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Progress in Measuring the Hubble Constant*

Jeremy Mould

Mount Stromlo and Siding Spring Observatories, Institute of Advanced Studies, Australian National University, Canberra, ACT 0200, Australia.

Abstract

The Hubble Space Telescope is breaking a long-standing impasse in physical cosmology. The distances of galaxies sufficiently remote for their random velocities to be negligible can now be measured in two steps, the first using Cepheid variable stars as standard candles to approximately 20 Mpc, the second using a variety of secondary distance indicators to distances 10 times larger. The present key project on the Hubble Constant aims to measure H_0 to 10%. Current results with approximately 20% uncertainty suggest that cosmologists will be offered a dilemma: an open Universe or a vacuum energy dominated Universe.

1. Hubble Constant

The Hubble Constant is the fundamental parameter that tells us to first order how the scale of the Universe changes:

$$H^2 = (\dot{a}/a)^2 = 8\pi G\rho/3 \pm K/a^2 + \Lambda/3\,.$$

The variables in this equation have their conventional meanings (Peebles 1993) with a the scale factor, K the curvature parameter and Λ the cosmological constant. The kinetic energy in the expansion is compared with the gravitational potential energy, and so the Hubble Constant is important in defining the critical density (and also the baryonic density of the Universe),

$$\Omega_0 = 8\pi G\rho/3H_0^2\,.$$

The Hubble Constant also specifies the expansion time scale

$$t = 1/H_0\,.$$

Together with an age constraint from another astrophysical 'clock', this allows the second of the cosmological parameters to be determined, e.g. Ω or Λ.

2. The Hubble Space Telescope

Measuring the Hubble Constant to 10% accuracy was declared a Key Project for HST in 1984 (see Aaronson and Mould 1986). The first three years of HST

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enabled the Key Project Team to resolve variable stars and measure the distances of the relatively nearby galaxies M81 and M100, but full implementation of the program had to await the First Hubble Servicing Mission in December 1993.

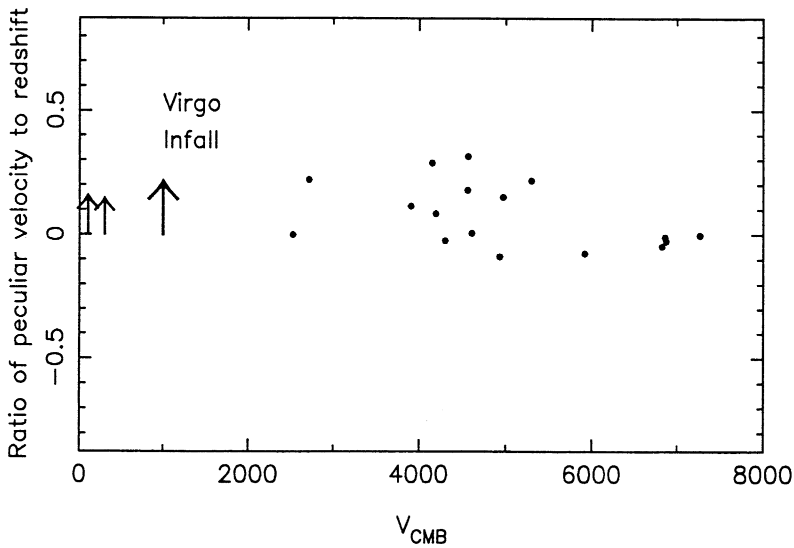

Fig. 1. Solid symbols show peculiar velocities of clusters of galaxies from Table 5 of Mould *et al.* (1993). The arrows are local velocity perturbations scaled to the cosmological redshift of the Virgo cluster. At large distances velocity perturbations do not present a problem for the determination of H_0.

From the discovery of Cepheid variable stars in the galaxy M100 Freedman *et al.* (1994) have determined the distance of the Virgo cluster. The Virgo cluster's recession velocity is of the same order as the dipole anisotropy of the microwave background (Smoot *et al.* 1991). Mould *et al.* (1995) employ measurements of distance ratios of objects with larger recession velocities to constrain the Hubble Constant in the following six ways:

(1) From the recession velocity of Virgo: $H_0 = 81 \pm 11$ km/s/Mpc.

Fig. 1 illustrates one of the significant problems in the determination of the Hubble Constant: the existence of peculiar velocities of galaxies of the order of hundreds of $\mathrm{km\,s^{-1}}$. These peculiar velocities are motions unrelated to the expansion of the Universe. If the Hubble Constant can be measured as the asymptotic ratio of recession velocity to distance at large redshift, the effect of these peculiar velocities can be minimised. Fig. 1 shows that the ratio of peculiar velocity to distance does indeed decline with increasing distance. Local velocity perturbations represent significant corrections to the redshift/distance ratio of the Virgo cluster, however. These perturbations include the velocity of the Sun with respect to the Galactic Centre, the velocity of the Milky Way with respect to the Local Group and the velocity of the Local Group with respect to the Virgo cluster in the comoving frame. These are shown as arrows in Fig. 1 scaled to the cosmological redshift of Virgo. The need to determine these quantities accurately is a limitation in the measurement of H_0 by this technique (Sandage and Tammann 1995; Aaronson and Mould 1986).

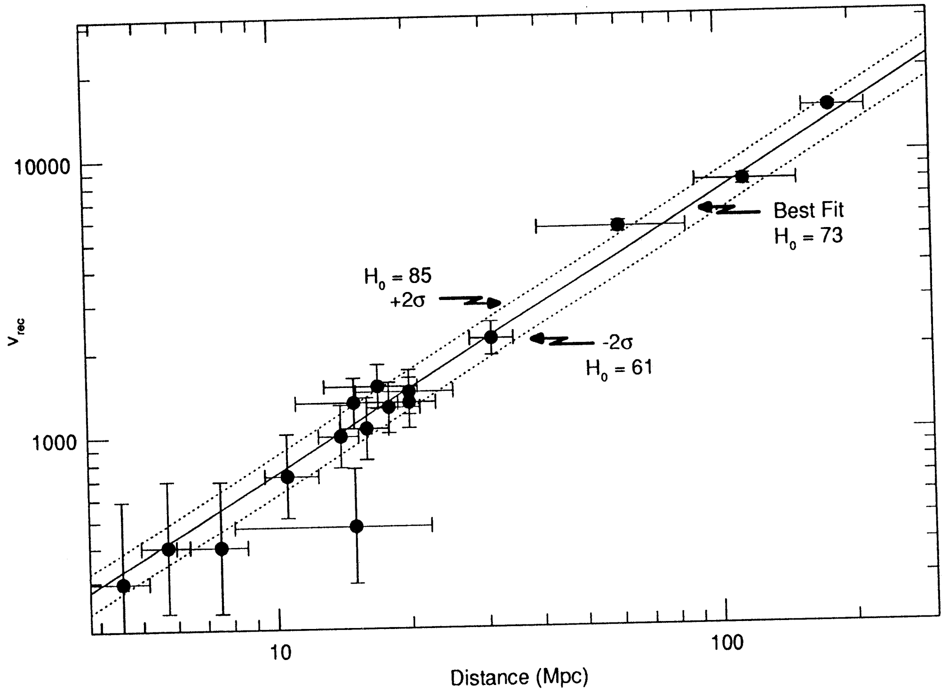

Fig. 2. Hubble flow from EPM distances (Schmidt *et al.* 1994).

(2) From the expanding photospheres of type II supernovae:* $H_0 = 73 \pm 11$ km/sec/Mpc.

The expanding photospheres method (EPM) allows distances to galaxies to be determined from the inferred radii of the photospheres of exploding massive stars. The distance information comes fundamentally from integration of their radial velocities of expansion and requires careful spectroscopic monitoring of the supernovae and detailed modelling of the radiative transfer in the expanding envelope. Fig. 2 shows the current state of the data. The distance of SN1979C is confirmed by the recent HST Cepheid results.

(3) From 'standard candle' type Ia supernovae:† $H_0 = 71 \pm 10$ km/sec/Mpc.

Type I supernovae are believed to be carbon deflagration events in massive degenerate stars. Their reliability as 'standard candles' has been extensively debated but, if we study the database of type Ia events by Tammann and Sandage (1995), we see a well-defined relation (Fig. 3) between recession velocity and distance. Distances have been calculated by assuming that all events have the same maximum luminosity in visible light as the mean Virgo cluster SNIa with the HST Cepheid distance of Virgo (Mould *et al.* 1995).

(4) From the Tully–Fisher relation for clusters: $H_0 = 82 \pm 11$ km/sec/Mpc.

The luminosities and rotation velocities of spiral galaxies are correlated for dynamical reasons (Tully and Fisher 1977; Aaronson *et al.* 1979). Mould *et al.*

* This result (Schmidt *et al.* 1994) is independent of the assumption that M100 is a member of the Virgo cluster.

† Mould *et al.* (1995) have shown this for galaxies with redshifts beyond 4000 km s^{-1}.

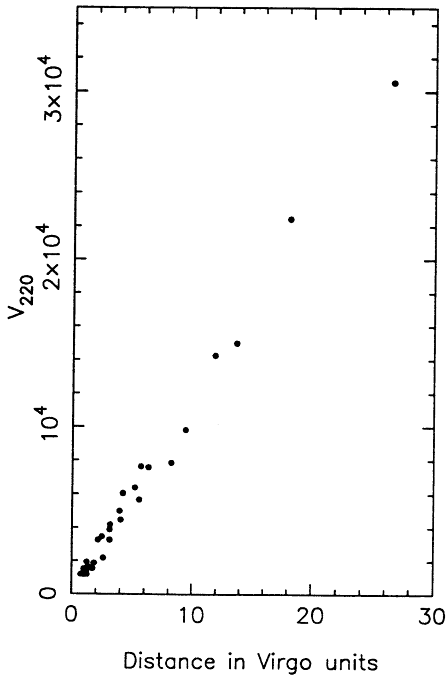

Fig. 3. Hubble flow as seen in the distances of type Ia supernovae. Recession velocities have been corrected for a Virgocentric flow model with an amplitude of 220 km s^{-1} locally.

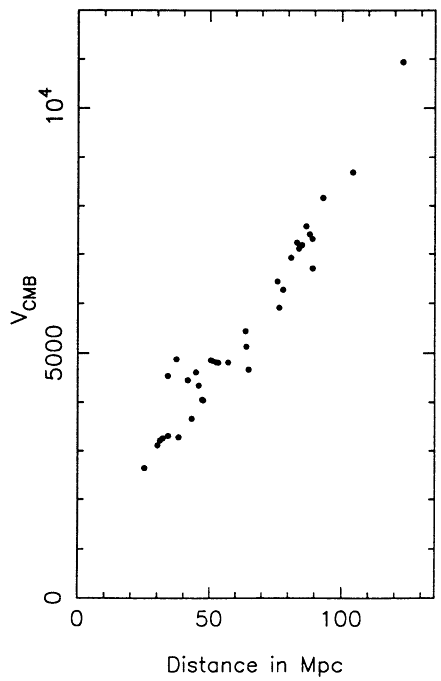

Fig. 4. A plot of the velocities of clusters of galaxies in the frame in which the microwave dipole anisotropy vanishes, against the Tully–Fisher distances of these clusters.

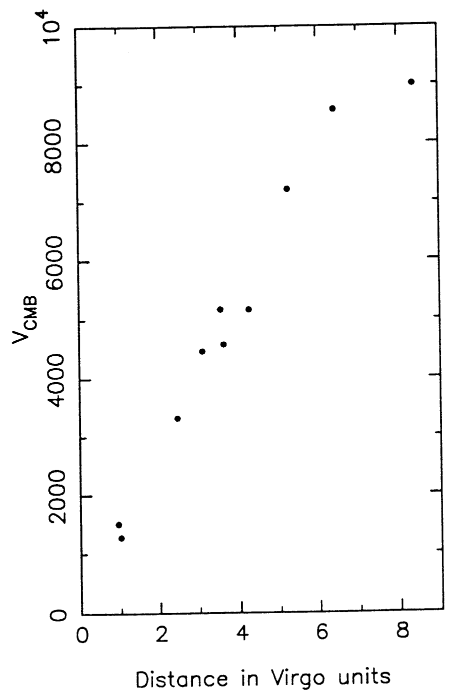

Fig. 5. All groups of galaxies studied by the Seven Samurai (Faber *et al.* 1985) with more than five observed members are shown in a plot of recession velocity versus distance.

(1995) recalibrated the Tully–Fisher relation using the HST Cepheid distance of Virgo. This is shown in Fig. 4. The cluster data are combined from Aaronson *et al.* (1985, 1989), Han and Mould (1992) and Mould *et al.* (1991, 1993).

(5) From the distance of the Coma cluster: $H_0 = 76 \pm 10$ km/sec/Mpc.

The diameters and velocity dispersions of elliptical galaxies are correlated (Faber *et al.* 1985). Fig. 5 shows the relation between recession velocity in the reference frame of the cosmic microwave background radiation and the distance of groups of elliptical galaxies in units of the distance of one of those groups, the Virgo cluster. The Coma cluster is the point approximately five times as far away as Virgo.

(6) From surface brightness fluctuations of galaxies: $H_0 = 84 \pm 16$ km/sec/Mpc.

Tonry (1991) has pointed out a very simple relation between the resolvability of galaxies and their distances.

It is easy to be misled by the rather good consistency of these measurements. The uncertainty in H_0 is dominated by systematic errors in distance, and the full uncertainty of the distance of Virgo affects each of the estimates of H_0 listed above. The principal uncertainty at this stage in the project is due to the very extended nature of the Virgo cluster. Measuring one galaxy does not strongly constrain the cluster centroid. Further work is required to reduce the uncertainty in H_0 to 10%.

3. Consequences

Our new results do put rather useful limits on the Hubble Constant, however. Mould *et al.* (1995) have shown that $50 < H < 100$ with 95% confidence. Sandage and Tammann (1995) find $H_0 = 55 \pm 2$ km/sec/Mpc, which lies just within these bounds.

According to Carroll *et al.* (1992), the relationship between the age of the Universe and H_0 is

$$t_0 \approx \tfrac{2}{3} H_0^{-1} \frac{\sinh^{-1}(\sqrt{|1 - \Omega_a|/\Omega_a})}{\sqrt{|1 - \Omega_a|}},$$

where

$$\Omega_a = \Omega_M - 0{\cdot}3\Omega_{tot} + 0{\cdot}3,$$

Ω_M being the matter density of the Universe and Ω_{tot} being the total density, including the vacuum density associated with a cosmological constant, which we can denote as Ω_Λ. These quantities are expressed in units of the critical density.

A range of models is consistent with the new results. We assume $t_0 = 15 \pm 2$ Gyr based on the ages of globular clusters (VandenBerg 1991):

(1) For $\Omega_{tot} = 1$, $\Omega_M = 0{\cdot}1 \pm 0{\cdot}1$ and $\Omega_\Lambda = 0{\cdot}9 \pm 0{\cdot}1$; such a Universe is vacuum energy dominated with a trace of matter.

(2) An open Universe $\Omega_{tot} \approx \Omega_{baryon} = 0{\cdot}02$ (Steigman and Tosi 1992) is not excluded with the present 20% uncertainty in H_0 and 13% uncertainty in t_0.

(3) Since $H_0 > 50$ km/sec/Mpc, then

$$H_0 t_0 > \tfrac{15}{20} > \tfrac{2}{3}.$$

For $\Omega_{tot} = 1$, $\Lambda = 0$ is excluded at some level which, because of the t_0 uncertainty, falls short of the 95% confidence level. But the elegant Einstein–de Sitter model is now an unlikely cosmological solution (see also Hogan 1994).

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