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On the Effects of Gravitational Fields on the Electrical Properties of Matter*

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Abstract

A discussion of the electrical state of a conducting solid in a static gravitational field is presented. The analysis of the stress–gravitational force balance inside the solid is complicated; however, outside the solid, in the evanescent electron field, the analysis of such a balance simplifies greatly. As a consequence of this external analysis, an expression for the electric field external to the body is presented which includes the direct effect of gravity on the electrons, as well as the indirect effect due to the stress induced by the action of gravity on the bulk solid. Such fields are an important determinant of the gravitational motion of charged particles within metallic shields.

1. Introduction

An understanding of the effects of the Earth's gravitational field on the electrical properties of matter is an essential ingredient in the design of experiments to study the free fall of charged particles and antiparticles. The effect of gravity on any shield around such falling particles is to destroy its electrical neutrality, i.e. it is no longer an equipotential. The problem of including the gravitational potential together with appropriate boundary and initial conditions within a quantum theory of condensed matter is not at all straightforward. In spite of the smallness of gravitational effects, this problem is not amenable to a solution by the usual versions of perturbation theory. Further, what work has been done on the problem begins with a particular model of a solid. As the issues are subtle, there are always nagging doubts about the model-dependent status of these theories. For an extensive review and bibliography of these theoretical and experimental problems, see Darling *et al.* (1992).

In this paper, the problem of matter in a gravitational field is formulated in a relatively model-free way. The solution to the problem of the modification of the bulk structure of a solid and its electric field by gravity is not presented here—that will be the substance of a future paper. Rather, it is shown that the balancing of stress within the external evanescent electron field of a gravitating solid suffices to determine the external electrical field. Fortunately, this is all that is needed for the charged-particle-fall experiments. The electrical field outside

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the solid is found to consist of two terms which agree with those of Schiff and Barnhill (1966) and Dessler *et al.* (1968).

A quantum-mechanical model of matter in a gravitational and electrical field is given by the following Lagrangian, expressed non-relativistically in terms of the Fock fields:

$$L = \frac{\mathrm{i}\hbar}{2}\left(\psi^\dagger\frac{\partial\psi}{\partial t} - \frac{\partial\psi^\dagger}{\partial t}\psi\right) - \frac{\hbar^2}{2m}\nabla\psi^\dagger\,.\,\nabla\psi + eV\psi^\dagger\psi - m\phi\psi^\dagger\psi$$
$$+\frac{\mathrm{i}\hbar}{2}\left(\Psi^\dagger\frac{\partial\Psi}{\partial\Psi} - \frac{\partial\Psi^\dagger}{\partial t}\Psi\right) - \frac{\hbar^2}{2M}\nabla\Psi^\dagger\,.\,\nabla\Psi - ZeV\Psi^\dagger\Psi - M\phi\Psi^\dagger\Psi$$
$$+\tfrac{1}{2}(\nabla V)^2\,, \tag{1}$$

where ϕ is the anticommuting Fock field of the electrons of mass m and charge $-e$, and Ψ is the Fock field of the nuclei treated as elementary particles of a single species of mass M and charge Ze. The field commutes or anticommutes depending on the nuclear spin. Here V is the electrostatic potential, which is taken as a C-number field in the Coulomb gauge, and ϕ is the gravitational potential. This Lagrangian omits special relativistic and magnetic effects, neither of which is germane to the physics and either of which may readily be included.

Varying the Lagrangian with respect to ψ, Ψ and V yields the following equations of motion respectively:

$$\mathrm{i}\hbar\partial\psi/\partial t = -(\hbar^2/2m)\nabla^2\psi - eV\psi + m\phi\psi\,, \tag{2a}$$

$$\mathrm{i}\hbar\partial\Psi/\partial t = -(\hbar^2/2M)\nabla^2\Psi + ZeV\Psi + m\phi\Psi\,, \tag{2b}$$

$$\nabla^2 V = -(Ze\Psi^\dagger\Psi - e\psi^\dagger\psi)\,. \tag{2c}$$

The Lagrangian (1) yields the Hamiltonian

$$H = \int\left\{\frac{\hbar^2}{2m}\nabla\psi^\dagger\,.\,\nabla\psi - eV\psi^\dagger\psi + m\phi\psi^\dagger\psi + \frac{\hbar^2}{2M}\nabla\Psi^\dagger\,.\,\nabla\Psi + ZeV\Psi^\dagger\Psi + M\phi\Psi^\dagger\Psi - \tfrac{1}{2}(\nabla V)^2\right\}\mathrm{d}^3x\,. \tag{3}$$

The Lagrangian also yields the stress tensor $\mathbf{T}$, whose components T_{ij} are given by

$$T_{ij} = T^{\mathrm{e}}_{ij} + T^{\mathrm{n}}_{ij} + T^{\mathrm{v}}_{ij}\,, \tag{4a}$$

$$T^{\mathrm{e}}_{ij} = -\frac{\hbar^2}{4m}\left\{\psi^\dagger(\partial_i\partial_j\psi) + (\partial_i\partial_j\psi^\dagger)\psi - (\partial_i\psi^\dagger)(\partial_j\psi) - (\partial_j\psi^\dagger)(\partial_i\psi)\right\}, \tag{4b}$$

$$T^{\mathrm{n}}_{ij} = -\frac{\hbar^2}{4M}\left\{\Psi^\dagger(\partial_i\partial_j\Psi) + (\partial_i\partial_j\Psi^\dagger)\Psi - (\partial_i\Psi^\dagger)(\partial_j\Psi) - (\partial_j\Psi^\dagger)(\partial_i\Psi)\right\}, \tag{4c}$$

$$T^{\mathrm{v}}_{ij} = +\left\{(\partial_i V)(\partial_j V) - \tfrac{1}{2}\delta_{ij}(\nabla V)^2\right\}, \tag{4d}$$

where $\mathbf{T}^{\mathrm{e}}$, $\mathbf{T}^{\mathrm{n}}$ and $\mathbf{T}^{\mathrm{v}}$ are the electronic, nuclear and electrostatic stress tensors, respectively, and $\partial_i \equiv \partial/\partial x^i$.

The electron momentum density is given by the operator expression

$$P_i^{\mathrm{e}}(\mathbf{x}) = \frac{\hbar}{\mathrm{i}}\left[\psi^\dagger(\partial_i\psi) - (\partial_i\psi^\dagger)\psi\right], \tag{5}$$

for which the Heisenberg equation of motion reads

$$\begin{aligned}\frac{\partial P_i^{\mathrm{e}}}{\partial t} &= \frac{1}{\mathrm{i}\hbar}\left[P_i^{\mathrm{e}}(\mathbf{x}), H\right] \\ &= -\,\partial_j T_{ij}^{\mathrm{e}} - e\psi^\dagger\psi E_i + m\psi^\dagger\psi g_i\,,\end{aligned} \tag{6}$$

where the electric and gravitational fields are given by

$$E_i = -\,\partial_i V\,, \tag{7a}$$

$$g_i = -\,\partial_i\phi\,. \tag{7b}$$

Equation (6) is the principal momentum balance equation.

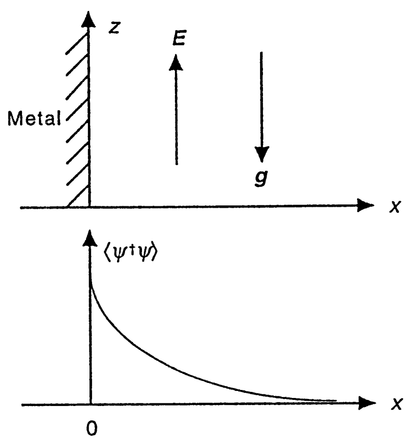

Fig. 1. The electric and gravitational fields outside a conductor, and the evanescent electron probability density. Electron exchange effects may reverse the sense of the electric field, as discussed in the text.

2. Equilibrium

In an energy eigenstate or in thermal equlibrium, the expectation value of equation (6) yields

$$-\partial_j\,\langle T_{ij}^{\mathrm{e}}\rangle - e\langle\psi^\dagger\psi E_i\rangle + m\langle\psi^\dagger\psi g_i\rangle = 0\,. \tag{8}$$

If we apply this equation to the region *outside* the solid, where the electron density decays evanescently (see Fig. 1), we find that

$$
\begin{aligned}
E_z &= -\,mg/e - \partial_j \langle T^{\mathrm{e}}_{zj} \rangle / e \langle \psi^\dagger \psi \rangle \\
&= E_z^{\mathrm{SB}} + E_z^{\mathrm{DMRT}} \,.
\end{aligned} \tag{9}
$$

The first term in (9) was discovered by Schiff and Barnhill (1966) and the second by Dessler *et al.* (1968).

3. Evaluation and Conclusion

The second term on the right-hand side of (9) may be evaluated using a quantum-mechanical model of the solid. In the case of a free-electron model, for large distances from the surface, the wavefunctions of electrons near the Fermi surface will predominate, yielding (very roughly)

$$
\begin{aligned}
E^{\mathrm{DMRT}} &\approx -\,(\hbar^2/me)\,\partial k_z^2/\partial z \\
&\approx +\,(2E_{\mathrm{F}}/3e)\epsilon_{zz} \\
&\approx (2E_{\mathrm{F}}/3e)(\rho g/Y)\,.
\end{aligned} \tag{10}
$$

In (10) E_{F} is the kinetic energy at the Fermi surface, ϵ_{zz} is the strain of the solid in the gravitational field, ρ is its mass density and Y is Young's modulus.

For copper, we estimate from (10)

$$
E^{\mathrm{DMRT}} = -3{\cdot}2 \times 10^3\, E^{\mathrm{SB}}\,. \tag{11}
$$

Consideration of the atomic theory of Young's modulus shows that the ratio of these two electric fields should be of order $\sim -(M/m)$.

If the surface dipole layer is taken into account as well as the effects of exchange, as in a jellium model, E^{DMRT} may have the opposite sign and be an order of magnitude smaller (Rossi 1991).

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