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Effect of Streaming and Finite Geometry on a Resonance-like Phenomenon in Beam–Plasma Systems

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Abstract

We have analysed the effect of finite geometry and streaming on a resonance-like phenomenon in a beam–plasma system placed in an infinite magentic field. The resonance-like phenomenon is displayed through a variation of the amplitude of the soliton with respect to α (the ratio of electron to ion density). It is shown that this event is very prominent in the case of an infinite plasma, but for a bounded system the effect is not so significant. As such, an effect of this type will be difficult to observe experimentally unless the dimension of the containment system is considerable. Furthermore, the peak of the resonance varies considerably with the streaming velocity and other parameters of the plasma. The whole phenomenon is crucially dependent on the phase velocity of the solitary wave, whose variation is also considered in detail. Lastly, it is demonstrated that the existence of a resonance-like phenomenon can also be ascertained from an analysis of the linear dispersion relation.

1. Introduction

The propagation of ion–acoustic solitary waves in different plasma systems has been of considerable interest (Jeffrey and Katutani 1972; Ikezi 1973; Washimi and Taniuti 1966). The behaviour and characteristics of such solitary waves have been studied extensively by various theoretical methods. In this respect various types of plasma, consisting of hot electrons and cold ions (Washimi and Taniuti 1966), hot electrons and hot ions (Tappert 1972) and two electron populations, have been dealt with. On the other hand, situations have been considered where the mass variation of the electron plays an important role, that is, when the electron is relativistic (Das and Paul 1985; Roy Chowdhury *et al.* 1988). A completely different scenario is seen to evolve when the finite geometry of the containment system is taken into account (Das and Ghosh 1988; Ghosh and Das 1986, 1987). Also, the finite geometry effect has been found to be of considerable importance in other situations (Hubin and Klein 1990).

Several important classes of events have been analysed in the context of the beam–plasma interaction (Yahma *et al.* 1983; Yajima *et al.* 1978). Here we study an interesting resonance phenomenon between the beam electron and the plasma itself, manifested through the behaviour of the amplitude of the soliton, when the whole system is placed in a static magnetic field. This phenomenon is found to be dependent on the streaming velocity of the system, the ratio θ of electron

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temperature of the beam to that of the plasma, the normalised ion temperature σ and, most important of all, the size of the confining system. The effect of the finite geometry of the system is very prominent. We show below that the resonance phenomenon is significant only when the size of the confining system is quite large.

2. Formulation

Here we consider the case of a hot plasma placed in an infinite magnetic field, with the axis of the field pointing along the x-axis (the axis of the wave-guide), and traversed by a hot electron beam. An infinite magnetic field means that the field is constant in magnitude but infinite in spatial extent. Such a field compels the particles to move in the longitudinal x-direction only (Sayal and Sharma 1990*a*, 1990*b*; Ghosh and Das 1985; Rasmussen 1978). We further assume that the usual hydrodynamic description is possible. As such, the equations governing the system are

$$n_{\mathrm{e}}^{\mathrm{p}}\frac{\partial \phi}{\partial x} - \frac{\partial n_{\mathrm{e}}^{\mathrm{p}}}{\partial x} = 0\,, \tag{1}$$

$$\frac{\partial n_{\mathrm{i}}}{\partial t} + \frac{\partial}{\partial x}(n_{\mathrm{i}}\, v_{\mathrm{i}}) = 0\,, \tag{2}$$

$$\frac{\partial v_{\mathrm{i}}}{\partial t} + v_{\mathrm{i}}\frac{\partial v_{\mathrm{i}}}{\partial x} + \frac{\partial \phi}{\partial x} + \frac{3\sigma}{(1+\alpha)^2}\, n_{\mathrm{i}}\, \frac{\partial n_{\mathrm{i}}}{\partial x} = 0\,, \tag{3}$$

$$\frac{\partial n_{\mathrm{e}}^{\mathrm{b}}}{\partial t} + \frac{\partial}{\partial x}(n_{\mathrm{e}}^{\mathrm{b}}\, v_{\mathrm{e}}^{\mathrm{b}}) = 0\,, \tag{4}$$

$$\mu n_{\mathrm{e}}^{\mathrm{b}}\, \frac{\partial v_{\mathrm{e}}^{\mathrm{b}}}{\partial t} + \mu n_{\mathrm{e}}^{\mathrm{b}}\, v_{\mathrm{e}}^{\mathrm{b}}\, \frac{\partial v_{\mathrm{e}}^{\mathrm{b}}}{\partial x} - n_{\mathrm{e}}^{\mathrm{b}}\, \frac{\partial \phi}{\partial x} + \theta\, \frac{\partial n_{\mathrm{e}}^{\mathrm{b}}}{\partial x} = 0\,, \tag{5}$$

$$\frac{\partial^2 \phi}{\partial z^2} + \frac{\partial^2 \phi}{\partial x^2} = n_{\mathrm{e}}^{\mathrm{p}} + n_{\mathrm{e}}^{\mathrm{b}} - n_{\mathrm{i}}\,. \tag{6}$$

These equations have been written as a simple extension of those given by Sayal and Sharma (1990*a*) for a beam–plasma system.

Here $n_{\mathrm{e}}^{\mathrm{p}}$ and $n_{\mathrm{e}}^{\mathrm{b}}$ denote the electron density in the plasma and the beam, n_{i} is that for the ions; $v_{\mathrm{e}}^{\mathrm{b}}$ and $v_{\mathrm{e}}^{\mathrm{p}}$ denote the corresponding velocities of the two types of electrons, v_{i} is that for the ions; and ϕ denotes the electrostatic potential. We have normalised the time to the inverse of the ion plasma frequency $\omega_{\mathrm{i}}^{\mathrm{p}} = (4\pi m_{\mathrm{e}}\, e^2/m_{\mathrm{i}})^{1/2}$, where e is the charge of the electron and m_{i} the mass of the ion species. The densities have been normalised to $n_{\mathrm{e}0}$, the equilibrium electron density. The space coordinate x has been normalised to the electron Debye length $\lambda_{\mathrm{D}} = (T_{\mathrm{e}}^{\mathrm{p}}/4\pi m_{\mathrm{e}0}\, e^2)^{1/2}$, where $T_{\mathrm{e}}^{\mathrm{p}}$ is the electron temperature. Furthermore, all velocities have been normalised to the sound velocity of the plasma, $c_{\mathrm{s}} = (T_{\mathrm{e}}^{\mathrm{p}}/m_{\mathrm{i}})^{1/2}$, and ϕ to $T_{\mathrm{e}}^{\mathrm{p}}/e$. In the following, the subscript zero indicates an unperturbed value. Let us denote the ratio of beam to plasma electron densities as $\alpha = n_{\mathrm{e}}^{\mathrm{b}}/n_{\mathrm{e}}^{\mathrm{p}}$, with $\theta = T_{\mathrm{e}}^{\mathrm{b}}/T_{\mathrm{e}}^{\mathrm{p}}$, and for the ion temperature $\sigma = T_{\mathrm{i}}/T_{\mathrm{e}}^{\mathrm{p}}$; lastly, μ denotes the ratio $m_{\mathrm{e}}/m_{\mathrm{i}}$.

To use the reductive perturbation technique (Verheest 1988; Tagare and Das 1975; Watanabe 1984; Konno and Taniuti 1978) we stretch the basic space–time coordinates (x, t) as

$$\xi = \epsilon^{1/2}(x - \lambda t), \qquad \eta = \epsilon^{3/2} x\,, \tag{7}$$

where ϵ is an arbitrarily small parameter. We further set

$$\begin{aligned} n_{\mathrm{e}}^{\mathrm{p}} &= n_{\mathrm{e}}^{\mathrm{p}(0)} + \epsilon n_{\mathrm{e}}^{\mathrm{p}(1)} + \epsilon^2 n_{\mathrm{e}}^{\mathrm{p}(2)} + \dots, \\ n_{\mathrm{i}} &= n_{\mathrm{i}}^{(0)} + \epsilon n_{\mathrm{i}}^{(1)} + \epsilon^2 n_{\mathrm{i}}^{(2)} + \dots, \\ n_{\mathrm{e}}^{\mathrm{b}} &= n_{\mathrm{e}}^{\mathrm{b}(0)} + \epsilon n_{\mathrm{e}}^{\mathrm{b}(1)} + \epsilon^2 n_{\mathrm{e}}^{\mathrm{b}(2)} + \dots, \\ v_{\mathrm{i}} &= v_{\mathrm{i}}^{(0)} + \epsilon v_{\mathrm{i}}^{(1)} + \epsilon^2 v_{\mathrm{i}}^{(2)} + \dots, \\ v_{\mathrm{e}}^{\mathrm{b}} &= v_{\mathrm{e}}^{\mathrm{b}(0)} + \epsilon v_{\mathrm{e}}^{\mathrm{b}(1)} + \epsilon^2 v_{\mathrm{e}}^{\mathrm{b}(2)} + \dots, \end{aligned} \tag{8}$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \dots. \tag{9}$$

Substituting (8) and (9) in equations (1) to (6) we get the following information regarding different perturbed quantities:

$$n_{\mathrm{i}}^{(1)} = \frac{(1+\alpha)^2 n_{\mathrm{i}}^{(0)}}{(1+\alpha)^2(\lambda - v_{\mathrm{i}}^{(0)})^2 - 3\sigma n_{\mathrm{i}}^{(0)2}}\,\phi^{(1)}\,, \tag{10}$$

$$v_{\mathrm{i}}^{(1)} = \frac{(\lambda - v_{\mathrm{i}}^{(0)})(1+\alpha)^2}{[(1+\alpha)^2(\lambda - v_{\mathrm{i}}^{(0)})^2 - 3\sigma n_{\mathrm{i}}^{(0)}]^2}\,\phi^{(1)}\,, \tag{11}$$

$$n_{\mathrm{e}}^{\mathrm{b}(1)} = \frac{n_{\mathrm{e}}^{\mathrm{b}(0)}}{e - \mu(\lambda - v_{\mathrm{e}}^{\mathrm{b}(0)})^2}\,\phi^{(1)}\,, \tag{12}$$

$$v_{\mathrm{e}}^{\mathrm{b}(1)} = \frac{\lambda - v_{\mathrm{e}}^{\mathrm{b}(0)}}{\theta - \mu(\lambda - v_{\mathrm{e}}^{\mathrm{b}(0)})^2}\,\phi^{(1)}\,, \tag{13}$$

$$\frac{\partial^2 \phi^{(1)}}{\partial z^2} = \phi^{(1)} + n_{\mathrm{e}}^{\mathrm{b}(1)} - n_{\mathrm{i}}^{(1)}\,. \tag{14}$$

Eliminating all other quantities in favour of $\phi^{(1)}$ we get

$$\frac{\partial^2 \phi^{(1)}}{\partial z^2} + q^2 \phi^{(1)} = 0\,, \tag{15}$$

where

$$q^2 = \frac{(1+\alpha)^2\, n_{\mathrm{i}}^{(0)}}{(1+\alpha)^2(\lambda - v_{\mathrm{i}}^{(0)})^2 - 3\sigma n_{\mathrm{i}}^{(0)2}} - \frac{n_{\mathrm{e}}^{\mathrm{b}(0)}}{\theta - \mu(\lambda - v_{\mathrm{e}}^{(0)})^2} - 1\,. \tag{16}$$

We consider a solution of (15) in the form

$$\phi^{(1)} = f(\xi,\, \eta)\,\mathrm{sin}qz$$

and impose the condition that $\phi^{(1)}$ should vanish on the boundary of the container, assumed to be a rectangular one. Thus we have $\phi^{(1)} = 0$ for $z = 0$ and $z = b$, so that we at once get $q = n\pi/b$. Combined with (16) this gives

$$\frac{n^2\pi^2}{b} = \frac{(1+\alpha)^2 n_{\mathrm{i}}^{(0)}}{(1+\alpha)^2(\lambda - v_{\mathrm{i}}^{(0)})^2 - 3\sigma n_{\mathrm{i}}^{(0)2}} - \frac{n_{\mathrm{e}}^{\mathrm{b}(0)}}{\theta - \mu(\lambda - v_{\mathrm{e}}^{(0)})^2} - 1\,. \tag{17}$$

Proceeding now to terms of the order of ϵ^2 we get

$$-(\lambda - v_{\mathrm{i}}^{(0)})\frac{\partial n_{\mathrm{i}}^{(2)}}{\partial \xi} + \frac{\partial}{\partial \xi}(n_{\mathrm{i}}^{(1)}\, v_{\mathrm{i}}^{(1)}) + n_{\mathrm{i}}^{(0)}\,\frac{\partial v_{\mathrm{i}}^{(2)}}{\partial \xi} + n_{\mathrm{i}}^{(0)}\,\frac{\partial v_{\mathrm{i}}^{(1)}}{\partial \eta} + v_{\mathrm{i}}^{(0)}\,\frac{\partial n_{\mathrm{i}}^{(1)}}{\partial \eta} = 0\,, \tag{18}$$

$$\begin{aligned} &-(\lambda - v_{\mathrm{i}}^{(0)})\frac{\partial v_{\mathrm{i}}^{(2)}}{\partial \xi} + v_{\mathrm{i}}^{(1)}\,\frac{\partial v_{\mathrm{i}}^{(1)}}{\partial \xi} + v_{\mathrm{i}}^{(0)}\,\frac{\partial v_{\mathrm{i}}^{(1)}}{\partial \eta} \\ &\quad + \frac{\partial \phi^{(2)}}{\partial \xi} + \frac{\partial \phi^{(1)}}{\partial \eta} + \frac{3\sigma}{(1+\alpha)^2}\,n_{\mathrm{i}}^{(0)}\,\frac{\partial n_{i}^{(2)}}{\partial \xi} \\ &\quad + \frac{3\sigma}{(1+\alpha)^2}\,n_{\mathrm{i}}^{(1)}\,\frac{\partial n^{(1)}}{\partial \xi} + \frac{3\sigma}{(1+\alpha)^2}\,n_{\mathrm{i}}^{(1)}\,\frac{\partial n_{\mathrm{i}}^{(1)}}{\partial \eta} = 0\,, \end{aligned} \tag{19}$$

$$\begin{aligned} &-(\lambda - v_{\mathrm{e}}^{\mathrm{b}(0)})\frac{\partial n_{\mathrm{e}}^{\mathrm{b}(2)}}{\partial \xi} + n_{\mathrm{e}}^{\mathrm{b}(0)}\frac{\partial v_{\mathrm{e}}^{\mathrm{b}(2)}}{\partial \xi} + \frac{\partial}{\partial \xi}(n_{\mathrm{e}}^{\mathrm{b}(1)}\, v_{\mathrm{e}}^{\mathrm{b}(1)}) \\ &\quad + n_{\mathrm{e}}^{\mathrm{b}(0)}\,\frac{\partial v_{\mathrm{e}}^{\mathrm{b}(1)}}{\partial \eta} + v_{\mathrm{e}}^{\mathrm{b}(0)}\,\frac{\partial n_{\mathrm{e}}^{\mathrm{b}(1)}}{\partial \eta} = 0\,, \end{aligned} \tag{20}$$

$$\begin{aligned} &-\mu n_{\mathrm{e}}^{\mathrm{b}(0)}(\lambda - v_{\mathrm{e}}^{\mathrm{b}(0)})\frac{\partial v_{\mathrm{e}}^{\mathrm{b}(2)}}{\partial \xi} - \mu\lambda n_{\mathrm{e}}^{\mathrm{b}(1)}\,\frac{\partial v_{\mathrm{e}}^{\mathrm{b}(1)}}{\partial \xi} \\ &\quad + \mu n_{\mathrm{e}}^{\mathrm{b}(0)}\,v_{\mathrm{e}}^{\mathrm{b}(1)}\,\frac{\partial v_{\mathrm{e}}^{\mathrm{b}(1)}}{\partial \xi} + \mu n_{\mathrm{e}}^{\mathrm{b}(1)}\,v_{\mathrm{e}}^{\mathrm{b}(0)}\,\frac{\partial v_{\mathrm{e}}^{\mathrm{b}(1)}}{\partial \xi} \\ &\quad + \mu n_{\mathrm{e}}^{\mathrm{b}(0)}\,v_{\mathrm{e}}^{\mathrm{b}(0)}\,\frac{\partial v^{\mathrm{b}(1)}}{\partial \eta} - n_{\mathrm{e}}^{\mathrm{b}(0)}\,\frac{\partial \phi^{(2)}}{\partial \xi} - n_{\mathrm{e}}^{\mathrm{b}(1)}\,\frac{\partial \phi^{(1)}}{\partial \xi} \\ &\quad - n_{\mathrm{e}}^{\mathrm{b}(0)}\,\frac{\partial \phi^{(1)}}{\partial \eta} + \theta\,\frac{\partial n_{\mathrm{e}}^{\mathrm{b}(2)}}{\partial \xi} + \theta\,\frac{\partial n_{\mathrm{e}}^{\mathrm{b}(1)}}{\partial \eta} = 0\,, \end{aligned} \tag{21}$$

$$\frac{\partial^2\phi^{(2)}}{\partial z^2}+\frac{\partial^2\phi^{(1)}}{\partial\xi^2}=\phi^{(2)}+\frac{\phi^{(1)2}}{2}+n_{\rm e}^{{\rm b}(2)}-n_{\rm i}^{(2)}\,. \tag{22}$$

Again eliminating other variables, we get an equation of the form

$$\frac{\partial^2}{\partial z^2}\left(\frac{\partial\phi^{(2)}}{\partial\xi}\right)+q^2\,\frac{\partial\phi^{(2)}}{\partial\xi}=Q(f)\,, \tag{23}$$

where $Q(f)$ is given by

$$Q(f)\;=\;\alpha_1\,{\rm sin}qz\,\frac{\partial^3 f}{\partial\xi^3}+\alpha_2\,{\rm sin}^2qzf\,\frac{\partial f}{\partial\xi}+\alpha_3\,{\rm sin}qz\,\frac{\partial f}{\partial\eta}\,,$$

and where

$$\begin{aligned}
\alpha_1 \;&=\; 1\,,\\
\alpha_2 \;&=\; B\Big/\left[[\theta-\mu(\lambda-v_{\rm e}^{{\rm b}(0)})^2]\left(\frac{3\sigma}{(1+\alpha)^2}\,n_{\rm i}^{(0)2}-(\lambda-v_{\rm i}^{(0)})^2\right)\right],\\
\alpha_3 \;&=\; C\Big/\left[[\theta-\mu(\lambda-v_{\rm e}^{{\rm b}(0)})^2]\left(\frac{3\sigma}{(1+\alpha)^2}\,n_{\rm i}^{(0)2}-(\lambda-v_{\rm i}^{(0)})^2\right)\right],\\
B \;&=\; \left(\frac{3\sigma}{(1+\alpha)^2}\,n_{\rm i}^{(0)2}-(\lambda-v_{\rm i}^{(0)})^2\right)\left(\frac{2n_{\rm e}^{{\rm b}(0)}\,\mu(\lambda-v_{\rm e}^{(0)})^2}{[\theta-\mu(\lambda-v^{{\rm b}(0)})^2]^2}\right.\\
&\quad+\frac{\mu n_{\rm e}^{{\rm b}(0)}\,v_{\rm e}^{{\rm b}(0)}(\lambda-v_{\rm e}^{{\rm b}(0)})}{[\theta-\mu(\lambda-v_{\rm e}^{{\rm b}(0)})^2]^2}-\frac{\mu\lambda n_{\rm e}^{{\rm b}(0)}(\lambda-v_{\rm e}^{{\rm b}(0)})}{[\theta-\mu(\lambda-v_{\rm e}^{{\rm b}(0)})^2]^2}\\
&\quad\left.+\frac{\mu n_{\rm e}^{{\rm b}(0)}(\lambda-v_{\rm e}^{{\rm b}(0)})^2}{[\theta-\mu(\lambda-v_{\rm e}^{{\rm b}(0)})^2]^2}-\frac{n_{\rm e}^{{\rm b}(0)}}{\theta-\mu(\lambda-v_{\rm e}^{{\rm b}(0)})^2}\right)\\
&\quad-[\theta-\mu(\lambda-v_{\rm e}^{{\rm b}(0)})^2]\left(\frac{2(1+\alpha)^4\,n_{\rm i}^{(0)}(\lambda-v_{\rm i}^{(0)})^2}{(1+\alpha)^2(\lambda-v_{\rm i}^{(0)})^2-3\sigma n_{\rm i}^{(0)2}}\right.\\
&\quad+\frac{n_{\rm i}^{(0)}(\lambda-v_{\rm i}^{(0)})^2(1+\alpha)^4}{[(1+\alpha)^2(\lambda-v_{\rm i}^{(0)})^2-3\sigma n_{\rm i}^{(0)2}]^2}\\
&\quad\left.+\frac{3\sigma(1+\alpha)^2\,n_{\rm i}^{(0)3}}{[(1+\alpha)^2(\lambda-v_{\rm i}^{(0)})^2-3\sigma n_{\rm i}^{(0)}]^2}\right)\\
&\quad-[\theta-\mu(\lambda-v_{\rm e}^{{\rm b}(0)})^2]\left(\frac{3\sigma}{(1+\alpha)^2}\,n_{\rm i}^{(0)2}-(\lambda-v_{\rm i}^{(0)})^2\right),
\end{aligned}$$

$$C = \left(\frac{3\sigma}{(1+\alpha)^2}\,n_{\rm i}^{(0)2} - (\lambda - v_{\rm i}^{(0)})^2\right)\left(\frac{\mu n_{\rm e}^{\rm b(0)}}{\theta - \mu(\lambda - v_{\rm e}^{\rm b(0)})^2}\right.$$
$$\times\,(\lambda - v_{\rm e}^{\rm b(0)})^2 + \frac{2\mu v_{\rm e}^{\rm b(0)}\,n_{\rm e}^{\rm b(0)}(\lambda - v_{\rm e}^{\rm b(0)})}{\theta - \mu(\lambda - v_{\rm e}^{\rm b(0)})^2}$$
$$\left. - \frac{\theta n_{\rm e}^{\rm b(0)}}{\theta - \mu(\lambda - v_{\rm e}^{\rm b(0)})^2} - n_{\rm e}^{\rm b(0)}\right) - [\theta - \mu(\lambda - v_{\rm e}^{\rm b(0)})^2]$$
$$\times\left(\frac{n_{\rm i}^{(0)}(\lambda - v_{\rm i}^{(0)})^2(1+\alpha)^2}{(1+\alpha)^2(\lambda - v_{\rm i}^{(0)})^2 - 3\sigma n_{\rm i}^{(0)2}} + n_{\rm i}^{(0)}\right.$$
$$+ \frac{2n_{\rm i}^{(0)}\,v_{\rm i}^{(0)}(\lambda - v_{\rm i}^{(0)})(1+\alpha)^2}{(1+\alpha)^2(\lambda - v_{\rm i}^{(0)})^2 - 3\sigma n_{\rm i}^{(0)2}}$$
$$\left. + \frac{3\sigma n_{\rm i}^{(0)3}}{(1+\alpha)^2(\lambda - v_{\rm i}^{(0)})^2 - 3\sigma n_{\rm i}^{(0)2}}\right).$$

In the present situation we take the following unperturbed values for physical parameters: $n_{\rm i}^{(0)} = 1+\alpha$, $v_{\rm i}^{(0)} = 0$, $v_{\rm e}^{\rm b(0)} = v_0$ (the bulk velocity) and $n_{\rm e}^{\rm b(0)} = \alpha$. Now multiplying both sides of equation (23) by $\sin qz$ and integrating over z, we get the KdV equation

$$\frac{\partial f}{\partial \eta} + \beta f\,\frac{\partial f}{\partial \xi} + \gamma\,\frac{\partial^3 f}{\partial \xi^3} = 0\,; \tag{24}$$

$$\beta = \int_0^b \alpha_2 \sin^3 qz \,{\rm d}z \bigg/ \int_0^b \alpha_3 \sin^2 qz\,{\rm d}z\,,$$

$$\gamma = \int_0^b \alpha_1 \sin^2 qz \,{\rm d}z \bigg/ \int_0^b \alpha_3 \sin^2 qz\,{\rm d}z\,.$$

3. Soliton Solutions

It is now easy to observe that the KdV equation (24) possesses a solitary wave solution of the form

$$f = A\,{\rm sech}^2\left(\frac{\xi - v\eta}{\delta}\right), \tag{25}$$

with

$$A = 3v/\beta \quad \text{and} \quad \delta = \sqrt{4\gamma/v}\,. \tag{26}$$

It is interesting to note that in the present case the soliton amplitude A and width δ both depend in a complicated way on the phase velocity through β, γ and α_i ($i = 1$, 2, 3). The phase velocity is actually determined from equation (17) which can be rewritten as

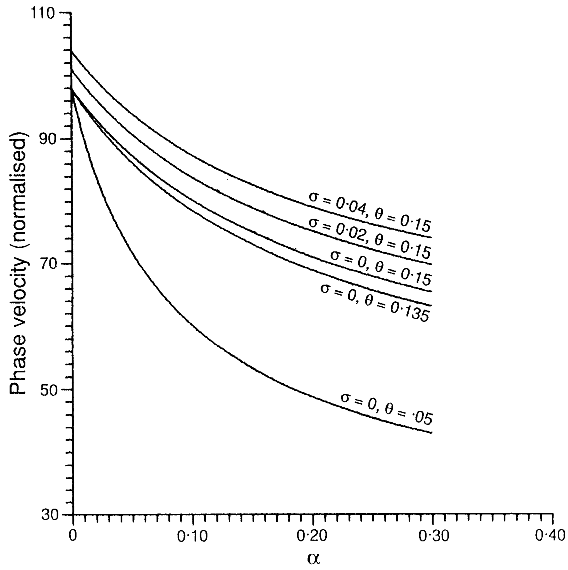

Fig. 1. Variation of phase velocity λ with α for $b = 15$ and $v_0 = 0{\cdot}5$.

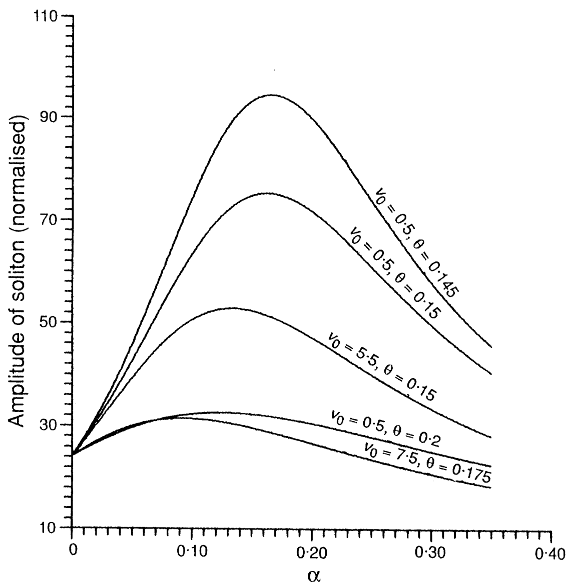

Fig. 2. Variation of the soliton amplitude showing the resonance phenomenon, for $\sigma = 0$ and $b = 15$.

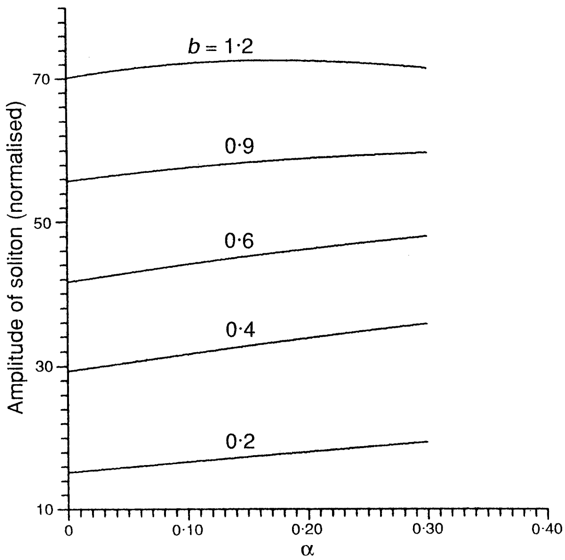

Fig. 3. Soliton amplitude for small values of b and for $\sigma = 0$, $v_0 = 0\cdot5$ and $\theta = 0\cdot145$. Note the disappearance of the sharp peak.

$$\lambda^4 - 2v_0\,\lambda^3 + (\delta_2 - \delta_1)\lambda^2 + 2v_0\,\delta_1\,\lambda - \delta_1\,\delta_2 - \frac{\alpha(1+\alpha)}{\mu(1+n^2\pi^2/b^2)^2} = 0\,, \tag{27}$$

where

$$\delta_1 = \frac{1+\alpha}{1+n^2\pi^2/b^2} + 3\sigma\,,$$

$$\delta_2 = v_0^2 - \frac{\theta}{\mu} - \frac{\alpha}{\mu(1+n^2\pi^2/b^2)}\,.$$

Equation (27) is biquadratic with general coefficients and cannot be solved analytically. We analysed the possible solutions of (27) numerically by the Newton–Raphson method on a fast computer. Several solutions of λ pertaining to different sets of values of the physical constants (v_0, θ, σ, b) are shown in Fig. 1. These values of λ were then used in the expressions for the soliton amplitude and width (26). We have plotted the amplitude as a function of the parameter $\alpha = n_e^b/n_e^b$, i.e. the ratio of the electron densities, in Figs 2–5 for different values of (v_0, θ, b, σ). Before further discussion, we note that the soliton amplitude clearly shows a peak in each case, but the peak height is very prominent when the size of the containing system is very large (see Fig. 2 for $b = 15$). If the size is smaller, however, for example $b = 3$ or even less, then the peak is not so distinct. Hence the dimension of the containment system has a very important effect on the resonance phenomenon between the incoming beam

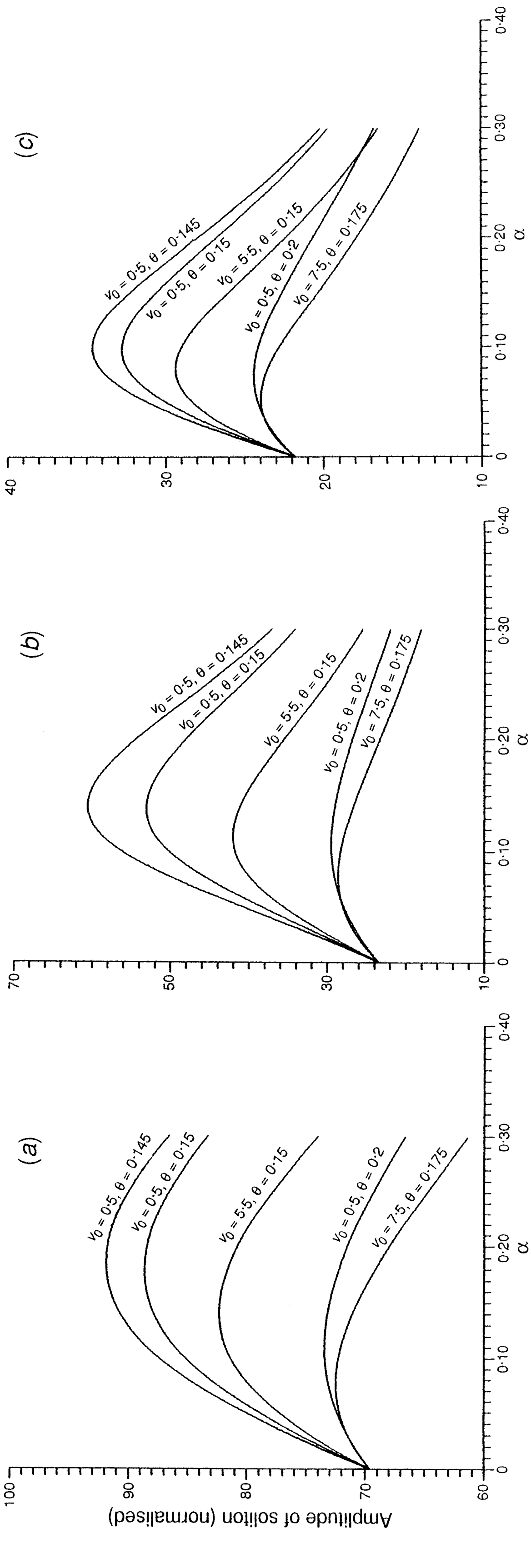

Fig. 4. Variation of soliton amplitude for (a) $b = 3$ and $\sigma = 0$, (b) $b = 15$ and $\sigma = 0{\cdot}01$ and (c) $b = 15$ and $\sigma = 0{\cdot}04$. In (b), note the reduction in the height of the peak.

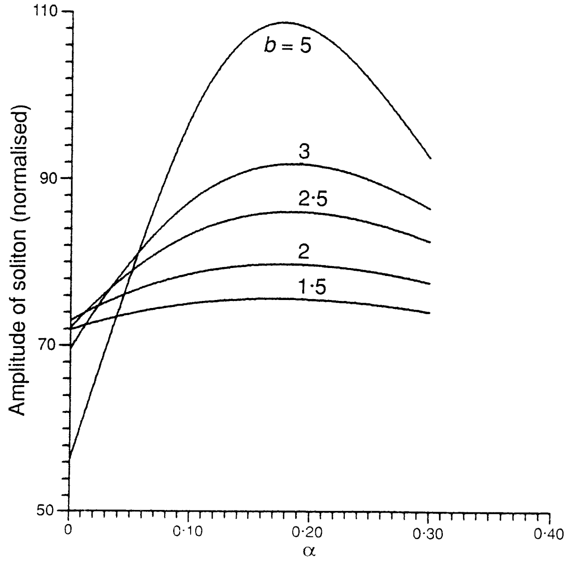

Fig. 5. Relative height of the peaks for different b values for $\sigma = 0$, $\theta = 0 \cdot 145$ and $v_0 = 0 \cdot 5$.

electrons and the plasma, as displayed through the peak in the variation of the soliton amplitude.

In short, it can be noted that since in the majority of experimental plasma arrangements it is difficult to enlarge the size of the confining system, this resonance phenomenon may not be easily detected. Of course the peak in the soliton amplitude depends on the streaming velocity v_0, as well as θ and σ. It is interesting to note (see Fig. 2) that the peak in the soliton amplitude increases with θ while all other parameters, including the streaming velocity v_0, are kept fixed. Such a situation is also exhibited in Fig. 4*a* where we show the phenomenon for the small value of $b = 3$ and $\sigma = 0$. It should be noted that this value of b can be considered marginal for the observation of this resonance phenomenon; if b is reduced further, it is difficult to observe the peak (see Fig. 2). On the other hand, as we increase σ the height of the peak is reduced. For the width of the soliton, we observe that the usual pattern is followed, as displayed in Fig. 6.

4. Linear Analysis and Resonance

In our analysis we have observed an interesting resonance phenomenon through the behaviour of the amplitude of the soliton, an essentially nonlinear mode in the plasma. Now we want to show that insight into this phenomenon can be obtained through a conventional linear analysis. Let us consider equations (1) and (6) and set

$$\begin{aligned} n_{\rm i} &= n_{\rm i0} + \delta n_{\rm i}; \qquad v_{\rm i} = v_{\rm i}^0 + \delta v_{\rm i}\,, \\ n_{\rm e}^{\rm b} &= n_{\rm e}^{\rm b(0)} + \delta n_{\rm e}^{\rm b}, \qquad v_{\rm e}^{\rm b} = v_{\rm e}^{\rm b(0)} + \delta v_{\rm e}^{\rm b}, \qquad \text{etc.}, \end{aligned} \tag{28}$$

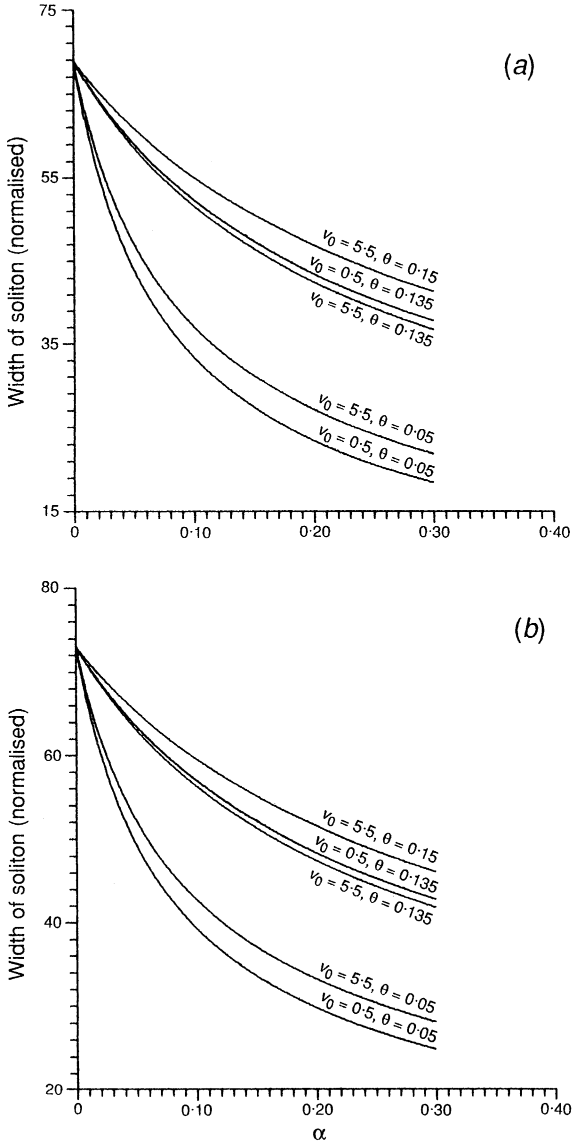

Fig. 6. Width of the soliton for $(b = 15)$: (a) $\sigma = 0 \cdot 4$ and (b) $\sigma = 0$.

with the equilibrium values as noted previously; we neglect higher-order terms in δn_{i}, δv_{i}, $\delta n_{\mathrm{e}}^{\mathrm{b}}$, etc. Hence we obtain a set of linear equations for these variations. We then make the usual assumptions that each such variation can be represented as

$$\delta v_{\mathrm{i}} = \bar{v}_{\mathrm{i}}(z)\, \mathrm{e}^{\mathrm{i}(kx-\omega t)}; \qquad \delta n_{\mathrm{e}}^{\mathrm{b}} = \bar{n}_{\mathrm{e}}^{\mathrm{b}}(z)\, \mathrm{e}^{\mathrm{i}(kx-\omega t)}; \quad \text{etc.}$$

As a result we obtain

$$-\,\omega \bar{n}_{\mathrm{i}}(z) + (1+\alpha)k\bar{v}_{\mathrm{i}}(z) = 0\,, \tag{29}$$

$$- \omega\overline{v}_{\mathrm{i}}(z) + k\overline{\phi}(z) + \frac{3\sigma k}{1+\alpha}\overline{n}_{\mathrm{i}}(z) = 0\,, \tag{30}$$

$$- \omega\overline{n}_{\mathrm{e}}^{\mathrm{b}}(z) + \alpha k\overline{v}_{\mathrm{e}}^{\mathrm{b}}(z) + v_0\, k\overline{n}_{\mathrm{e}}^{\mathrm{b}}(z) = 0\,, \tag{31}$$

$$- \omega\mu\alpha\overline{v}_{\mathrm{e}}^{\mathrm{b}}(z) + \mu\alpha v_0(z)\,\overline{v}_{\mathrm{e}}^{\mathrm{b}}(z) - \alpha k\overline{\phi}(z) + \theta k\overline{n}_{\mathrm{e}}^{\mathrm{b}}(z) = 0\,, \tag{32}$$

$$\frac{\partial^2\phi(z)}{\partial z^2} = (k^2+1)\overline{\phi}(z) + \overline{n}_{\mathrm{e}}^{\mathrm{b}}(z) - \overline{n}_{\mathrm{i}}(z)\,. \tag{33}$$

Eliminating all other variables in favour of $\overline{\phi}$ we get

$$\frac{\partial^2\phi}{\partial z^2} + \left(\frac{1+\alpha}{(\omega^2/k^2) - 3\sigma^2} - \frac{\alpha}{\theta - 2\mu v_0(\omega/k) - \mu v_0^2 - \mu(\omega^2/k^2)} - (k^2+1)\right)\overline{\phi} = 0\,. \tag{34}$$

Solving this equation with the boundary condition $\phi = 0$ on $z = 0$ and $z = b$, we arrive at the sixth-order dispersion relation

$$A_0\, k^6 + B_0\, k^5 + C_0\, k^4 + D_0\, k^3 + E_0\, k^2 + F_0\, k + G_0 \;=\; 0\,, \tag{35}$$

where the coefficients are

$$\begin{aligned}
A_0 &= 3\sigma\mu v_0^2 - 3\sigma\theta\,,\\
B_0 &= 6\sigma\mu v_0\,\omega\,,\\
C_0 &= -\,\theta(1+\alpha) + \mu v_0^2(1+\alpha) - 3\sigma\alpha \;+\; \theta\omega^2 - \mu v_0^2\,\omega^2\\
&\quad -\,3\sigma\theta(1+n^2\pi^2/b^2) + 3\sigma\mu(1+n^2\pi^2/b^2)v_0^2\,,\\
D_0 &= 2\mu v_0(1+\alpha)\omega - 2\mu v_0\,\omega^3 + 6\sigma\mu(1+n^2\pi^2/b^2)v_0\,\omega\,,\\
E_0 &= \mu(1+\alpha)\omega^2 + \alpha\omega^2 + \theta(1+n^2\pi^2/b^2)\omega^2\\
&\quad -\,\mu(1+n^2\pi^2/b^2)v_0^2\,\omega^2 + 3\sigma(1+n^2\pi^2/b^2)\mu\omega^2\,,\\
F_0 &= -\,2\mu v_0\,\omega^3(1+n^2\pi^2 b^2)\,,\\
G_0 &= -\,\mu(1+n^2\pi^2/b^2)\omega^4\,.
\end{aligned}$$

We have solved equation (35) numerically and have observed the variation of a particular mode of k for a fixed value of ω. Here also we looked at the variation with respect to α. The variation of k shows sharp discontinuous changes near values of α where we observe a peak in the amplitude of the soliton (see Fig. 7). So here again we are observing the resonance phenomenon in another guise.

5. Discussion

In our analysis we have studied an interesting resonance phenomenon between beam electrons and plasma, from the point of view of both linear and nonlinear

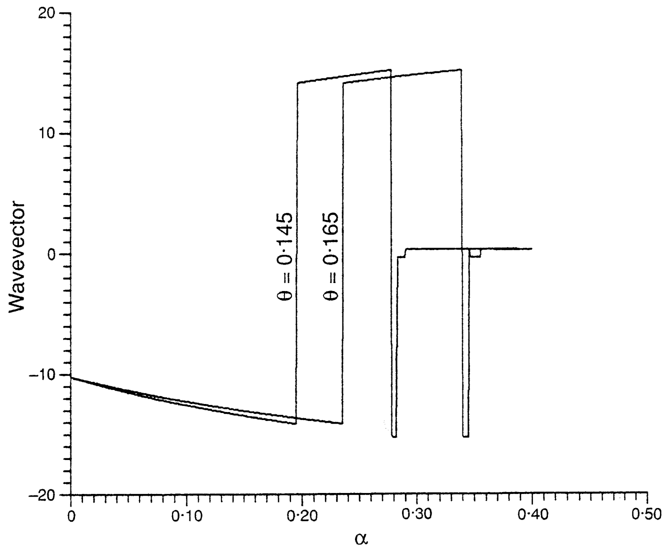

Fig. 7. Solution of the linear dispersion relation showing the sharp change in k as an indication of the resonance, with $b = 15$, $\sigma = 0$, $\omega = 0 \cdot 001$ and $v_0 = 0 \cdot 5$.

theory. The analysis shows that the phenomenon is greatly influenced by the dimensions of the containment system and other plasma parameters. It also indicates the fact that the streaming velocity has an important role to play in the enhancement of the phenomenon.

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