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Scattering of Solitons from the Density Gradient of a Relativistic Bounded Plasma with Variable Streaming

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Abstract

We study a relativistic bounded plasma consisting of hot ions and electrons with density gradient and variable streaming. It is observed that the nonlinear wave generated inside is governed by a variable coefficient KdV equation, which we analyse numerically. Furthermore, the solitary wave is scattered at the discontinuity due to inhomogeneity, and the characteristics of the left and right moving wave become different. Such scattering usually takes place in the solar wind, interplanetary shocks, and in the environment of a comet where, as the relativistic effect gives rise to streaming, the non-uniformity of the plasma leads to a density variation. By evaluating a suitable form of such variations we compute the amplitude, velocity and width of the soliton before and after the scattering. The amplitude of the reflected wave is seen to decrease as in previous observations.

1. Introduction

The properties of nonlinear waves generated in a plasma hold a central place in theoretical plasma research. Of late the importance of relativisitic effects has been realised, since they are solely responsible for the phenomenon of mass variation and streaming (Nejoh 1987; Roy Chowdhury *et al.* 1988; Mukhopadhyay *et al.* 1992; Das and Paul 1985). These two physical phenomena have an important influence on various aspects of nonlinear waves. Experiments in plasma are usually performed in a very small volume so that the effects of the finite boundary should be taken into account. Already some results are available regarding the influence of such boundaries (Das and Ghosh 1988; Mukhopadhyay *et al.* 1992). Another important aspect of a real plasma is that its density seldom remains uniform. The effect of a density gradient has been considered in a few cases by Buti (1991) and Nishikawa and Kau (1975). An important consequence of the density gradient is that the solitary wave is scattered in such a region. It is important to note that scattering of solitons in plasma has been observed by Dahiya *et al.* (1978) and reflection of an ion-acoustic soliton from a negatively biased grid has been studied by Papa and Oertl (1983). Kuehl (1983) investigated theoretically the reflection of the ion-acoustic solition and showed that the amplitude of the reflected wave is much smaller than that of the incident. Recent experimental findings on the scattering of solitons by Nishida (1984) support this observation.

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In theoretical analysis so far the effects of the finite boundary (which is essential for any experimental observation) and relativity have been neglected. Here in this paper we study the scattering of solitons in a relativistic bounded plasma, consisting of hot ions and electrons, by treating the soliton on the left and right sides of the impurity separately. Due to the inhomogeneity and relativistic mass variation, the streaming velocity turns out to be variable and becomes a function of distance. Our paper is organised as follows. In the first section we deduce the variable coefficient KdV equation by the reductive perturbation technique, modified due to the density gradient and presence of the finite boundary (Mukhopadhyay *et al.* 1994). Next we discuss the technique to solve such an equation to obtain the solitary wave solution explicitly and then discuss how the amplitude and velocity depend on the variable streaming velocity, relativisitic mass variation, and the radius of the boundary. A modified form of scaled variable is introduced to describe the scattering of the solitary waves from the inhomogeniety.

2. Formulation

Let us consider a one-dimensional collisionless relativistic weakly inhomogeneous plasma having a spatial gradient in the ion density. The electrons are assumed to be isothermal because of their large thermal conductivity. The electron temperature is assumed to be much greater than the ion temperature which is assumed to be constant. As a consequence, the effect of Landau damping becomes small. In the absence of solitons the plasma is considered to be in a time independent steady state, which arises as a result of the loss of ions and electrons in the walls of the boundary and their production due to ionisation. The frequency of ionisation is small and therefore the condition of quasi-neutrality is a good approximation over a region extending from the plasma centre to the transionic layer. In the present analysis we assume that the scale length of plasma inhomogeneity is large compared with the soliton width. Under this assumption the soliton retains its shape and its amplitude, width and speed are functions of position. Neglecting transport properties such as heat conduction and viscosity and assuming a Boltzmann distribution for the electrons, we can write the one-dimensional ion continuity and momentum equations, the equation of state, the electron Boltzmann distribution and Poisson's equation in the following form (Mukherjee and Roy Chowdhury 1994; Murthy *et al.* 1984):

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) = 0\,, \tag{1a}$$

$$n\,\frac{\partial v_\alpha}{\partial t} + nv\,\frac{\partial v_\alpha}{\partial x} + \frac{\partial p}{\partial x} + n\,\frac{\partial \phi}{\partial x} = 0\,, \tag{1b}$$

$$\frac{\partial p}{\partial t} + v\,\frac{\partial p}{\partial x} + 3p\,\frac{\partial v_\alpha}{\partial x} = 0\,, \tag{1c}$$

$$\frac{\partial^2\phi}{\partial z^2} + \frac{\partial^2\phi}{\partial x^2} = n_e - n\,, \tag{1d}$$

$$n_e = \exp(\phi)\,, \tag{1e}$$

with

$$v_\alpha = \gamma v \approx (1 + v^2/2c^2)\, v\,.$$

Here we have used an approximate form for the Lorentz factor. Further, we have normalised the ion and electron densities by the zeroth order ion density n_0 at an arbitrary reference point in the plasma, which we choose to be $x = 0$. Also, v is the ion velocity normalised by the ion-acoustic speed $(kT_e/m)^{1/2}$, ϕ is the electric potential normalised by kT_e/e and p is the ion pressure normalised by a characteristic electron pressure $n_0\,kT_e$. The space coordinate x and time t are normalised respectively by the electron Debye length and ion plasma period at $x = 0$.

3. Reductive Perturbation Analysis

In order to obtain the nonlinear partial differential equation governing the propagation of the right and left moring waves we carry out a reductive pertubation analysis of equations (1a)–(1e).

(3a) Right Moving Wave

The set of stretched coordinates for a right moving wave (positive x direction) in a spatially inhomogeneous plasma is

$$\xi = \epsilon^{1/2}\left(\int^x \frac{\mathrm{d}x}{\lambda_0(x)} - t\right), \qquad \eta = \epsilon^{3/2}\,. \tag{2}$$

Such a modified version of the stretching of the coordinates was initially suggested by Asano (1974) and later used by Nisikawa and Kaw (1975). It may be observed that when $\lambda(x)$ is constant this become identical to the usual stretched coordinates used in plasmas. The functional dependence of λ is used to take care of the inhomogeneity of the plasma. The physical quantities describing the plasma such as (n, v, ϕ, p) all are expanded as follows (Tagare 1974):

$$\Psi = \Psi_0(x) + \epsilon\Psi_1(x,\, t) + \epsilon^2\Psi_2(x,\, t) + \dots\,, \tag{3}$$

where Ψ stands for n, v, ϕ or p, and the subscript zero indicates the unperturbed quantity which is a slowly varying function of the position coordinate x. Substituting (2) and (3) in (1) we get

$$\frac{\partial\lambda_0}{\partial\xi} = \frac{\partial\Psi_0}{\partial\xi} = 0\,, \tag{4}$$

along with

$$\frac{\partial}{\partial\eta}(n_0\, v_0) = 0\,,$$

$$n_0\, v_0\, \frac{\partial v_0}{\partial\eta} + n_0\, \frac{\partial\phi_0}{\partial\eta} + \frac{\partial p_0}{\partial\eta} = 0\,,$$

$$v_0\, \frac{\partial p_0}{\partial\eta} + 3p_0\, \frac{\partial v_0}{\partial\eta} = 0\,, \tag{5}$$

$$\frac{\partial^2\phi_0}{\partial\eta^2} = n_{e0} - n_0\,, \qquad n_{e0} = \exp(\phi_0).$$

Also from terms of first order in ϵ, we get

$$-\frac{\partial n_1}{\partial\xi} + \frac{n_0}{\lambda_0}\frac{\partial v_1}{\partial\xi} + \frac{v_0}{\lambda_0}\frac{\partial n_1}{\partial\xi} = 0\,,$$

$$-n_0\, \frac{\partial}{\partial\xi}\left[v_1\left(1 + \frac{3v_0^2}{2c^2}\right)\right] - n_1\, \frac{\partial}{\partial\xi}\left[v_0\left(1 + \frac{v_0^2}{2c^2}\right)\right] + \frac{1}{\lambda_0}\frac{\partial p_1}{\partial\xi}$$

$$+ \frac{n_0\, v_0}{\lambda_0}\, \frac{\partial}{\partial\xi}\left[v_1\left(1 + \frac{3v_0^2}{2c^2}\right)\right] + \frac{n_0}{\lambda_0}\frac{\partial\phi_1}{\partial\xi} = 0\,,$$

$$-\frac{\partial p_1}{\partial\xi} + \frac{v_0}{\lambda_0}\frac{\partial p_1}{\partial\xi} + \frac{3p_0}{\lambda_0}\left(1 + \frac{3v_0^2}{2c^2}\right)\frac{\partial v_1}{\partial\xi} = 0\,,$$

$$\frac{\partial^2\phi}{\partial z^2} = \phi_1 - \frac{n_1}{n_0}\,. \tag{6}$$

Integrating these equations with the boundary conditions gives

$$n_1 = v_1 = \phi_1 \qquad v_0,\, \phi_0,\, p_0 \to 0\,,$$

$$n_0,\, \lambda_0 \to 1, \qquad \text{as } |\,\xi\,| \to \infty\,. \tag{7}$$

Since the plasma is assumed to be confined to a region bounded by the planes $z = 0$ and $z = b$, which we assume to be perfectly conducting, we should impose the condition that $\phi = 0$ on the $z = 0$ and $z = b$ planes. Such a procedure was also followed by Das and Ghosh (1988). The condition is satisfied by

$$\phi_1 = f(\xi,\, \eta)\sin[q(\eta)z]\,, \tag{8}$$

when $q(\eta) = n\pi/b$, along with

$$q^2 = \frac{n_0}{(1 + 3v_0^2/2c^2)[n_0\,(\lambda_0 - v_0)^2 - 3p_0]}\,. \tag{9}$$

We also get

$$\lambda_0 = v_0 + \frac{n_0 + 3p_0\,(1 + n^2\pi^2/b^2)\,(1 + 3v_0^2/2c^2)^{1/2}}{n_0(1 + n^2\pi^2/b^2)\,(1 + 3v_0^2/2c^2)}\,. \tag{10}$$

Using these results in equations (6) and (8) we at once obtain

$$\begin{aligned} n_1 &= n_1(\eta)\, f(\xi,\, \eta)\sin[q(\eta)z]\,, \\ v_1 &= v_1(\eta)\, f(\xi,\, \eta)\sin[q(\eta)z]\,, \\ p_1 &= p_1(\eta)\, f(\xi,\, \eta)\sin[q(\eta)z]\,, \end{aligned} \tag{11}$$

where

$$\begin{aligned} n_1(\eta) &= \frac{n_0^2}{(1 + 3v_0^2/2c^2)[n_0(\lambda_0 - v_0)^2 - 3p_0]}\,, \\ v_1(\eta) &= \frac{n_0(\lambda_0 - v_0)}{(1 + 3v_0^2/2c^2)[n_0\,(\lambda_0 - v_0)^2 - 3p_0]}\,, \\ p_1(\eta) &= \frac{3p_0\, n_0}{n_0(\lambda_0 - v_0)^2 - 3p_0}\,. \end{aligned} \tag{12}$$

Proceeding now to second order terms in ϵ^2, after using the results in equations (10) and (11) we get

$$\frac{\partial^2}{\partial z^2}\left(\frac{\partial \phi_2}{\partial \xi}\right) + \left(\frac{n_0^2}{C_1(\eta)} - 1\right)\frac{\partial \phi_2}{\partial \xi} + \frac{1}{\lambda_0^2}\sin[q(\eta)z]$$

$$\times\ \frac{\partial^3 f}{\partial \xi^3} + \frac{A_1(\eta)\lambda_0}{C_1(\eta)}\sin[q(\eta)z]\frac{\partial f}{\partial \eta} + \left(\frac{A_2(\eta)}{C_1(\eta)}\lambda_0 - 1\right)$$

$$\times\ \sin^2[q(\eta)z]f\,\frac{\partial f}{\partial \xi} + \frac{A_3(\eta)\lambda_0}{C_1(\eta)}\,\frac{\partial}{\partial \eta}\{\sin[q(n)z]\}f + \frac{A_4(n)\lambda_0}{C_1(n)}$$

$$\times \frac{\partial v_1(\eta)}{\partial \eta} \sin[q(\eta)z]f + \frac{A_5(\eta)\lambda_0}{C_1(\eta)} \frac{\partial v_0}{\partial n} \sin[q(\eta)z]f$$

$$+ \frac{A_6(\eta)\lambda_0}{C_1(\eta)} \frac{\partial p_1(\eta)}{\partial \eta} \sin[q(\eta)z] + \frac{A_7(\eta)\lambda_0}{C_1(\eta)} \frac{\partial p_0}{\partial \eta}$$

$$\times \sin[q(\eta)z]f + \frac{\lambda_0 v_0}{\lambda_0 - v_0} \frac{\partial n_1(\eta)}{\partial \eta} \sin[q(\eta)z]f = 0\,, \tag{13}$$

where the coefficients occurring in the above equation are defined as follows:

$$C_1(\eta) = [n_0(\lambda_0 - v_0)^2 - 3p_0](1 + 3v_0^2/2c^2)(\lambda_0 - v_0)\,,$$

$$A_1(\eta) = \frac{2n_0^3\lambda_0(\lambda_0 - v_0)^2}{(1 + 3v_0^2/2c^2)[n_0(\lambda_0 - v_0)^2 - 3p_0]}\,,$$

$$\begin{aligned} A_2(\eta) = {} & n_0(\lambda_0 - v_0)\left[\left(1 + \frac{3v_0^2}{2c^2}\right)n_1(\eta)\,v_1(\eta) - \frac{3n_0\,v_0}{c^2}\,v_1^2(\eta)\right. \\ & + \frac{3n_0\,v_0^2}{\lambda_0\,c^2}\,v_1^2(\eta) + \frac{n_0(1 + \frac{3}{2}v_0^2/c^2)}{\lambda_0}\,v_1^2(\eta) \\ & \left. + \frac{v_0}{\lambda_0}\left(1 + \frac{3v_0^2}{2c^2}\right)n_1(\eta)\,v_1(\eta) + \frac{n_1(\eta)}{\lambda_0}\right] \\ & + \frac{n_0}{\lambda_0}\,v_1(\eta)\,p_1(\eta) + \frac{9p_0\,v_0\,n_0}{c_2}\,v_1^2(\eta) + \frac{3n_0}{\lambda_0}\,p_1(\eta)\,v_1(\eta) \\ & \times\left(1 + \frac{3v_0^2}{2c^2}\right) - 2[3p_0 - n_0(\lambda_0 - v_0)^2]\left(1 + \frac{3v_0^2}{2c^2}\right)n_1(\eta)\,v_1(\eta), \end{aligned}$$

$$A_3(\eta) = \frac{2n_0^3\lambda_0(\lambda_0 - v_0)^2}{(1 + 3v_0^2/2c^2)[n_0(\lambda_0 - v_0)^2 - 3p_0]}\,,$$

$$\begin{aligned} A_4(\eta) = {} & n_0^3(\lambda_0 - v_0)(1 + 3v_0^2/2c^2) + 3p_0\,n_0(1 + 3v_0^2/2c^2) \\ & - n_0[3p_0 - n_0(\lambda_0 - v_0)^2](1 + 3v_0^2/2c^2)\,, \end{aligned}$$

$$A_5(\eta) = n_0(\lambda_0 - v_0)\left[\frac{3n_0\,v_0^2}{c^2}\,v_1(\eta) + n_0\,v_1(\eta)\left(1 + \frac{3v_0^2}{2c^2}\right)\right.$$

$$+ v_0\, n_1(\eta)\left(1 + \frac{3v_0^2}{2c^2}\right)\Bigg] + \frac{q n_0\, p_0\, v_0}{c^2}\, v_1(\eta)$$

$$+ 3\left(1 + \frac{3v_0^2}{2c^2}\right) n_0\, p_1(\eta) - [3p_0 - n_0(\lambda_0 - v_0)^2]$$

$$\times\ (1 + 3u_0^2/2c^2) n_1(\eta)\,,$$

$$A_6(\eta) = n_0\, \lambda_0, \qquad A_7(\eta) = n_0\, v_1(\eta)\,. \tag{14}$$

Multiplying both sides of equation (13) by sin $[q(\eta)z]$ and integrating between the limits 0 to b, we get

$$\frac{\partial f}{\partial \eta} + \frac{B_2(\eta)}{B_1(\eta)} \sigma f \frac{\partial f}{\partial \xi} + \sigma_1 \frac{\partial^3 f}{\partial \xi^3} + \sigma_2 f = 0\,, \tag{15}$$

where

$$\sigma = \int_0^b \sin^3(qz)\, \mathrm{d}z \Big/ \int_0^b \sin^2(qz)\, \mathrm{d}z, \qquad \sigma_1 = \frac{1}{\lambda_0^2 B(\eta)}\,,$$

$$\sigma_2 = \frac{B_3(\eta)}{B_1(\eta)} \left(\int_0^b \sin(qz)\, \mathrm{d}z \Big/ \int_0^b \sin^2(qz)\, \mathrm{d}z \right)$$

$$\times\ \frac{\partial}{\partial \eta} \sin[q(\eta)z] + \frac{B_4(\eta)}{B_1(\eta)}\,,$$

$$B_1(\eta) = \frac{A_1(\eta)\lambda_0}{C_1(\eta)}, \qquad B_2(\eta) = \frac{A_2(\eta)\lambda_0}{C_1(\eta)}\,,$$

$$B_3(\eta) = \frac{A_3(\eta)\lambda_0}{C_1(\eta)}\,,$$

$$B_4(\eta) = \frac{\lambda_0}{C_1(\eta)} \left(A_4(\eta) \frac{\partial v_1(\eta)}{\partial \eta} + A_5(\eta) \frac{\partial v_0}{\partial \eta} + A_6(\eta) \right.$$

$$\left. \times\ \frac{\partial p_1(\eta)}{\partial \eta} + A_7(\eta) \frac{\partial p_0}{\partial \eta} + \frac{\lambda_0\, v_0}{\lambda_0 - v_0} \frac{\partial n_1}{\partial \eta} \right). \tag{16}$$

Equation (15) is the required nonlinear KdV equation with variable coefficients describing the propagation of the right moving wave. Due to the variable nature of the coefficients we seek a solution of the form

$$f_{\mathrm{R}} = A(\eta)\,\mathrm{sech}^2\left(\frac{\xi - v(\eta)}{\delta(\eta)}\right), \tag{17}$$

where $A(\eta)$, $v(\eta)$ and $\delta(\eta)$ are the variable amplitude, velocity and width, governed by the equations

$$\frac{\partial A(\eta)}{\partial \eta} + D_3(\eta)\,A(\eta) = 0\,,$$

$$\delta^2(\eta) = \frac{4D_2(\eta)b}{D_1(\eta)\,A(\eta)}\,, \tag{18}$$

$$A(\eta)\,\delta^2(\eta)\,\frac{\partial v(\eta)}{\partial \eta} + [A(\eta)\,\delta(\eta) \,-\, A(\eta)v(\eta)\eta]\frac{\partial \delta}{\partial \eta}$$

$$+A(\eta)\,\delta^2(\eta)\,v(\eta) - \frac{D_1(\eta)}{b}\,A^2(\eta)\,\delta^3(\eta)\; + 8A(\eta)\,D_2(\eta) = 0\,.$$

(3b) Left Moving Wave

Due to the inhomogeneity of the plasma the left and right moving waves behave differently. The appropriate set of variables for the left waves is

$$\xi = \epsilon^{1/2}\left(-\int^x \frac{\mathrm{d}x}{\lambda_0(x)} - t\right), \qquad \eta = -\epsilon^{3/2}\,. \tag{19}$$

Proceeding similarly as in the previous case we arrive at

$$\frac{\partial}{\partial \eta}(n_0\,v_0) = 0\,,$$

$$n_0\,v_0\,\frac{\partial v_0}{\partial \eta} + n_0\,\frac{\partial \phi_0}{\partial \eta} + \frac{\partial p_0}{\partial \eta} = 0\,,$$

$$v_0\frac{\partial p_0}{\partial \eta} + 3p_0\,\frac{\partial v_0}{\partial \eta} = 0\,,$$

$$\frac{\partial^2 \phi_0}{\partial \eta^2} = n_{e0} - n_0, \qquad n_0 = \exp(\phi_0)\,. \tag{20}$$

Terms of first order in ϵ lead to

$$
\begin{aligned}
v_1 &= -\frac{\lambda_0+v_0}{n_0}\,n_1\,,\\
p_1 &= -\frac{3p_0}{\lambda_0+v_0}\left(1+\frac{3v_0^2}{2c^2}\right)v_1\,,
\end{aligned}
$$

$$
n_0(\lambda_0+v_0)v_1\left(1+\frac{3v_0^2}{2c^2}\right) = -p_1 - n_0\,\phi_1\,,
$$

$$
\frac{\partial^2\phi_1}{\partial z^2} = \phi_1 - \frac{n_1}{n_0}\,, \tag{21}
$$

with boundary conditions

$$
q(\eta) = \frac{n\pi}{b}, \qquad q^2 = \frac{n_0}{(1+3v_0^2/2c^2)n_0(\lambda_0+v_0)^2-3p_0}\,. \tag{22}
$$

Without going into details of the computation for terms of higher order in ϵ (which is similar to the previous case but differs in detail only), we give the final results:

$$
\lambda_0 = -v_0 + \left(\frac{n_0+3p_0(1+n^2\pi^2/b^2)(1+3v_0^2/2c^2)}{n_0(1+n^2\pi^2/b^2)(1+3v_0^2/2c^2)}\right)^{1/2},
$$

along with

$$
\begin{aligned}
n_1 &= n_{11}(\eta)\,f(\xi,\,n)\sin[q(n)z]\,,\\
v_1 &= v_{11}(\eta)\,f(\xi,\,n)\sin[q(n)z]\,,\\
p_1 &= p_{11}(\eta)\,f(\xi,\,n)\sin[q(n)z]\,,
\end{aligned}
$$

where

$$
\begin{aligned}
n_{11}(\eta) &= \frac{n_0^2}{(1+3v_0^2/2c^2)[n_0(\lambda_0+v_0)^2-3p_0]}\,,\\
v_{11}(\eta) &= \frac{n_0(\lambda_0+v_0)}{(1+3v_0^2/2c^2)[n_0(\lambda_0+v_0)^2-3p_0]}\,,\\
p_{11}(\eta) &= \frac{3p_0\,n_0}{n_0(\lambda_0+v_0)^2-3p_0}\,.
\end{aligned}
$$

Taking into account terms of higher order in ϵ, we can again obtain another KdV equation which can be written as

$$
\frac{\partial f}{\partial\eta} + \frac{\tilde{D}_1(\eta)}{b}\,f(\xi,\,n)\,\frac{\partial f}{\partial\xi} + \tilde{D}_2(n)\,\frac{\partial^3 f}{\partial\xi^3} + \tilde{D}_3\,f = 0\,. \tag{23}
$$

We now follow the same technique as in the case of the right moving wave, that is we assume a modulated form for the solitary wave and obtain the differential equations for the amplitude, velocity, etc. by substituting this assumed form in equation (22).

The form of the soliton may be written

$$f_{\mathrm{L}} = a(\eta)\,\mathrm{sech}^2\left(\frac{\xi - u(\eta)\eta}{\omega(\eta)}\right), \tag{24}$$

where $a(\eta)$, $v(\eta)$ and $\omega(\eta)$ are respectively the amplitude, velocity and width of the left moving wave, governed by the following system of equations:

$$\frac{\partial a(\eta)}{\partial \eta} + \tilde{D}_3(\eta)\,a(\eta) = 0\,,$$

$$\omega^2(n) = \frac{4\tilde{D}_2(\eta)b}{\tilde{D}_1(\eta)\,a(\eta)}\,, \tag{25}$$

$$a(\eta)\,\omega^2(\eta)\,\frac{\partial v}{\partial \eta} + [a(\eta)\,u(\eta) \;-a(\eta)\,v(\eta)\eta]\frac{\partial \omega}{\partial \eta}$$

$$+a(\eta)\,\omega^2(\eta)\,v(\eta) \;-\; \frac{\tilde{D}_1(\eta)}{b}\,a^2(\eta)\,\omega^2(\eta) \;+8a(\eta) \;+8a(\eta)\tilde{D}_2(\eta) = 0\,.$$

The coefficients $D_i(\eta)$ and $\tilde{D}_i(\eta)$ are given in the Appendix.

4. Analysis and Inferences

Since the equations governing the motion of our solitary waves have variable coefficients, numerical methods are to be adopted for the integration of the sets (18) and (25). To do this information is needed about the zeroth order terms such as v_0, p_0 and n_0, which are governed by equation (5). We observe that a possible set of solution is obtained as follows. We set $n_0 = 1{+}\eta$ so that as $\eta \to 0$, then $n_0 \to 1$, the equilibrium value. With this information we can now solve (5) and obtain

$$v_0 = \frac{k_1}{n_0}, \qquad p_0 = \frac{k^2}{v_0^3}\,,$$

where k_1 and k_2 are certain constants. For carrying out the numerical integration of equations (18) and (25), it is necessary to fix a range of η values. Since v_0 varies with η, the relativistic or nonrelativistic nature of the system is automatically fixed along with the range of η. For large values of η, we have $v_0 \to 0$ and so we are in the nonrelativistic region, whereas for small η, values v_0 may become comparable to c. Simultaneously, it should be kept in mind that for large η, p_0 is large and for η small, p_0 tends to be small. We now utilise these forms of p_0, v_0 and n_0 in the differential equations for the phase velocity and the amplitude of the left and right moving soliton, given in equations (18) and (25). These are then integrated numerically for various values of the plasma parameters. All these results are displayed in Figs 1 to 6.

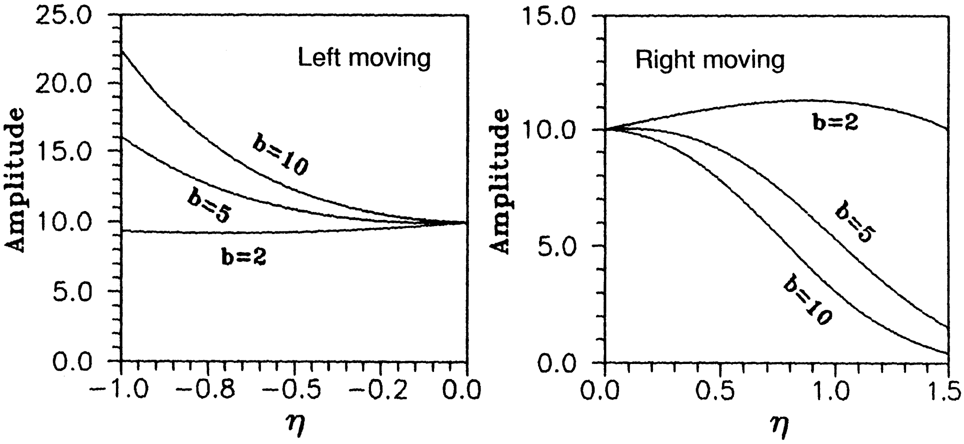

Fig. 1. Amplitude of the right and left moving soliton after scattering from the inhomogeneity for different b and small η.

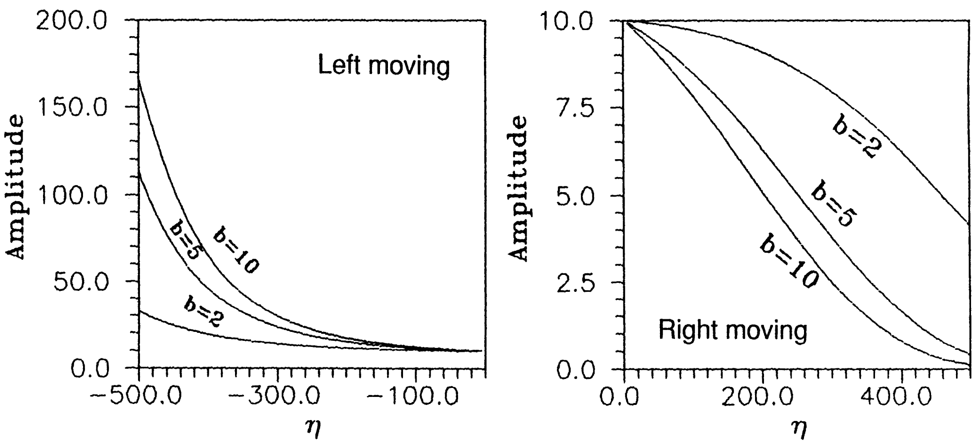

Fig. 2. Amplitude of the right and left moving soliton after scattering from the inhomogeneity for different b and large η.

In each figure we show the cases of left and right moving waves separately. An important role is seen to be played by the dimension of the bounding surface. In a previous analysis (Mukhopadhyay *et al.* 1994) we showed that b has an important influence on the formation of solitons in plasma. So here we have

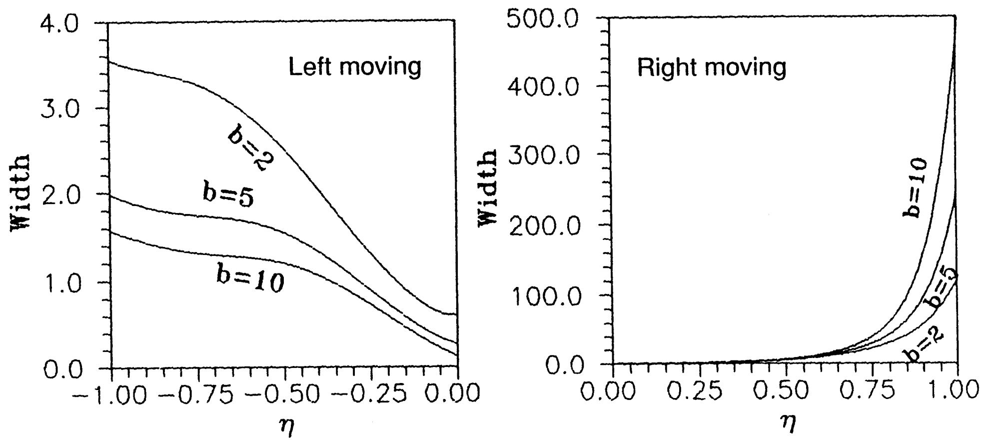

Fig. 3. Width of the right and left moving soliton after scattering from the inhomogeneity for different b and small η.

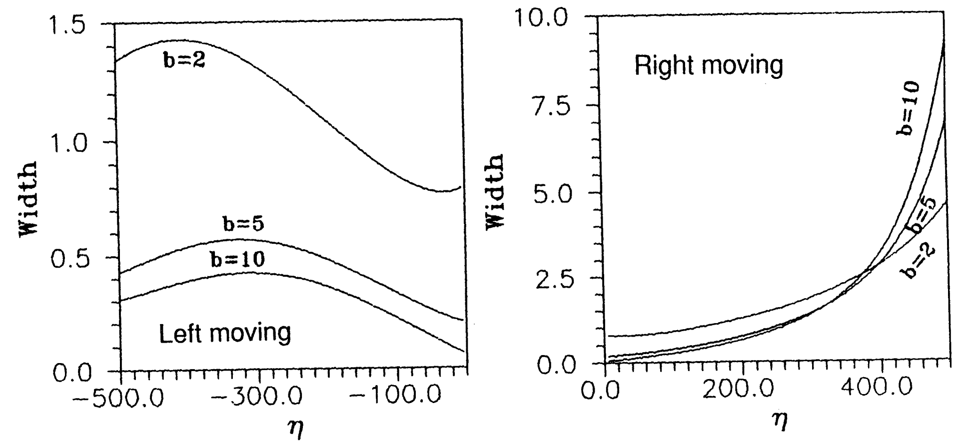

Fig. 4. Width of the right and left moving soliton after scattering from the inhomogeneity for different b and large η.

considered three ranges, $b = 10$, $b = 5$ and $b = 2$; large, medium and small. In Figs 1 and 2 the amplitudes of the left and right moving waves are shown. Fig. 1 gives the relativistic situation, whereas Fig. 2 yields the nonrelativisitic case (remember these are determined by the range of η). Though the trend of the curves in these two regions both for the left and right moving waves remains the same to some extent, they differ in finer details. It is important to note that for the various values of b their variation follows a definite pattern. Next in Figs 3 and 4 we plot the width of the solitary waves. In this case for the left moving wave the behaviour changes drastically in passing from the relativistic to the nonrelativistic regime, but the right wave shows a similar behaviour. Lastly in Figs 5 and 6 we exhibit the velocities of the two solitons. The nature of these curves mimics that of the amplitude, which is reasonable because in the case of ordinary solitary waves these are proportional to each other. The important

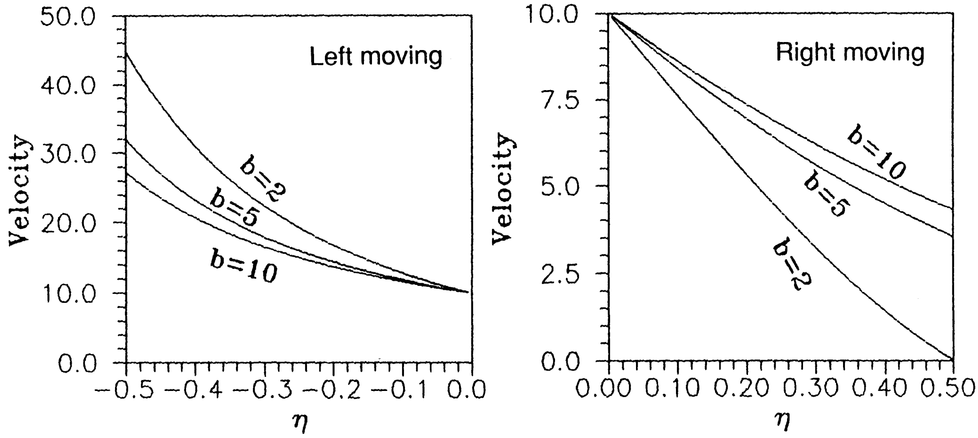

Fig. 5. Velocity of the right and left moving soliton after scattering from the inhomogeneity for different b and small η.

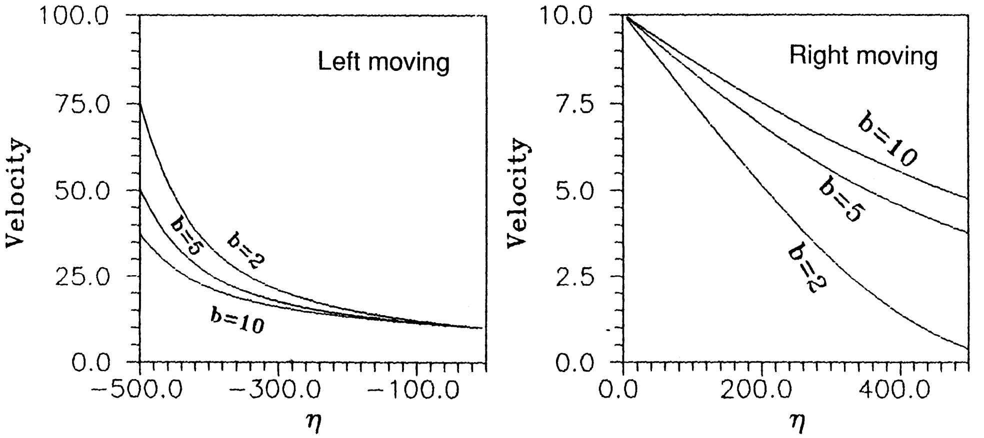

Fig. 6. Velocity of the right and left moving soliton after scattering from the inhomogeneity for different b and large η.

feature that emerges out of the above analysis is that due to the imhomogeniety the soliton is scattered and its behaviour changes completely. Though still not feasible, it may in the future become possible to measure the changes in the nature of the soliton by scattering in a relativistic plasma. Then, many physical characteristics of a plasma might be determined by using soliton scattering such as has been discussed here.

One may note that even in the presence of relativistic effects and a finite boundary, the basic observation of Mukhopadhyay *et al.* (1994) still holds; that is, the amplitude of the reflected soliton is less than that of the incident one. Only the magnitudes vary due to the variations in size of the boundary. Another important feature of our analysis is that the variation of the ion temperature is automatically taken care of due to the dependence of p_0 on η and it is not at all a parametric feature.

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Appendix

In the following we give detailed expressions for the coefficients occuring in equation (25) and a few more relations which have been proved to be useful in the analysis:

$$D_{11}(\eta) = (-1)^{n+2}\,\frac{4B_{21}(\eta)}{3q(\eta)\,B_{11}(\eta)} - A_1\,,$$

$$D_{21}(\eta) = \frac{1}{\lambda_0^2}\,B_{11}(\eta)\,,$$

$$D_{31}(\eta) = \frac{B_{31}(\eta)}{2B_{11}(\eta)\,q(\eta)}\,\frac{\partial q(\eta)}{\partial\eta} - \frac{B_{41}(\eta)}{B_{11}(\eta)}\,,$$

$$B_{11}(\eta) = \frac{A_{11}(\eta)}{C_{11}(\eta)}\,\lambda_0\,,$$

$$B_{21}(\eta) = \frac{A_{21}(\eta)\lambda_0}{C_{11}(\eta)} - 1\,,$$

$$B_{31}(\eta) = \frac{A_{31}(\eta)\lambda_0}{C_{11}(\eta)}\,,$$

$$B_{41}(\eta) = \frac{\lambda_0}{C_{11}(\eta)}\left(A_{41}(\eta)\frac{\partial v_{11}(\eta)}{\partial \eta} + A_{51}(\eta)\frac{\partial v_0}{\partial \eta}\right.$$

$$\left. + A_{61}(\eta)\frac{\partial p_{11}(\eta)}{\partial \eta} + A_{71}(\eta)\frac{\partial p_0}{\partial \eta} + \frac{\lambda_0 v_0}{\lambda_0 + v_0}\frac{\partial n_{11}(\eta)}{\partial \eta}\right),$$

$$C_{11}(\eta) = [n_0(\lambda_0 + v_0)^2 - 3p_0]\left(1 + \frac{3v_0^2}{2c^2}\right)(\lambda_0 + v_0),$$

$$A_{11}(\eta) = \frac{2n_0^3\lambda_0(\lambda_0 + v_0)^2}{(1 + 3v_0^2/2c^2)[n_0(\lambda_0 + v_0)^2 - 3p_0]},$$

$$A_{21}(\eta) = n_0(\lambda_0 + v_0)\left[\left(1 + \frac{3v_0^2}{2c^2}\right)n_{11}(\eta)v_{11}(\eta)\right.$$

$$- \frac{3n_0\, v_0}{c^2}\, v_{11}^2(\eta) + \frac{3n_0\, v_0^2}{\lambda_0\, c^2}\, v_{11}^2(\eta)$$

$$+ \frac{n_0(1 + 3v_0^2/2c^2)}{\lambda_0}\, v_{11}^2(n) + \frac{v_0}{\lambda_0}\left(1 + \frac{3v_0^2}{2c^2}\right)n_{11}(\eta)\, v_{11}(\eta)$$

$$\left. + \frac{n_{11}(\eta)}{\lambda_0}\right] + \frac{n_0}{\lambda_0}\, v_{11}(\eta)\, p_{11}(\eta) + \frac{qp_0\, v_0\, n_0}{\lambda_0\, c^2}\, v_{11}^2(\eta)$$

$$+ 3n_0\, p_{11}(\eta)\, v_{11}(\eta) + \frac{(1 + \frac{3}{2}v_0^2/c^2)}{\lambda_0} + 2[n_0(\lambda_0 + v_0)^2$$

$$- 3p_0]\left(1 + \frac{3v_0^2}{2c^2}\right)n_{11}(\eta)v_{11}(\eta),$$

$$A_{31}(\eta) = \frac{2n_0^3\, \lambda_0(\lambda_0 + v_0)^2}{(1 + 3v_0^2/2c^2)[n_0(\lambda_0 + v_0)^2 - 3p_0]},$$

$$A_{41}(\eta) = n_0^3(\lambda_0 + v_0)\left(1 + \frac{3v_0^2}{2c^2}\right) + 3p_0\, n_0\left(1 + \frac{3v_0^2}{2c^2}\right)$$

$$+ n_0[n_0(\lambda_0 + v_0)^2 - 3p_0]\left(1 + \frac{3v_0^2}{2c^2}\right),$$

$$
\begin{aligned}
A_{51}(\eta) &= n_0(\lambda_0 - v_0)\left[\frac{3n_0^2 v_0}{c^2} v_{11}(\eta) + n_0 v_{11}(\eta)\right. \\
&\quad \times \left(1 + \frac{3v_0^2}{2c^2}\right) + v_0\, n_{11}(\eta)\left.\left(1 + \frac{3v_0^2}{2c^2}\right)\right] + \frac{q n_0\, p_0\, v_0}{c^2} \\
&\quad \times v_{11}(n) + 3\left(1 + \frac{3v_0^2}{2c^2}\right) n_0\, p_{11}(\eta) \\
&\quad - [3p_0 - n_0(\lambda_0 + v_0)^2]\left(1 + \frac{3v_0^2}{2c^2}\right) n_{11}(\eta)\,, \\
A_{61}(\eta) &= \lambda_0\, n_0\,, \qquad A_{71}(\eta) = n_0\, v_{11}(\eta)\,.
\end{aligned}
$$

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