

The Distribution of Electrons in a Uniform Electric Field

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Abstract

Experiments are under way at the University of New England to measure the optical absorption of excited gas particles in a pre-breakdown discharge. Such measurements can be used to deduce the number density of electrons in the discharge. By comparing this experimental density map with the predictions of theory, electron transport parameters may be determined. In this paper, new theoretical expressions are derived for the number density distributions of electrons in a uniform electric field. These are found by solving the electron diffusion equation in a plane parallel electrode geometry with a radially symmetric cathodic current source. The contribution of ion-induced secondary current is included, and problems posed by non-equilibrium conditions near the electrodes are addressed. Techniques of data reduction are discussed with a particular emphasis on the avoidance of these problems.

1. Introduction

Electron transport parameters have been determined with success by measuring the light emitted by excited atoms or molecules in a pre-breakdown discharge (Blevin and Fletcher 1992). The rate of excitation of these gaseous particles has been shown to be approximately proportional to the number density distribution of electrons in the discharge. A disadvantage of this 'luminous flux' method is the time delay between the excitation of a state and its fluorescent decay (Fletcher and Reid 1980). At the University of New England experiments are under way that seek to avoid this problem by the direct measurement of number densities of excited particles in a pre-breakdown discharge. It is hoped that by mapping the optical absorption of the discharge at selected transition frequencies, the rates of excitation, and therefore the distribution of electrons, may be more accurately deduced.

In order to analyse these measurements, theoretical electron distributions must first be calculated. Previous authors (e.g. Lucas 1964) have calculated number density profiles of electrons in a one-dimensional discharge; three-dimensional density distributions have also been published for electron swarms and streams between finite plane electrodes in geometries that include a point source of electrons (Huxley 1972) and an array of guard rings between cathode and anode (Lucas 1965). However, discharge chambers in use at the University of New England have been designed to examine changes in the current passed by the discharge as the electrode separation is varied (see e.g. Folkard and Haydon 1971). In order to

preserve the uniformity of the electric field at small electrode separations, it has been found necessary to generate the primary current photoelectrically by the external illumination of a conducting but transparent window in the centre of the cathode. Because the current density available from a photoelectric source is low, the window must occupy a significant fraction of the cathode in order to maximise the total primary current. Huxley's theory becomes a poor approximation in this case. Guard rings have not been used in these chambers because of the difficulty of maintaining the spacing between rings as the electrodes are moved. As far as the author knows, there is no published work on the spatial distribution of electrons that is appropriate to our chamber geometries. The purpose of the present paper is to calculate such distributions.

The theory as given below represents a compromise between two conflicting aims: to make the theory as complete as possible while retaining a degree of mathematical simplicity. Many approximations have been made, for example, in the treatment of the secondary current. This approach is justified in the final section, where it is shown that useful information about the transport parameters can be gained from an analysis that is independent of the distribution and size of the cathode current.

2. Discharge Geometry and Processes

The geometry of the discharge volume is depicted in Fig. 1. The discussion in this paper will be restricted to gases which are ionisable but non-attaching. Electron-ion pairs are generated in such a discharge by collisions between drifting electrons and gas molecules at a rate equal to $w\alpha_1n$, where w is the electron drift velocity, α_1 is Townsend's primary ionisation coefficient and $n(r, z)$ is the number of electrons per unit volume. Secondary electrons may be ejected from the cathode by the incidence of ions and also by excited gas particles or resonant photons. Excited particles can be divided into short- and long-lived (i.e. metastable) types, depending on whether a significant fraction decay to the ground state before diffusing to the cathode. In this paper the contribution of excited states and photons will be neglected, for reasons given below.

The typical efficiency γ of ejection of a secondary electron by an energetic photon is low. A simple experiment performed by the author showed that approximately $100\ \mu\text{W}$ of $254\ \text{nm}$ light incident on a clean copper cathode produced $150\ \text{pA}$ of photocurrent, giving a γ value of about 10^{-5} for these photons. In contrast, Haydon and Williams (1973*b*, 1976) determined the total efficiency of ejection of secondaries from a clean gold electrode by molecular nitrogen ions plus resonant photons to be about 2×10^{-3} . Fletcher and Blevin (1981) found that atomic ions in nitrogen and hydrogen discharges were several times more efficient at ejecting electrons from the cathode than were resonant photons [although the molecular ion H_3^+ , the dominant species in a hydrogen discharge at low values of E/N (the applied electric field divided by the gas number density), was found to be a less efficient producer of secondaries than hydrogen-resonant photons].

The number densities of short-lived states, and therefore the flux of these particles into the cathode, can, by definition, be expected to be negligible. The exception to this rule arises in the case of a strong radiative coupling between the excited and ground states. The effective lifetime of these states may then be kept artificially high by the phenomenon of radiation trapping. The problem

of finding the distributions of resonant states under these circumstances is an intractable one (Holstein 1947) and will not be attempted here. However, the difficulty may be avoided if the effective lifetime of the excited state is significantly longer than the time required for ions to drift between the electrodes. Amies and Fletcher (1983) found this to be true in helium, for example. In this case the respective contributions to the secondary current of ions and excited states may be distinguished by the technique described in the following paragraph.

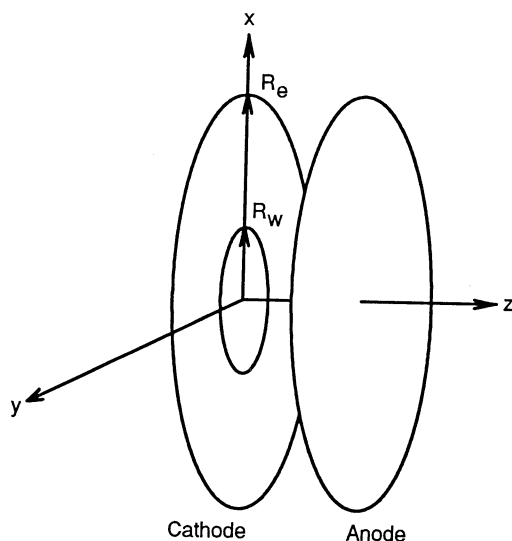


Fig. 1. The discharge geometry and coordinate axes.

Metastable excited states may be present in large numbers and may have a significant probability of ejecting an electron upon striking the cathode. Their contribution is, however, separable from that of the ions. This is because ions and electrons drift much faster than the speed of diffusion of neutral gas particles. An experiment using a 'chopped' source of primary current therefore exhibits phenomena upon two different timescales (see e.g. Haydon and Williams 1976). The electron distribution attains a quasi-equilibrium state within a few microseconds of each activation of the primary source. Nearly all the electrons leaving the cathode at this time have come either from the primary source or from the impact of ions on the cathode; there has not been sufficient time for significant numbers of metastable particles to diffuse back to the cathode. The rise in current due to metastable-induced secondaries is typically slower by two to three orders of magnitude. Therefore the distribution of electrons immediately after turn-on may be calculated to a good approximation by solving the steady-state electron diffusion equation without including the effect of metastable particles.

3. Validity and Boundary Conditions

The Boltzmann equation for the density of electron states can be approximated by a second-order continuity equation in the electron density $n(\mathbf{r})$ (e.g. Skullerud 1974):

$$\mathbf{D} : \nabla \nabla n - \mathbf{w} \cdot \nabla n + w \alpha_i n = \frac{\partial n}{\partial t},$$

where $\mathbf{D}$ is the electron diffusion tensor. In cylindrical polar coordinates, the steady state form of this becomes

$$D \left(\frac{\partial^2 n}{\partial r^2} + \frac{1}{r} \frac{\partial n}{\partial r} \right) + D_L \frac{\partial^2 n}{\partial z^2} - w \frac{\partial n}{\partial z} + w \alpha_i n = 0, \quad (1)$$

where the notation $D = D_{xx} = D_{yy}$ and $D_L = D_{zz}$ has been used to conform with previous work (Huxley and Crompton 1974). The radial diffusion coefficient D has been distinguished from D_L as it is well established that they are unequal (e.g. Wagner *et al.* 1967). The usual route to equation (1) has been via an expansion of the electron distribution function $f(\mathbf{v}, \mathbf{r})$ in spherical harmonics within velocity space. Generally only the first two terms are kept. Lowke *et al.* (1977) used this approach without any further approximations to derive a continuity equation in one spatial dimension, similar in form to equation (1) except that the coefficients D and w were found to be functions of $\mathbf{r}$ through a dependence on the first term in the expansion of f . These authors concluded that the use of constant values of these coefficients was an acceptable approximation only where ∇n was small; in particular, the approximation was shown to fail badly near an absorbing boundary. The thickness of this non-equilibrium boundary layer was found to be $\approx D/w$. Skullerud (1974) reached similar conclusions.

At first sight it seems that equation (1) is not likely to be of any use in predicting the variation in electron number density throughout a discharge because sensible boundary conditions cannot be imposed. However, it has been found that solutions of equation (1) conform closely in shape to experimentally determined electron distributions away from the electrode boundary layers (see e.g. Blevin *et al.* 1976*a*, 1976*b*, 1978). It therefore seems likely that the main effect of these layers is to make it impossible to use equation (1) to determine with accuracy the amplitude of the number density function. The emphasis in this paper has therefore been placed upon the determination of transport parameters by the use of ratios in, rather than absolute values of, the electron number density.

The next problem to address is the selection of physically reasonable boundary conditions. This paper will follow the practice of previous authors (Huxley 1972; Lucas 1965) in adopting, at the anode, the boundary condition $n(d) = 0$ (where d is the electrode separation). Whereas the anode might be expected to exert a purely local perturbation upon the electron distribution, the effect of the cathode is more fundamental. This is because the streaming of electrons from cathode to anode ensures that a large fraction of the electrons making up the bulk of the discharge have come straight from the cathode. The electron distribution throughout the entire discharge volume thus depends crucially on the distribution of current leaving the cathode. These electrons are ejected from the cathode surface with a highly non-thermal velocity distribution, their average energy being generally lower than the equilibrium value (Haydon and Williams 1973*b*). The lower energy of the electrons in the cathode layer results in a variation in the ionisation rate α_i within this layer. A method of accounting for this variation will be examined in Section 4*b*.

Another problem at the cathode is that the current at a small distance from this electrode is less than the current of electrons actually ejected from its surface. This is because some of the ejected electrons are immediately reflected by gas molecules back to the surface, where they are resorbed. The total current density $j(r, z)$ in the equilibrium region far from the electrodes is

$$j(r, z) = e \left[wn(r, z) - D_L \frac{\partial n}{\partial z} \right], \quad (2)$$

where e is the electronic charge. It is tempting to identify the backscattered electrons with the diffusive part of the current density defined in equation (2) and therefore to equate the ejected current density to the drift current density $ewn(r, 0)$. This was essentially the approach adopted by Huxley (1972), who used a dipole source of electrons at the centre of the cathode. This type of source ensures that $n(r, 0) = 0$ for $r > 0$. This approach cannot be used if the current source is extended across the cathode, however, because there may then be places on the cathode where $(\partial n / \partial z)|_{z=0}$ is negative; equation (2) then predicts a local current density near the cathode which is *larger* than the amount ejected from the adjacent surface. To avoid this impossibility, the cathode boundary condition adopted throughout the present work is that $j(r, 0)$ given by equation (2) is equal to some constant fraction of the flux of electrons ejected from the cathode. In other words, let

$$j(r, 0) = q[j_{uv}(r) + e\gamma_i \phi_i(r, 0)], \quad (3)$$

where q is the constant of proportionality, $j_{uv}(r)$ is the primary (photo-) current density and γ_i and $\phi_i(r, 0)$ are respectively the electron ejection efficiency and cathodic flux of the ions.

4. Results

(4a) Solution of the Diffusion Equation

For discharge regimes where the electron number density at the electrode edges is negligible, the discharge may be modelled by a space bounded by infinite plane electrodes but with the primary current restricted to a circularly symmetrical region in the centre of the cathode. Solutions using this model have been found for a point source of primary current (Huxley 1972), but none have been proposed for an extended current source. Such a solution is developed in this section.

The above geometry lends itself to use of the zero-order Hankel transform (Sneddon 1972):

$$\mathcal{H}[f(r) : r \rightarrow \eta] = \int_0^\infty f(r) J_0(\eta r) r \, dr$$

to solve equation (1). [$J_n(x)$ is the Bessel function of order n .] Although it is generally difficult to back-transform the solutions analytically to obtain closed-form expressions for $n(r, z)$, this can be done numerically if desired.

One advantage in using the Hankel transform arises from the identity

$$\mathcal{H}[f(r) : r \rightarrow \eta] = \mathcal{F}_c\{\mathcal{A}[f(r) : r \rightarrow x] : x \rightarrow \eta\}, \quad (4)$$

where $\mathcal{F}_c\{\}$ and $\mathcal{A}[\]$ are the Fourier cosine and Abel transforms respectively. Many techniques for measuring the densities of molecular species in a cylindrically symmetrical space return Abel-transformed data; in these cases, the application of a Fourier cosine inversion to the Hankel transformed solutions presented below enables the comparison with experiment to be made without the necessity for smoothing and Abel inversion of the data values.

The transformed steady-state electron distribution $N(\eta, z) = \mathcal{H}[n(r, z) : r \rightarrow \eta]$ obeys the differential equation

$$\frac{\partial^2 N}{\partial z^2} - 2\lambda \frac{\partial N}{\partial z} + [\lambda^2 - u^2(\eta)]N = 0, \quad (5)$$

where

$$\lambda = w/2D_L$$

and

$$u^2(\eta) = \lambda^2 - 2\lambda\alpha_i + \eta^2 D/D_L. \quad (6)$$

There are two cases to consider: either $u^2(0) > 0$ or $u^2(0) \leq 0$. The first case only will be considered here, since $\lambda \gg 2\alpha_i$ in most discharge regimes of interest. In this case the general solution to equation (5) is

$$N(\eta, z) = A(\eta) \exp[(\lambda - u)z] + B(\eta) \exp[(\lambda + u)z].$$

Application of the electrode boundary conditions described in Section 3 gives the particular solution

$$N(\eta, z) = \frac{\zeta(\eta, 0)}{eD_L\beta(\eta)} \{ \exp[(\lambda - u)z] - \exp[(\lambda + u)z - 2ud] \}, \quad (7)$$

where

$$\zeta(\eta, 0) = \mathcal{H}[j(r, 0) : r \rightarrow \eta]$$

and

$$\beta(\eta) = \lambda + u - (\lambda - u) \exp(-2ud).$$

The electron distribution can be found by back-transforming, i.e.

$$n(r, z) = \int_0^\infty N(\eta, z) J_0(\eta r) \eta d\eta. \quad (8)$$

In the pre-breakdown regime most of the cathode current $j(r, 0)$ will be concentrated within the window area of radius R_w . This is because the current within this area arises from both primary and secondary contributions whereas the current density at greater radius is due only to secondary sources. The

function $j(r, 0)$ may therefore be expected to have a shape that is peaked at $r = 0$, decreasing asymptotically to zero as $r \rightarrow \infty$. The width of this function at half maximum will be of the order of R_w . Recall that the space-bandwidth product, found by multiplying the respective half-maximum widths of a function and its Fourier transform, is approximately equal to 2π . Because the Hankel-transformed cathode current $\zeta(\eta, 0)$ represents a radial 'slice' through the two-dimensional Fourier transform of $j(r, 0)$ (Sneddon 1972), one might expect the amplitude of $\zeta(\eta, 0)$ to be negligible at values of η much greater than $2\pi/R_w$. A good approximation to the solution $n(r, z)$ may therefore be obtained by truncating the integral in equation (8) at some value $\eta' > 2\pi/R_w$. Equation (8) could then be integrated numerically using a discrete Hankel transform. An example solution calculated using the discrete transform algorithm of Siegman (1977) is displayed in Fig. 2. The form used for $j(r, 0)$ was

$$j(r, 0) = qj_{uv}(r) = \begin{cases} qj_{uv}, & r < R_w \\ 0, & r > R_w. \end{cases} \quad (9)$$

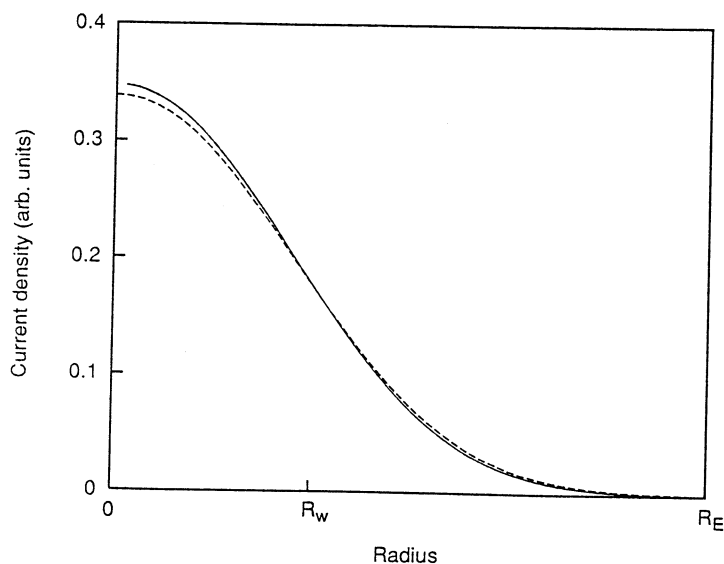


Fig. 2. Distributions of current across the anode. The solid curve is the exact solution obtained by the Hankel inversion of equation (7); the dashed curve is the approximate solution given by equation (17). The small gap in the solid curve near the vertical axis is an artefact of the Hankel transform algorithm.

The transport parameters used to calculate the curves in Fig. 2 are similar to those measured by Purdie and Fletcher (1989) for a discharge in nitrogen at a reduced electric field E/N of 100 Td ($1 \text{ Td} = 10^{-21} \text{ V m}^2$). An electrode separation of 25 mm and a gas number density of $1.609 \times 10^{22} \text{ m}^{-3}$ ($=0.5 \text{ Torr}$ at 300 K) were assumed. The values used for R_w and R_E were, respectively, 10 and 30 mm. For the sake of the example secondary processes were neglected.

It is possible to further manipulate equation (7) so that only the distribution of primary current need be known *a priori*. The first step in this procedure is to

consider the flux of ions onto the cathode. This is easy to calculate if the radial diffusion of the ions can be neglected. Consider a cloud of ions originating at a point source on the anode. The radial spread of this cloud may be estimated by calculating the mean radial displacement $|\bar{r}|$ of the ions. McDaniel (1964) gave as the ratio between $|\bar{r}|$ and the electrode separation d ,

$$\frac{|\bar{r}|}{d} = \frac{0.172}{\sqrt{V}},$$

where V is the potential difference between the electrodes. Clearly the sideways diffusion of ions can be neglected for values of V larger than about 5 V. Hence we may write

$$\begin{aligned}\Phi_i(\eta, 0) &= w\alpha_i \int_0^d N(\eta, z) dz \\ &= \frac{2\lambda\alpha_i \zeta(\eta, 0)}{e(\lambda^2 - u^2)} \left\{ \frac{2u}{\beta} \exp[(\lambda - u)d] - 1 \right\},\end{aligned}\quad (10)$$

where Φ_i is the transformed cathodic ion flux density. Equation (10), when inserted into the Hankel transform of equation (3), allows the reformulation of equation (7) as

$$N(\eta, z) = \frac{qS\zeta_{uv}(\eta, 0)}{eD_L\beta} \{ \exp[(\lambda - u)z] - \exp[(\lambda + u)z - 2ud] \}, \quad (11)$$

where

$$\frac{1}{S(\eta)} = \left\{ 1 - \frac{2q\lambda\alpha_i\gamma_i}{\lambda^2 - u^2} \left[\frac{2u}{\beta} \exp[(\lambda - u)d] - 1 \right] \right\}. \quad (12)$$

If $u(\eta) = \lambda$ for any η , $S(\eta)$ becomes, for that value of η ,

$$\frac{1}{S(\eta)} = \left\{ 1 - \frac{q\alpha_i\gamma_i}{2\lambda} [(2\lambda d + 1) \exp(-2\lambda d) - 1] \right\}.$$

In the case that the primary current is generated by the backlighting of a circular window, as described in Section 1, $j_{uv}(r)$ will have the form given in equation (9). The Hankel transform of this function is

$$\zeta_{uv}(\eta, 0) = j_{uv} R_w J_1(\eta R_w)/\eta. \quad (13)$$

Note that S as given by equation (12) is singular if

$$\frac{2q\lambda\alpha_i\gamma_i}{\lambda^2 - u^2} \left\{ \frac{2u}{\beta} \exp[(\lambda - u)d] - 1 \right\} = 1.$$

It can be shown that S is nonsingular for all η at electrode separations d less than some value d_{bd} ; at this separation, the singularity occurs at $\eta = 0$. No solution exists at this separation because the inverse transform in equation (8)

does not converge in this case. This singularity corresponds to the electrical breakdown of the gas.

(4b) *Effects of the Variation of α_i near the Cathode*

As mentioned in Section 3, the effective ionisation coefficient may vary in the cathode non-equilibrium layer. The above analysis could be amended to include this effect by allowing α_i in equation (5) to vary with z . This dependence of α_i on z can be approximated, in many discharge regimes, by the step function

$$\alpha_i(z) = \begin{cases} 0, & z < d_0 \\ \alpha_i, & z > d_0 \end{cases}$$

without significant error (Haydon and Williams 1973a, 1973b). An analytical solution is possible in this approximation, albeit at the expense of greater mathematical complexity. The solution N becomes

$$N(\eta, z) = A(\eta) \begin{cases} B(\eta) \exp[(\lambda - u')z] + C(\eta) \exp[(\lambda + u')z], & z < d_0 \\ \exp[(\lambda - u)z] - \exp[(\lambda + u)z - 2ud], & z > d_0, \end{cases}$$

where

$$(u')^2 = \lambda^2 + \eta^2 D/D_L,$$

The coefficients A , B and C can be evaluated by appropriate use of the boundary conditions at $z = 0$, d_0 and d . When secondary current is included, an expression similar to equation (11) is obtained. The parameter S' that has the same role as the S in equation (11) is given by

$$S'(\eta) = \left[1 - \frac{2q\lambda\alpha_i\gamma_i}{\beta'(\eta)} W(\eta) \right]^{-1},$$

where

$$\beta'(\eta) = B(\eta)(\lambda + u') + C(\eta)(\lambda - u'),$$

$$W(\eta) = 2u \exp[(\lambda - u)d]$$

$$- \{(\lambda + u) - (\lambda - u) \exp[-2u(d - d_0)]\} \exp[(\lambda - u)d_0].$$

In Fig. 3 three distributions of anode current are plotted. The transport parameters used here are similar to those measured by Blevin *et al.* (1976b, 1978) for a discharge in hydrogen at 200 Td and $1.609 \times 10^{22} \text{ m}^{-3}$ ($=0.5$ Torr at 300 K). Folkard and Haydon (1971) measured the thickness of the cathode non-equilibrium layer to be about 16 mm at this field and pressure. An electrode separation of 25 mm has been used in the present example, the secondary ionisation coefficient γ_i being arbitrarily set to one half of the value required to produce electrical breakdown of the gas at this distance. All the other parameters are the same as those used to calculate the current distributions displayed in Fig. 2.

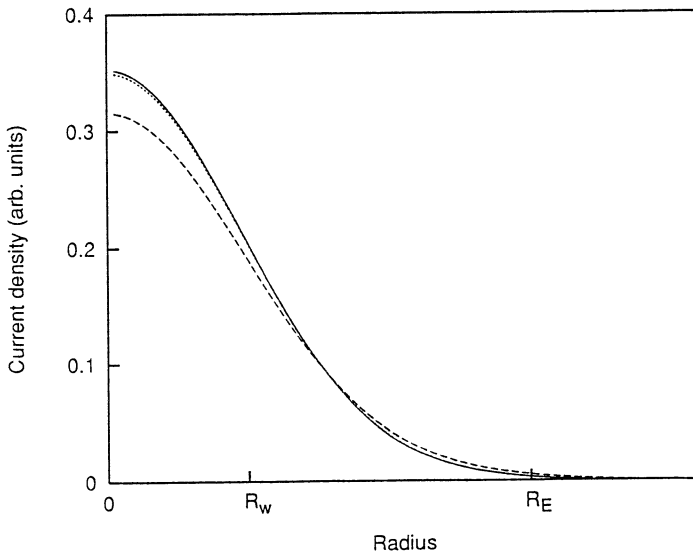


Fig. 3. The effect of the variation of α_i in the cathode non-equilibrium layer upon the anode current distribution. The solid curve represents the distribution calculated using the step-function approximation for α_i , with $d_0 = d/4$. The dashed curve is the current distribution in the approximation $d_0 = 0$. The dotted curve was calculated using the same approximation, but with an effective value of γ_i' given by equation (15).

The solid curve in Fig. 3 represents the distribution of anode current calculated using the step-function approximation for α_i , with a value for d_0 of $d/4$. The dashed curve is the solution obtained using the cruder approximation that $d_0 = 0$. The two curves have been normalised to the same total anode current but are clearly different in shape. However, most of this difference arises from the spread of the discharge across the cathode as the proportion of secondary to primary current increases in the approach to breakdown. In fact the error in the shape of the $d_0 = 0$ solution can be largely corrected by using an effective value of γ_i scaled so that the ratio between the total primary cathode current and the total secondary cathode current is the same for the two cases. This can be achieved by setting $S(0)$ equal to $S'(0)$. A fair approximation to this condition can be obtained by neglecting the influence of longitudinal diffusion, giving a value for the effective coefficient γ_i' of

$$\gamma_i' = \gamma_i \frac{\exp(\alpha_i d - \alpha_i d_0) - 1}{\exp(\alpha_i d) - 1}. \quad (14)$$

A third solution was calculated, using the (simpler) $d_0 = 0$ analysis, but with an effective γ_i calculated using equation (14). When the resulting current density is plotted, normalised again to the same value of total anode current as the $d_0 = d/4$ solution, these two curves vary by less than 2%.

Note that the parameters necessary to apply equation (14) can be independently calculated by measurements of the total current passed by the discharge at varying d (see e.g. Haydon and Williams 1976).

(4c) Approximate Solutions

Equation (6) can be rewritten as

$$u^2(\eta) = u_0^2 \left(1 + \frac{\eta^2 D}{u_0^2 D_L} \right),$$

where the shorthand $u_0 = u(0)$ has been used. Expanding $u(\eta)$ in terms of η gives

$$u(\eta) = u_0 + \frac{\eta^2 D}{2u_0 D_L} + O(\eta^4).$$

This series converges provided that

$$\eta < u_0 \sqrt{\frac{D_L}{D}}.$$

The transformed number density N given by equation (7) may therefore be approximated at small values of η by

$$N(\eta, z) \approx \frac{\zeta(\eta, 0)}{e D_L \beta_0} Z_0(z) \exp\left(-\frac{\eta^2 D z}{2u_0 D_L}\right), \quad (15)$$

where

$$Z_0(z) = \exp[(\lambda - u_0)z] - \exp[(\lambda + u_0)z - 2u_0 d].$$

In Section 4a it was shown that N becomes negligible at values of η greater than $2\pi/R_w$. Equation (15) can, therefore, be used in place of equation (7), provided that

$$\frac{2\pi}{R_w} < u_0 \sqrt{\frac{D_L}{D}}.$$

Use of the assumption made in Section 4a that $u_0 \approx \lambda$ changes this condition to

$$w R_w > 4\pi \sqrt{D D_L}.$$

This condition is satisfied by many monatomic and diatomic gases at values of R_w of around 10 mm, if the gas number density is greater than about $3 \times 10^{22} \text{ m}^{-3}$ (equivalent to roughly 1 Torr at 300 K). The solution of equation (15) is obtained formally by a reverse transform:

$$n(r, z) \approx \frac{Z_0(z)}{e D_L \beta_0} \int_0^\infty \zeta(\eta, 0) \exp\left(-\frac{\eta^2 D z}{2u_0 D_L}\right) J_0(\eta r) \eta d\eta. \quad (16)$$

In the small-signal limit, $j(r, 0)$ has the step function form described by equation (9). The Hankel transform of this current distribution is given in

equation (13). If $\zeta(\eta, 0)$ has this form, the right-hand side of equation (16) can be expressed in terms of a tabulated function. Inserting equation (13) into equation (16) and using the replacements

$$x^2 = \frac{u_0 D_L}{2zD} R_w^2,$$

$$y^2 = \frac{u_0 D_L}{2zD} r^2$$

and

$$t^2 = \frac{zD}{2u_0 D_L} \eta^2$$

produces

$$n(r, z) \approx \frac{qj_{uv} Z_0(z)}{eD_L \beta_0} \left[2x \int_0^\infty \exp(-t^2) J_0(2yt) J_1(2xt) dt \right]. \quad (17)$$

The term in square brackets is the function $x^2 \sqrt{\pi} F(x, y)$ described by Luke (1962) and references therein. It is tabulated as $P(x\sqrt{2}, y\sqrt{2})$ in Masters (1955).

A plot of the anode current obtained using equation (17) is compared in Fig. 2 with the exact solution derived in Section 4a. The same transport parameters were used. The two curves agree well, indicating that the approximation given in equation (16) is a valid one.

5. Data Reduction

The proposed experiment uses a 'chopped' ultraviolet light source to generate bursts of primary current. Changes in the number density of excited states will be monitored by measuring the optical absorption integrated along a path through the discharge. The number density n_j of an excited state obeys a diffusion equation of the form

$$\frac{\partial n_j}{\partial t} = w\alpha_j(\mathbf{r}, t) n(\mathbf{r}) + D_j \nabla^2 n_j - \frac{n_j}{\tau_j},$$

where α_j is the total rate of electronic excitation, D_j is the diffusion coefficient and τ_j the lifetime of the state. The time dependence of the excitation rate arises from the cascaded contributions from states of higher energy (Fletcher and Reid 1980). Provided that the maximum lifetime τ of these states is significantly shorter than τ_j , an intermediate time scale $\tau < t < \tau_j$ can be defined. Within this timescale the rate of increase of n_j may be expected to be approximately constant and given by

$$\frac{\partial n_j}{\partial t} \approx w\alpha_j(\mathbf{r}) n(\mathbf{r}).$$

The initial rate of rise $\dot{\kappa}(x, z)$ of the total optical absorption along a line parallel to the y axis will therefore be given, for $\tau < t < \tau_j$, by

$$\dot{\kappa}(x, z) = \frac{1}{I} \frac{dI}{dt} = w\sigma \int_{-\infty}^{\infty} \alpha_j(\mathbf{r}) n(\mathbf{r}) dy,$$

where σ is the absorption cross section. In deriving this expression it has been assumed that the total absorption is small and that the light is monochromatic and tuned to a transition of the j th excited state. The value of α_j depends on the mean energy of the electrons. The conditions under which this is approximately spatially invariant have been discussed in Section 3. Where this invariance holds, it follows that

$$\dot{\kappa}(x, z) \propto \int_{-\infty}^{\infty} n(\mathbf{r}) dy.$$

From the identity given in equation (4),

$$\mathcal{F}_c[\dot{\kappa}(x, z) : x \rightarrow \eta] \propto N(\eta, z) = \mathcal{H}[n(r, z) : r \rightarrow \eta].$$

The Hankel-transformed electron number density N can therefore be approximated by sampling $\dot{\kappa}$ at several values of x and then applying a discrete Fourier transform. Let the function G be defined as

$$G(\eta) = \frac{\mathcal{F}_c[\dot{\kappa}(x, z_1) : x \rightarrow \eta]}{\mathcal{F}_c[\dot{\kappa}(x, z_2) : x \rightarrow \eta]} = \frac{N(\eta, z_1)}{N(\eta, z_2)}.$$

Use of the approximation in equation (15) shows that

$$G(\eta) \approx \exp \left[\left(\lambda - u_0 - \frac{D\eta^2}{2D_L u_0} \right) (z_1 - z_2) \right].$$

The parameters $\lambda - u_0$ and $D/D_L u_0$ can therefore be determined by evaluating G from measurements of $\dot{\kappa}$ at two values of z and then fitting a Gaussian to the result. These parameters can be cast in a more familiar form by use of equation (6). This gives

$$\lambda - u_0 = \lambda(1 - \sqrt{1 - 4\alpha_i D_L/w});$$

for $\alpha_i D_L \ll w$ this reduces to

$$\lambda - u_0 \approx \alpha_i(1 + \alpha_i D_L/w).$$

In the same limit

$$\frac{D}{D_L u_0} \approx \frac{2D}{w} \left(1 + \frac{2\alpha_i D_L}{w} \right).$$

Note that

$$\dot{\kappa}(0, z) = 2w\alpha_i \sigma \int_0^\infty N(\eta, z) d\eta.$$

The parameter u_0 can be estimated by fitting the integral in this equation to samples of κ taken near the anode.

The techniques described above permit the determination of α_i and the ratios w/D_L and D/D_L . It does not appear to be possible to deduce the value of w using steady-state measurements. A more complete experimental technique would require the use of a short, intense source of primary current such as the excimer laser used by Purdie and Fletcher (1989).

Acknowledgments

I would like to extend my most sincere thanks to Dr M. P. Fewell for his patient advice during the protracted gestation of this paper.

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