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^{8}Be Level Properties from the Low-energy ^{7}Li(p, γ_0)^{8}Be Cross Section

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Abstract

R-matrix fits to ^{7}Li(p, γ_0)^{8}Be cross section data for $E_{\rm p} \leq 1500$ keV give reduced width amplitudes of the 1^{+} levels of ^{8}Be at 17·64 and 18·15 MeV having signs in agreement with shell model calculations, contrary to previous fits to less-extensive data.

1. Introduction

There has recently been renewed interest in the low-energy ^{7}Li(p, γ_0)^{8}Be cross section, following work by Chasteler *et al.* (1994) which gave evidence for a substantial p-wave strength, of uncertain origin. They pointed out that this could significantly reduce the zero-energy astrophysical S factor obtained by assuming pure s-waves, and suggested that a similar phenomenon might be present in the ^{7}Be(p, γ_0)^{8}B reaction, which is of importance in the solar neutrino problem. Several papers have sought to explain the Chasteler *et al.* data, which is for $E_{\rm p} \leq 80$ keV (Rolfs and Kavanagh 1994; Zahnow *et al.* 1995; Weller and Chasteler 1995; Barker 1995; Godwin *et al.* 1996).

Earlier, ^{7}Li(p, γ_0)^{8}Be data in the energy range $E_{\rm p} = 380$–960 keV (Mainsbridge 1960; Schlueter *et al.* 1964; Ulbricht *et al.* 1977) had been fitted using elements of the transition matrix as the adjustable parameters (Barker 1979). Interpretation of these matrix elements in terms of ^{8}Be level parameters suggested relative signs of the reduced width amplitudes of the 1^{+} levels at 17·64 and 18·15 MeV different from shell model predictions.

The similar fit (Barker 1995) to the low-energy data of Chasteler *et al.* required relative signs different from those obtained in the higher-energy fits, and possibly different from the shell model values.

The aim of the present work is to obtain a consistent fit to all the data, including some that has become available recently (Zahnow *et al.* 1995), and, if this is possible, to see if the signs are necessarily in conflict with shell model values. The main difference in approach from the earlier work is that here the fitting parameters are the level parameters themselves rather than the transition matrix elements.

The R-matrix formulae for the ^{7}Li(p, γ_0)^{8}Be total cross section and for the angular distribution and analysing power coefficients are given in the next section.

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The data are in Section 3. Section 4 gives the fitting procedure and results, and finally these results are discussed.

2. Formulae

The differential cross section for the ^{7}Li(p, γ_0)^{8}Be reaction can be expressed as

$$\sigma(\theta) = (\sigma_{tot}/4\pi)W(\theta)\,, \qquad (1)$$

where σ_{tot} is the total cross section, and the angular distribution $W(\theta)$ is given by

$$W(\theta) = 1 + \sum_{k=1} a_k P_k(\cos\theta)\,. \qquad (2)$$

The analysing power is likewise given by

$$A_y(\theta) = [W(\theta)]^{-1} \sum_{k=1} b_k P_k^1(\cos\theta)\,. \qquad (3)$$

Here σ_{tot}, a_k and b_k are energy-dependent quantities, which may be expressed in terms of the complex transition matrix elements $U^{J_i}_{s\ell,LpJ_f}$, where the radiative transition is from an initial ^{8}Be state J_i, formed from ^{7}Li + p with channel spin s and relative orbital angular momentum ℓ, to a final state J_f, and L is the multipolarity of the transition with $p = 0$ (1) for electric (magnetic) radiation. Since $J_f = 0$ here, we have $L = J_i$ and $p = mod(\mid J_i - 1 - \ell \mid, 2)$, so that the matrix element may be abbreviated to $U^{J_i}_{s\ell}$. In the present analysis, for proton energies $E_p \leq 1{\cdot}5$ MeV, we restrict the radiations to $E1$ and $M1$, so that $J_i = 1$ and J_i may be omitted as an index. We also assume $\ell \leq 2$. Then there are five matrix elements $U_{s\ell}$: U_{10}, U_{11}, U_{21}, U_{12} and U_{22}, and one has* (Seyler and Weller 1979)

$$\sigma_{tot} = (3\pi/8k^2)T\,, \qquad (4)$$

* Equivalent formulae (without the $\ell = 2$ terms) were given by Ulbricht *et al.* (1977). Barker (1979) pointed out that these contained errors (see also Seyler and Weller 1979). The equations (4)–(8) of Ulbricht *et al.* were derived from equations (33·8) and (33·11) of Baldin *et al.* (1961). Of these, however, equation (33·11) is incorrect and should be multiplied by the factor $(-1)^{J_1+s_1'+\ell_1'-J_2-s_2'-\ell_2'+1}$. This factor appears to arise from an error in the derivation by Baldin *et al.* of their equation (31·8) from (31·6), in that (31·8) should have an additional factor $(-1)^{J_1+s_1'+\ell_1'-J_2-s_2'-\ell_2'+1+2S}$. Correction of this error would cause a change of sign of the first term in the expression for B_1 in equation (7) of Ulbricht *et al.* Furthermore the coefficients of all terms in B_1 and B_2 should be doubled and the signs of all these terms should be changed (the relation $P_y = -A_y$ given by Ulbricht *et al.* is correct if the same y-axis is used for both the polarisation and the analysing power measurements; however, if the y-axis in each measurement is chosen in accordance with the Basel convention, as is the case for both Ulbricht *et al.* and Baldin *et al.*, then one has $P_y = A_y$).

where

$$T = | U_{10} |^2 + | U_{11} |^2 + | U_{21} |^2 + | U_{12} |^2 + | U_{22} |^2 \tag{5}$$

and

$$a_1 = T^{-1}[-2 \cdot 449 Re(U_{10}^* U_{11}) - 1 \cdot 732 Re(U_{11}^* U_{12})$$

$$- 2 \cdot 324 Re(U_{21}^* U_{22})], \tag{6}$$

$$a_2 = T^{-1}[1 \cdot 414 Re(U_{10}^* U_{12}) + 0 \cdot 5 \mid U_{11} \mid^2 - 0 \cdot 1 \mid U_{21} \mid^2 - 0 \cdot 5 \mid U_{12} \mid^2$$

$$+ 0 \cdot 5 \mid U_{22} \mid^2], \tag{7}$$

$$b_1 = T^{-1}[0 \cdot 612 Im(U_{10}^* U_{11}) + 1 \cdot 369 Im(U_{10}^* U_{21}) + 0 \cdot 866 Im(U_{11}^* U_{12})$$

$$+ 0 \cdot 775 Im(U_{21}^* U_{12}) - 1 \cdot 162 Im(U_{21}^* U_{22})], \tag{8}$$

$$b_2 = T^{-1}[-0 \cdot 354 Im(U_{10}^* U_{12}) - 0 \cdot 354 Im(U_{10}^* U_{22}) - 0 \cdot 224 Im(U_{11}^* U_{21})$$

$$+ 0 \cdot 5 Im(U_{12}^* U_{22})]. \tag{9}$$

The total cross section is usually expressed as an astrophysical S factor

$$S = E \sigma_{tot} e^{2\pi\eta}, \tag{10}$$

where E is the ^{7}Li + p c.m. energy and η is the Sommerfeld parameter.

Expressions for the transition matrix elements $U_{s\ell}$ in terms of level parameters may be obtained from R-matrix theory (Lane and Thomas 1958). Although the standard R-matrix theory is based on assumptions that are not justified for radiative capture reactions, such as the present ^{7}Li(p, γ_0)^{8}Be, modified formulae including channel contributions may be obtained for these reactions; these formulae reduce to the standard form if the final state is strongly bound (Barker and Kajino 1991), as is the case here ($Q = 17 \cdot 25$ MeV). Thus we assume R-matrix formulae of standard form, involving real constant reduced-width amplitudes for the photon channels. For the $E1$ component, Zahnow *et al.* (1995) assumed only s-wave direct capture, but they had to use very large values of the spectroscopic factor in order to fit the data ($C^2S = 6 \cdot 8$ for γ_0 and 27 for $\gamma_0 + \gamma_1$).

We assume that two 1^+ levels of ^{8}Be contribute to the $M1$ capture; these are the known levels at $E_x = 17 \cdot 64$ and $18 \cdot 15$ MeV (Ajzenberg-Selove 1988), labelled $\lambda = 1$ and 2 respectively. Then one has

$$U_{s1} = 2ie^{i(\omega_1 - \phi_1)}(P_1 P_\gamma)^{\frac{1}{2}} \sum_{\lambda\mu} \gamma_{\lambda s1} \gamma_{\mu\gamma}^1 A_{\lambda\mu} \qquad (s = 1,\, 2), \tag{11}$$

where

$$(\mathbf{A}^{-1})_{\lambda\mu} = (E_\lambda^1 - E)\delta_{\lambda\mu} - \sum_s \gamma_{\lambda s1}\gamma_{\mu s1}(S_1 - B_1 + iP_1)\,. \qquad (12)$$

Here S_ℓ, P_ℓ and $-\phi_\ell$ are the energy-dependent shift factor, penetration factor and hard-sphere phase shift, which are expressible in terms of Coulomb functions evaluated at the channel radius a, B_ℓ is the constant boundary condition parameter, and ω_ℓ is the Coulomb phase, all for the ^{7}Li(g.s.) + p channel (Lane and Thomas 1958). Also we take $P_\gamma = (E_\gamma/E_{\gamma_0})^3$, with $E_{\gamma_0} = 17{\cdot}64$ MeV. The eigenenergies E_λ^p are labelled by p to distinguish the 1^+ and 1^- levels, and the $\gamma_{\lambda s\ell}$ and $\gamma_{\lambda\gamma}^p$ are reduced width amplitudes. In equation (12), contributions to the summation from neutron channels and from proton channels to excited states of ^{7}Li are expected to be small and are neglected.

We also assume that two 1^- levels contribute to the $E1$ capture. Of these one ($\lambda = 1$) is taken to be the giant dipole resonance (GDR) based on the ^{8}Be ground state, which has been observed (Fisher *et al.* 1976) at $E_x = 21{\cdot}6$ MeV. This may be identified with the second 1^-, $T = 1$ shell model state of ^{8}Be, which has a large $E1$ matrix element to the ground state; it is predicted at $18{\cdot}26$ MeV, or $3{\cdot}24$ MeV above the lowest negative-parity state, which is 2^-, $T = 0$ and identified with the level observed at $18{\cdot}9$ MeV (van Hees and Glaudemans 1983, 1984). A 1^-, $T = 0$ level has been observed at $19{\cdot}4$ MeV (Ajzenberg-Selove 1988). Barker (1977) found that the ^{7}Li + n scattering lengths suggest a low-lying 1^- level of ^{8}Li, and hence a 1^-, $T = 1$ level of ^{8}Be at about $20{\cdot}3$ MeV. Van Hees and Glaudemans predict the lowest 1^-, $T = 1$ state $1{\cdot}88$ MeV below the GDR, and 1^-, $T = 0$ states $4{\cdot}50$ and $2{\cdot}09$ MeV below. Nominally $T = 0$ 1^- states would contribute to the ^{7}Li(p, γ_0)^{8}Be reaction only through isospin mixing with $T = 1$ states. Ulbricht *et al.* (1977) claimed evidence for a 1^- level at $17{\cdot}70$ MeV, but the justification for this has been contested (Barker 1979; Arnold and Seyler 1979). Thus the other assumed 1^- level ($\lambda = 2$) could represent an actual $1^-, T = 1$ level, or an isospin-mixed $T = 0$ level, or more generally some background contribution.

The R-matrix formulae for the transition matrix elements due to the 1^- levels can then be written

$$U_{s\ell} = 2ie^{i(\omega_\ell - \phi_\ell)}(P_\ell P_\gamma)^{\frac{1}{2}} \sum_{\lambda\mu} \gamma_{\lambda s\ell}\gamma_{\mu\gamma}^0 A_{\lambda\mu}\,, \qquad (13)$$

where

$$(\mathbf{A}^{-1})_{\lambda\mu} = (E_\lambda^0 - E)\delta_{\lambda\mu} - \sum_{s\ell} \gamma_{\lambda s\ell}\gamma_{\mu s\ell}(S_\ell - B_\ell + iP_\ell)\,, \qquad (14)$$

with $s\ell$ taking the values 10, 12 and 22.

There are therefore 18 adjustable parameters (four E_λ^p, ten $\gamma_{\lambda s\ell}$ and four $\gamma_{\lambda\gamma}^p$), in addition to a and B_ℓ. In order to obtain starting values for these parameters, we use one-level approximations to fit observed level energies and widths, and shell model calculations to obtain signs and ratios of reduced width amplitudes. For the particular level $\lambda(p)$, with observed resonance energy E_r, half width $\Gamma_{1/2}$

and radiation width Γ_γ, we take

$$E_\lambda^p = E_r + \sum_{s\ell} \gamma_{\lambda s\ell}^2 [S_\ell(E_r) - B_\ell] \,, \tag{15}$$

and identify the widths with the calculated 'observed widths' in the Thomas approximation

$$\Gamma_{1/2} \equiv \Gamma_\lambda^0 = \frac{2\sum_{s\ell} \gamma_{\lambda s\ell}^2 P_\ell(E_r)}{1 + \sum_{s\ell} \gamma_{\lambda s\ell}^2 [dS_\ell/dE]_{E_r}} \,, \tag{16}$$

$$\Gamma_\gamma \equiv \Gamma_{\lambda\gamma}^0 = \frac{2(\gamma_{\lambda\gamma}^p)^2 P_\gamma(E_r)}{1 + \sum_{s\ell} \gamma_{\lambda s\ell}^2 [dS_\ell/dE]_{E_r}} \,, \tag{17}$$

provided $B_\ell = S_\ell(E_r)$. The proton reduced width amplitudes are related to spectroscopic amplitudes $\mathcal{S}_{\lambda s\ell}^{\frac{1}{2}}$ by

$$\gamma_{\lambda s\ell} = \theta_{\lambda s\ell} (\hbar^2/Ma^2)^{\frac{1}{2}} \,, \tag{18}$$

$$\theta_{\lambda s\ell} = \theta_\ell^0 \mathcal{S}_{\lambda s\ell}^{\frac{1}{2}} \,, \tag{19}$$

where the single-particle dimensionless reduced width amplitude is given by

$$\theta_\ell^0 = \left(\frac{a}{2}\right)^{\frac{1}{2}} u_\ell(a) \Big/ \left[\int_0^a u_\ell^2(r)dr\right]^{\frac{1}{2}} \tag{20}$$

with $r^{-1}u_\ell(r)$ the radial wave function at energy E_r. We calculate the $\mathcal{S}_{\lambda s\ell}^{\frac{1}{2}}$ using the shell model code OXBASH (Brown *et al.* 1986). In order to obtain the signs correctly, allowance must be made for the different coupling order assumed in OXBASH and in the formulae of Seyler and Weller (1979), and for the different form of e.m. operator (Y_ℓ as compared with $i^\ell Y_\ell$).

For comparison with the analysis of Zahnow *et al.* (1995), we have also made fits in which the $E1$ contribution to the ^{7}Li(p,γ_0)^{8}Be reaction is not attributed to 1^- levels of ^{8}Be, but is taken from the s-wave direct-capture formula given by Zahnow *et al.* Then we get

$$U_{10} = [(16M/3\pi\hbar^2)S_{DC}(E_\mathrm{p})\exp(-88 \cdot 12E^{-0\cdot 5})]^{\frac{1}{2}} e^{i(\xi-\phi_0)} \,, \tag{21}$$

while $U_{12} = U_{22} = 0$. Here M is the reduced mass (in AMU), $S_{DC}(E_\mathrm{p})$ is as given by Zahnow *et al.*, E is in keV, and ξ is an arbitrary constant phase. Then U_{10} contains two adjustable parameters, $(C^2S)^{\frac{1}{2}}$ and ξ.

3. Data

We are interested in the ^{7}Li(p, γ_0)^{8}Be cross section at energies well below the GDR, which is at $E_\mathrm{p} \approx 5$ MeV. We therefore consider data for energies

$E_{\rm p} \leq 1\cdot 5$ MeV, say. In this energy range, Zahnow *et al.* (1995) have recently given values of the astrophysical S factor, for $E_{\rm p} \gtrsim 100$ keV. They also give values of the forward–backward anisotropy $I_{\gamma_0}(0°)/I_{\gamma_0}(150°)$. Earlier, Schlueter *et al.* (1964) had given values of the relative total cross section for $E_{\rm p} \gtrsim 400$ keV, as well as angular distribution coefficients a_k for $E_{\rm p} \gtrsim 800$ keV. Fisher *et al.* (1976) gave absolute values of $\sigma(90°)$ for $E_{\rm p} \gtrsim 800$ keV. Values of a_k may also be obtained from Table 1 of Mainsbridge (1960) (who actually gave the coefficients for an expansion in powers of $\cos\theta$) for $E_{\rm p} = 321 - 1100$ keV.

The coefficients B_k ($k = 1, 2$) given by Ulbricht *et al.* (1977) for $E_{\rm p} = 381-960$ keV are related to the quantities in the preceding section by $B_k = 3Tb_k$. Ulbricht *et al.* must originally have measured values of b_k; they then calculated values of B_k by taking values of T from Schlueter *et al.* (1965), normalised in the region $E_{\rm p} \approx 1$ MeV to the absolute values of Fisher *et al.* (1976). The values they used for T are, however, not very certain, particularly in the region of the 441 keV peak, which Ulbricht *et al.* took to have a width of $12\cdot 2$ keV, because of the large thickness of the target used by Schlueter *et al.* (32 keV at $E_{\rm p} = 441$ keV). Thus we have derived values of b_k and their uncertainties by fitting the values of the analysing power $A_y(\theta)$ given in Fig. 4 of Ulbricht *et al.*, using equation (3), with $W(\theta)$ taken from the measurements of Mainsbridge (1960).

Cecil *et al.* (1992) have given values of the S factor and angular distribution $W(\theta)$ at low energies, $E_{\rm p} = 40 - 170$ keV, and Chasteler *et al.* (1994) have measured the angular distribution and analysing power $A_y(\theta)$ at $E_{\rm p} \approx 70$ keV. These values are not included in our fits, but the predictions of the fits to the higher-energy data are compared with these low-energy values. The same applies to angular distribution measurements by Hahn *et al.* (1996), which were published after these calculations were completed.

In our fits, we use the values and assigned errors of the S factor and the forward–backward anisotropy from Zahnow *et al.* (1995). The a_k values are taken from Mainsbridge (1960) and Schlueter *et al.* (1964). Schlueter *et al.* gave the errors in their values. Mainsbridge gave only a constant error $\pm 0\cdot 036$ in his coefficient of $\cos^2\theta$, so we take an error $\pm 0\cdot 025$ in the a_2 values, and assume an error $\pm 0\cdot 05$ for a_1. The errors in the b_k values that we obtain from fits to the data of Ulbricht *et al.* (1977) correspond to an increase in χ^2 by χ^2/ν, where ν is the number of degrees of freedom.

4. Procedure and Results

The 18 adjustable level parameters in the R-matrix formulae of Section 2 are varied to give a least-squares best fit to the data of Section 3. We use the conventional value for the channel radius $a = 1\cdot 45(7^{\frac{1}{3}} + 1)\text{fm} = 4\cdot 22$ fm. Starting values of some of the parameters are obtained by fitting measured properties (energies, widths, radiative widths) of observed levels (Ajzenberg-Selove 1988) and by using results of shell model calculations (van Hees and Glaudemans 1983, 1984).

For the two 1^+ levels, identified as the $17\cdot 64$ and $18\cdot 15$ MeV levels of ^{8}Be, we choose $B_1 = S_1(E_1^1)$, which makes E_1^1 equal to the resonance energy ($0\cdot 386$ MeV) of the $17\cdot 64$ MeV level. Then a value of $\gamma_{111}^2 + \gamma_{121}^2$ is obtained by fitting the observed width of $10\cdot 7$ keV, using equation (16), and a value of $(\gamma_{1\gamma}^1)^2$ comes from fitting Γ^0_γ=16·7 eV, using equation (17). Values of γ^2_{111} and γ^2_{121} separately are obtained by using the measured channel spin ratio, $\gamma^2_{121}/\gamma^2_{111} = 3{\cdot}2$. Likewise starting values of $E^1_2, \gamma^2_{211}, \gamma^2_{221}$ and $(\gamma^1_{2\gamma})^2$ are obtained by fitting the energy, width, radiative width and channel spin ratio (taken as $\gamma^2_{221}/\gamma^2_{211} = 1{\cdot}5$, from Barker 1995) of the 18·15 MeV level, and allowing for the change of parameter values with change of B_1 (Barker 1972). The relative signs of the $\gamma_{\lambda s1}$ and $\gamma^1_{\lambda\gamma}$ are obtained from the shell model calculations (see Barker 1995); then $\gamma_{111}, \gamma_{121}, \gamma_{221}$ and $\gamma^1_{2\gamma}$ are all positive, while γ_{211} and $\gamma^1_{1\gamma}$ are negative. Some level properties are not well determined by the present data; these include the energy and width of the 18·15 MeV level, the best values of which ($E_r = 0{\cdot}896 \pm 0{\cdot}004$ MeV and $\Gamma = 138 \pm 6$ keV) come from the ^{7}Li(p, p$'\gamma$)^{7}Li and ^{10}B(d, α)^{8}Be reactions. Thus in most of our fits, including those described below, we do not change the values of E^1_2 and $\gamma^2_{211} + \gamma^2_{221}$.

Similarly for the two 1^- levels, one level ($\lambda = 1$) is identified as the GDR, and we choose $B_0 = S_0(E^0_1)$ and $B_2 = S_2(E^0_1)$. From the observed properties of the GDR, $E_x = 21{\cdot}6$ MeV, $\Gamma = 5{\cdot}3$ MeV and $\sigma_{tot} = 33\ \mu$b (Fisher *et al.* 1976), we obtain a value of the parameter E^0_1 and two relations between the parameters $\gamma^2_{110}, \gamma^2_{112} + \gamma^2_{122}$ and $(\gamma^0_{1\gamma})^2$. Since the GDR lies far above the range of the present data, these data do not determine well the GDR properties, so we do not vary the value of E^0_1 and we use the relations to determine the values of γ^2_{110} and $(\gamma^0_{1\gamma})^2$ in terms of $\gamma^2_{112} + \gamma^2_{122}$. The ratios and relative signs of these $\gamma_{1s\ell}$ are initially obtained from the shell model calculation, assuming for simplicity equal single-particle reduced widths for $\ell = 0$ and $\ell = 2$. The other 1^- level is initially assumed to be the lowest 1^-, $T = 1$ shell model state, lying 1·88 MeV below the GDR, but the parameter E^0_2 as well as $\gamma_{210}, \gamma_{2s2}$ and $\gamma^0_{2\gamma}$ are all allowed to vary in the fits. At this energy, this level is also a 'background' level.

In fitting the data, the parameters of the 1^+ levels are determined fairly well. The same is not true for the 1^- level parameters. With the energy E^0_2 of the second 1^- level kept fixed, the best fit ($\chi^2 = 833$ for a total of 127 data points) is obtained with unrealistically-large magnitudes for all the reduced width amplitudes of this level ($\gamma_{210}, \gamma_{2s2}$ and $\gamma^0_{2\gamma}$). With these reduced width amplitudes restricted, rather subjectively, to 'reasonable' values, the best fit gives $\chi^2 = 849$. Allowing E^0_2 to vary does not lead to significantly better fits. The parameter values for this best fit are given in Table 1, and the fits to the data are shown by the solid curves in Figs 1–4.

Table 1. Values of level parameters for the best fit to ^{7}Li(p, γ_0) ^{8}Be data, with two-level R-matrix approximation for both 1^+ and 1^- levels

$a = 4{\cdot}22$ fm, $B_\ell = S_\ell\ (E^p_1)$

J^π	p	λ	E^p_λ (MeV)	$\gamma^p_{\lambda\gamma}$ (MeV$^{\frac{1}{2}}$)	ℓ	$\gamma_{\lambda 1\ell}$ (MeV$^{\frac{1}{2}}$)	$\gamma_{\lambda 2\ell}$ (MeV$^{\frac{1}{2}}$)
1^+	1	1	0·385	−0·00305	1	0·284	0·516
		2	1·036	0·000864	1	−0·598	0·510
1^-	0	1	4·350	0·0164	0	−1·236	
					2	−0·743	−1·493
		2	2·472	0·00118	0	2·0	
					2	−2·318	−1·217

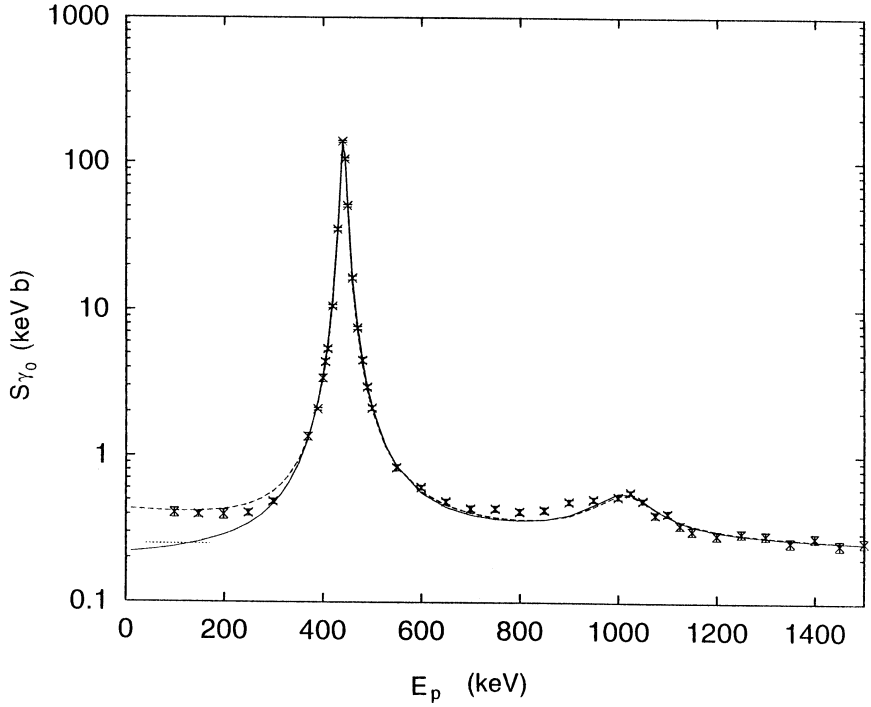

Fig. 1. The ^{7}Li(p, γ_0)^{8}Be S factor as a function of proton energy. The experimental points are from Zahnow *et al.* (1995). The solid curve corresponds to the parameter values in Table 1, giving the best simultaneous fit to the S factor, angular distribution and analysing power coefficients, and forward–backward anisotropy, when the $E1$ contribution is given by the R-matrix two-level approximation. The dashed curve, corresponding to Table 2, is a similar fit in which the $E1$ contribution is taken as s-wave direct capture. The dotted line between 40 and 170 keV indicates the experimental values of Cecil *et al.* (1992).

Although it may not be obvious at first glance, by far the largest χ^2 per data point comes from the b_1 fit (Fig. 3a); this is apparently due to the smallness of the errors compared with the change of b_1 between adjacent points. The next-largest values of χ^2 per data point come from S (Fig. 1) and a_2 (Fig. 2b). The negative excursion of the calculated a_2 values between 600 and 800 keV is due to the approximate vanishing of U_{11} in this energy range (see equation 7), and this is caused by the destructive interference between the contributions to U_{11} from the two 1^+ levels. This destructive interference was discussed previously (Barker 1979, 1995); it is due to the opposite signs of γ_{111} and γ_{211}, and of $\gamma^1_{1\gamma}$ and $\gamma^1_{2\gamma}$ (see Table 1). These signs are consistent with shell model calculations. From Fig. 1, it is seen that the calculated S factor at low energy strongly disagrees with the measurements of Zahnow *et al.* (1995). This difficulty is due to the increase with energy of the calculated $E1$ contribution to S from the 1^- levels, so that a good fit cannot be found to the data for both $E_p \lesssim 200$ keV and $E_p \gtrsim 1200$ keV. On the other hand, there is good agreement with the low-energy values of Cecil *et al.* (1992), which were not included in the fit.

There is also good agreement with the low-energy values of a_1, a_2 and b_1 obtained in earlier fits (Chasteler *et al.* 1994; Barker 1995) to the data of Chasteler

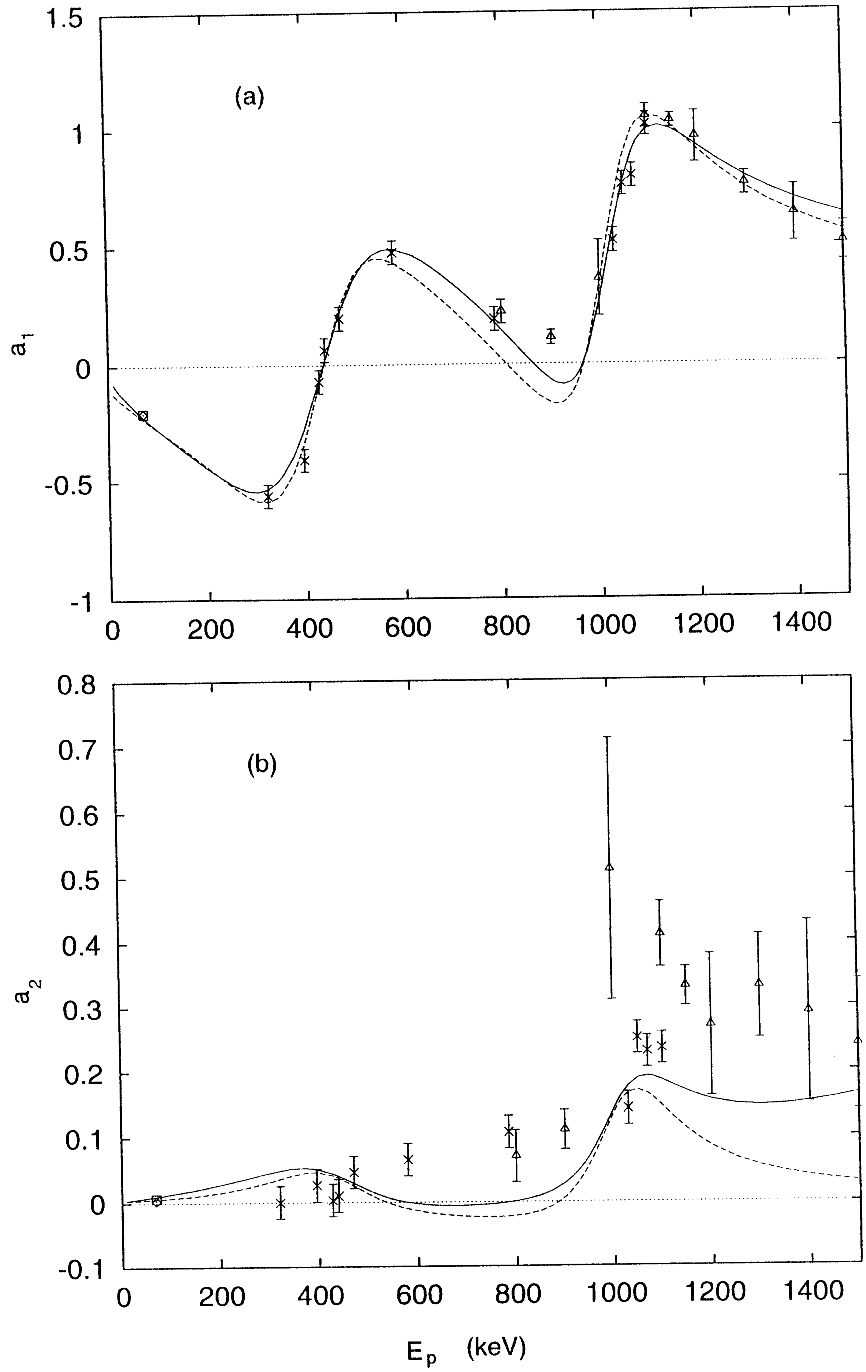

Fig. 2. Values of the ^{7}Li(p, γ_0)^{8}Be angular distribution coefficients (*a*) a_1 and (*b*) a_2 as functions of proton energy. The experimental points are from Mainsbridge (1960) (crosses) and Schlueter *et al.* (1964) (triangles). The curves are as in Fig. 1. The points at 70 keV are from previous fits to the data of Chasteler *et al.* (1994); they are the best fit for all transition matrix elements independently adjustable (Chasteler *et al.* 1994, as revised in Barker 1995) (square), and for the $M1$ contribution attributed to the 17·64 and 18·15 MeV levels of ^{8}Be, as in Fig. 1 of Barker (1995) (diamond).

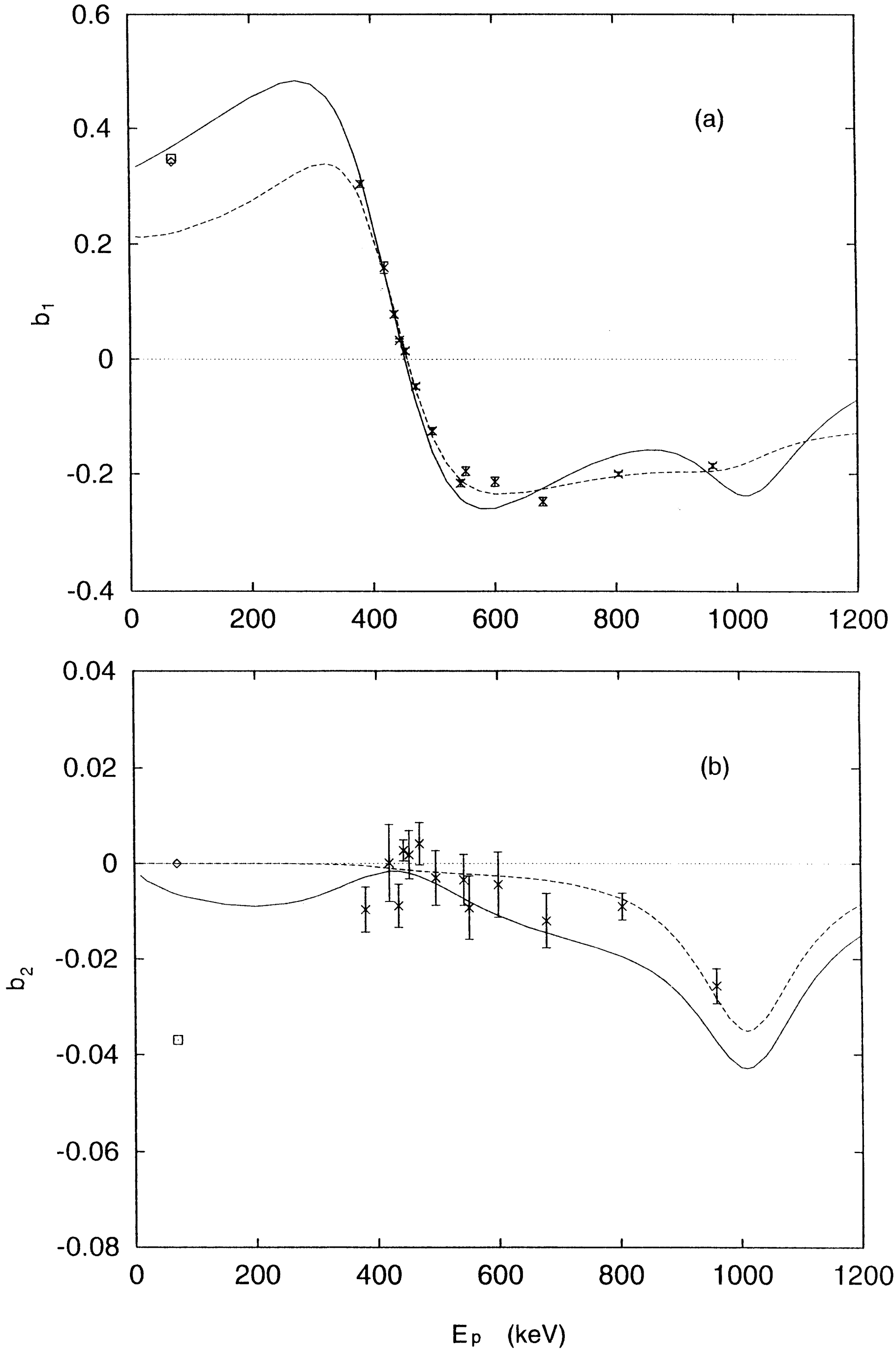

Fig. 3. Values of the ^{7}Li(p, γ_0)^{8}Be analysing power coefficients (*a*) b_1 and (*b*) b_2 as functions of proton energy. The experimental points are from Ulbricht *et al.* (1977) (crosses). The curves are as in Fig. 1, the 70 keV points as in Fig. 2.

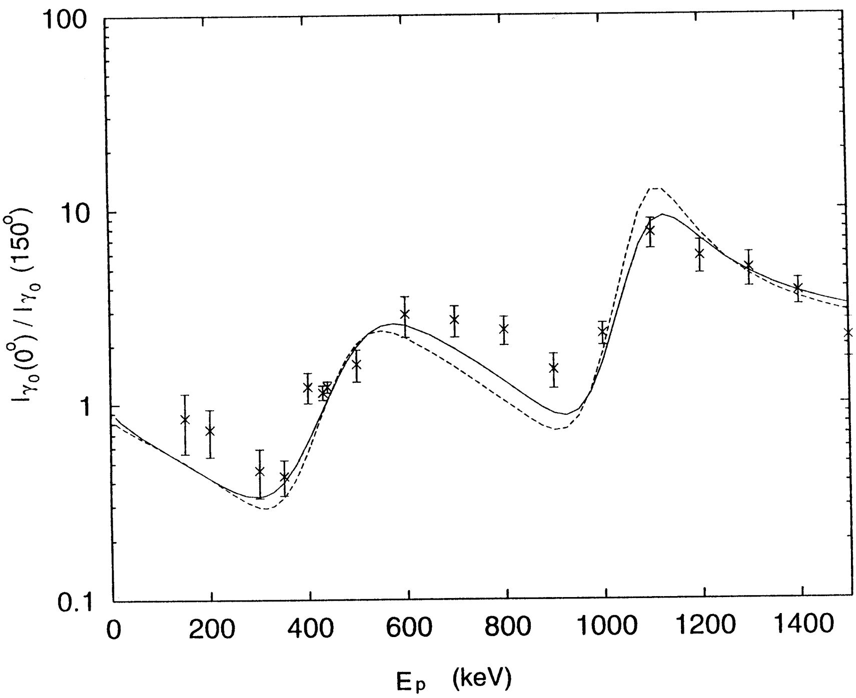

Fig. 4. Values of the ^{7}Li(p, γ_0)^{8}Be forward–backward anisotropy as a function of proton energy. The experimental points are from Zahnow *et al.* (1995). The curves are as in Fig. 1.

et al. at $E_p \approx 70$ keV, while b_2 lies within the range obtained in the earlier fits. The present parameter values give a fit to the Chasteler *et al.* data with $\chi^2 = 24\cdot38$ (no adjustable parameters, $\nu = 17$ degrees of freedom). This is to be compared with the best fits with four adjustable parameters (Barker 1995; Godwin *et al.* 1996) giving $\chi^2 = 18\cdot76$ ($\nu = 13$). Thus χ^2/ν is in fact smaller for the present fit. Using $R_\lambda = \gamma_{\lambda 21}^2/\gamma_{\lambda 11}^2$, we have $R_1 = 3\cdot29$ for the $17\cdot64$ MeV level and $R_2 = 0\cdot70$ for the $18\cdot14$ MeV level [for $B_1 = S_1(E_2^1)$]; these may be compared with the values $R_1 = 4\cdot4$ and $R_2 = 1\cdot5$ assumed in the previous comparable fit (Barker 1995), which led to $\chi^2 = 29\cdot8$. For comparison with that fit, we note that $r_1 = 0\cdot195, r_2 = 0\cdot254, \psi_1 = \psi_2 = 60\cdot8°$, giving $R = 1\cdot70$ and an $M1$ strength of $9\cdot3\%$. Godwin *et al.* (1996) say that there is only 'a 10% statistical chance that the correct solution contains less than 10% $M1$', but this neglects the change in the number of degrees of freedom.

Similar fits to the data of Section 3 have also been made using formulae in which the $E1$ contribution to the ^{7}Li(p, γ_0)^{8}Be cross section is attributed to s-wave direct capture rather than to transitions through 1^- levels of ^{8}Be. There are then 10 adjustable parameters; the best-fit values of these are given in Table 2, and the corresponding fits to the data are shown by the dashed curves in Figs 1–4. This fit has $\chi^2 = 705$, appreciably less than the value 849 obtained in the preceding fit. The reduction is due to improved fits to S, b_1 and b_2, although the fits to the other quantities are poorer. The improved fit to S is possible because the direct-capture $E1$ contribution to S decreases with

Table 2 Values of level parameters and direct-capture parameters for the best fit to ^{7}Li(p, γ_0)^{8}Be data, with the $E1$ contribution attributed to direct capture

$a = 4\cdot22$ fm, $B_1 = S_1$ (E_1^1)

J^π	λ	E_λ^1 (MeV)	$\gamma_{\lambda\gamma}^1$ (MeV$^{\frac{1}{2}}$)	$\gamma_{\lambda 11}$ (MeV$^{\frac{1}{2}}$)	$\gamma_{\lambda 21}$ (MeV$^{\frac{1}{2}}$)	$(C^2S)^{\frac{1}{2}}$	ξ (deg)
1^+	1	0·385	−0·00301	0·271	0·498	2·77	196·3
	2	1·036	0·000907	−0·659	0·428		

Table 3. Comparison of measured and predicted angular distribution coefficients for ^{7}Li(p, γ_0)^{8}Be

E_p (keV)	Measured[A]		Predicted			
			R-matrix fit		Direct-capture fit	
	a_1	a_2	a_1	a_2	a_1	a_2
73	−0·165±0·040		−0·222	0·010	−0·230	0·006
396	−0·236±0·037	0·022±0·057	−0·274	0·051	−0·334	0·046
442	0·079±0·019	−0·002±0·024	0·034	0·039	0·033	0·037

[A] Hahn *et al.* (1996).

increasing energy (see Fig. 4*a* of Zahnow *et al.* 1995). The low-energy S factor measurements of Cecil *et al.* (1992) are not supported. This best fit gives values of a_1, a_2 and b_2 at $E_\mathrm{p} = 70$ keV in agreement with those obtained in the earlier fits to the Chasteler *et al.* (1994) data, but there is significant disagreement for b_1, leading to $\chi^2 = 51\cdot55$. As in the fit of Zahnow *et al.* (1995), the value $C^2S = 7\cdot7$ is very large.

Values of a_1 and a_2 measured by Hahn *et al.* (1996) were not included in our fits. These values are compared with those predicted in the best fits in Table 3. There is reasonable agreement.

For both the R-matrix and direct-capture descriptions of the $E1$ contributions, attempts have been made to fit the data using different relative signs of the reduced width amplitudes for the 1^+ levels, the main aim being to avoid the destructive interference between the contributions to U_{11} from the two 1^+ levels, in the region between the levels, and so improve the fit to a_2 in this region. With starting values obtained as before, except that the sign of γ_{211} is taken as positive, the fitting procedure changes this sign and leads back to the previous best fits. If γ_{211} is restrained to remain positive, the a_2 data can be better fitted in the region between the 1^+ levels, but at the expense of a poorer fit in the region of the upper level and above, while the fits to a_1 and b_1 in this higher-energy region are much worse.

5. Discussion

Previous analyses of the low-energy ^{7}Li(p, γ_0)^{8}Be data by Chasteler *et al.* (1994) and Rolfs and Kavanagh (1994) and of the higher-energy data by Ulbricht *et al.* (1977) have been discussed above (Sections 2 and 3) and in Barker (1979, 1995). Weller and Chasteler (1995) also pointed out deficiencies in the Rolfs and Kavanagh analysis, and concluded that the low-energy data required at least an order-of-magnitude more p-wave strength than can be expected from the tails of the $E_p = 441$ and 1030 keV resonances. This conclusion was contradicted by the results of Barker (1995), who found acceptable fits to the data with the p-wave strength attributed to these two levels.

Zahnow *et al.* (1995) fitted their own data (values of the S factor and the forward–backward anisotropy), but used a formula for S that did not treat contributions from different channel spins in an adequate way, while their formula for the angular distribution was admittedly a simplified approximation. They did not include any analysing power data in their fits. As mentioned above, their assumption that the $E1$ component is due entirely to s-wave direct capture leads to very large spectroscopic factors. The criticism by Weller and Chasteler (1995) of the analysis by Zahnow *et al.* seems to be based on the incorrect belief that Zahnow *et al.* included a direct-capture $M1$ component.

In our earlier work (Barker 1995), the fit to the low-energy data (Chasteler *et al.* 1994) led to relative signs of the reduced width amplitudes of the ^{8}Be 1^+ levels at 17·64 and 18·15 MeV that were different from those obtained in the fit (Barker 1979) to the higher-energy data (Ulbricht *et al.* 1977), and the latter fit in particular suggested signs different from shell model values. Here we have found satisfactory fits to all the higher-energy data, including the recent measurements of Zahnow *et al.* (1995), using two approaches in which the $E1$ component of the ^{7}Li(p, γ_0)^{8}Be cross section is attributed to either 1^- levels of ^{8}Be or direct capture (Figs 1–4). The former is consistent with the low-energy data of Chasteler *et al.* and of Cecil *et al.* (1992). The parameter values for the two 1^+ levels are similar in the two approaches (Tables 1 and 2). The relative signs of the reduced width amplitudes for the 1^+ levels agree with shell model predictions.

The present fit differs from our earlier fits (Barker 1979, 1995) in taking the adjustable parameters as the level parameters themselves rather than the transition matrix elements. The main reason, however, for the present result being different from that obtained in fitting the higher-energy data (Barker 1979) appears to lie in the range of the data fitted. The two 1^+ levels are at $E_p \approx 441$ and 1030 keV. In the earlier fit, which found constructive interference in the $s = 1$ transition matrix element in the region between the levels, contrary to the shell model prediction, the data extended only up to $E_p = 960$ keV. Constructive interference in the region between the levels implies destructive interference above the upper level; this possibility is, however, ruled out by the additional data fitted here, which extend up to $E_p = 1500$ keV.

The present analysis confirms that the p-wave strength in the low-energy ^{7}Li(p, γ_0)^{8}Be cross section need not be as large as Chasteler *et al.* (1994) thought, and that it can be attributed to known 1^+ levels of ^{8}Be, with parameter values agreeing with shell model calculations. For the reaction ^{7}Be(p, γ)^{8}B, similar considerations would attribute any low-energy p-wave strength to the tail of the 1^+ first excited state of ^{8}B, and this is calculated to be negligible (see for example Fig. 7 of Filippone *et al.* 1983).

Godwin *et al.* (1996) found very small low-energy p-wave strength in the ^{7}Li(p, $\gamma_{16\cdot6}$)^{8}Be* reaction to the 2^+ state of ^{8}Be at 16·6 MeV, and say that this supports the conclusion of negligible p-wave strength in the ^{7}Be(p, γ)^{8}B reaction. The connection between these two reactions is, however, not very close. In ^{7}Be(p, γ)^{8}B, the $E1$ γ-transition is necessarily from an initial $T = 1$ state to a final $T = 1$ state. In ^{7}Li(p, $\gamma_{16\cdot 6}$)^{8}Be*, although the final 16·6 MeV state is a mixture of $T = 0$ and $T = 1$ components, with the $T = 1$ part being the isospin analogue of the ^{8}B ground state, the $E1$ transition to this $T = 1$ part can come only from $T = 0$ initial states (since $E1$ transitions in self-conjugate nuclei are forbidden between states of the same isospin). Likewise, transitions from initial $T = 1$ states can populate only the $T = 0$ part of the 16·6 MeV state.

6. Conclusion

The present R-matrix fits to the ^{7}Li(p, γ_0)^{8}Be cross section data for $E_p \leq 1500$ keV find parameter values in agreement with shell model calculations, and are satisfactory in most respects. They suggest the desirability of new measurements of the S factor at low energies, where the two presently-available measurements are inconsistent, and of the angular distribution coefficient a_2 in the energy region between the two 1^+ levels and above; the latter would be an extension to higher energies of the work of Hahn *et al.* (1996) (see Table 3).

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