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Short-range Correlation Effect on p–p Elastic Scattering with the Composite Model

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Abstract

Using a representation of the quark distribution inside the proton, where a short-range correlation between quarks is considered, we can describe the p–p elastic scattering differential cross section with experimental agreement in the range $0 \leq q^2 \leq 10\ (\mathrm{GeV}/c)^2$. Evaluations of the proton core and quark radii are obtained. From our analysis, we see that the suggested configuration of quarks inside the proton can be considered as a nearly realistic distribution even with a non-relativistic wavefunction.

1. Introduction

Using p–p scattering we try to learn about the internal structure of the proton, specially about the quark–quark short-range correlation. In quantum chromodynamics, the nucleon is represented in the form of a central part in which the three valence quarks are concentrated and surrounded by a meson cloud (Baldin 1967; Islam 1975; Bogolyubskii 1982). This picture is used in many works—with or without the cloud part—to describe the proton in high-energy hadron–proton scattering in the framework of the non-relativistic Glauber theory (Kuroda and Miyazawa 1973; Wakaizumi 1978; Goloskokov *et al.* 1982).

In this introduction we try to answer the question: can we use the non-relativistic wavefunction for the proton ground state to represent short-range properties? Since in scattering problems the large distance properties of the incident and target particle wavefunction play an important role at small momentum-transfer, while the short distance feature of the wavefunction is dominant at large values of momentum transfer between the coliding particles, then this question can be reformulated as follows: can we use a non-relativisitic proton wavefunction to describe high-energy p–p scattering for large values of the momentum transfer?

Therefore, we note that by using the non-relativistic wavefunction of the proton ground state a good fit with the p–p elastic scattering differential cross section at high energies is obtained over a wide range of momentum-transfer squared—for example, Harrington and Pagnamenta (1968) at $p_{\mathrm{L}} = 15\ \mathrm{GeV}/c$ (Fig. 1), Wakaizumi and Tanimoto (1977) and Wakaizumi (1978) at $p_{\mathrm{L}} = 1500\ \mathrm{GeV}/c$ (Figs 2*a* and 2*b*). Also, the relativistic wavefunction results (Goloskokov *et al.* 1981) are not better than the results (Wakaizumi 1978) of the more realistic non-relativistic wavefunction (the exponential form—see Figs 2*b* and 2*c*).

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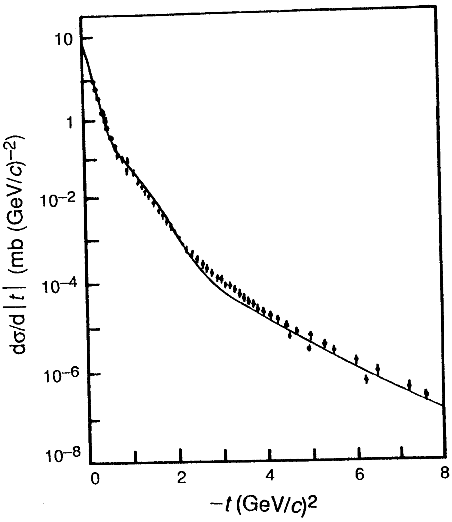

Fig. 1. The p–p elastic scattering differential cross section at $p_L = 15$ GeV/c for a non-relativistic Gaussian wavefunction for the proton (Harrington and Pagnamenta 1968).

Thus, from these works, the non-relativistic ground state wavefunction of the proton can be used to describe the p–p elastic scattering differential cross section, at least, in the range $0 \le q^2 < 6$ $(\mathrm{GeV}/c)^2$.

However, a non-relativistic Gaussian wavefunction is used to calculate the electromagnetic form factors of nucleons based on the quark model (Fujimura *et al.* 1970). The results are inconsistent with the experimental data except at very small values of momentum-transfer squared $q^2 < 1$ $(\mathrm{GeV}/c)^2$(see curve 1 in Fig. 3). However, the relativistic Gaussian wavefunction can produce a form factor which is consistent with the experimental data for small and large values of q^2 (curve 2 in Fig. 3). Also, Chung and Coester (1991) used a relativistic constituent-quark model to describe the experimental data on electromagetic nucleon form factors for $0 \le q^2 \le 6$ $(\mathrm{GeV}/c)^2$. Therefore, for $0 \le q^2 \le 6$ $(\mathrm{GeV}/c)^2$, it seems that we must use a relativistic wavefunction to calculate the hadronic form factor of the proton and the p–p elastic scattering differential cross section.

However, it is well known that, from the present point of view of quark–parton models, part of the hadronic matter and in particular gluons—which play an important role in the dynamics of the strong interaction—are neutral particles. Their distributions are not directly measured in electromagnetic interactions, and so hadronic matter may not be distributed everywhere like electromagnetic matter, i.e. the hadronic and electromagnetic form factors may not be identical (Bogolyubskii 1982). Thus, the conclusion about the electromagnetic nucleon from factor cannot be generalised simply and automatically to the hadronic form factor.

The non-relativistic quark model is also used to obtain a model wavefunction of the nucleon–nucleon system with the possibility of a six-quark bag which presents some kind of short-range feature of the wavefunction (Kurihara and Faessler 1987) and to study the short-range nucleon–nucleon interaction where the distances are of the order 0·4 fm (Warke and Shanker 1979). Our final conclusion is that we can use a non-relativistic wavefunction for the proton to reflect some kind of short-range correlations of the quark–quark interaction.

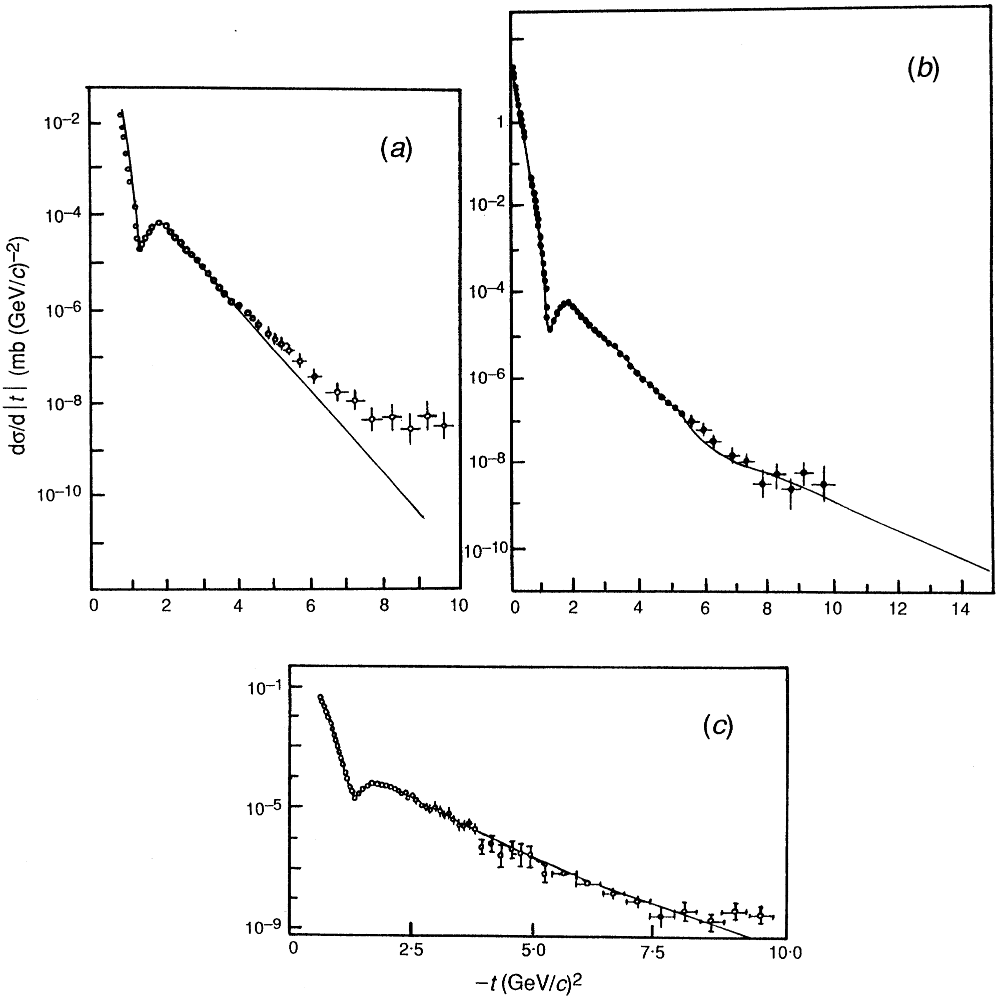

Fig. 2. The p–p elastic scattering differential cross section at $\sqrt{s} = 53$ GeV for (a) the Gaussian wavefunction (Wakaizumi 1978), (b) a more realistic wavefunction (Wakaizumi 1978) and (c) a relativistic wavefunction for the proton (Goloskokov *et al.* 1981).

This wavefunction can at least be considered as an effective distribution of quarks inside the proton. One purpose of this work is to examine a proposed proton wavefunction containing a short-range correlation factor.

2. A Proposed Proton Wavefunction

The short-range correlation from a non-relativistic quark model point of view is mainly studied in the two or many nucleon system to evaluate the contribution of the six-quark bag, nine-quark bag, etc. in the two or many nucleon wavefunction and its effect on calculations of the binding energy and collision cross sections (Kurihara and Faessler 1987; Ableev *et al.* 1983; Kopeliovich and Zakharov 1984; Zakharov and Kopeliovich 1985; Anchishkin and Kobushkin 1985; Dijk and Bakker 1991). Also it gives an interpretation of the repulsive core in the

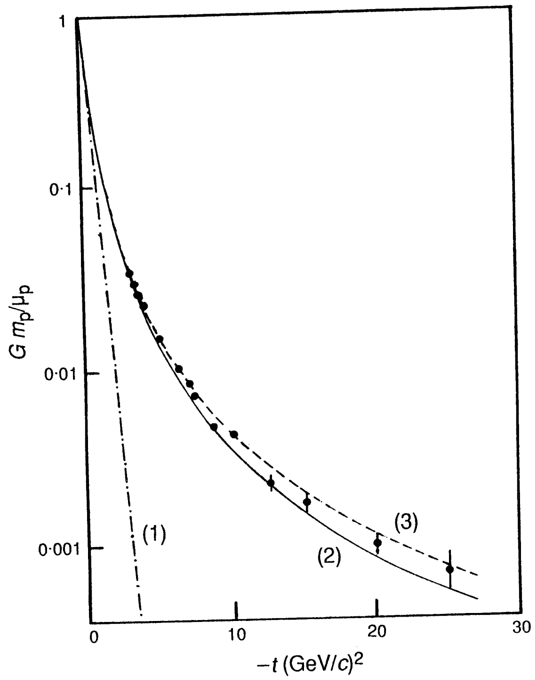

Fig. 3. Electromagnetic form factor where the dipole formula and the non-relativistic and relativistic Gaussian forms are shown respectively by the dashed, chain and solid curves (Fujimura *et al.* 1970).

nucleon–nucleon interaction (Warke and Shanker 1979; Babutsidze *et al.* 1981). In all these studies, the short-range quark–quark interaction is assumed to be attractive and asymptotically free at very small distances and the quark is considered as a point-like particle. On the other hand, the finite size of quarks is studied in hadron–nucleon elastic scattering (Bogolyubskii 1982; Kuroda and Miyazawa 1973; Franco 1987).

Therefore, it is interesting to learn about the short-range correlation between quarks inside the nucleon and the size of the quark. Of course, as a logical beginning, we use the usual form of the nucleon–nucleon short-range correlation factor in the quark–quark case. To do this, we propose a non-relativistic wavefunction for the ground state of the nucleon, where it is considered as a three point-like quark system. Using the proposed wavefunction, we calculate the p–p elastic scattering differential cross section at different energies, and try to determine short-range properties of the quark–quark interaction. This function can be considered as an effective distribution of quarks inside the nucleon.

Since a non-relativistic Gaussian wavefunction gives good agreement with experimental data on the high-energy p–p elastic scattering differential cross section, at least up to $q^2 \approx 6\,(\mathrm{GeV}/c)^2$, and this wavefunction represents independent motion of the three quarks inside the proton, we try to introduce some kind of short-range correlation factor into the Gaussian wavefunction to constrain this motion represented by the Gaussian form. Therefore, the proposed wavefunction is considered as a modified Gaussian form with conditions on the quark motion at small distances from each other.

The short-range correlation factor is usually used in the nucleon–nucleon case to represent a repulsive core at short distances. But in the quark–quark case, the same factor can be interpreted as follows. Since the quarks are massive elementary particles ($M_{\mathrm{q}} \approx 333$ MeV) and at the same time asymptotically free at short distances then, if we consider the quarks as extended objects, they are moving such that one cannot pass through the other, i.e. the minimum distance between the mass centres of two quarks approximately equals the diameter of the quark. In other words, elementary quarks with finite size cannot make a new sub-quark bag, as in the nucleon–nucleon system where a six-quark bag can be considered. Thus, if we do not take the quark matter distribution into account, we can at least suppose that the point-like quarks are moving such that the distance between them have a nonzero value.

For simplicity, we consider this configuration and propose the following proton wavefunction:

$$\psi_{\mathrm{p}}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3) = C(1 - \mathrm{e}^{-\gamma r_{21}^2})(1 - \mathrm{e}^{-\gamma r_{32}^2})(1 - \mathrm{e}^{-\gamma r_{31}^2})\mathrm{e}^{-\frac{1}{2}\beta(r_1^2+r_2^2+r_3^2)}\,, \tag{1}$$

where $\boldsymbol{r}_i$ is the position vector of ith quark with respect to the centre-of-mass of the proton, $\boldsymbol{r}_{ij}$ is the relative position vector of the ith quark and jth quark, $1-\mathrm{e}^{-\gamma r_{ij}^2}$ represents the correlation factor and γ is a correlation radius parameter. The normalisation constant C is obtained from the normalisation condition

$$\int |\,\psi_{\mathrm{p}}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3)\,|^2\, \delta\!\left(\frac{\boldsymbol{r}_1 + \boldsymbol{r}_2 + \boldsymbol{r}_3}{3}\right) \mathrm{d}\boldsymbol{r}_1\, \mathrm{d}\boldsymbol{r}_2\, \mathrm{d}\boldsymbol{r}_3 = 1\,. \tag{2}$$

Thus we get

$$\begin{aligned} C^{-1} = \left(\frac{1}{3\sqrt{3}\,\pi^3}\right)^{\frac{1}{2}} \Bigg(& \frac{12}{(\beta+5\gamma)^{\frac{3}{2}}(\beta+3\gamma)^{\frac{3}{2}}} - \frac{6}{(\beta+4\gamma)^{\frac{3}{2}}(\beta+6\gamma)^{\frac{3}{2}}} \\ & - \frac{12}{(\beta^2+6\beta\gamma+6\gamma^2)^{\frac{3}{2}}} + \frac{12}{(\beta+\gamma)^{\frac{3}{2}}(\beta+3\gamma)^{\frac{3}{2}}} \\ & + \frac{3}{(\beta+2\gamma)^{\frac{3}{2}}(\beta+6\gamma)^{\frac{3}{2}}} + \frac{3}{\beta^{\frac{3}{2}}(\beta+4\gamma)^{\frac{3}{2}}} - \frac{6}{\beta^{\frac{3}{2}}(\beta+2\gamma)^{\frac{3}{2}}} \\ & + \frac{1}{\beta^3} - \frac{8}{(\beta+3\gamma)^3} + \frac{1}{(\beta+6\gamma)^3}\Bigg)^{\frac{1}{2}}. \end{aligned} \tag{3}$$

The parameters β and γ are related to the rms distance $\langle r^2\rangle^{\frac{1}{2}}$ of the quark from the c.m. of the proton by the relation

$$\begin{aligned} \langle r^2\rangle = 3\sqrt{3}\,\pi^3|\,C\,|^2\Bigg(& \frac{1}{\beta^4} - \frac{8}{(\beta+3\gamma)^4} + \frac{1}{(\beta+6\gamma)^4} + \frac{3\beta+6\gamma}{\beta^{\frac{5}{2}}(\beta+4\gamma)^{\frac{5}{2}}} \\ & - \frac{6(\beta+\gamma)}{\beta^{\frac{5}{2}}(\beta+2\gamma)^{\frac{5}{2}}} + \frac{4}{(\beta+\gamma)^4} + \frac{4(2\beta+5\gamma)}{(\beta+\gamma)^{\frac{5}{2}}(\beta+3\gamma)^{\frac{5}{2}}} + \frac{1}{(\beta+2\gamma)^4} \end{aligned}$$

$$+\frac{2(\beta+5\gamma)}{(\beta+2\gamma)^{\frac{5}{2}}(\beta+\gamma)^{\frac{5}{2}}}-\frac{12(\beta+3\gamma)}{(\beta^2+6\beta\gamma+6\gamma^2)^{\frac{5}{2}}}-\frac{6(\beta+5\gamma)}{(\beta+4\gamma)^{\frac{5}{2}}(\beta+6\gamma)^{\frac{5}{2}}}$$

$$+\frac{12(\beta+4\gamma)}{(\beta+3\gamma)^{\frac{5}{2}}(\beta+5\gamma)^{\frac{5}{2}}}\Bigg), \tag{4}$$

where

$$\langle r^2\rangle=\int r_i^2\,|\psi_{\rm p}|^2\delta\left(\frac{\boldsymbol{r}_1+\boldsymbol{r}_2+\boldsymbol{r}_3}{3}\right)\mathrm{d}\boldsymbol{r}_1\,\mathrm{d}\boldsymbol{r}_2\,\mathrm{d}\boldsymbol{r}_3\,,\qquad i=1,2,3\,. \tag{5}$$

We consider γ as a fitting parameter and the corresponding value of β is obtained from formula (4).

3. Proton–Proton Elastic Scattering Amplitude

For very high-energy hadron collisions, where the wavelength of the incident particle is very small, the internal structure of the particles is more significant and must be taken into account in calculations of the collision cross section. By considering the proton as a composite system of three quarks (Gell-Mann 1964) in the framework of Glauber (1959) high-energy theory, we can write the p–p elastic scattering amplitude in terms of the quark–quark profile function $\Gamma_{j\ell}$ as (Harrington and Pagnamenta 1968; Czyz and Maximon 1969; Wakaizumi 1969)

$$T_{\rm p\text{-}p}(s,\,t)=\frac{i}{2\pi}\int \mathrm{d}^2\boldsymbol{b}\,\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{b}}\langle f\,|\left[1-\prod_{j=1}^{3}\prod_{\ell=4}^{6}\{1-\Gamma_{j\ell}(\boldsymbol{b}-\boldsymbol{s}_j+\boldsymbol{s}'_\ell,\,s)\}\right]|\,i\rangle\,, \tag{6}$$

where s is the c.m. energy squared, $-t=q^2$, and the initial state $|i\rangle$ and final state $|f\rangle$ are substituted by a product of ground state wavefunctions of the two composite protons. The quark–quark profile function $\Gamma_{j\ell}(\boldsymbol{b}-\boldsymbol{s}_j+\boldsymbol{s}'_\ell,\,s)$ is related to the quark–quark phase shift $\chi_{j\ell}(\boldsymbol{b}-\boldsymbol{s}_j+\boldsymbol{s}'_\ell,\,s)$ by

$$\Gamma_{j\ell}(\boldsymbol{b}-\boldsymbol{s}_j+\boldsymbol{s}'_\ell,\,s)=1-\mathrm{e}^{i\chi_{j\ell}(b-S_j+S'_l,S)}\,. \tag{7}$$

The total phase shift for p–p elastic scattering, from the assumption of additivity of the phase shifts, is given by (Harrington and Pagnamenta 1968; Wakaizumi 1969; Takada 1968)

$$\chi_{\rm p\text{-}p}(\boldsymbol{b}-\boldsymbol{s}_j+\boldsymbol{s}'_\ell,\,s)=\sum_{j,\ell}\chi_{j\ell}(\boldsymbol{b}-\boldsymbol{s}_j+\boldsymbol{s}'_\ell,\,s)\,. \tag{8}$$

The summation is taken over all pairs of constituents in the two protons, and the scattering force between constituents is assumed to be a two-body one.

By incorporating in (8) the important property of hadron reactions (geometrical scaling), discovered at CERN-ISR energies (Dias de Deus 1973), the total phase shift is

$$\chi_{\text{p-p}}(\boldsymbol{b}-\boldsymbol{s}_j+\boldsymbol{s}'_\ell,\, s)=\sum_{j,l}\chi_{j\ell}[(\boldsymbol{b}-\boldsymbol{s}_j+\boldsymbol{s}'_\ell,\, s)/R_0(s)]\,, \tag{9}$$

where $R_0(s)\equiv(\sigma_{\text{inel}}(s)/\pi)^{\frac{1}{2}}$ is the inelastic radius of the proton and $\sigma_{\text{inel}}(s)$ is the total inelastic cross section for p–p collisions.

Considering the geometrical scaling property, represented by equation (9), and introducing the inverse Fourier transformation

$$\Gamma_{j\ell}(\boldsymbol{b})=\frac{1}{2\pi\mathrm{i}}\int \mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{b}}t_{\text{qq}}(\boldsymbol{q})\,\mathrm{d}^2\boldsymbol{q} \tag{10}$$

into (6), the following expression for $T_{\text{p-p}}(s,t)$ is obtained in terms of the quark–quark elastic scattering amplitude $t_{\text{qq}}(\boldsymbol{q}^0)$ (Wakaizumi 1978):

$$\begin{aligned}T_{\text{p-p}}(s,\,t)/R_0^2 &= 9t_{\text{qq}}(\boldsymbol{q}^0)[f_1(\boldsymbol{q}^0)]^2+\frac{9\mathrm{i}}{\pi}\int \mathrm{d}\boldsymbol{q}_1^0\,\mathrm{d}\boldsymbol{q}_2^0\,t_{\text{qq}}(\boldsymbol{q}_1^0)\,t_{\text{qq}}(\boldsymbol{q}_2^0)\\ &\times\,\delta(\boldsymbol{q}_1^0+\boldsymbol{q}_2^0-\boldsymbol{q}^0)[f_2(\boldsymbol{q}_1^0,\,\boldsymbol{q}_2^0)]^2-\frac{6}{(2\pi)^2}\int \mathrm{d}\boldsymbol{q}_1^0\,\mathrm{d}\boldsymbol{q}_2^0\,\mathrm{d}\boldsymbol{q}_3^0\,t_{\text{qq}}(\boldsymbol{q}_1^0)\\ &\times\,t_{\text{qq}}(\boldsymbol{q}_2^0)\,t_{\text{qq}}(\boldsymbol{q}_3^0)\,\delta(\boldsymbol{q}_1^0+\boldsymbol{q}_2^0+\boldsymbol{q}_3^0-\boldsymbol{q}^0)\\ &\times\,[f_3(\boldsymbol{q}_1^0,\boldsymbol{q}_2^0,\boldsymbol{q}_2^0)]^2+\ldots,\end{aligned} \tag{11}$$

where $\boldsymbol{q}_1^0$, $\boldsymbol{q}_2^0$, ... are nondimensional momenta transferred between the two colliding consitutents, i.e. $\boldsymbol{q}^0=R_0(s)\boldsymbol{q}$ is a scaling variable.

In (11) the first term represents single scattering of the constituents, the second term represents double scattering, and so on. According to Abarbanel (1976), we neglect terms in double and triple scattering in which the constituents in one proton repeatedly collide with the same constituent in another. In fact, these terms turn out to damp t more rapidly than the terms left in (11). The functions $f_1(\boldsymbol{q}^0)$, $f_2(\boldsymbol{q}_1^0,\,\boldsymbol{q}_2^0)$, $f_3(\boldsymbol{q}_1^0,\,\boldsymbol{q}_2^0,\,\boldsymbol{q}_3^0)$, ... etc. represent the hadronic form factors related to each multiple scattering, where

$$f_1(\boldsymbol{q}^0)\;=\;\int \mathrm{e}^{-\mathrm{i}\boldsymbol{q}^0\cdot\boldsymbol{r}_1}|\,\psi_{\text{p}}(\boldsymbol{r}_1,\,\boldsymbol{r}_2,\,\boldsymbol{r}_3)\,|^2\,\delta\!\left(\frac{\boldsymbol{r}_1+\boldsymbol{r}_2+\boldsymbol{r}_3}{3}\right)\mathrm{d}\boldsymbol{r}_1\,\mathrm{d}\boldsymbol{r}_2\,\mathrm{d}\boldsymbol{r}_3\,, \tag{12}$$

$$\begin{aligned}f_2(\boldsymbol{q}_1^0,\,\boldsymbol{q}_2^0)\;&=\;\int \mathrm{e}^{-\mathrm{i}(\boldsymbol{q}^0\cdot\boldsymbol{r}_1+\boldsymbol{q}_2^0\cdot\boldsymbol{r}_2)}|\,\psi_{\text{p}}(\boldsymbol{r}_1,\,\boldsymbol{r}_2,\,\boldsymbol{r}_3)\,|^2\\ &\times\,\delta\!\left(\frac{\boldsymbol{r}_1+\boldsymbol{r}_2+\boldsymbol{r}_3}{3}\right)\mathrm{d}\boldsymbol{r}_1\,\mathrm{d}\boldsymbol{r}_2\,\mathrm{d}\boldsymbol{r}_3\,,\end{aligned} \tag{13}$$

and so on. Here $\psi_{\text{p}}(\boldsymbol{r}_1,\,\boldsymbol{r}_2,\,\boldsymbol{r}_3)$ is the wavefunction of the proton constructed from three constituents (three quarks).

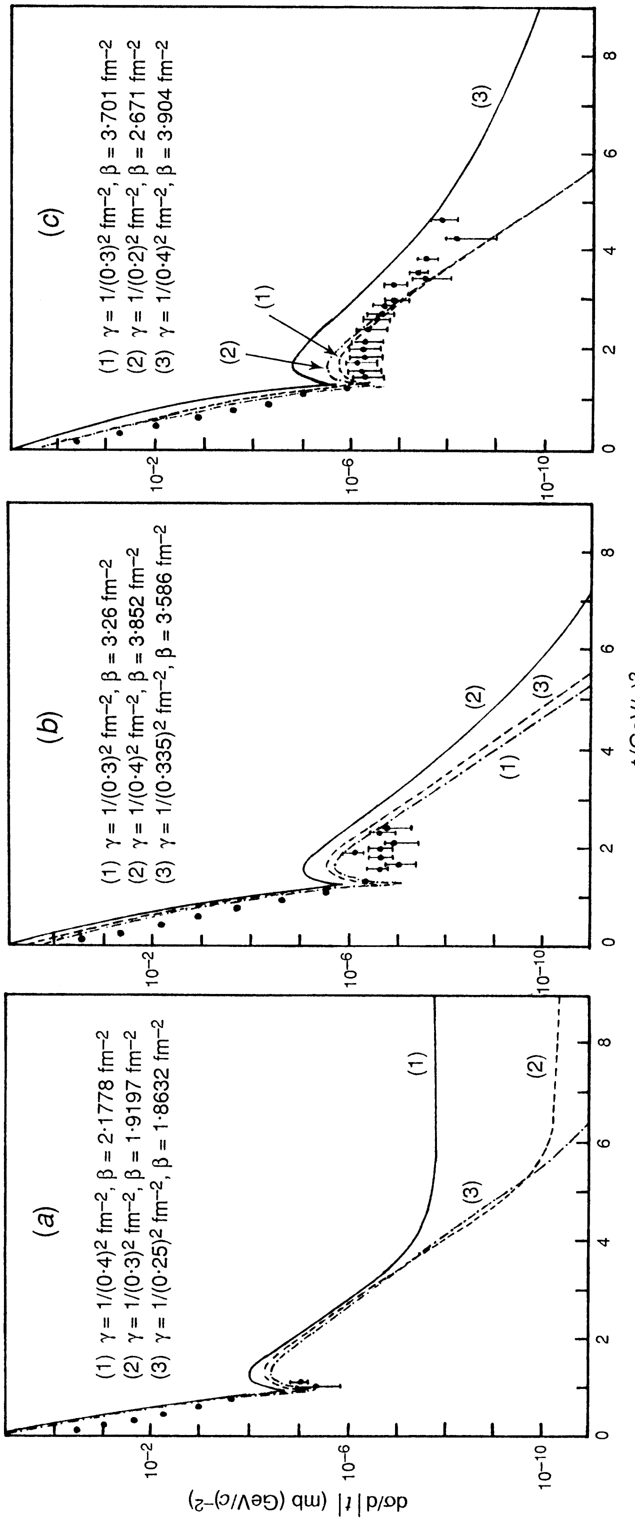

Fig. 4. The p–p elastic scattering differential cross section using the proton wavefunction (1), with different values of γ and β, at (a) $p_{\mathrm{L}} = 290$ GeV/c, (b) 500 GeV/c and (c) 1070 GeV/c. The data are from Bouras and Dias de Deus (1974).

Since the effects of the multiple scattering terms of higher order than in double scattering are in general small, and due to the complicated form of the wavefunction used which leads to an enormous amount of calculation with small effect, we shall for simplicity neglect these terms. However, with the following form of the quark–quark scattering amplitude (Wakaizumi 1978):

$$t_{\rm qq}(\boldsymbol{q}^0) = \frac{\mathrm{i}+\alpha}{9}\,\frac{A}{4\pi}\,\mathrm{e}^{-\frac{1}{2}a_0 \boldsymbol{q}^{02}}\,, \tag{14}$$

where A and a_0 are dimensionless quantities and α is the ratio of the real to imaginary parts of the quark–quark amplitude in the forward direction, the final form of $T_{\rm p\text{-}p}$ for the proposed wavefunction is very long (available from the authors on request). For the Gaussian wavefunction only,

$$\psi_{\rm p}(\boldsymbol{r}_1,\,\boldsymbol{r}_2,\,\boldsymbol{r}_3) = \left(\frac{\beta^3}{3\sqrt{3}\,\pi^3}\right)^{\frac{1}{2}} \mathrm{e}^{-\frac{1}{2}\beta(r_1^2+r_2^2+r_3^2)}\,, \tag{15}$$

and with our simplification a simple form of $T_{\rm p\text{-}p}$ can be given here as

$$T_{\rm p\text{-}p}(s,\,t) = (\mathrm{i}+\alpha)\,R_0^2\,\frac{A}{4\pi}\left[\exp\left\{-\left(\frac{a_0}{2}+\frac{1}{3\beta R_0^2}\right)\boldsymbol{q}^{02}\right\}\right.$$

$$\left.+\,\frac{\mathrm{i}(\mathrm{i}+\alpha)A}{36\pi}\left(\frac{\beta R_0^2}{a_0\,R_0^2\,\beta+1}\right)\exp\left\{-\left(\frac{a_0}{4}+\frac{1}{12\beta R_0^2}\right)\boldsymbol{q}^{02}\right\}\right]\,, \tag{16}$$

where, in this case, $\beta = 1/\langle r^2\rangle$.

4. Results and Discussion

The p–p elastic scattering differential cross section at laboratory momenta $p_{\rm L} = 290$, 500, 1070 and 1500 GeV/c, using the proposed wavefunction (1) and simplifications in the previous section, is calculated on the basis of (11). The results are presented for different values of both the momentum and parameter γ in Figs 4 and 5. The values of the rms distance $\langle r^2\rangle^{\frac{1}{2}}$ and the quark–quark parameters α, A and a_0 are given in Table 1. These values are obtained by comparing the theoretical results of the p–p elastic scattering differential cross section for the Gaussian wavefunction with the experimental data at the momenta used (Youssef 1993).

Table 1. Quark–quark parameters and $\langle r^2\rangle^{\frac{1}{2}}$ (from Youssef 1993)

$p_{\rm L}$ (GeV/c)	α	A	a_0	$\langle r^2\rangle^{\frac{1}{2}}$ (GeV)$^{-1}$
290	0·181	1·14	0·032	3·78
500	0·065	0·70	0·130	3·02
1070	0·110	0·65	0·090	3·15
1500	0·181	1·10	0·033	3·15

From these figures we can see that the p–p elastic scattering differential cross section at small values of q^2 is relatively weakly dependent, as expected, on the short-range parameter γ. Its sensitivity with respect to this parameter is clear after the first minimum. It is obvious that the value of the cross section increases when γ decreases. However, the correlation effect at very low q^2 values is related to the small value of $\langle r^2\rangle^{\frac{1}{2}}$—for example, see Figs 4*a* and 4*c* where $\langle r^2\rangle^{\frac{1}{2}} = 3\cdot 78$ and $3\cdot 15\ \mathrm{GeV}^{-1}$ respectively. The dependence on γ increases when $\langle r^2\rangle^{\frac{1}{2}}$ decreases.

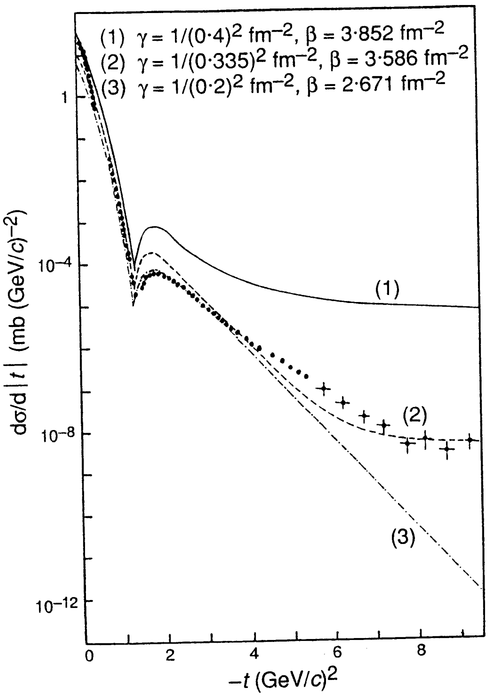

Fig. 5. Same as Fig. 4 but for $p_{\mathrm{L}} = 1500\ \mathrm{GeV}/c$ and the data from De Kerret *et al.* (1976, 1977).

The value of γ used to describe the experimental data at $\sqrt{s} = 53$ GeV in the wide range of momentum-transfer squared $[0 \leq q^2 \leq 10\ (\mathrm{GeV}/c)^2]$ is of order $(0\cdot 34)^{-2}\ \mathrm{fm}^{-2}$ (see Fig. 5). The values of γ at the other values of $\sqrt{s}$ are of the same order as shown in Fig. 4. We note that the proposed wavefunction gives the general features of the data, and we obtain a good fit with the experimental data especially for $q^2 > 6\ (\mathrm{GeV}/c)^2$ as in Fig. 5. The good fit with experimental data for $q^2 > 3\ (\mathrm{GeV}/c)^2$ at $\sqrt{s} = 53$ GeV means that the proposed wavefunction gives a good description of the quark distribution at short distances.

Thus, we can conclude that the quarks inside the proton are distributed from each other and the minimum distance equals $0\cdot 34$ fm. However, there is no repulsive core between quarks at short distances and they are asymptotically free. Therefore, the quark must be an elementary particle with finite size, having a radius of order $0\cdot 34/2 = 0\cdot 17$ fm. This means that the linear dimension of the core of the proton where the quarks can be concentrated is larger than or equal to $0\cdot 34$ fm (see Fig. 6). These results are consistent with the results of Kuroda and Miyazawa (1973) for the quark radius obtained from the analysis of high-energy p–p elastic scattering, using the geometrical impact parameter representation where the diameter of an extended quark is taken to be 0·31 fm. Also, our prediction for the proton core radius is consistent with the results of (Islam 1975; Goloskokov *et al.* 1982) for the hadron core which equals 0·48 fm.

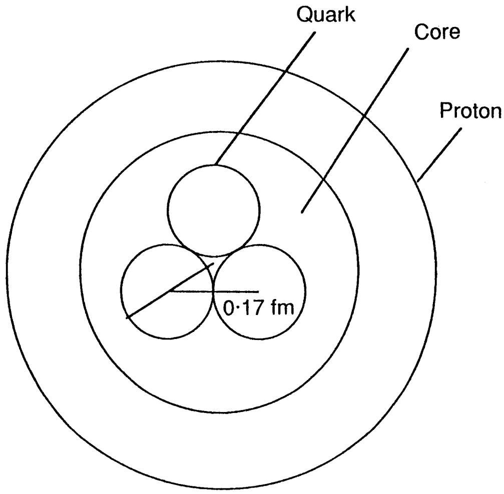

Fig. 6. Schematic of the core and quark radii.

On the other hand, evaluations of the quark radius (Bogolyubskii 1982; Franco 1987), which are of order 0·35 fm, are double our value. However, we believe that our results are more realistic than these results. If we consider the radius of a quark to be 0·35 fm, the radius of the proton core must be larger than 0·7 fm, which is inconsistent with the general estimation of 0·4–0·5 fm. Also, the treatment of quarks as point-like particles is more consistent with the value 0·17 fm than 0·35 fm.

Finally, the p–p elastic scattering differential cross section at the momenta used, for the Gaussian and proposed wavefunctions, is presented in Figs 7 and 8. It is clear that the difference between the two wavefunctions at small momentum transfer, $q^2 < 1\cdot 4\ (\mathrm{GeV}/c)^2$ (the first minimum), can be neglected. At the second maximum, the results of our wavefunction are higher than those of the Gaussian which agree with the experimental data. But as q^2 increases, the results of the Gaussian wavefunction lead to disagreement with the data. At the same time, our wavefunction leads to closer agreement. We can interpret an increase in the cross section for the proposed wavefunction as follows.

The short-range correlation factor in wavefunction means that the distances between quarks must not be less than a definite value of order 0·34 fm. This increases the effective cross section more than in the Gaussian wavefunction case, where any distance between two quarks is possible. Also, the effect of this short-range correlation and the effect of the Lorentz contraction of the proton, which is the possible origin of the dipole formula (Fujimura *et al.* 1970) considered in the relativistic Gaussian wavefunction (Amelin *et al.* 1983), have something common. Both effects separate the quarks at a distance larger than the case where these effects are neglected, especially if we consider the finite size of the quark. This common effect between the proposed wavefunction and Lorentz contraction of the proton—as a whole not as a three-quark system—may be used to explain the agreement of our calculations with the experimental data.

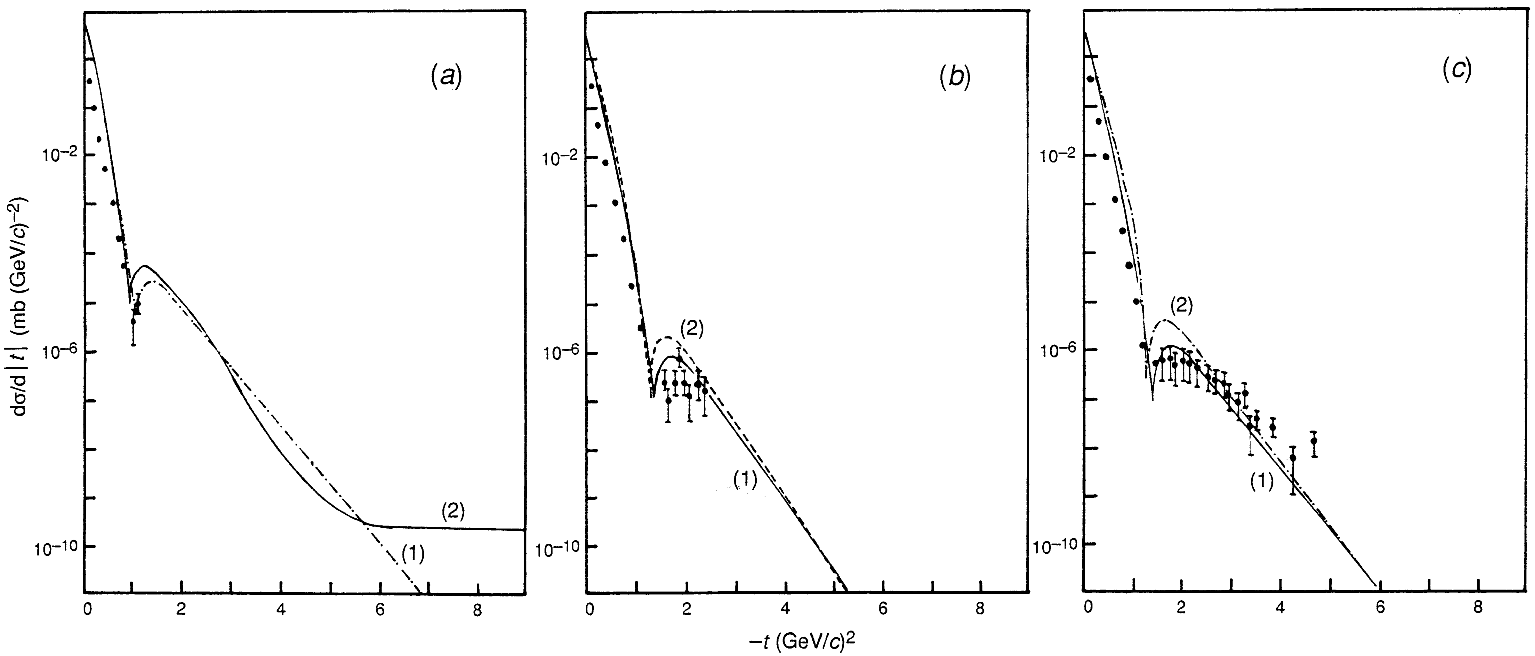

Fig. 7. The p–p elastic scattering differential cross section for (1) the Gaussian wavefunction and (2) the wavefunction of equation (1) for (*a*) $p_L = 290$ GeV/c, (*b*) 500 GeV/c and (*c*) 1070 GeV/c. The data are from Bouras and Dias de Deus (1974).

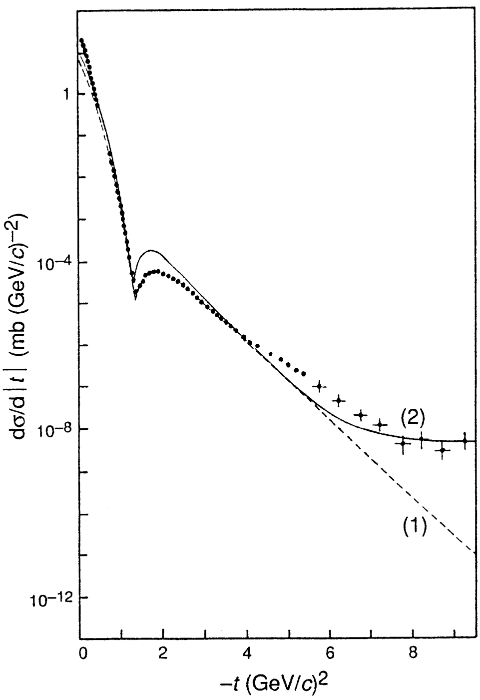

Fig. 8. Same as Fig. 7 but for $p_{\rm L} = 1500$ GeV/c and the data from De Kerret *et al.* (1976, 1977).

Thus, using a representation of the quark distribution inside the proton, where a short-range correlation between quarks is considered, we can describe the p–p elastic scattering differential cross section and its agreement with experimental data in the range $0 \leq q^2 \leq 10$ (GeV/c)^2. Estimations of the proton core and quark radii are obtained. From our analysis, we see that the suggested configuration of quarks inside the proton can be considered as a reasonably realistic distribution even with a non-relativistic wavefunction.

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