Aust. J. Phys., 1996, **49**, 613–22

Physical Implications of the Gravitational Fluxoid

M. Agop,[A,C] *C. Ciubotariu,*[A] *C. Gh. Buzea,*[B] *C. Buzea*[B] *and B. Ciobanu*[A]

[A] Department of Physics, Technical 'Gh. Asachi' University, Iasi 6600, România.
[B] Institute of Technical Physics, B-dul D. Mangeron, Iasi 6600, România.
[C] Mailing address: Ph.D. Agop Maricel, Str. Moara de Foc nr. 10, bl. 406, et VIII, ap. 35, Iasi 6600, România.

Abstract

A physical model which considers a U_1 gauge invariant Lagrangian with spontaneous symmetry-breaking and minimally coupled to a gravitoelectromagnetic field is proposed. Its topological charge is identified with the gravitational fluxoid. In this paper we analyse the spin-gravitomagnetic field interaction and the kinetic moment-gravitomagnetic field interaction.

1. Introduction

The concept of a gravitational field (or at least some components of the gravitational field) analogous to the electromagnetic field is supported by the following observations (Ciubotariu 1991). For the case of weak fields:

(i) Newton's law is analogous to Coulomb's law.
(ii) The linearised Einstein equation has the same form as Maxwell's equations (Balasz and Bertotti 1963; Peng 1983, 1990).
(iii) The geodesic equation has the same form as the Lorentz equation of motion (Fuchs 1981).

For the case of strong fields:

(i) The autofocusing of gravitational waves (Ferrari 1988*a*, 1988*b*) is similar to that of laser radiation in nonlinear optics (Askar'Jan 1962).
(ii) The gravitational field of axially symmetric and reflection-symmetric systems, in the near-field approximation, has a structure very similar to the electric-type solutions of electromagnetic theory (Morgan 1971).
(iii) There exists a gravitational analogue of the electromagnetic Faraday rotation (Piran and Safiev 1985).

Recently, Peng (1983, 1990) discussed a set of Maxwell-like equations that arise in the slow-motion $v/c \ll 1$ weak-field limit of Einstein's field equations. In this case, the equations of general relativity can be written in terms of separate space–time coordinates, and one can introduce the gravitoelectric (E_{g})

0004-9506/96/030613$05.00

and gravitomagnetic (B_g) fields (Ciubotariu *et al.* 1993). The set of governing equations may be called gravitoelectromagnetic and are written as

$$\nabla \times \boldsymbol{B}_g = -4\pi \boldsymbol{j}_m + \frac{\partial \boldsymbol{E}_g}{\partial t}, \tag{1}$$

$$\nabla \times \boldsymbol{E}_g = -\frac{\partial \boldsymbol{B}_g}{\partial t}, \tag{2}$$

$$\nabla \cdot \boldsymbol{E}_g = -4\pi\rho_m, \tag{3}$$

$$\nabla \cdot \boldsymbol{B}_g = 0, \qquad \boldsymbol{B}_g = \nabla \times \boldsymbol{A}_g, \tag{4}$$

where $\boldsymbol{j}_m = \rho_m \boldsymbol{v}$ is the mass current density, $\rho_m = nm$ is the proper mass density, with n the number density of particles of rest mass m, and $\boldsymbol{A}_g$ is the gravitomagnetic vector potential.

For instance, the gravitomagnetic field induced by the rotation of the Earth, at a geographical latitude of 45°, is given by (Ljubicic and Logan 1992)

$$\boldsymbol{B}_g = \frac{2GI}{cR^3}\left(\frac{3(\omega \cdot \boldsymbol{R})\boldsymbol{R}}{R^2} - \omega\right) \approx 10^{-14}\ \mathrm{s}^{-1}, \tag{5}$$

where G is the gravitational constant, R is the radius of the Earth, I is the moment of inertia of the Earth about an axis through its centre and ω is the angular velocity associated with the Earth's rotation.

Since in this approximation the mass is the unique source of the gravitational field, we have

$$T^{\alpha,\beta} = \rho_m u^\alpha u^\beta, \quad T^{00} \approx \rho_m c^2, \quad T^{0i} \approx \rho_m v^i = j^i_m, \quad T^{ij} \approx 0, \tag{6}$$

$$T^\alpha = T^{0\alpha} = (\rho_m, j^i_m); \quad \alpha, \beta, \ldots = 0, 1, 2, 3; \quad i, j, \ldots = 1, 2, 3,$$

$$\frac{\partial \rho_m}{\partial t} \nabla \cdot \boldsymbol{j}_m = 0 \quad \text{(equation of continuity)} \tag{7}$$

and the equation of motion are geodesics, i.e.

$$u^\beta \nabla_\beta u^\alpha = \frac{\mathrm{d}^2 x^\alpha}{\mathrm{d}\tau^2} + \left\{ \begin{matrix} \alpha \\ \beta\gamma \end{matrix} \right\} \frac{\mathrm{d}x^\beta}{\mathrm{d}\tau}\frac{\mathrm{d}x^\gamma}{\mathrm{d}\tau} = 0. \tag{8}$$

This is analogous to the equation for the Lorentz force acting on an electrical charged particle, that is, (8) is of the form (Peng 1983; Ciubotariu 1991)

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \boldsymbol{E}_g + 4\,\boldsymbol{v} \times \boldsymbol{B}_g. \tag{9}$$

Now, if we choose $h_{\alpha\beta}$ of the form (Ho and Morgan 1994)

$$h_{00} = -\frac{V}{c^2}; \quad h_{ij} = -\delta_{ij}\,\frac{V}{c^2}; \quad h_{0j} = h_{j0} = \frac{A_{gj}}{c} \quad (i,\,j = 1,\,2,\,3)\,, \tag{10}$$

so that

$$2\boldsymbol{E}_{\mathrm{g}} = -\nabla V - \frac{\partial \boldsymbol{A}_{\mathrm{g}}}{\partial t}; \qquad \boldsymbol{B}_g = \nabla \times \boldsymbol{A}_{\mathrm{g}}\,, \tag{11}$$

where $h_{\alpha\beta}$ is a small perturbation to the metric term $g_{\alpha\beta}$

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}; \qquad |h_{\alpha\beta}| \ll 1 \tag{12}$$

and $\eta_{\alpha\beta}$ is the Minkowski metric, then the gravitoelectromagnetic 4-vector (Ho and Morgan 1994) can be introduced:

$$A_\mu = \left(-\frac{V}{c},\, \boldsymbol{A}_{\mathrm{g}} \right). \tag{13}$$

In the present paper we propose a physical model which considers a U_1 gauge invariant Lagrangian with spontaneous symmetry-breaking minimally coupled to a gravitoelectromagnetic field. The topological charge is identified with the gravitational fluxoid and in this context some physical effects are studied.

2. Mathematical Model

We consider the local U_1 gauge invariant Lagrangian L with spontaneous symmetry-breaking:

$$L = \tfrac{1}{2}(\nabla_\mu \phi)^*(\nabla^\mu \phi) - \tfrac{1}{4} f\left(\phi^*\phi - \frac{\lambda^2}{f}\right)^2 - \tfrac{1}{4} F_{\mu\nu}\,F^{\mu\nu}\,, \tag{14}$$

where $\nabla_\mu = \partial_\mu - i(4m/\hbar)A_\mu$ is the covariant derivative, A_μ is the gravitoelectromagnetic 4-vector field, ϕ is the Higgs field coupled minimally to the gravitoelectromagnetic field, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the gravitoelectromagnetic tensor field, $4m/\hbar$ is the coupling constant, $\hbar$ is the reduced Planck constant and $f > 0$ is the self-interaction constant.

Using the Euler–Lagange equations for the Lagrangian (14), the field equations are

$$\partial^\mu F_{\mu\nu} = 2i\,\frac{m}{\hbar}(\phi^*\partial_\mu \phi - \phi\partial_\mu \phi^*) + \frac{16m^2}{\hbar^2}\,A_\mu\,\phi^*\phi\,, \tag{15}$$

$$\nabla_\mu \nabla^\mu \phi = -f\phi\left(\phi\phi^* - \frac{\lambda^2}{f}\right). \tag{16}$$

These equations have the solution (Chaichian and Nelipa 1984)

$$F_{\mu\nu} = 0, \qquad \nabla_\mu \phi = 0, \qquad |\phi| = \lambda/\sqrt{f}, \tag{17}$$

which corresponds to the absolute minimum of the energy functional. The model gives rise to a continuous degenerate vacuum which is represented graphically as a circle of radius $R = \lambda/\sqrt{f}$ in the complex plane ϕ (Chaichian and Nelipa 1984). It is noted that all particles are massive, the mass of the 4-vector field being $(4\,m/\hbar)\lambda/\sqrt{f}$ and the mass of the scalar field is $\lambda\sqrt{2}$ (Chaichian and Nelipa 1984). There is no residual symmetry in the model.

One is not in a position to find analytical solutions to the field equations (15) and (16). We shall look for a solution with a symmetric form:

$$A_0 = 0, \qquad \boldsymbol{A}_\mathrm{g} = A(r)\,\boldsymbol{u}, \qquad \phi = R(r)\,\mathrm{e}^{in\theta}, \tag{18}$$

where

$$r^2 = x^2 + y^2 \tag{19}$$

and θ is the rotation angle (phase), $\boldsymbol{u}$ is the unit vector normal to the radius vector and n is an integer. With these conditions equations (15) and (16) become

$$-\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(rA)\right] + \left(\frac{16\,m^2}{\hbar^2}A - \frac{4nm}{r\hbar}\right)R^2 = 0\,, \tag{20}$$

$$-\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}R}{\mathrm{d}r}\right) + \left[\left(\frac{n}{r} - \frac{4m}{\hbar}A\right)^2 + f\left(R^2 - \frac{\lambda^2}{f}\right)\right]R = 0\,. \tag{21}$$

The two-dimensional soliton solutions are called vortices (Chaichian and Nelipa 1984).

A particular solution to equation (20) is the function

$$A = \frac{n\hbar}{4mr}\,. \tag{22}$$

Then equation (21) has the form

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}R}{\mathrm{d}R}\right) - (\lambda^2 - fR^2)R = 0\,. \tag{23}$$

Because $R(r)$ is a finite function for $r \to \infty$, the vacuum value ϕ_ν can be expressed, by virtue of (17), as

$$\phi_\nu = |\phi_\nu|\mathrm{e}^{in\theta}\,. \tag{24}$$

Combining (24) with (23) we find that

$$R(r) = \lambda/\sqrt{f}\,. \tag{25}$$

3. Topological Considerations

The model gives rise to a discrete homotopy group $\pi_1(S^1)$ with an infinite number of elements, so that this admits an infinite number of different soliton solutions (Chaichian and Nelipa 1984). The associated topological charge is (Chaichian and Nelipa 1984)

$$\phi_{\rm g} = \int_{R^2}\int \boldsymbol{B}(\boldsymbol{r})\,.\,{\rm d}\boldsymbol{S} = \int_{S^1=\partial R^2} \boldsymbol{A}\,.\,{\rm d}\boldsymbol{l} = \frac{n\hbar}{4m}\int_0^{2\pi} {\rm d}\theta = n\,\frac{\pi\hbar}{2m}\,, \tag{26}$$

where R^2 is the region bounded by S^1, the circumference of the circle of radius R. Equation (26) defines the gravitomagnetic (quantum) flux

$$\phi_{\rm g} = n\phi_{\rm g0}\,, \tag{27}$$

where

$$\phi_{\rm g0} = \frac{\pi\hbar}{2m} = \int_{S_0}\int \boldsymbol{B}\,.\,{\rm d}\boldsymbol{S} \tag{28}$$

is the gravitational flux quantum (gravitational fluxoid), $S_0 = 4\pi R_0^2$ is the elementary fluxoid surface and n is an integer identified with the number of flux quanta.

These results allow us to conjecture that vortices will appear in a superfluid placed in an external gravitomagnetic field. If the area of a vortex is $A = \pi R^2$, where R is the vortex radius, then the number of vortices per unit surface area is given by

$$N = \frac{1}{\pi R^2}\,. \tag{29}$$

On the other hand, from (26) we can express the vortex density as

$$N = \frac{2m}{n\pi\hbar}|\nabla \times \boldsymbol{A}|\,. \tag{30}$$

Now, substituting (30) into (29) we get

$$R = \left(n\frac{\hbar}{2m}\frac{1}{|\boldsymbol{B}_{\rm g}|}\right)^{\frac{1}{2}} = \sqrt{n}\,R_0\,, \tag{31}$$

where R_0 is the elementary fluxoid radius, given by

$$R_0 = \left(\frac{\hbar}{2m}\frac{1}{|\boldsymbol{B}_{\rm g}|}\right)^{\frac{1}{2}} \tag{32}$$

The result (31) indicates that the vortex radius is also quantised.

For a superfluid in which the generic particle has the mass $m_{\rm He} \approx 6\cdot 64\times 10^{-27}$ kg (helium), the radius of the elementary fluxoid in the terrestrial gravitomagnetic field is $R_0 \sim 10^3$ m, and for the electron–positron vacuum, for which the generic particle has a mass of $m_e \approx 10^{-31}$ kg, then $R_0 \sim 10^5$ m.

Evidently, in terrestrial laboratories such an experiment will be hard to realise. A pulsar (rotating neutron star) provides a better relativistic laboratory. Indeed, by evaluating the gravitomagnetic field of the pulsar NP0532 in the Crab Nebulae (period $T_p = 0 \cdot 033$ s, mass $M_p = 2 \times 10^{30}$ kg, radius $R_p = 10^4$ m) based on the relation $B_g \approx 2\pi G M_p / c^2 R_p T_p$ (Forder 1984), we find the value $B_g \approx 28 \cdot 2\ \mathrm{s}^{-1}$. Under these circumstances for an electron–positron vacuum, placed in the gravitomagnetic field of the pulsar, the elementary fluxoid radius becomes $R_0 \approx 4 \times 10^{-3}$ m. At the end of this section we discuss some topological considerations. We emphasise that at the moment we cannot assert that all the topological properties of the space–time associated with the linearised gravitational field equations are identical to those of a general Riemannian space–time corresponding to the nonlinear Einstein field equations. But since we use only the topological properties of the tangent space extended to a patch on curved space–time, the linearised theory provides a satisfactory description of the topological properties.

4. Interaction Spin Gravitomagnetic Field

Let σ be the spin operator for the electron of mass m. Then, the Dirac identity is (Titeica 1984)

$$H = \frac{1}{2m} (\hat{\sigma} \cdot \hat{p})^2 , \tag{33}$$

with $\hat{p}$ denoting the momentum operator; in the presence of the gravitomagnetic field this becomes

$$H' = \frac{1}{2m} (\hat{\sigma} \cdot \hat{P})^2 , \tag{34}$$

where

$$\hat{P}_a = \frac{\hbar}{\mathrm{i}} \nabla_\alpha = \partial_\alpha - 4m\boldsymbol{A}_g . \tag{35}$$

Since

$$\hat{\boldsymbol{P}} \times \hat{\boldsymbol{P}} = -\frac{\hbar}{\mathrm{i}} 4m\boldsymbol{B}_g , \tag{36}$$

equation (34) becomes

$$H' = \frac{1}{2m} \hat{\boldsymbol{P}} \cdot \hat{\boldsymbol{P}} - 2\hbar\hat{\sigma} \cdot \boldsymbol{B} . \tag{37}$$

We now define the spin gravitational current density via the expression (Titeica 1984)

$$\boldsymbol{j}_{gs} = 2\hbar \, \mathrm{curl}(\psi^* \hat{\sigma} \psi) , \tag{38}$$

where ψ is the microparticle wave function. From (38), we can obtain the expression for the gravitomagnetic moment (Titeica 1984)

$$\boldsymbol{\mu}_{gs} = \tfrac{1}{2}\int (\boldsymbol{r} \times \boldsymbol{j}_{gs})\,\mathrm{d}V = -\hbar \int [\boldsymbol{r} \times \mathrm{curl}(\psi^* \hat{\sigma}\psi)]\,\mathrm{d}V\,. \tag{39}$$

Integration over the volume V gives

$$\boldsymbol{\mu}_{gs} = -2\hbar \int (\psi^* \hat{\sigma}\psi)\,\mathrm{d}V\,. \tag{40}$$

Therefore, the spin gravitomagnetic moment is the mean value of the operator

$$\hat{\mu}_{gs} = -2\hbar\hat{\sigma}\,. \tag{41}$$

The elementary gravitomagnetic moment (gravitational magneton) is given by the quantity

$$\mu_{g0} = 2\hbar \tag{42}$$

or, taking into account the gravitational fluxoid, we can write

$$\mu_{g0} = \pi^{-1} 4m\phi_{g0}\,, \tag{43}$$

With these conditions, the second term in the Hamiltonian (37) refers to the interaction of the spin gravitomagnetic moment with the exterior field $\boldsymbol{B}_g$.

From (41) it is seen that a body placed in an external gravitomagnetic field (created by a spinning body) can be gravitationally 'magnetised'. This arises from the gravitationally induced alignment of the gravitomagnetic spin moments in the external gravitomagnetic field B_g.

This phenomenon may be observed in an Einstein–de Haas gravitational experiment. To this end let us imagine a body made up of a large number of isolated gravitomagnetic moments (gyroscopes), oriented parallel to each other and 'suspended' inside the body. If the body is suspended by a torsion thread, as in the Einstein–de Haas experiment, in an external gravitomagnetic field, then according to (41) the body must turn. The rotation angle α_g is computed by equating the rotational energy $W_r = C\alpha_g^2/2$, C being the torsion constant of the thread, with the gravitomagnetic energy

$$W_g = N_0\,|\hat{\mu}_{gs} \cdot \boldsymbol{B}_g|\,, \tag{44}$$

where N_0 is the total number of gyroscopes contained in the body. This yields

$$\alpha_g = \left(\frac{2N_0\,|\hat{\mu}_{gs} \cdot \boldsymbol{B}_g|}{C}\right)^{\frac{1}{2}} \tag{45}$$

The rotational angle α_g is large for bodies containing a large number of 'gyroscopes' placed in an intense external gravitomagnetic field. On a cosmic scale these conditions may be fulfilled, thus allowing us to conjecture that, on this scale, such bodies will rotate. From (41) and (45) we get

$$\alpha_g = \left(\frac{4N_0\,\hbar|\hat{\sigma}\,.\,\boldsymbol{B}_g|}{C}\right)^{\frac{1}{2}}. \tag{46}$$

Such an experiment may be used to determine Planck's constant. In this context, h is a fundamental constant, not only of the electromagnetic interaction, but also of the gravitational interaction. It is defined, except for factor π^{-1}, as the quantum of the gravitomagnetic (flux) moment.

5. Interaction Kinetic Moment Gravitomagnetic Field

If $\boldsymbol{B}_g$ is uniform, then for $\boldsymbol{A}_g$ one can choose the expression (Titeica 1984)

$$\boldsymbol{A}_g = \tfrac{1}{2}(\boldsymbol{B}_g \times \boldsymbol{r})\,. \tag{47}$$

In this case the Hamiltonian

$$\hat{H} = \frac{1}{2m}(\hat{p} - 4m\boldsymbol{A}_g)^2 \tag{48}$$

in the weak field approximation becomes

$$\hat{H} \approx \frac{1}{2m}\hat{p}\,.\,\hat{p} - 2\hbar\hat{L}\,.\,\boldsymbol{B}_g\,. \tag{49}$$

We now define the gravitational current density

$$\boldsymbol{j}_g = 4m\boldsymbol{j}_p\,, \tag{50}$$

with

$$\boldsymbol{j}_p = \frac{\hbar}{2mi}(\psi^*\nabla\psi - \psi\nabla\psi^*)\,. \tag{51}$$

The expression for $\boldsymbol{\mu}_g$ (Titeica 1984) becomes

$$\boldsymbol{\mu}_g = \tfrac{1}{2}\int(\boldsymbol{r} \times \boldsymbol{j}_g)\,\mathrm{d}V\,. \tag{52}$$

After integration over the volume we obtain

$$\hat{\mu}_g = \hbar(\langle\psi|\hat{L}\psi\rangle + \langle\hat{L}\psi|\psi\rangle)\,. \tag{53}$$

As the kinetic moment operator $\hat{L}$ is self-adjoint, the last term in (53) becomes

$$\hat{\mu}_g = 2\hbar\langle\psi|\hat{L}\psi\rangle\,. \tag{54}$$

Consequently, the gravitomagnetic moment is the mean value of the operator

$$\hat{\mu}_{\rm g} = 2\hbar \hat{L}\,. \tag{55}$$

In this case, the second term in the Hamiltonian (49) refers to the energy of the gravitomagnetic moment $\hat{\mu}_{\rm g}$ in the exterior field $\hat{B}_{\rm g}$, i.e.

$$\Delta E = \hat{\mu}_{\rm g} \, . \, \hat{B}_{\rm g} = 2\hbar \hat{L} \, . \, \hat{B}_{\rm g} = \frac{4}{\pi} m\phi_{\rm g0} \, \hat{L} \, . \, \hat{B}_{\rm g}\,. \tag{56}$$

Choosing the z axis as the direction for the gravitomagnetic field, i.e. $B_x = B_y = 0$ and $B_z = B_g$, equation (56) becomes

$$\Delta E = 2\hbar m_L \, B_{\rm g}\,, \tag{57}$$

where m_L is the projection of the kinetic moment in the gravitomagnetic field direction. The expression (57) shows that, in the presence of an external gravitomagnetic field, a spectral line is split into three distinct lines, according to the selection rule $\Delta m_L = 0, \pm 1$; thus

$$\Delta E = 0 \quad \text{or} \quad \pm 2\hbar B_{\rm g}\,, \tag{58}$$

the width of the splitting is

$$\Delta E_{\rm g} = \hbar \omega_{L{\rm g}}\,, \tag{59}$$

where

$$\omega_{L{\rm g}} = 2B_{\rm g} \tag{60}$$

is the gravitational Larmor frequency. We refer to this phenomenon as the gravitational Zeeman effect.

In the Earth's gravitomagnetic field the width is $\Delta E_{\rm g} \approx 6{\cdot}62 \times 10^{-48}$ J, which corresponds to a wavelength of $\lambda_{\rm g} \approx 10^{22}$ m. Since the ratio of optical wavelengths to $\lambda_{\rm g}$ is typically 10^{-29}, the gravitational Zeeman effect is unlikely to be experimentally observed. However, on a cosmic scale, in the vicinity of bodies with intense gravitomagnetic fields, the phenomenon may be evident.

6. Conclusions

We have constructed a U_1 gauge invariant Lagrangian with spontaneous symmetry breaking coupled to a gravitoelectromagnetic field. The topological charge of this physical model is identified with the gravitational fluxoid. Some physical effects were studied; namely

(i) the spin gravitomagnetic field interaction which leads to a gravitational Einstein–de Haas effect,

(ii) the kinetic moment gravitomagnetic field interaction which leads to a gravitational Zeeman effect.

In Section 3 we have shown that vortices will appear when a superfluid is placed in an external gravitomagnetic field; both the gravitomagnetic flux and the radius of the vortices are quantised. Terrestrial experiments cannot realise such an effect, but a pulsar provides a good relativistic laboratory.

The analysis in Section 4 shows that a body made up of a large number of colinear 'gyroscopes', placed in an external gravitomagnetic field, will rotate. The phenomenon may be observable on a cosmic scale for celestial bodies in intense gravitomagnetic fields. In this context, Planck's constant (except for a factor) is identified with the quantum of the gravitomagnetic moment.

The gravitational Zeeman effect was discussed in Section 5 and may lead to observable effects in highly intense gravitomagnetic fields.

Acknowledgment

This paper was substantially improved based on observations by a referee, to whom the authors are indebted.

References

Askar'Jan, G. A. (1962). *Sov. Phys. JETP* **42**, 1567.
Balsz, N. S., and Bertotti, B. (1963). *Nuovo Cimento* **27**, 1087.
Chaichian, M., and Nelipa, F. (1984). 'Introduction in the Gauge Theory' (Springer: Berlin).
Ciubotariu, C. (1991). *Phys. Lett.* A **158**, 27.
Ciubotariu, C, Gottlieb, I., Simaciu, I., Griga, V., Ciubotariu, C. I., and Croitoru, S. (1993). *Tensor* **52**, 138.
Ferrari, V. (1988*a*). *Phys. Rev.* D **37**, 3061.
Ferrari, V. (1988*b*). *Proc. R. Soc. London* A **417**, 417.
Forder, P. W. (1984). *J. Phys.* D **17**, 665.
Fuchs, B. (1981). *Phys. Lett.* A **82**, 285.
Ho, Vu. B., and Morgan, M. J. (1994). *Aust. J. Phys.* **47**, 245.
Ljublicic, A., and Logan, B. A. (1992). *Phys. Lett.* A **172**, 3.
Morgan, T. A. (1971). *Phys. Rev.* D **3**, 800.
Peng, H. (1983). *Gen. Rel. Grav.* **15**, 725.
Peng, H. (1990). *Gen. Rel. Grav.* **22**, 609.
Piran, T., and Safier, P. N. (1985). *Nature* **318**, 271.
Titeica, S. (1984). 'Mecanica Cuantica' (Ed. Academiei: Bucuresti).

Manuscript received 5 July, accepted 18 December 1995