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Study of Squeezing Properties in a Two-level System

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Abstract

We have studied the squeezing properties of a field and atom in a two-level system. The influence of nonlinear interactions (i.e. the arbitrary intensity-dependent coupling of a single-mode field to a single two-level atom, the nonlinear interaction of the field with a nonlinear Kerr-like medium) on the squeezing is discussed in detail in the rotating wave approximation (RWA). We show numerically that the effect of the virtual-photon field suppresses dipole squeezing predicted in the RWA and leads to an increased squeeze revival period; the suppressed squeezing can be revived due to the presence of the nonlinear Kerr-like medium.

1. Introduction

Recent achievements with highly excited Rydberg atoms in high-Q microwave cavities (Brune *et al.* 1987) have made it possible to investigate experimentally the dynamical features of the atom–field interaction which depend explicitly on the quantum nature of the electromagnetic field. The revival (Sukumar and Buck 1981; Buck and Sukumar 1981; Buzek 1989*a*, *b*; Buzek and Jex 1990) of interest in the Jaynes–Cummings model (1963) (JCM) is due to the observation of some non-classical features in the laboratory, such as squeezing of the radiation field (Slusher *et al.* 1985).

The JCM without the RWA is not solvable by the usual techniques since the eigenstates of the Hamiltonian cannot be found in closed form. The first solution of this model was due to Graham and Höhnerbach (1984) in which a numerical method was employed. It has been shown that the virtual-photon field (counter-rotating wave terms) is the source of the Lamb shift (Compagno *et al.* 1983), that it can ensure causality in atom–field coupling systems (Compagno *et al.* 1989), and that it has a significant effect on the atomic inversion (Zaheer and Zubairy 1988) and photon antibunching (Xie *et al.* 1995), even under the condition that the RWA is considered valid. Recently, Lais and Steimle (1990) have shown numerically that the counter-rotating wave terms suppress the squeezing of the radiation field predicted in the JCM. All studies have shown that it is worth while to investigate the quantum dynamical properties in the atom–field coupling system, where the counter-rotating wave terms are included.

The squeezing of the radiation field has been studied theoretically (Walls and Zoller 1981; Meystre and Zubairy 1982; Buzek 1989*b*) and experimentally (Slusher

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et al. 1985) due to its potential application (Yuen and Shapiro 1978) in optical communication and gravitational wave detection. Meanwhile, the squeezing of fluctuations of the atomic dipole variables has also received considerable interest (Walls and Zoller 1981; Wodkiewicz *et al.* 1987; Hu and Aravind 1989; Zhou and Peng 1991). It has been shown by Wodkiewicz *et al.* (1987) that an atom exhibiting reduced dipole fluctuations can radiate the squeezed light. Hu and Aravind (1989) have indicated that there exists a striking symmetry between the squeezing of the field and that of the atom in the vacuum field JCM. In recent years, some authors (Wodkiewicz *et al.* 1987; Zhou and Peng 1991) have even formulated a new class of squeezed states (i.e. the superposition squeezed states) of the field and atoms with a simple implementation of the fundamental linear superposition principle of quantum mechanics. In the mixed JCM and anharmonic oscillator model (Buzek 1989*b*), Buzek and Jex (1990) have shown that the interaction between the field and the nonlinear Kerr-like medium leads to the deterioration of the squeezing of the radiation field in weak nonlinearity regimes. Buzek (1989*a*) has also found that the squeezing of the field exhibits periodic revivals in the intensity-dependent-coupling JCM (Buck and Sukumar 1981).

Brune *et al.* (1987) have reported the first two-photon quantum oscillator by employing Rydberg atoms in a high-Q superconducting microwave cavity. Scully *et al.* (1988) have predicted that a two-photon correlated-emission laser can produce the stable squeezed light. Zhou and Peng (1991) have studied dipole squeezing in the two-photon JCM with superposition state preparations. In the present paper, we consider the two-photon JCM, where various forms of intensity-dependent nonlinear coupling of the field to the atom are supposed and a nonlinear Kerr-like medium is introduced. The purpose is to investigate the influence of the above nonlinear interactions on the squeezing of the field and atom. In addition, the influence of virtual-photon processes on dipole squeezing will be discussed through a numerical method (Graham and Höhnerbach 1984).

The paper is organised as follows. The theoretical model and its solution in the RWA are given in Section 2. Then we study, in Section 3, the squeezing properties of the atom in this model with and without the RWA. In Section 4, light squeezing is studied in detail in the model with the RWA. Finally, we give our summary in Section 5.

2. The Model and Its Solution in the RWA

Here we consider a two-level system, where a single two-level atom is surrounded by a nonlinear Kerr-like medium contained inside a single-mode cavity, and the cavity mode is also coupled to the Kerr-like medium (Buzek 1989*b*), as well as to the atom. If cavity damping and the effect of the thermal field are neglected and if an arbitrary intensity-dependent nonlinear coupling between the atom and the cavity exists, then the total Hamiltonian of the system can be written as

$$\begin{aligned} H &= \Omega_a(a^+a+\tfrac{1}{2})+\Omega_b b^+b+qb^{+2}b^2+\lambda(a^+b+b^+a) \\ &\quad +\omega S_z+G(R^++R)(S_-+S_+)\,. \end{aligned} \tag{1}$$

In the adiabatic limit, the total Hamiltonian (1) can be transformed to a form involving only the photon and atomic operators:

$$H_{\text{eff}} = \Omega(a^+a + \tfrac{1}{2}) + \omega S_z + \chi a^{+2}a^2 + G(R^+ + R)(S_- + S_+)\,, \tag{2}$$

where S_z and $S_\pm$ are operators of atomic inversion and transition, respectively, having the following commutation relations:

$$[S_z, S_\pm] = \pm S_\pm\,, \qquad [S_+, S_-] = 2S_z\,. \tag{3}$$

Here ω is the atomic transition frequency, a^+ (a) is the creation (annihilation) operator of the cavity mode with the frequency Ω_a and the commutation relation $[a, a^+] = 1$, while b^+ (b) is the creation (annihilation) operator of the Kerr-like medium with the frequency Ω_b and the commutation relation $[b, b^+] = 1$, and G is the atom–field coupling constant. The operators R^+ and R are the generalisation of the definition (Buck and Sukumar 1981) in the following way:

$$R^+ = V(N)a^{+2}\,, \qquad R = a^2V(N)\,, \tag{4}$$

where $N = a^+a$ is the photon number operator and $V(N)$ is an arbitrary function of N and reflects arbitrary intensity-dependent coupling between the atom and the field of the Hamiltonian (1). The new frequency Ω and the nonlinear coupling constant χ (i.e. the dispersive part of the third-order nonlinearity of the Kerr-like medium) are given by (Agarwal and Puri 1989)

$$\chi = q\lambda^4/(\Omega_b - \Omega_a)^4\,, \tag{5}$$

$$\Omega = \Omega_a - \lambda^2/(\Omega_b - \Omega_a)\,. \tag{6}$$

If the RWA is explicitly used, the effective Hamiltonian (2) can be written as

$$H_{\text{eff}}^{\text{RWA}} = \Omega(a^+a + \tfrac{1}{2}) + \omega\ S_z + \chi a^{+2}a^2 + G(R^+S_- + RS_+)\,. \tag{7}$$

Throughout we employ units with $\hbar = c = 1$. The dynamics of the effective Hamiltonian (7) can be solved explicitly as follows. In the representation of the atomic eigenstates

$$|+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } |-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}\,,$$

we have obtained the time-dependent unitary transformation operator as follows:

$$U(t,0) = \exp(-iH_{\text{eff}}^{\text{RWA}}t) = \begin{pmatrix} U_{++}, U_{+-} \\ U_{-+}, U_{--} \end{pmatrix}\,, \tag{8}$$

where

$$\begin{aligned}
U_{++} &= \exp\{-i[\Omega(a^+a+\tfrac{3}{2})+\chi((a^+a)^2+a^+a+1)]t\}\{\cos(At) \qquad (9)\\
&\quad -i(\Delta/2-\chi(2a^+a+1))\sin(At)/A\}\,,\\
U_{--} &= \exp\{-i[\Omega(a^+a-\tfrac{1}{2})+\chi((a^+a)^2+a^+a+1)]t\}\{\cos(Bt)\\
&\quad +i(\Delta/2+\chi(2a^+a+1))\sin(Bt)/B\}\,, \qquad (10)\\
U_{+-} &= -iG\exp\{-i[\Omega(a^+a+\tfrac{3}{2})+\chi((a^+a+2)^2\\
&\quad +a^+a+3)]t\}R\sin(Bt)/B\,, \qquad (11)\\
U_{-+} &= -iG\exp\{-i[\Omega(a^+a-\tfrac{1}{2})+\chi((a^+a-2)^2\\
&\quad +a^+a-1)]t\}R^+\sin(At)/A\,, \qquad (12)\\
A &= \sqrt{[\Delta/2-\chi(2a^+a+1)]^2+G^2V^2(N)a^2a^{+2}}\,, \qquad (13)\\
B &= \sqrt{[\Delta/2+\chi(2a^+a+1)]^2+G^2a^{+2}a^2V^2(N)}\,, \qquad (14)\\
\Delta &= \omega-2\Omega\,. \qquad (15)
\end{aligned}$$

Hence, if the initial state $\mid\psi(t{=}0)\rangle$ of the system is given, using equation (8), we can obtain the state $\mid\psi(t)\rangle$ of the system at time t. As an example, in the present paper we assume that the system is initially prepared in the coherent superposition state

$$\mid\beta\rangle=\begin{pmatrix}\eta_+\\ \eta_-\end{pmatrix}$$

of the two-level atom (Wodkiewicz *et al.* 1987; Hu and Aravind 1989; Zhou and Peng 1991) and the superposition (Meystre and Zubairy 1982; Wodkiewicz *et al.* 1987; Zhou and Peng 1991) of the photon number states $\mid n\rangle$, i.e.

$$\mid\psi(0)\rangle=\begin{pmatrix}\eta_+\\ \eta_-\end{pmatrix}\otimes\sum_{n=0}^{\infty}f_n\mid n\rangle\,. \qquad (16)$$

Then the time-dependent state $\mid\psi(t)\rangle$ of the system at the time t is

$$\mid\psi(t)\rangle=U(t,0)\mid\psi(0)\rangle=\begin{pmatrix}\mid\psi_1(t)\rangle\\ \mid\psi_2(t)\rangle\end{pmatrix}, \qquad (17)$$

where

$$
\begin{aligned}
|\psi_1(t)\rangle &= \sum_{n=0}^{\infty} \eta_+ f_n \exp\{-i[\Omega(n+\tfrac{3}{2}) + \chi(n^2+n+1)]t\}\{\cos(\Omega_n t) \\
&\quad - i[\Delta/2 - \chi(2n+1)]\frac{\sin(\Omega_n t)}{\Omega_n}\}|n\rangle - iG\sum_{n=P}^{\infty} \eta_- f_n \exp\{i[\Omega(n-\tfrac{1}{2}) \\
&\quad + \chi(n^2+n+1)]t\}V(n)\sqrt{\frac{n!}{(n-2)!}}\frac{\sin(\omega_n t)}{\omega_n} \mid n-2\rangle\,, \qquad (18)
\end{aligned}
$$

$$
\begin{aligned}
\mid \psi_2(t)\rangle &= \sum_{n=0}^{\infty} \eta_- f_n \exp\{-i[\Omega(n-\tfrac{1}{2}) + \chi(n^2+n+1)]t\}\{\cos(\omega_n t) \\
&\quad + i[\Delta/2 + \chi(2n+1)]\frac{\sin(\omega_n t)}{\omega_n}\} \mid n\rangle - iG\sum_{n=0}^{\infty} \eta_+ f_n \exp\{-i[\Omega(n+\tfrac{3}{2}) \\
&\quad + \chi(n^2+n+1)]t\}V(n+2)\sqrt{\frac{(n+2)!}{n!}}\frac{\sin(\Omega_n t)}{\Omega_n} \mid n+2\rangle\,, \qquad (19)
\end{aligned}
$$

$$
\Omega_n = \sqrt{[\Delta/2 - \chi(2n+1)]^2 + G^2V^2(n+2)\frac{(n+2)!}{n!}}\,, \qquad (20)
$$

$$
\omega_n = \sqrt{[\Delta/2 + \chi(2n+1)]^2 + G^2V^2(n)\frac{n!}{(n-2)!}}\,, \qquad (21)
$$

with $\sum_{n=0}^{\infty} \mid f_n \mid^2 = 1$ and $\mid \eta_+ \mid^2 + \mid \eta_- \mid^2 = 1$. Here $V(n)$ is an arbitrary function of photon number n. Obviously, when $\chi = 0$ and $V(n) = 1$, equation (17) is in agreement with those obtained by Zhou and Peng (1991).

3. Squeezing Properties of an Atom

In order to investigate the squeezing properties of the atom, we consider two hermitian conjugate operators S_x and S_y (Walls and Zoller 1981),

$$
S_x = \tfrac{1}{2}(S_+ + S_-); \qquad S_y = \frac{1}{2i}(S_+ - S_-)\,, \qquad (22)
$$

which correspond to the in-phase and out-of-phase components of the amplitude of the atomic polarisation (Walls and Zoller 1981), respectively, and obey the commutation relation

$$
[S_x, S_y] = iS_z\,. \qquad (23)
$$

Then, we have the Heisenberg uncertainty relation

$$
\langle(\Delta S_x)^2\rangle\langle(\Delta S_y)^2\rangle \geq \tfrac{1}{4}\langle S_z\rangle^2\,, \qquad (24)
$$

where

$$\langle(\Delta S_x)^2\rangle = \langle S_x^2\rangle - \langle S_x\rangle^2; \qquad \langle(\Delta S_y)^2\rangle = \langle S_y^2\rangle - \langle S_y\rangle^2 \tag{25}$$

is the variance of S_x and S_y respectively. If the following condition is satisfied,

$$F_1 = \langle(\Delta S_x)^2\rangle - \tfrac{1}{2} \mid \langle S_z\rangle \mid < 0 \ or \ F_2 = \langle(\Delta S_y)^2\rangle - \tfrac{1}{2} \mid \langle S_z\rangle \mid < 0\,, \tag{26}$$

then the fluctuations in the component S_x or S_y of the atomic dipole are squeezed (Walls and Zoller 1981).

(3a) Dipole Squeezing in the RWA

In this section we assume that the atom is initially prepared in the ground state and the field is in a superposition (Wodkiewicz *et al.* 1987; Zhou and Peng 1991) of the vacuum state $\mid 0\rangle$ and two-photon state $\mid 2\rangle$, i.e.

$$\mid \psi(0)\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes (f_0 \mid 0\rangle + f_2 \mid 2\rangle)\,. \tag{27}$$

Using equation (17), we obtain the time evolution of the functions F_1 and F_2 as

$$F_1(t) = \tfrac{1}{4} - 2G^2V^2(2) \mid f_0 \mid^2 \mid f_2 \mid^2 \left(\frac{\sin(\omega_2 t)\sin(\frac{1}{2}\Delta t + 7\chi t + \delta\phi)}{\omega_2}\right)^2 - \tfrac{1}{2} \mid \langle Sz\rangle \mid\,, \tag{28}$$

$$F_2(t) = \tfrac{1}{4} - 2G^2V^2(2) \mid f_0 \mid^2 \mid f_2 \mid^2 \left(\frac{\sin(\omega_2 t)\cos(\frac{1}{2}\Delta t + 7\chi t + \delta\phi)}{\omega_2}\right)^2 - \tfrac{1}{2} \mid \langle Sz\rangle \mid\,, \tag{29}$$

where

$$\langle S_z\rangle = -\tfrac{1}{2}\left(\mid f_2 \mid^2 \frac{(\frac{1}{2}\Delta + 5\chi)^2 + 2V^2(2)G^2\cos(2\omega_2 t)}{\omega_2^2} + \mid f_0 \mid^2\right), \tag{30}$$

$$\omega_2 = \sqrt{(\tfrac{1}{2}\Delta + 5\chi)^2 + 2V^2(2)G^2}\,, \tag{31}$$

with $\mid f_0 \mid^2 + \mid f_2 \mid^2 = 1$. Here $\delta\phi$ is the relative phase between the vacuum and two-photon state. Clearly, when $\Delta = 0$ and $\chi = 0$, equations (28) and (29) are in agreement with those obtained by Zhou and Peng (1991).

Now we study the squeezing properties of the fluctuations of the atomic dipole variables. When the field is initially in the vacuum state (i.e. $f_0 = 1$) or in the two-photon state (i.e. $f_2 = 1$), it can be seen from (28) and (29) that S_x or S_y cannot be squeezed. Does the squeezing of the atom exist when the field is initially in the superposition state? We shall answer this question in the following.

Dipole squeezing for $\chi = 0$. Here we study the role of arbitrary intensity-dependent coupling of the field to the atom on dipole squeezing. For simplicity, we take $\chi = \Delta = 0$ and $\delta\phi = \pi/2$. Obviously, $F_2 \geq 0$, so the fluctuations in the component S_y of the atomic dipole cannot be squeezed; however, we have verified that the fluctuations in S_x are squeezed under the following condition: when $\sqrt{2}/2 < | f_2 | < 1$, $F_1(t) < 0$ during the time

$$\frac{2k\pi + C_A}{2\sqrt{2}V(2)G} < t < \frac{2k\pi + 2\pi - C_A}{2\sqrt{2}V(2)G} \qquad (k = 0, 1, 2, \ldots), \tag{32}$$

where

$$C_A = \arccos\frac{3 \mid f_2 \mid^2 - 2 \mid f_2 \mid^4 - 2}{3 \mid f_2 \mid^2 - 2 \mid f_2 \mid^4}. \tag{33}$$

It means that S_x is squeezed periodically with the squeeze duration

$$T_{\text{sque}} = \frac{\pi - C_A}{\sqrt{2}V(2)G}, \tag{34}$$

and the squeeze revival period

$$T_{\text{R}} = \frac{\pi}{\sqrt{2}V(2)G}. \tag{35}$$

Meanwhile, the maximum squeeze amplitude

$$A_{\text{max}} = \mid F_1 < 0 \mid_{\text{max}} = (\mid f_2 \mid^2 - \tfrac{1}{2})(1 - \mid f_2 \mid^2) \tag{36}$$

appears at the time

$$t_{\text{max}} = \frac{k\pi + \pi/2}{\sqrt{2}V(2)G} \qquad (k = 0, 1, 2, \ldots). \tag{37}$$

We can see that $A_{\text{max}} = 0{\cdot}0625$ at $| f_2 | = \sqrt{3}/2$ is the largest one among those at $\sqrt{2}/2 < | f_2 | < 1$.

When $| f_2 | = \sqrt{2}/2$, $F_1 = 0$, S_x is not squeezed. When $0 < | f_2 | < \sqrt{2}/2$, S_x is squeezed almost at all times except t_0 at which $F_1(t_0) = 0$:

$$t_0 = \frac{k\pi}{\sqrt{2}V(2)G} \qquad (k = 0, 1, 2, \ldots). \tag{38}$$

In this case, the squeeze duration and its revival period are

$$T_{\text{sque}} = T_{\text{R}} = \frac{\pi}{\sqrt{2}V(2)G}, \tag{39}$$

and the maximum squeeze amplitude

$$A_{\text{max}} = \mid F_1 < 0 \mid_{\text{max}} = \mid f_2 \mid^2 (\tfrac{1}{2} - \mid f_2 \mid^2) \tag{40}$$

appears at

$$t_{\max} = \frac{k\pi + \pi/2}{\sqrt{2}V(2)G} \qquad (k = 0, 1, 2, ...) \,. \tag{41}$$

Also $A_{\max} = 0{\cdot}0625$ at $\mid f_2 \mid = \frac{1}{2}$ is the largest among those at $0 < \mid f_2 \mid < \sqrt{2}/2$. In addition, we find from equation (28) that whether the fluctuations in S_x are squeezed or not also depends on the phase $\delta\phi$. For example, when $\delta\phi = 0$, $F_1 \geq 0$, the fluctuations in S_x cannot be squeezed [in this case $\delta\phi = 0$ we can see from equation (29) that the fluctuations in S_y are squeezed and the results are similar to those discussed above].

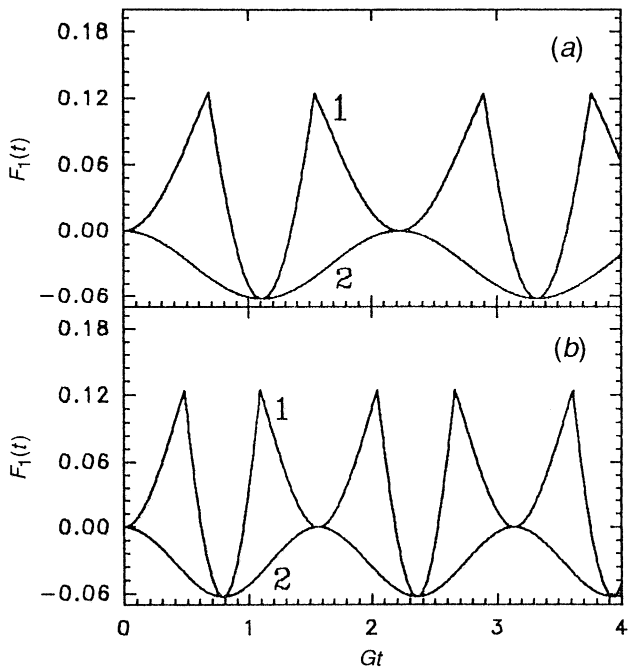

Fig. 1. Time evolution of F_1 with the RWA for $G = 0{\cdot}05$, $\delta\phi = \pi/2$, $\Delta = 0$, $\chi = 0$ and $V(N) = N^r$ for (*a*) $r = 0$; (*b*) $r = \frac{1}{2}$, where curve 1 is for $\mid f_2 \mid = \sqrt{3}/2$ and curve 2 for $\mid f_2 \mid = \frac{1}{2}$.

When $\delta\phi = \pi/2$, we can see from above analysis that T_{sque}, T_{R} and $t_{\max}$ all are inversely proportional to $V(2)G$, and the maximum squeeze amplitude $A_{\max}$ is only related to $\mid f_2 \mid$ (note $\mid f_0 \mid^2 + \mid f_2 \mid^2 = 1$). These reflect that the forms of arbitrary intensity-dependent coupling between the atom and the field only influence the squeeze duration T_{sque}, the squeeze revival period T_{R} and the time $t_{\max}$ of appearance of the maximum squeeze amplitude $A_{\max}$. As an example, we assume $V(N) = N^r (r \geq 0)$ ($r = \frac{1}{2}$ is the definition given by Buck and Sukumar 1981). We have plotted the time evolution of F_1 in Fig. 1 for $r = 0, \frac{1}{2}$ and $\mid f_2 \mid = \frac{1}{2}$, $\sqrt{3}/2$, where it can be seen clearly that T_{sque}, T_{R} and $t_{\max}$ all decrease with an increase of the factor r, and that the maximum squeeze amplitude $A_{\max}$ is not influenced by the forms of arbitrary intensity-dependent coupling. Interestingly, if there exist the forms with $V(N) = N^{-r}$ $(r \geq 0)$, longer T_{sque}, T_{R} and $t_{\max}$ values are obtained with an increase in the factor r.

Dipole squeezing for $\chi \neq 0$. For simplicity, we let $\chi = -\Delta/10$. In the following, we take the component S_x of the atomic dipole and the cases with $\delta\phi = \pi/2$ and $0 < | f_2 | < \sqrt{2}/2$ as an example. We have verified that the fluctuation in S_x is squeezed under the following conditions.

(1) In the cases where

$$\frac{|\chi|}{G} > E\,, \tag{42}$$

where

$$E = \frac{V(2)\sqrt{2}D}{2\pi}\,, \tag{43}$$

$$D = \arccos\left(\frac{| f_2 |^2}{1-| f_2 |^2}\right) \tag{44}$$

Here S_x is the squeezed duration for

$$\frac{2k\pi + \pi - D}{4|\chi|} < t < \frac{2k\pi + \pi + D}{4|\chi|} \qquad (k = 0, 1, 2, \ldots)\,. \tag{45}$$

Clearly, the squeeze duration is

$$T_{\text{sque}} = \frac{D}{2|\chi|}\,, \tag{46}$$

the squeeze revival period is

$$T_{\text{R}} = \frac{\pi}{2|\chi|}\,, \tag{47}$$

and if $2 | \chi | /\sqrt{2}GV(2)$ is an odd number, we observe the maximum squeeze amplitude

$$A_{\max} = | F_1 < 0 |_{\max} = | f_2 |^2 \left(\tfrac{1}{2} - | f_2 |^2\right) \tag{48}$$

at time

$$t_{\max} = \frac{(2k+1)\pi}{4|\chi|} \qquad (k = 0, 1, 2, \ldots)\,. \tag{49}$$

(2) In the cases where

$$\frac{\chi}{G} \leq E\,, \tag{50}$$

we have verified that S_x is squeezed almost completely during the following, except at t_0 at which $\sin(\sqrt{2}GV(2)t_0) = 0$,

$$\frac{2k\pi + \pi - D}{4 \mid \chi \mid} < t < \frac{2k\pi + \pi + D}{4 \mid \chi \mid} \qquad (k = 0, 1, 2, ...)\,, \tag{51}$$

where the squeeze duration (here we have included the time t_0) is

$$T_{\text{sque}} = \frac{D}{2 \mid \chi \mid}\,, \tag{52}$$

the squeeze revival period is

$$T_{\text{R}} = \frac{\pi}{2 \mid \chi \mid}\,, \tag{53}$$

and if $\sqrt{2}GV(2)/2 \mid \chi \mid$ is an odd number, the maximum squeeze amplitude $A_{\max}$ (given by equation 48) appears at the time $t_{\max}$ (given by equation 49).

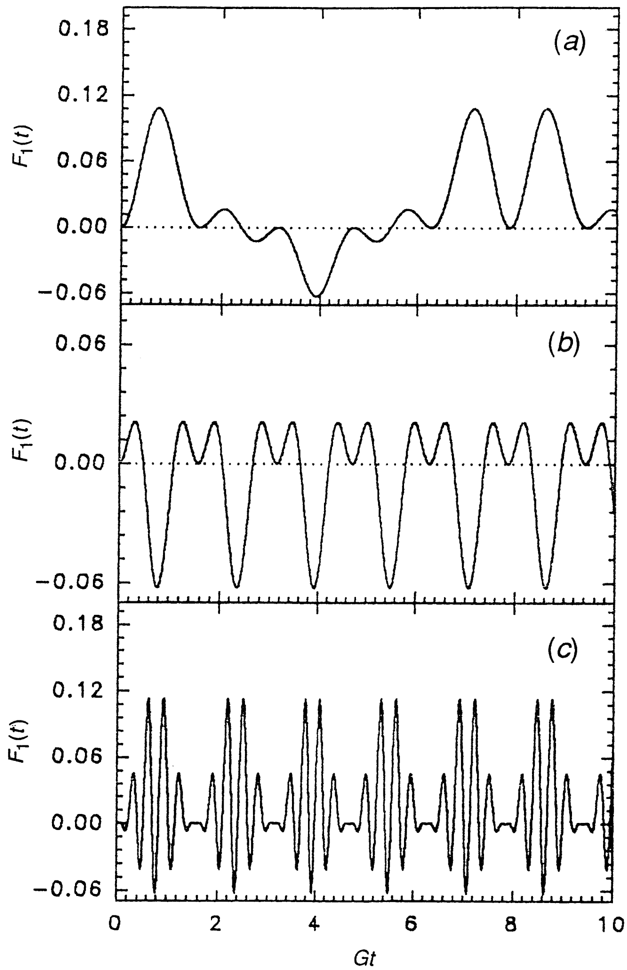

Fig. 2. Time evolution of F_1 with the RWA for $\mid f_2 \mid = \frac{1}{2}$, $G = 0{\cdot}05$, $\delta\phi = \pi/2$, $\Delta = 0$, $V(N) = \sqrt{N}$ and (a) $\chi/G = \frac{1}{5}$; (b) $\chi/G = 1$; and (c) $\chi/G = 5$.

From the above analysis, we can see that $T_{\rm sque}$ and $T_{\rm R}$ are inversely proportional to χ, and the maximum squeeze amplitude $A_{\rm max}$ is related to χ, V and G as well as to $\mid f_2 \mid$. These reflect that the squeeze duration $T_{\rm sque}$, the squeeze revival period $T_{\rm R}$ and the maximum squeeze amplitude $A_{\rm max}$ all are influenced due to the nonlinear interaction of the field with the medium. In addition, the forms of intensity-dependent coupling between the atom and the field only influence the maximum squeeze amplitude $A_{\rm max}$. In Figs 2*a* and 2*c*, we show the time evolution of F_1 with $V(N) = \sqrt{N}$, $\mid f_2 \mid = \frac{1}{2}$ and $\chi/G = \frac{1}{5}$, 1, 5 (in this case, $E \approx 0{\cdot}4$). These plots clearly show periodical squeezing below the dotted line, where the squeeze duration and the squeeze revival period decrease with an increase of χ/G. In addition, we can see that $\sqrt{2}GV(2)/2 \mid \chi \mid$ in Fig. 2*a* and $2 \mid \chi \mid / \sqrt{2}GV(2)$ in Figs 2*b* and 2*c* are odd numbers, so $A_{\rm max} = 0{\cdot}0625$ appears at $t_{\rm max}$ (given by equation 49).

(3b) Dipole Squeezing without the RWA: Numerical Results

Not much work has been done on the influence of the virtual-photon processes on squeezing. In recent years Lais and Steimle (1990) have numerically shown, using a continue fraction method, that the counter-rotating wave terms suppress the squeezing of the radiation field. In this section, we want to examine numerically the role of the virtual-photon field on dipole squeezing.

Numerical method. Taking $\mid m,n\rangle$ as our basis, where $S_z \mid m,\, n\rangle = m \mid m,\, n\rangle$, $m = \pm\frac{1}{2}$ and $a^{+}a \mid m,\, n\rangle = n \mid m,\, n\rangle$ $(n = 0,1,2,...)$, we can obtain the eigenfunction $\mid \phi_i\rangle$ and the energy eigenvalue E_i $(i{=}1,2,...)$ of the effective Hamiltonian (2) by truncating the infinite matrix to finite order (Graham and Höhnerbach 1984), then taking the initial condition (27) we can obtain the expected value of the variable $S_l(l{=}x,\, y,\, z)$ at t as

$$\begin{aligned}\langle S_l\rangle &= \langle\psi(t) \mid S_l \mid \psi(t)\rangle \\ &= \sum_{i,j}\langle\psi(0) \mid \phi_i\rangle\langle\phi_i \mid S_l \mid \phi_j\rangle\langle\phi_j \mid \psi(0)\rangle \exp\left(-i(E_j - E_i)t\right). \end{aligned} \tag{54}$$

By substituting into equation (26), we can investigate the squeezing properties of atomic dipole variables as follows. The efficiency of the numerical method used in this section is adequate, since the RWA corresponds to a perturbation treatment to first order and the cases considered in the following satisfy the RWA condition (Graham and Höhnerbach 1984) (i.e. $\mid \omega - 2\Omega \mid \ll \omega$ and $G \ll \omega$).

It should be stressed that an analytical explanation of the physical origin of our numerical results is still lacking but will appear in a future paper. In the following, we only describe our numerical results (we have only considered the resonance case $\Delta = 0$ in our numerical work).

Numerical results. In Fig. 3 we plot the time evolution of F_1 for $\mid f_2 \mid = \frac{1}{2}$, $\chi = 0$, $\delta\phi = \pi/2$ and $r = 0, \frac{1}{2}$. It can be seen that periodical revival squeezing appears; however, comparing these results with those in the RWA (see Figs 1*a* and 1*b* for $\mid f_2 \mid = \frac{1}{2}$ and $r = 0$, $\frac{1}{2}$, respectively), we found that the interference between the real-photon and virtual-photon processes suppresses the squeezing of some time regions predicted in the RWA and leads to an increased revival squeeze period. In addition, as in the RWA, we can see from Fig. 3 that the squeeze revival period decreases with an increase in r.

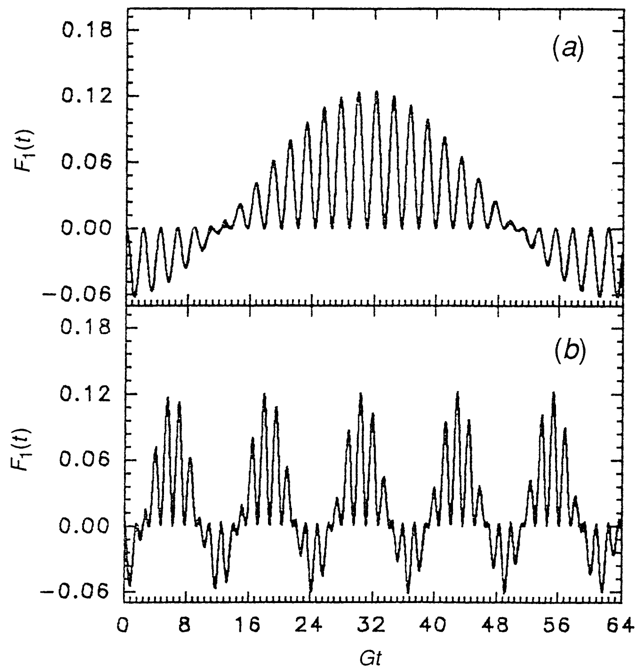

Fig. 3. Time evolution of F_1 without the RWA with $| f_2 | = \frac{1}{2}$, $G = 0{\cdot}05$, $\delta\phi = \pi/2$, $\Delta = 0$, $\chi = 0$ and $V(N) = N^r$ for (a) $r = 0$; and (b) $r = \frac{1}{2}$.

Finally, we plot the time evolution of F_1 in Fig. 4 for $| f_2 | = \frac{1}{2}$, $\delta\phi = \pi/2$ and $\chi/G = 0{\cdot}01,\ 0{\cdot}02,\ 0{\cdot}04$. We can see that the suppressed squeezing in Fig. 3a is gradually revived with an increase of χ. These numerical results show that the destructive effect of the counter-rotating wave terms on the squeezing may be weakened due to the presence of the nonlinear Kerr-like medium. An analytical explanation for these results would be very interesting.

(3c) Conclusions

If the field is initially prepared as a superposition of the vacuum and two-photon state and the atom is initially in the ground state, we reach the following conclusions: (i) when $\chi = 0$, $\Delta = 0$ and $\delta\phi = \pi/2$, we have shown that dipole squeezing is observed in the cases with $\sqrt{2}/2 < | f_2 | < 1$ or $0 < | f_2 | < \sqrt{2}/2$, where the forms of arbitrary intensity-dependent coupling between the atom and the cavity mode only influence the squeeze duration $T_{\rm sque}$, the squeeze revival period $T_{\rm R}$ and the time $t_{\rm max}$ of appearance of the maximum squeeze amplitude $A_{\rm max}$; (ii) dipole squeezing is phase insensitive; (iii) taking $\chi = -\Delta/10$, $\delta\phi = \pi/2$ and $0 < | f_2 | < \sqrt{2}/2$, we have shown that $T_{\rm sque}$, $T_{\rm R}$ and $A_{\rm max}$ all are influenced by the nonlinear coupling of the field to the medium, while the forms of arbitrary intensity-dependent coupling of the field with the atom only influence $A_{\max}$; and (iv) the counter-rotating wave terms suppress dipole squeezing predicted in the RWA and lead to an increased squeeze revival period—this suppressed effect may be weakened by taking into account the nonlinear Kerr-like medium.

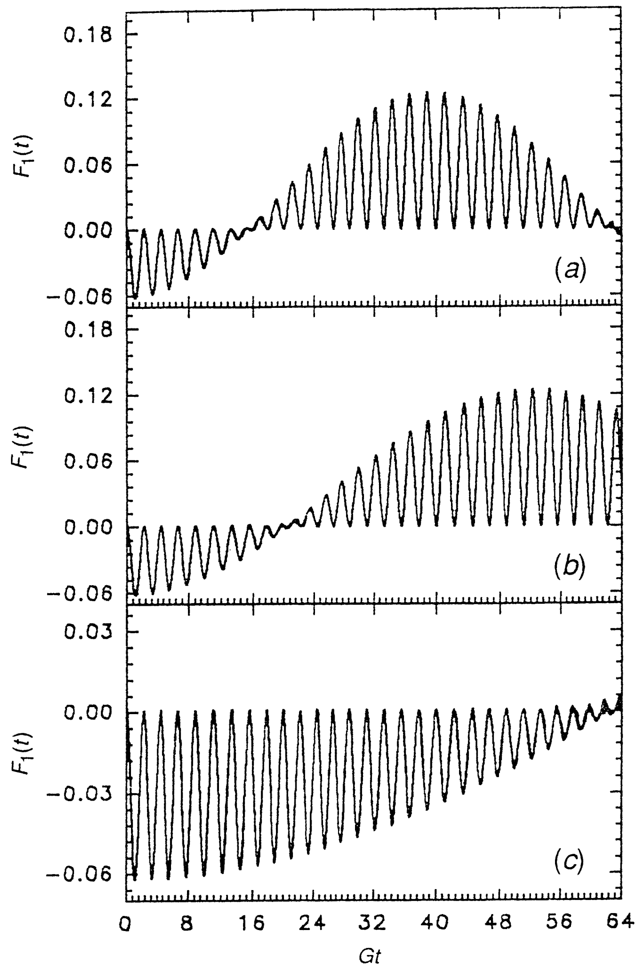

Fig. 4. Time evolution of F_1 without the RWA for $\mid f_2 \mid = \frac{1}{2}$, $G = 0{\cdot}05$, $\delta\phi = \pi/2$, $\Delta = 0$ and (*a*) $\chi/G = 0{\cdot}01$; (*b*) $\chi/G = 0{\cdot}02$; and (*c*) $\chi/G = 0{\cdot}04$.

4. Squeezing of Light in the RWA

Recently Buzek (1989*a*) has studied light squeezing in the intensity-dependent-coupling JCM (Buck and Sukumar 1981). In this section we shall investigate light squeezing in arbitrary intensity-dependent-coupling JCM described by the effective Hamiltonian (7). Meanwhile, we also wish to indicate the relation between the field and atomic squeezing.

In order to investigate the squeezing properties of the light, two quadrature field operators a_1 and a_2 are defined as (Meystre and Zubairy 1982)

$$a_1 = \tfrac{1}{2}(a + a^+); \qquad a_2 = \frac{1}{2i}(a - a^+). \tag{55}$$

They obey the commutation relation

$$[a_1, a_2] = i\tfrac{1}{2}. \tag{56}$$

The Heisenberg uncertainty relation is

$$\langle(\Delta a_1)^2\rangle\langle(\Delta a_2)^2\rangle \geq \tfrac{1}{16}, \tag{57}$$

where

$$\langle(\Delta a_{\mathrm{i}})^2\rangle = \langle a_{\mathrm{i}}^2\rangle - \langle a_{\mathrm{i}}\rangle^2 \qquad (i = 1, 2). \tag{58}$$

It is convenient to define two functions

$$h_1 = \langle(\Delta a_1)^2\rangle - \tfrac{1}{4}; \qquad h_2 = \langle(\Delta a_2)^2\rangle - \tfrac{1}{4}. \tag{59}$$

The squeezing condition is $h_i < 0$ ($i = 1$ or 2).

Taking the initial condition (27) and using (17), we obtain the time evolution of h_1 and h_2 as (here $\chi = 0$)

$$\begin{aligned} h_1(t) &= |f_2|^2 \left(\cos^2(\omega_2 t) + (\tfrac{1}{2}\Delta)^2 \frac{\sin^2(\omega_2 t)}{\omega_2^2}\right) + \frac{\sqrt{2}}{2} |f_0||f_2| \\ &\quad \times \left(\cos(\omega_2 t)\cos(\tfrac{1}{2}\Delta t + \delta\phi) + \frac{\Delta \sin(\omega_2 t)}{2\omega_2} \sin(\tfrac{1}{2}\Delta t + \delta\phi)\right), \end{aligned} \tag{60}$$

$$\begin{aligned} h_2(t) &= |f_2|^2 \left(\cos^2(\omega_2 t) + (\tfrac{1}{2}\Delta)^2 \frac{\sin^2(\omega_2 t)}{\omega_2^2}\right) - \frac{\sqrt{2}}{2} |f_0||f_2| \\ &\quad \times \left(\cos(\omega_2 t)\cos(\tfrac{1}{2}\Delta t + \delta\phi) + \frac{\Delta \sin(\omega_2 t)}{2\omega_2} \sin(\tfrac{1}{2}\Delta t + \delta\phi)\right), \end{aligned} \tag{61}$$

with

$$\omega_2 = \sqrt{(\tfrac{1}{2}\Delta)^2 + 2V^2(2)G^2} \tag{62}$$

and $|f_0|^2 + |f_2|^2 = 1$. Here $\delta\phi$ is the relative phase between the vacuum and two-photon state.

Now we study the squeezing properties of the field. When the field is initially in the vacuum state (i.e. $f_0 = 1$) or two-photon state (i.e. $f_2 = 1$), it can be seen from (60) and (61) that a_1 or a_2 cannot be squeezed. Does the squeezing exist when the field is initially in the coherent superposition state? In the following we shall take h_1 as an example and answer the question. For simplicity, we assume the field to be resonant with the atomic frequency (i.e. $\Delta = 0$) and take $\delta\phi = 0$. Clearly, when $0 < |f_2| < \sqrt{3}/3$, light squeezing can be observed during the time (i.e. $h_1(t) < 0$)

$$\frac{2k\pi + \pi/2}{\sqrt{2}V(2)G} < t < \frac{2k\pi + 3\pi/2}{\sqrt{2}V(2)G}, \tag{63}$$

where the squeeze duration is

$$T_{\text{sque}} = \frac{\pi}{\sqrt{2}V(2)G}, \tag{64}$$

the squeeze revival period is

$$T_{\text{R}} = \frac{2\pi}{\sqrt{2}V(2)G}, \tag{65}$$

and

$$A_{\max} = | f_2 |^2 \left(\frac{\sqrt{2} | f_0 |}{2 | f_2 |} - 1 \right) \tag{66}$$

appears at $t_{\max}$ which satisfies $\cos(\sqrt{2}V(2)Gt_{\max}) = -1$.

When $\sqrt{3}/3 < | f_2 | < 1$, light squeezing can be observed during (i.e. $h_1(t) < 0$)

$$\frac{2k\pi + \pi/2}{\sqrt{2}V(2)G} < t < \frac{2k\pi + C_L}{\sqrt{2}V(2)G}, \tag{67}$$

or

$$\frac{2k\pi + 2\pi - C_L}{\sqrt{2}V(2)G} < t < \frac{2k\pi + 3\pi/2}{\sqrt{2}V(2)G}, \tag{68}$$

where

$$C_L = \arccos\left(-\frac{\sqrt{2} | f_0 |}{2 | f_2 |} \right). \tag{69}$$

It can be seen that

$$T_{\text{sque}} = \frac{C_L - \pi/2}{\sqrt{2}V(2)G}, \tag{70}$$

$$T_{\text{R}} = \frac{2\pi}{\sqrt{2}V(2)G}. \tag{71}$$

We can show that

$$A_{\max} = | h_1 < 0 |_{\max} = -\frac{| f_0 |^2}{8} \tag{72}$$

appears at $t_{\max}$ which satisfies $\cos(\sqrt{2}V(2)Gt_{\max}) = -\sqrt{2} | f_0 | /4 | f_2 |$.

From the above analysis, we can see that when $\delta\phi = 0$ and $\Delta = 0$, light squeezing appears for $0 < \mid f_2 \mid < 1$, where $T_{\rm sque}$, $T_{\rm R}$ and $t_{\rm max}$ are inversely proportional to $V(2)G$, and $A_{\rm max}$ is only related to $\mid f_2 \mid$. These mean that only $T_{\rm sque}$, $T_{\rm R}$ and the time $t_{\rm max}$ of appearance of $A_{\rm max}$ are influenced by the forms of intensity-dependent coupling between the atom and cavity mode. As an example, we assume $V(N) = N^r (r \geq 0)$, and plot the time evolution of h_1 in Fig. 5 for $r = 0$, $\frac{1}{2}$ and $\mid f_2 \mid = \frac{1}{10}, \sqrt{3}/3, \frac{1}{2}$. It can be seen from these plots that $T_{\rm sque}$, $T_{\rm R}$ and $t_{\rm max}$ decrease with an increase in r, and $A_{\rm max}$ is not influenced by the forms of intensity-dependent coupling.

When $\Delta = \delta\phi = \chi = 0$, as addressed above, the fluctuations in S_y are squeezed for $0 < \mid f_2 \mid < \sqrt{2}/2$ or $\sqrt{2}/2 < \mid f_2 \mid < 1$. Surely, for these parameters, light squeezing appears. Moreover, these show the relation between the field and atomic squeezing: the squeezed atom can radiate the squeezed light.

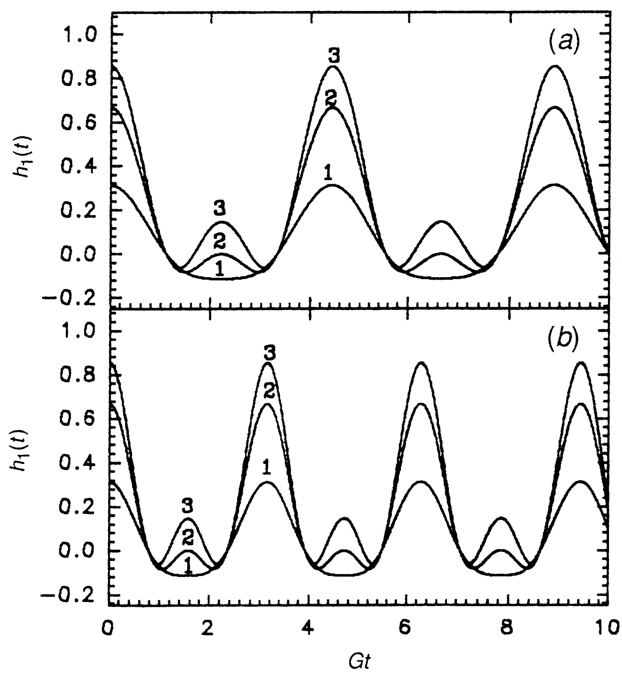

Fig. 5. Time evolution of h_1 with the RWA for $G = 0{\cdot}05$, $\delta\phi = 0$, $\Delta = 0$, $\chi = 0$ and $V(N) = N^r$: (*a*) $r = 0$; (*b*) $r = \frac{1}{2}$, where curve 1 is for $\mid f_2 \mid = \frac{1}{10}$, curve 2 for $\mid f_2 \mid = \sqrt{3}/3$ and curve 3 for $\mid f_2 \mid = \frac{1}{2}$.

5. Summary

In this paper we have dealt with a generalisation of the two-photon JCM, where arbitrary intensity-dependent coupling between the atom and field has been taken into account and a nonlinear Kerr-like medium has been introduced. The general time-dependent state of the system within the RWA has been analytically obtained when the system is initially restricted to the coherent superposition state of the atom and that of the photon number state. We have analytically investigated the role of the nonlinear interaction of the cavity mode with Kerr-like medium and the arbitrary intensity-dependent nonlinear coupling of the field to the atom on squeezing of the field and atom in the system with the RWA. Also, we have numerically examined the influence of the counter-rotating wave terms on dipole squeezing predicted in the RWA.

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References

Agarwal, G. S., and Puri, R. R. (1989). *Phys. Rev.* A **39**, 2969.
Brune, M., Raimond, J. M., Goy, P., Davidovich, L., and Haroche, S. (1987). *Phys. Rev. Lett.* **59**, 1899.
Buck, B., and Sukumar, C. V. (1981). *Phys. Lett.* A **81**, 132.
Buzek, V. (1989*a*). *Phys. Rev.* A **39**, 5432.
Buzek, V. (1989*b*). *Phys. Rev.* A **39**, 3196.
Buzek, V., and Jex, I. (1990). *Opt. Commun.* **78**, 425.
Compagno, G., Palma, G. M., Passante, R., and Persico, F. (1989). *Europ. Phys. Lett.* **9**, 215.
Compagno, G., Passante, R., and Persico, F. (1983). *Phys. Lett.* A **98**, 253.
Graham, R., and Höhnerbach, M. (1984). *Phys. Lett.* A **101**, 61.
Hu, G., and Aravind, P. K. (1989). *J. Opt. Soc. Am.* B **6**, 1757.
Jaynes, E. T., and Cummings, F. W. (1963). *Proc. IEEE* **51**, 126.
Lais, P., and Steimle, T. (1990). *Opt. Commun.* **78**, 346.
Meystre, P., and Zubairy, M. S. (1982). *Phys. Lett.* A **89**, 390.
Scully, M. O., *et al.* (1988). *Phys. Rev. Lett.* **60**, 1832.
Slusher, R. E., Hollberg, W., Yurke, B., Mertz, J. C., and Valley, J. F. (1985). *Phys. Rev. Lett.* **55**, 2409.
Sukumar, C. V., and Buck, B. (1981). *Phys. Lett.* A **83**, 211.
Walls, D. F., and Zoller, P. (1981). *Phys. Rev. Lett.* **47**, 709.
Wodkiewicz, K., Knight, P. L., Buckle, S. J., and Barnett, S. M. (1987). *Phys. Rev.* A **35**, 2567.
Xie, R. H., Xu, G. O., and Liu, D. H. (1995). *Phys. Lett.* A **202**, 28.
Yuen, H. P., and Shapiro, J. H. (1978). *IEEE Trans. Inf. Theory* **IT 24**, 657.
Zaheer, K., and Zubairy, M. S. (1988). *Phys. Rev.* A **37**, 1628.
Zhou, P., and Peng, J. S. (1991). *Phys. Rev.* A **44**, 3331.

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