0\cdot53$, $a_2 = 1\cdot54$, $b_1 = -3\cdot39$, $b_2 = 0$ and $r_1 = 24\,m$.

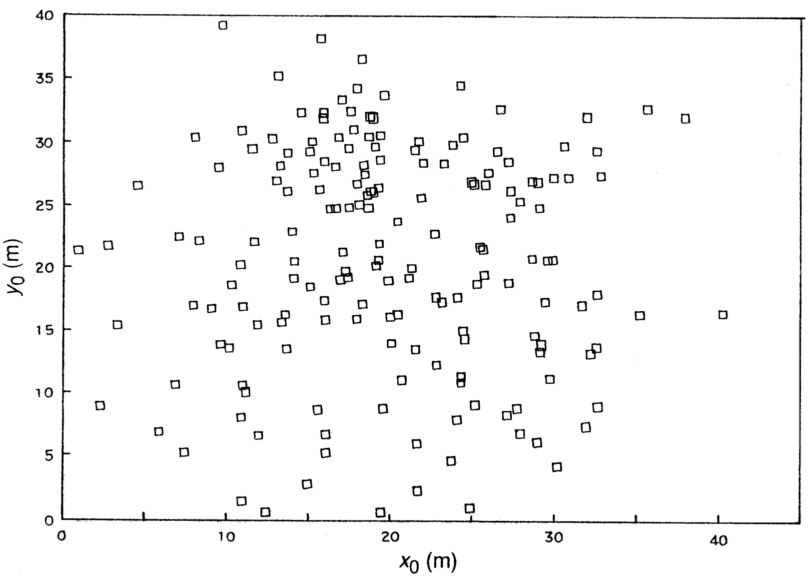

Fig. 1. Distribution of shower core location (x_0, y_0) for a group of 178 showers of $\overline{N}_e = 1\cdot2\times10^5$ with $\bar{s} = 1$.

The computed results on the shower core location (x_0, y_0) form a random distribution, as shown in Fig. 1 for a group of 178 showers of size $\overline{N}_e = 1\cdot2\times10^5$ with age $\bar{s} = 1$.

(2a) Determination of Radial Age Parameter

Using the Hillas–Lapikens (HL) structure function (1) and assuming that the normalisation constants do not vary much at two neighbouring radial points r_i and r_j measured from the core position (e.g. Fig. 1), we obtain for the radial age parameter, at radial location $r_i - r_j$,

$$S_{ij}(r) = \ln(F_{ij}\, X_{ij}^{2\cdot 07}\, Y_{ij}^{3\cdot 39})/1\cdot 54 \ln X_{ij}\,, \tag{2}$$

where $F_{ij} = f(r_i)/f(r_j)$, $X_{ij} = r_i/r_j$, $Y_{ij} = (1+x_i)/(1+x_j)$, with $x = r/r_1$. Substitution of the measured electron densities at the radial points r_i and r_j in the above formula gives $S_{ij}(r)$. With the Nishimura–Kamata–Greisen (NKG) function (Greisen 1960), the expression for $S_{ij}(r)$ under the same conditions as in (2) is

$$S_{ij}(r) = \ln(F_{ij}\, X_{ij}^{2}\, Y_{ij}^{4\cdot 5})/\ln(X_{ij}\, Y_{ij})\,. \tag{3}$$

Some representative results are shown in Table 1.

Table 1. Radial variation of shower age $S_{ij}(r)$ for three shower sizes

		(a) Shower size $N_e = 5\cdot 3\times 10^4$			
Radial distance interval (m)	2–5·5	5·5–8·5	8·5–12·5	12·5–17·5	17·5–22·5
HL	$1\cdot 481^{+0\cdot 075}_{-0\cdot 059}$	$1\cdot 535^{+0\cdot 026}_{-0\cdot 024}$	$1\cdot 592^{+0\cdot 034}_{-0\cdot 032}$	$1\cdot 464^{+0\cdot 015}_{-0\cdot 014}$	
NKG		$1\cdot 759^{+0\cdot 038}_{-0\cdot 034}$	$1\cdot 687^{+0\cdot 047}_{-0\cdot 044}$	$1\cdot 384^{+0\cdot 019}_{-0\cdot 018}$	$1\cdot 761^{+0\cdot 009}_{-0\cdot 007}$
		(b) Shower size $N_e = 1\cdot 2\times 10^5$			
Radial distance interval (m)	12·5–17·5	17·5–22·5	22·5–27·5	27·5–32·5	32·5–37·5
HL	$1\cdot 177^{+0\cdot 009}_{-0\cdot 008}$	$1\cdot 779^{+0\cdot 025}_{-0\cdot 023}$	$1\cdot 666^{+0\cdot 007}_{-0\cdot 007}$	$1\cdot 550^{+0\cdot 015}_{-0\cdot 014}$	$1\cdot 738^{+0\cdot 113}_{-0\cdot 123}$
NKG	$1\cdot 002^{+0\cdot 012}_{-0\cdot 011}$	$1\cdot 699^{+0\cdot 032}_{-0\cdot 030}$	$1\cdot 491^{+0\cdot 009}_{-0\cdot 009}$	$1\cdot 312^{+0\cdot 019}_{-0\cdot 017}$	$1\cdot 513^{+0\cdot 133}_{-0\cdot 144}$
		(c) Shower size $N_e = 1\cdot 2\times 10^6$			
Radial distance interval (m)	22·5–27·5	27·5–32·5	32·5–37·5	37·5–45	45–55
HL	$1\cdot 489^{+0\cdot 049}_{-0\cdot 069}$	$1\cdot 971^{+0\cdot 009}_{-0\cdot 163}$	$1\cdot 688^{+0\cdot 121}_{-0\cdot 096}$	$1\cdot 877^{+0\cdot 031}_{-0\cdot 112}$	$1\cdot 775^{+0\cdot 058}_{-0\cdot 052}$
NKG	$1\cdot 271^{+0\cdot 061}_{-0\cdot 085}$	$1\cdot 820^{+0\cdot 085}_{-0\cdot 197}$	$1\cdot 455^{+0\cdot 142}_{-0\cdot 114}$	$1\cdot 654^{+0\cdot 151}_{-0\cdot 128}$	$1\cdot 526^{+0\cdot 065}_{-0\cdot 058}$

The average age parameter of a shower at a particular size is given by

$$\bar{S} = \sum_{ij} \frac{2w_{ij}}{r_j^2 - r_i^2} \int_{r_i}^{r_j} S_{ij}(r) r\, dr\,, \tag{4}$$

where (r_i, r_j) is the radial distance interval within which the radial age parameters are measured experimentally, and w_{ij} is the statistical weight factor of that particular radial distance bin (i, j).

According to electron–photon cascade theory, the shower age is

$$S(\text{theor.}) = 3t/[t + 2\ln(E_0/\epsilon_0) + 2\ln z]\,, \tag{5}$$

where $\epsilon_0 = 0{\cdot}0842$ GeV is the critical energy of an electron in air, t is the air depth in radiation lengths, E_0 is the primary energy and $z = r/r_0$. The average values at different shower sizes are found in the following way:

$$\bar{S}(\text{theor.}) = \frac{6t}{z_2^2 - z_1^2}\int_{z_1}^{z_2} z\,\mathrm{d}z/[t + 2\ln(E_0/\epsilon_0) + 2\ln z]\,. \tag{6}$$

The average radial ages $\bar{S}$(HL) and $\bar{S}$(NKG), determined from (4) using equations (2) and (3) for the HL and NKG lateral structure functions, are compared with the theoretical average values in Table 2.

Table 2. Comparison of average radial ages with the theoretical average for three shower sizes

N_e	$5{\cdot}3\times10^4$	$1{\cdot}2\times10^5$	$1{\cdot}2\times10^6$
$\bar{S}$(HL)	1·494	1·493	1·733
$\bar{S}$(NKG)	1·703	1·324	1·476
$\bar{S}$(theor.)	1·517	1·434	1·325

(2b) Measurement of Age Parameter from Electron Density Data

The χ^2-minimisation technique using the gradient search method has been used to fit the measured electron densities of individual showers to the chosen interpolation function. The distribution of measured shower age for a sample of 8651 showers in the size range $10^{4\cdot3}$–$10^{6\cdot2}$ is shown in Fig. 2. Experimentally measured shower ages are compared with those of the Moscow group (quoted in Capdevielle and Gawin 1985) in Table 3.

3. Effect of Age Parameter on Lateral Structure

The radial electron densities ρ, measured at various radial points in a group of 893 showers in the size intervals $(5\text{–}5{\cdot}5)\times10^4$, $(1\text{–}1{\cdot}5)\times10^5$ and $(1\text{–}1{\cdot}5)\times10^6$, and with the age distribution shown in Fig. 2, are presented in Figs 3, 4 and 5. The fixed size showers of ages s differing by ~0·1 are distinguishable only in the data at small core distances, as is made evident in these radial electron density distributions.

The observed showers belonging to the age distribution in Fig. 2 are shown in Fig. 6 as a distribution of shower size $\bar{N}_e$ in shower age $\bar{s}$. The error bars represent the standard errors in the mean s. Standard deviations are shown in the same figure. The plot shows that the age of the electron cascade in a shower observed in a vertical direction at sea level decreases with an increase in shower size. The theoretical calculations on this feature given by Capdevielle and Gawin (1982) and the experimental results of the Akeno group (Hara *et al.* 1981) are shown in the same figure.

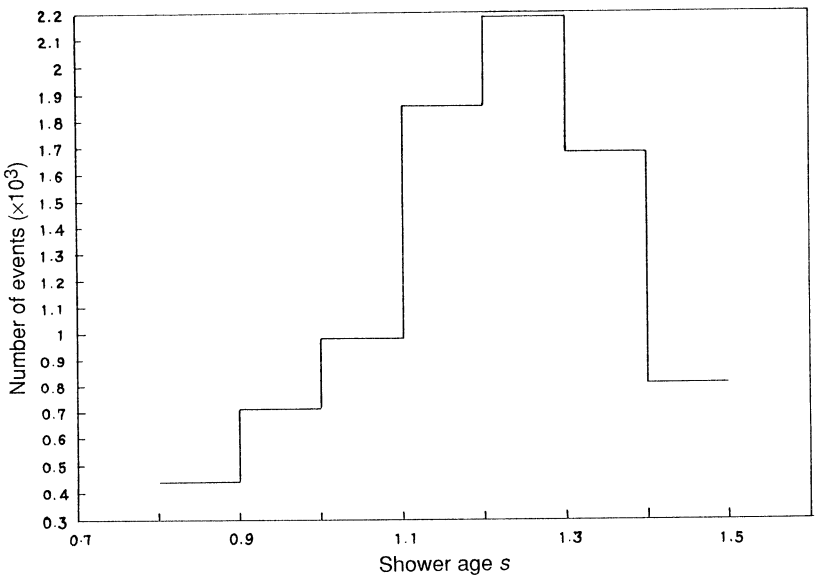

Fig. 2. Distribution of shower age measured by χ^2 minimisation (total 8651 showers).

Table 3. Comparison of present experimental shower ages with those of the Moscow group (quoted in Capdevielle and Gawin 1985)

N_e	$5{\cdot}3\times10^4$	$1{\cdot}2\times10^5$	$1{\cdot}2\times10^6$
Present	1·19	1·10	1·00
Moscow	1·126	1·068	0·924

4. Discussion

The age parameter of cosmic ray extensive air showers has been the subject of further study in recent years. It has been used (Idenden 1990; Cheung and MacKeown 1987; Samorski and Stamm 1983) to distinguish between ultra-high-energy photon-initiated showers and charged cosmic ray particle-initiated showers. Some workers (Hara *et al.* 1983; Sasaki 1971; Capdevielle and Gawin 1982, 1985; Dai *et al.* 1990) have used the radial age parameter in the shower analysis, in addition to the longitudinal age, to describe the longitudinal development of the shower in the atmosphere. In the present work, it has been shown that the average of the radial shower age at different radial distances over the whole shower disk is almost identical to the theoretical average value of the shower age, as given by electron–photon cascade theory. The age value of a particular shower group determined by the χ^2-minimisation technique is dependent on the shower-detecting area and the detector spacing. A comparison is shown in Table 3 for the present work and the Moscow experiment, which had nearly the same detecting area as in the present experiment.

The shower age measured by the χ^2-minimisation technique has been used as a parameter to show the measured radial electron density distributions in Figs 3, 4 and 5 for three shower size groups, each with different age values.

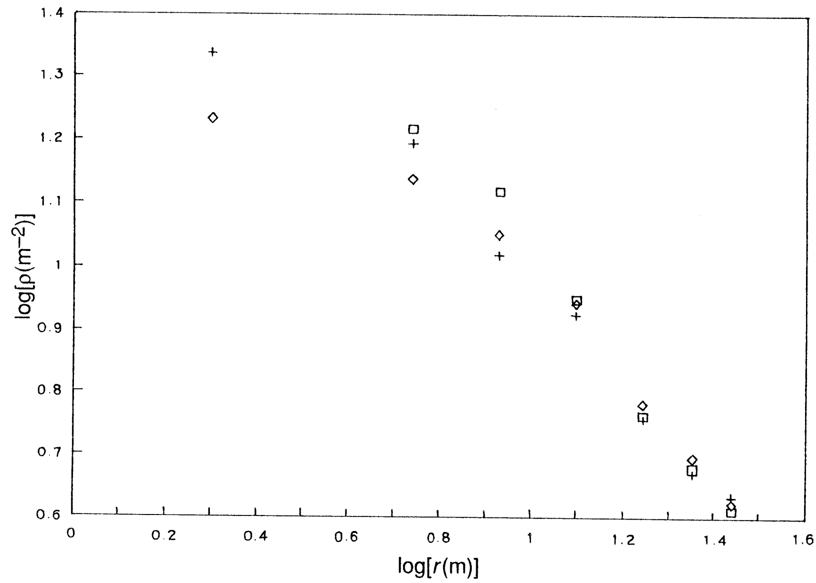

Fig. 3. Radial density distribution of the electron component for N_e in the range (5–5·5)×10^4: □ for $s = 0\cdot99$; + for $s = 1\cdot11$; ◇ for $s = 1\cdot19$.

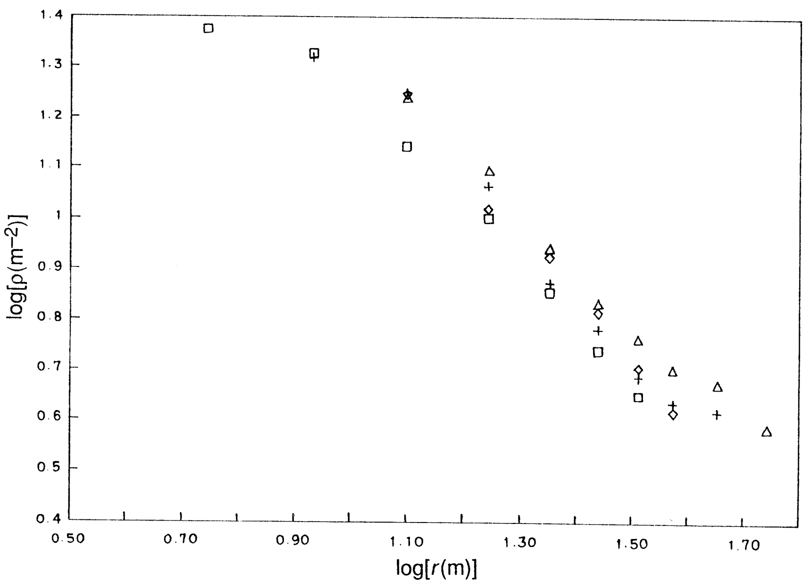

Fig. 4. Radial density distribution of the electron component for N_e in the range (1–1·5)×10^5: □ for $s = 0\cdot89$; + for $s = 1\cdot00$; ◇ for $s = 1\cdot10$; and △ for $s = 1\cdot20$.

These results are in agreement with expectation (Hillas and Lapikens 1977). A reconfirmation of the earlier results on the variation of the shower age measured by the minimisation technique with shower size published by the Akeno group (Hara *et al.* 1981) and Clay *et al.* (1981) is also given in the present work for

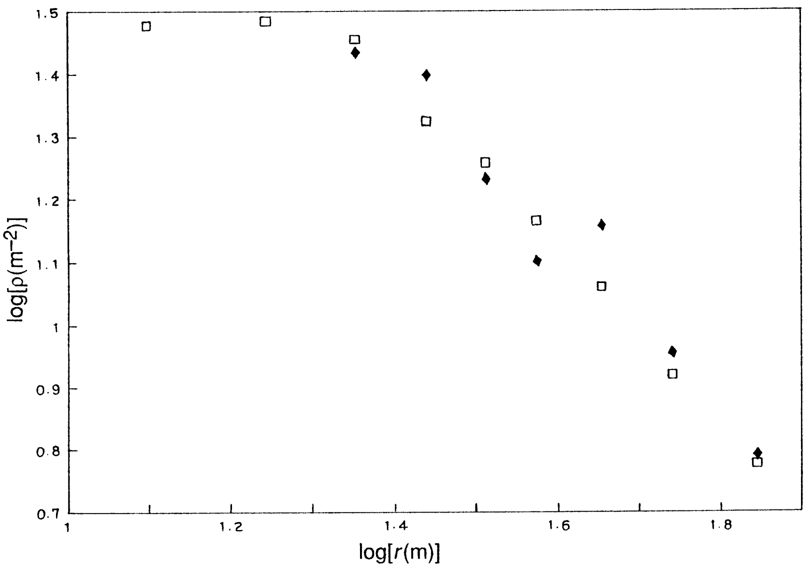

Fig. 5. Radial density distribution of the electron component for N_c in the range $(1\text{–}1\cdot5)\times10^6$: □ for $s=1\cdot00$; ◆ for $s = 1\cdot07$.

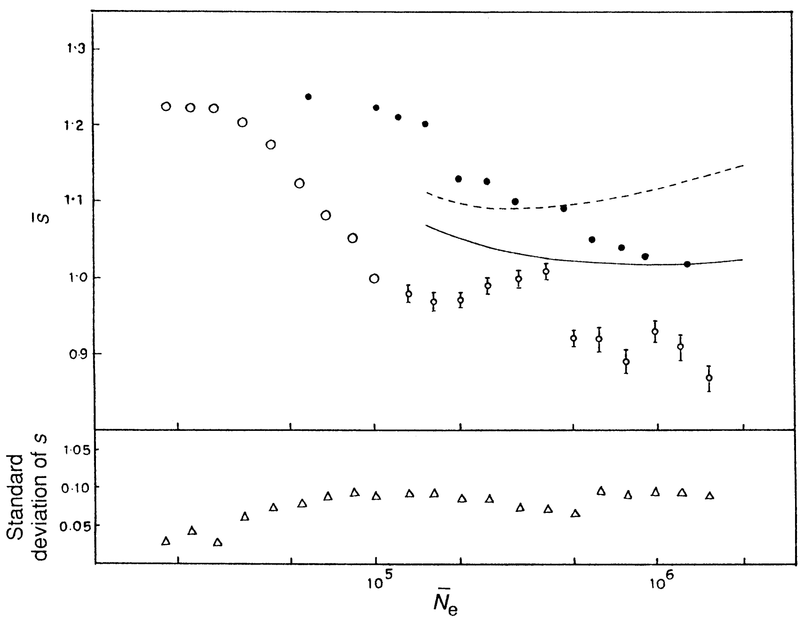

Fig. 6. Distribution of shower size $\overline{N}_e$ with shower age $\bar{s}$: ○, present experiment (sea level); ●, Akeno experiment (900 g cm^{-2}, Hara *et al.* 1981); simulation results (solid curve, scale breaking model; dashed curve, high multiplicity model) of Capdevielle and Gawin (1982). Triangles represent the standard deviation at each point.

vertically incident showers. As can be seen from Fig. 6, a shower of given size developing in the vertical direction over one attenuation length ($\bar{\Lambda} = 112\,\mathrm{g\,cm^{-2}}$ for $N_e \geq 5{\times}10^5$, Sasaki 1971) increases in age by $\sim$0·07. This result is in good agreement with that measured (0·06/100 $\mathrm{g\,cm^{-2}}$, by Clay *et al.* (1981) and that from the measurements of Hara *et al.* (1981) in the shower size range 10^5–10^6. A similar trend was obtained by Capdevielle and Gawin (1982) for two models, as shown also in Fig. 6. The present results on the variation of shower age with radial distance and with shower size are in accordance with the predictions of the electron–photon cascade theory.

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