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Gravitational Paramagnetism, Diamagnetism and Gravitational Superconductivity

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Abstract

In the weak field approximation to the gravitational field equations, we study gravitational paramagnetism and diamagnetism, the gravitational Meissner effect and gravitational superconductivity. The spontaneous symmetry breaking corresponds to crossing from closed geodesics to open ones, and to the existence of a critical temperature in the frame of a gauge model at finite temperature. In this later case one can obtain expressions giving the dependence of several superconducting parameters on temperature.

1. Introduction

Recent results (Peng 1983, 1990; Ciubotariu 1991; Ciubotariu *et al.* 1993) have shown that Einstein's equations in the weak field, low speed $(v \ll c)$ approximation reduce to Maxwell-type equations:

$$\nabla \times \boldsymbol{B}_{\mathrm{g}} = -4\pi \boldsymbol{j}_{\mathrm{m}} + \frac{\partial \boldsymbol{E}_{\mathrm{g}}}{\partial t}, \tag{1}$$

$$\nabla \times \boldsymbol{E}_{\mathrm{g}} = -\frac{\partial \boldsymbol{B}_{\mathrm{g}}}{\partial t}, \tag{2}$$

$$\nabla \cdot \boldsymbol{E}_{\mathrm{g}} = -4\pi \rho_{\mathrm{m}}, \tag{3}$$

$$\nabla \cdot \boldsymbol{B}_{\mathrm{g}} = 0, \tag{4}$$

$$\boldsymbol{B}_{\mathrm{g}} = \nabla \times \boldsymbol{A}_{\mathrm{g}}; \qquad \boldsymbol{E}_{\mathrm{g}} = -\frac{\partial \boldsymbol{A}_{\mathrm{g}}}{\partial t} - \nabla V_{\mathrm{g}}, \tag{5}$$

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where $\boldsymbol{B}_{\rm g}$ is the gravitomagnetic field, $\boldsymbol{E}_{\rm g}$ the gravitoelectric field, $\boldsymbol{A}_{\rm g}$ the vector potential, $V_{\rm g}$ the scalar potential, $\boldsymbol{j}_{\rm m} = \rho_{\rm m}\,\boldsymbol{v}$ the mass current density, and $\rho_{\rm m} = nm$ is the proper mass density with n the number density of particles having rest mass m. In the same approximation, the geodesic equation reduces to a Lorentz-type equation (Peng 1983; Ciubotariu 1991):

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \boldsymbol{E}_{\rm g} + 4(\boldsymbol{v} \times \boldsymbol{B}_{\rm g})\,. \tag{6}$$

The analogy between the gravitational and the electromagnetic field is almost perfect both in the weak field approximation (Peng 1983, 1990; Fuchs 1981; Ho and Morgan 1994; Agop *et al.* 1996) and in strong fields (Ferrari 1988*a*, 1988*b*; Morgan 1971; Piran and Safiev 1985). An exception is the lack of a gravitational Meissner-type effect (Ciubotariu and Agop 1996).

The 'scope' of the present work is to study several effects induced by the gravitomagnetic field—gravitational paramagnetism and diamagnetism, the gravitational Meissner effect, gravitational superconductivity—and to derive expressions for several superconducting parameters.

2. Quantum Gravitomagnetic Moment

For a particle of mass m, which moves in a gravitational field with 4-vector potential $(V_{\rm g}, \boldsymbol{A}_{\rm g})$, the Lagrangian is expressed as

$$L \approx \tfrac{1}{2}m\boldsymbol{v}^2 + 4m\boldsymbol{v}\,.\,\boldsymbol{A}_{\rm g} - mV_{\rm g}\,, \tag{7}$$

and the corresponding Hamiltonian is

$$H = \frac{(\boldsymbol{P} - 4m\boldsymbol{A}_{\rm g})^2}{2m} + mV_{\rm g}\,, \tag{8}$$

where the canonical momentum $\boldsymbol{P}$ is defined via the Lagrangian (7) as

$$\boldsymbol{P} = \frac{\partial L}{\partial \boldsymbol{v}} = m\boldsymbol{v} + 4m\boldsymbol{A}_{\rm g}\,. \tag{9}$$

Once the classical Hamiltonian (8) is known, the corresponding quantum mechanical equation is obtained by replacing the classical parameters with their respective quantum operators. Then, the Schrödinger equation becomes

$$\frac{\hbar}{\mathrm{i}}\frac{\partial \Psi}{\partial t} + \left[\frac{1}{2m}(\hat{P} - 4m\hat{A}_{\rm g})^2 + mV_{\rm g}\right]\Psi = 0\,. \tag{10}$$

Since

$$\hat{P}_\ell\,\hat{A}_{{\rm g}\ell} = \hat{A}_{{\rm g}\ell}\,\hat{P}_l + \frac{\hbar}{\mathrm{i}}\frac{\partial A_{{\rm g}\ell}}{\partial x_\ell}, \quad \ell = 1, 2, 3\,, \tag{11}$$

expression (10) when expanded becomes

$$\frac{\hbar}{\mathrm{i}}\frac{\partial\Psi}{\partial t}-\frac{\hbar^2}{2m}\nabla^2\Psi-\frac{4\hbar}{\mathrm{i}}\boldsymbol{A}_{\mathrm{g}}\nabla\Psi-\frac{2\hbar}{\mathrm{i}}(\nabla\boldsymbol{.}\boldsymbol{A}_{\mathrm{g}})\Psi+8m\boldsymbol{A}_{\mathrm{g}}^2\Psi+mV_{\mathrm{g}}\Psi=0\,, \tag{12}$$

and the complex conjugate equation is

$$\frac{\hbar}{\mathrm{i}}\frac{\partial\Psi^*}{\partial t}+\frac{\hbar^2}{2m}\nabla^2\Psi^*-\frac{4\hbar}{\mathrm{i}}\boldsymbol{A}_{\mathrm{g}}\nabla\Psi^*-\frac{2\hbar}{\mathrm{i}}(\nabla\boldsymbol{.}\boldsymbol{A}_{\mathrm{g}})(\Psi_{\mathrm{g}})\Psi^*-8m\boldsymbol{A}_{\mathrm{g}}^2\Psi^*-mV_{\mathrm{g}}\Psi^*=0\,. \tag{13}$$

By multiplying equation (12) with Ψ^*, (13) with Ψ and then by adding them, the continuity equation is obtained for the probability density $\rho_{\mathrm{p}}=\Psi^*\Psi$

$$\frac{\partial\rho_{\mathrm{p}}}{\partial t}+\nabla\boldsymbol{.}\boldsymbol{j}_{\mathrm{p}}=0\,, \tag{14}$$

where

$$\boldsymbol{j}_{\mathrm{p}}=\frac{\hbar}{2m\mathrm{i}}(\Psi^*\nabla\Psi-\Psi\nabla\Psi^*)-4\boldsymbol{A}_{\mathrm{g}}\Psi^*\Psi \tag{15}$$

defines the probability current.

If the particle has mass m, then one can associate the mass density $\rho_{\mathrm{m}}=4m\rho_{\mathrm{p}}$ with the probability density, and the mass current density, $\boldsymbol{j}_{\mathrm{m}}=4m\boldsymbol{j}_{\mathrm{p}}$, with the probability current density (Agop *et al.* 1996). These parameters satisfy a continuity equation of the form (14) which expresses mass conservation.

Under these circumstances, the quantum gravitomagnetic moment is given by (Titeica 1984)

$$\mu_{\mathrm{g}}=\tfrac{1}{2}\int(\boldsymbol{r}\times\boldsymbol{j}_{\mathrm{m}})\,\mathrm{d}V \tag{16}$$

or explicitly

$$\mu_{\mathrm{g}}=\hbar\int[\Psi^*(\boldsymbol{r}\times\mathrm{i}^{-1}\nabla\Psi)+(\boldsymbol{r}\times\mathrm{i}^{-1}\nabla\Psi)^*\Psi]\,\mathrm{d}V-8m\int\Psi^*(\boldsymbol{r}\times\boldsymbol{A}_{\mathrm{g}})\Psi\,\mathrm{d}V\,. \tag{17}$$

The first integral in (17) corresponds to the permanent gravitomagnetic moment of the moving particle (Agop *et al.* 1996), i.e.

$$\hat{\mu}_{0\mathrm{g}}=2\hbar\hat{L}\,, \tag{18}$$

where $\hat{L}$ is the kinetic moment operator. The second integral

$$\mu_{1\mathrm{g}}=-8m\int\Psi^*(\boldsymbol{r}\times\boldsymbol{A}_{\mathrm{g}})\Psi\,\mathrm{d}V \tag{19}$$

is the mean value on the Ψ state of the operator

$$\hat{\mu}_{1\mathrm{g}} = -8m(\boldsymbol{r} \times \boldsymbol{A}_{\mathrm{g}})\,. \tag{20}$$

We will call $\hat{\mu}_{1\mathrm{g}}$ the gravitomagnetic moment induced by the gravitomagnetic field. Therefore, a particle moving in a gravitomagnetic field develops a permanent gravitomagnetic moment, defined by (18), simultaneously with an induced gravitomagnetic moment, defined by (20). Thus the total gravitomagnetic moment (16) becomes

$$\mu_{\mathrm{g}} = \hat{\mu}_{0\mathrm{g}} + \hat{\mu}_{1\mathrm{g}}\,. \tag{21}$$

3. Gravitational Paramagnetism and Diamagnetism

Let us allow the particle of mass m to be subjected simultaneously to the static gravitoelectric field $\boldsymbol{E}_{\mathrm{g}}$ and to a homogeneous gravitomagnetic field $\boldsymbol{B}_{\mathrm{g}}$, given by

$$\boldsymbol{A}_{\mathrm{g}} = -\tfrac{1}{2}(\boldsymbol{r} \times \boldsymbol{B}_{\mathrm{g}})\,. \tag{22}$$

In this case, the Hamiltonian operator takes the form

$$\begin{aligned} \hat{H} &= \frac{1}{2m}(\hat{p} + 2m\boldsymbol{r} \times \boldsymbol{B}_{\mathrm{g}})^2 + mV_{\mathrm{g}} \\ &= \frac{\hat{p}\hat{p}}{2m} + 2\hat{p}(\boldsymbol{r} \times \boldsymbol{B}_{\mathrm{g}}) + 2m(\boldsymbol{r} \times \boldsymbol{B}_{\mathrm{g}})^2 + mV_{\mathrm{g}}\,. \end{aligned} \tag{23}$$

Since

$$\hat{p}(\boldsymbol{r} \times \boldsymbol{B}_{\mathrm{g}}) = -(\boldsymbol{r} \times \hat{p})\boldsymbol{B}_{\mathrm{g}} \quad \text{and} \quad (\boldsymbol{r} \times \boldsymbol{B}_{\mathrm{g}})^2 = \boldsymbol{r}^2\boldsymbol{B}_{\mathrm{g}}^2 - (\boldsymbol{r}\,.\,\boldsymbol{B}_{\mathrm{g}})^2\,, \tag{24}$$

then (23) becomes

$$\hat{H} = \frac{\hat{p}\hat{p}}{2m} - 2(\hat{r} \times \hat{p})\boldsymbol{B}_{\mathrm{g}} + 2m[\boldsymbol{r}^2\boldsymbol{B}_g^2 - (\boldsymbol{r}\,.\,\boldsymbol{B}_{\mathrm{g}})^2] + mV_{\mathrm{g}}\,. \tag{25}$$

We define the total gravitomagnetic moment (Titeica 1984) by

$$\hat{\mu}_{\mathrm{g}} = -\frac{\partial \hat{H}}{\partial \boldsymbol{B}_{\mathrm{g}}} = 2(\boldsymbol{r} \times \hat{p}) - 4m[\boldsymbol{r}^2\boldsymbol{B}_{\mathrm{g}} - (\boldsymbol{r}\,.\,\boldsymbol{B}_{\mathrm{g}})\boldsymbol{r}]\,. \tag{26}$$

Noting that $\boldsymbol{r} \times \boldsymbol{p} = \hbar\hat{L}$, the first term on the right side of (26) takes the value $\hat{\mu_{0\mathrm{g}}}$. When the external gravitomagnetic field is weak in the Hamiltonian (25), the first order term in $\boldsymbol{B}_{\mathrm{g}}$ is dominant (Agop *et al.* 1996):

$$-2(\boldsymbol{r} \times \hat{p})\boldsymbol{B}_{\mathrm{g}} = \hat{\mu}_{0\mathrm{g}}\,.\,\boldsymbol{B}_{\mathrm{g}}\,, \tag{27}$$

which represents the energy of the permanent moment $\hat{\mu}_{0\mathrm{g}}$ in the external field $\boldsymbol{B}_\mathrm{g}$. The macroscopic effect of this energy is expressed by the tendency of orienting the permanent gravitomagnetic moments of a macroscopic body containing many 'elementary gyroscopes' along the $\boldsymbol{B}_\mathrm{g}$ direction. Consequently, the body exhibits gravitational paramagnetism. For this feature to show, it is necessary that the permanent gravitomagnetic moment be non-zero, implying $\hat{L} \neq 0$.

In the states with $\hat{L} = 0$, the permanent gravitomagnetic moment is null, so that only the induced gravitomagnetic moment

$$\hat{\mu}_{1\mathrm{g}} = 8m\tfrac{1}{2}[\boldsymbol{r}\,.\,(\boldsymbol{r}\,.\,\boldsymbol{B}_\mathrm{g})] = 4m[\boldsymbol{r}(\boldsymbol{r}\,.\,\boldsymbol{B}_\mathrm{g}) - \boldsymbol{r}^2\boldsymbol{B}_\mathrm{g}] \tag{28}$$

remains, the average value of which is

$$\hat{\mu}_{1\mathrm{g}} = -4m\int[\boldsymbol{r}^2\boldsymbol{B}_\mathrm{g} - (\boldsymbol{r}\,.\,\boldsymbol{B}_\mathrm{g})\,.\,\boldsymbol{r}]\Psi^*\Psi\,\mathrm{d}V\,. \tag{29}$$

The $\hat{L} = 0$ states have a spherical symmetry, and therefore the function $\Psi^*\Psi$ depends only on the distance r. Calculating the integral (29) in spherical polar coordinates system and knowing that (Titeica 1984)

$$\int x_i x_j\,\mathrm{d}\Omega = 0 \quad (i \neq j); \qquad \int x_i^2\,\mathrm{d}\Omega = \tfrac{1}{3}\int r^2\,\mathrm{d}\Omega \quad (i = j = 1,\,2,\,3)\,, \tag{30}$$

one gets

$$\hat{\mu}_{1\mathrm{g}} = -4m\int(r^2 - x_1^2)\Psi^*\Psi\,\mathrm{d}V = -\tfrac{2}{3}4mK\boldsymbol{B}_\mathrm{g}\,, \tag{31}$$

where

$$K = \int r^2\Psi^*\Psi\,\mathrm{d}V \tag{32}$$

denotes a positive integral. The induced gravitomagnetic moment is proportional to the field $\boldsymbol{B}_\mathrm{g}$ and oriented antiparallel to it; thus

$$\hat{\mu}_{1\mathrm{g}} = -\alpha\boldsymbol{B}_\mathrm{g}\,. \tag{33}$$

The coefficient α, defined as the gravitational polarisability of the 'elementary gyroscope', is given by

$$\alpha = \tfrac{8}{3}m\int r^2\Psi^*\Psi\,\mathrm{d}V\,. \tag{34}$$

In this situation, the effect of the field $\boldsymbol{B}_\mathrm{g}$ on a macroscopic body involves induction of a gravitomagnetic moment antiparallel to the field. We call this property gravitational diamagnetism. One can notice that, even if the 'elementary gyroscopes' of the substance have permanent gravitomagnetic momwents, gravitational diamagnetism does not disappear but, since it is weak, it is hidden by the gravitational paramagnetism. Indeed in the terrestrial gravitomagnetic field we have $|\boldsymbol{B}_{\mathrm{g}}| \approx 10^{-14}\ s^{-1}$ (Ljubicic and Logan 1992), and by considering the atom as an 'elementary gyroscope' we get $|\hat{\mu}_{1\mathrm{g}}|/|\hat{\mu}_{0\mathrm{g}}| \approx 10^{-2}$.

Within this context, the gravitational Einstein–de Haas and gravitational Zeeman effects, vortices in a superfluid placed in an external gravitomagnetic field, have been also studied (Agop *et al.* 1996).

4. Wave Function Rigidity and the Absence of the Meissner Effect

Let the mass current density be

$$\boldsymbol{j}_{\mathrm{m}} = \frac{2\hbar}{\mathrm{i}}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*) - 16m\boldsymbol{A}_{\mathrm{g}}\,\Psi^*\Psi\,. \tag{35}$$

In the absence of the gravitomagnetic field we have $\boldsymbol{A}_{\mathrm{g}} = 0$ and, with $\Psi = \Psi_0$, the mass current density is zero, i.e.

$$\boldsymbol{j}_{0\mathrm{m}} = \frac{2\hbar}{\mathrm{i}}(\Psi_0^*\nabla\Psi_0 - \Psi_0\nabla\Psi_0^*) = 0\,. \tag{36}$$

If we allow the wave function Ψ to be 'rigid' (Balla and Deutsch 1970), meaning that it does not vary when a gravitomagnetic field is applied, then $\Psi \approx \Psi_0$ in all situations, i.e. the wave function is equal to this value in the absence of the gravitomagnetic field. Consequently, by considering both this feature and relation (35), we have

$$\boldsymbol{j}_{\mathrm{m}} = -16m\boldsymbol{A}_{\mathrm{g}}\,\rho_{\mathrm{p}} = -4\rho_{\mathrm{m}}\,\boldsymbol{A}_{\mathrm{g}} \tag{37}$$

or, by applying the curl operator with $\rho_{\mathrm{m}} \approx$ constant,

$$\nabla \times \boldsymbol{j}_{\mathrm{m}} = -4\rho_{\mathrm{m}}\,\boldsymbol{B}_{\mathrm{g}}\,. \tag{38}$$

When the driving current $\partial\boldsymbol{E}_{\mathrm{g}}/\partial t$ is absent, on applying the curl operator equation (1) takes the form

$$\nabla \times \nabla \times \boldsymbol{B}_{\mathrm{g}} = \nabla(\nabla\,.\,\boldsymbol{B}_{\mathrm{g}}) - \nabla^2\boldsymbol{B}_{\mathrm{g}} = -4\pi\nabla \times \boldsymbol{j}_{\mathrm{m}} \tag{39}$$

from which, taking into account (4) and (38), one obtains

$$\nabla^2\boldsymbol{B}_{\mathrm{g}} + 16\pi\rho_{\mathrm{m}}\,\boldsymbol{B}_{\mathrm{g}} = 0\,. \tag{40}$$

This result shows that $\boldsymbol{B}_{\mathrm{g}}$ does not 'induce' a gravitational Meissner effect, but orders space (Ciubotariu and Agop 1996) as a monocrystal, the gravitational lattice constant being $\lambda = (1/16\pi\rho_{\mathrm{m}})^{1/2}$. For $n = |\Psi|^2 = 10^{28}\ m^{-3}$, the gravitational lattice constant λ is about 10^{13} m, which is the same order of magnitude as the diameter of Pluto's orbit, a tiny distance compared to the cosmic scale (Peng 1990). Considerations based on very large currents up to 10^{21} A (Linet 1990) which can be carried by the cosmic string may ultimately give much smaller limits for λ. Consequently, the 'rigidity' of the wave function is responsible for the space's 'ordering'.

Obviously, the wave function is not completely 'rigid'. As a result, one can assume that relation (37) is only a first approximation. Under such conditions the difference between the electromagnetic and the linear gravitational field is evident; the wave functions 'rigidity' implies the presence of a Meissner effect for the electromagnetic field and its absence for the gravitational field.

5. Gravitational Superconductivity

Let us consider the Schrödinger-type Lagrangian

$$L = \tfrac{1}{2}(\nabla\Psi)^*\nabla\Psi - \tfrac{1}{4}\beta\left(|\Psi|^2 + \frac{\alpha}{\beta}\right)^2 - \tfrac{1}{2}\boldsymbol{B}_{\mathrm{g}}^2\,, \tag{41}$$

where $\nabla_\ell = \partial_\ell - i(4m/\hbar)A_{\mathrm{g}\ell}$ is the covariant derivative, Ψ is the wave function coupled minimally to the gravitomagnetic field, $\boldsymbol{B}_{\mathrm{g}}$ is the gravitomagnetic field intensity, $4m/\hbar$ is the coupling constant and $\beta > 0$ is the self-interaction constant.

Using the Euler–Lagrange equations for the Lagrangian (41), the field equations are

$$-\nabla\times\boldsymbol{B}_{\mathrm{g}} = \frac{2im}{\hbar}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*) + \frac{16m^2}{\hbar^2}\boldsymbol{A}_{\mathrm{g}}\,\Psi^*\Psi\,, \tag{42}$$

$$\nabla_\ell\nabla_\ell\Psi = -\beta\Psi\left(|\Psi|^2 + \frac{\alpha}{\beta}\right). \tag{43}$$

The energy minimum is obtained for $\alpha < 0$ and $\Psi = (-\alpha/\beta)^{1/2}$ (Chaichian and Nelipam 1984). For this value of Ψ, (42) becomes

$$-\nabla\times\boldsymbol{B}_{\mathrm{g}} = \frac{16m^2}{\hbar^2}\,\frac{\alpha}{\beta}\,\boldsymbol{A}_{\mathrm{g}}\,, \tag{44}$$

from which, by applying the curl operator and taking into account (4), we obtain

$$\nabla^2\boldsymbol{B}_{\mathrm{g}} - \frac{16m^2}{\hbar^2}\,\frac{\alpha}{\beta}\,\boldsymbol{B}_{\mathrm{g}} = 0\,. \tag{45}$$

This means that spontaneous symmetry breaking produces a gravitational Meissner effect, the penetration depth being

$$\lambda = \frac{\hbar}{4m}\left(\frac{\beta}{\alpha}\right)^{1/2}. \tag{46}$$

In the absence of spontaneous symmetry breaking, i.e. for $\alpha > 0$, relation (45) takes the form

$$\nabla^2\boldsymbol{B}_{\mathrm{g}} + \frac{16m^2}{\hbar^2}\,\frac{\alpha}{\beta}\boldsymbol{B}_{\mathrm{g}} = 0\,, \tag{47}$$

which reflects a space 'ordering' as a monocrystal where λ is the lattice constant.

By comparing the ordinary Schrödinger equation with the one derived from (43), we obtain $\alpha = 2mE/\hbar^2$, where E is the total energy of the test particle. Under these conditions, one can associate with spontaneous symmetry breaking a bonding state of the test particle ($E < 0$), and in the absence of spontaneous symmetry breaking we obtain free motion of the same test particle ($E > 0$). Therefore, any system of particles moving along the geodesics may be called a gravitational superconductor, since no external (either non-gravitational or gravitational) forces act on the particle. As an example, consider a dust filled Universe. When the particles move along open geodesics, there is no gravitational Meissner effect. However, when they move along closed geodesics, a gravitational Meissner effect occurs. Passing from one effect to another is achieved by spontaneous symmetry breaking.

Generally, speaking, one associates spontaneous symmetry breaking with a critical temperature in finite temperature field theories. The method is based on the following idea (Dariescu *et al.* 1992): the functional $Z[J]$, which expresses the amplitude of a vacuum-to-vacuum transition (at $T = 0$) will be redefined at a temperature $T \neq 0$ and identified with the quantum statistical partition function. One can associate a temperature $T \sim 1/\beta_0$ with the Ψ field. For $Z[J]$ to be invariant to 'temporal' translations in β_0, Ψ needs to be a time periodic function, with the period β_0. Making use of this procedure one can define α_{ef} as

$$\alpha_{\mathrm{ef}} = \alpha - \left(\beta + \frac{16m^2}{\hbar^2}\right)\delta T^2 , \tag{48}$$

with δ a dimensional constant, from which, by restricting $\alpha_{\mathrm{ef}} = 0$, we introduce the critical temperature

$$T_{\mathrm{c}} = \left[\alpha \Big/ \delta\left(\beta + \frac{16m^2}{\hbar^2}\right)\right]^{\frac{1}{2}} . \tag{49}$$

Dimensional analysis of (43) and (46) leads us to postulate the existence of a 'coherence length' $\xi = 1/\sqrt{\alpha}$, and a penetration depth λ, respectively, so that (49) becomes

$$T_{\mathrm{c}} = 1 \Big/ \left[\delta\xi^2 \frac{16m^2}{\hbar^2}\left(\frac{\lambda^2}{\xi^2} + 1\right)\right]^{\frac{1}{2}} . \tag{50}$$

If $\lambda/\xi \ll 1$, equation (50) takes the form

$$T_{\mathrm{c}}(1) = \frac{\hbar}{4m\xi}\frac{1}{\sqrt{\delta}} , \tag{51}$$

and if $\lambda/\xi \gg 1$, equation (50) becomes

$$T_{\mathrm{c}}(2) = \frac{\hbar}{4m\lambda}\frac{1}{\sqrt{\delta}} . \tag{52}$$

This means that there are two types of gravitational superconductor, corresponding to the critical temperatures in (51) and (52); this is analogous to type I and type II superconductors in electromagnetism (Burns 1992).

In this context, one can define the following temperature dependencies:

(i) The coherence length at temperature T,

$$\xi(T) = \frac{1}{\sqrt{\alpha_{\rm ef}(T)}} = \frac{\xi(0)}{\sqrt{1-(T/T_{\rm c})^2}}, \tag{53}$$

where $\xi(0) = 1/\sqrt{\alpha}$ is the coherence length at $T = 0$.

(ii) The energy gap at temperature T,

$$E(T) = E(0)\sqrt{1-(T/T_{\rm c})^2}, \tag{54}$$

where $E(0)$ is the energy gap at $T = 0$. This result is obtained on condition that (43) should reduce to the linear Schrödinger equation. Then, we have

$$\frac{2m}{\hbar^2}\, e(T) = \alpha_{\rm ef}; \qquad \frac{2m}{\hbar^2}\, E(0) = \alpha\,. \tag{55}$$

(iii) The critical gravitomagnetic field at temperature T,

$$B_{\rm gc}(T) = B_{\rm gc}(0)[1-(T/T_{\rm c})^2], \tag{56}$$

where $B_{\rm gc}(0)$ is the critical gravitomagnetic field at $T = 0$. This result comes from

$$B_{\rm gc}^2(T) = \alpha_{\rm ef}^2(T)/\beta \tag{57}$$

by making an analogy with the result in Burns (1992), whence we note $B_{\rm gc}(0) = \alpha(0)/\sqrt{\beta}$.

In terms of the way the gravitational field is defined, we interpret the previous results as follows:

(i) Equation (53) is the minimum distance for two neighbouring geodesics to be indiscernible. In this way, one can introduce the 'space quantum' $\xi(T) = \hbar/\sqrt{2mE}$, which depends on temperature.

(ii) Equation (54) corresponds to the minimum energy necessary for a test particle to pass from the bonding to the non-bonding state. This energy is lower as $T \to T_{\rm c}$.

(iii) Equation (56) corresponds to a critical gravitomagnetic field through which a test particle passes from a bonding to a non-bonding state. The value of this field is lower as $T \to T_{\rm c}$.

(iv) In the absence of the gravitomagnetic field, equation (49) reduces to $T_{\rm c} \sim (\alpha/\beta)^{1/2}$ (Vilenkin 1985), i.e. a critical temperature corresponding to the cosmological phase transition. As the Universe cools through the critical temperature, the 'field' acquires expectation values $+(\alpha/\beta)^{1/2}$ and $-(\alpha/\beta)^{1/2}$ at random in different regions of space. One can introduce a correlation length, $\xi = 1/\sqrt{\alpha}$, such that the values of $\langle\Psi\rangle$ are uncorrelated over distances greater than ξ (Vilenkin 1985).

(v) Cosmic dust condensing by a gravitational Meissner effect on the cosmic string forms the galactic nucleus. Along the z = constant planes (oriented parallel to the string axis), and in the absence of the gravitational Meissner effect, positively defined transverse stresses arise which, in principle, are responsible for the galactic arms (Vilenkin 1985).

6. Conclusions

The main results of this paper may be summarised as follows:

(i) A microparticle in a gravitomagnetic field has both a permanent and an induced gravitomagnetic moment. The permanent gravitomagnetic moment is responsible for the gravitational paramagnetism, and the induced one, for the gravitational diamagnetism.

(ii) The 'rigidity' of the wave function is interpreted as an 'ordering' of the space–time variety, i.e. the absence of a gravitational Meissner effect.

(iii) By using a nonlinear Schrödinger-type Lagrangian, spontaneous symmetry breaking can be reconciled with the gravitational Meissner effect. In this context the gravitational superconductivity was defined and the temperature dependencies of the superconducting parameters were found.

(iv) The dual properties of the microparticle are conditioned by the space 'ordering'. Indeed, for a free microparticle we have $E = p^2/2m$, and thus from (55), with $\alpha = 1/k^2$, $p = \hbar k$ results where k is the wave number.

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References

Agop, M., Ciubotariu, C., Buzea, Gh. C., Buzea, C., and Ciobanu, B. (1996). *Aust. J. Phys.* **49**, 613.

Balla, D., and Deutsch, R. V. (1970). 'Introducere in Fizica Temperaturilor Joase' (Ed. Academiei: Bucuresti).

Burns, G. (1992). 'High Temperature Superconductivity: An Introduction' (Academic: San Diego).

Chaichian, M., and Nelipa, F. (1984). 'Introduction to Gauge Theory' (Springer: Berlin).

Ciubotariu, C. (1991). *Phys. Lett.* A **158**, 27.

Ciubotariu, C., and Agop, M. (1996). *Gen. Rel. Grav.* **28**, 1.

Ciubotariu, C., Gottlieb, I., Simaciu, I., Griga, V., Ciubotariu, C. I., and Croitoru, S. (1993). *Tensor* **52**, 138.

Dariescu, M., Dariescu, C., and Agop, M. (1992). Mem. Sect. Stiint. ale Acad. Române S, IV, T15, **1**, 125.

Ferrari, V. (1988*a*). *Phys. Rev.* D **37**, 3061.

Ferrari, V. (1988*b*). *Proc. R. Soc. London* A **417**, 417.

Fuchs, B. (1981). *Phys. Lett.* A **82**, 285.

Ho, Vu. B., and Morgan, M. J. (1994). *Aust. J. Phys.* **47**, 245.

Linet, B. (1990). *Phys. Lett.* A **146**, 159.

Ljubicic, A., and Logan, B. A. (1992). *Phys. Lett.* A **172**, 3.

Morgan, T. A. (1971). *Phys. Rev.* D **3**, 800.

Peng, H. (1983). *Gen. Rel. Grav.* **15**, 725.

Peng, H. (1990). *Gen. Rel. Grav.* **22**, 609.

Piran, T., and Safiev, P. N. (1985). *Nature* **318**, 271.

Titeica, S. (1984). 'Mec. Cuantica' (Ed. Academiei: Bucuresti).
Vilenkin, A. (1985). *In* 'Cosmic Strings and Other Topological Defects in Quantum Gravity and Cosmology' (Eds H. Sato and T. Inami), p. 269 (World Scientific: Singapore).

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