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Laser Assisted Electron–Alkali Atom Collisions

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Abstract

Lasar assisted inelastic scattering of electrons by alkali atoms is studied theoretically. The non-perturbative quasi-energy method, which is generalised for many atomic states, is used to describe the laser–atom interaction, and the electron–atom interaction is treated within the first Born approximation. We have calculated the total cross section for the excitation of sodium atoms due to simultaneous electron–photon collisions. We show the effect of laser and collision parameters, e.g. laser intensity, polarisation and incident electron energy, on the excitation process.

1. Introduction

We present calculations of the collisionally-aided radiative excitation of sodium atoms due to electron impact in the presence of a laser field. Such a study is useful in the analysis of electron–atom collisions (Mittleman 1982) for a quantitative modelling of low and high temperature plasmas (Massey *et al.* 1984). The study of collision processes in the presence of a laser field has attracted considerable attention during the last few years. This is partly because of the availability of intense laser radiation sources and their application to populating resonantly excited states of atoms.

Mason and Newell (1982) reported experimental evidence of simultaneous electron–photon excitation of atoms. However, most experimental studies have been performed with noble gases (Wallbank *et al.* 1988, 1989, 1990), a recent one being with a Nd:YAG laser (Luan *et al.* 1991). On the other hand, theoretical studies are not easy to perform with these atoms and the hydrogen atom has been studied extensively (Rahman and Faisal 1978; Jetzke *et al.* 1984; Pundir and Mathur 1988; Frankcken *et al.* 1988; Bhattacharya *et al.* 1993). Sodium as a one electron atom is simple to deal with and it offers interesting information concerning the main features of the problem.

Also there have been many theoretical and experimental studies on electron impact excitation of sodium atoms in the absence of a laser field (Ganas 1985;

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Bray *et al.* 1991; Umakrishnan and Stump 1992). We want to show the effect of various collision and laser parameters on the collision process. This is the motivation of the paper. Furthermore, most theoretical studies for scattering of electrons by atoms in an intense radiation field are based on perturbation theory (Gersten and Mittleman 1976; Byron and Joachain 1984; Gravila *et al.* 1990). Starting with the well known Kroll and Watson (1976) work on the soft photon approximation, there exists only a few non-perturbative approaches for this problem. Shakeshaft (1983) formulated a non-perturbative method of coupled integral equations for calculating the scattering cross section by assuming the potential to be separable. Rosenberg (1981) applied the variational method for Coulomb scattering in a laser field using a low frequency approximation.

In this paper we used a recently developed non-perturbative method (Agre and Rapport 1982; Sharma and Mohan 1992) to study the laser assisted electron impact excitation of sodium atoms. We have used this method quite successfully to study various collisional problems (Sharma *et al.* 1993; Prasad *et al.* 1996). The reaction studied in the present work is

$$e^-(\boldsymbol{k}_\mathrm{i}) + \mathrm{Na}(i) + N(\omega,\, \hat{\epsilon}) \;\rightarrow\; e^-(\boldsymbol{k}_\mathrm{f}) + \mathrm{Na}(j) + (N \pm L)\gamma(\omega,\, \hat{\epsilon})\,,$$

representing the collision of an incoming electron with momentum $\boldsymbol{k}_\mathrm{i}$ with a sodium atom initially in the state i in the presence of a single mode laser beam moving to excited state j, with exchange of L photons between the electron and laser field. We have used the Born approximation, to treat the electron–atom interaction as it is simple enough to allow calculation of a larger number of transitions, and it becomes exact at high energies. In the alkalis all cross sections (zero field case) measured so far (Stump and Gallagher 1985) converge in the Born approximation at about 100 times the threshold energy (E_t). In Section 2 we present the general theory for the excitation process using the non-perturbative quasi-energy approach. The results are discussed in Section 3. We use atomic units throughout this paper, otherwise the units are mentioned.

2. Theory

We consider a collision between an electron and sodium atom in the presence of a laser field. The field is assumed to be purely monochromatic with angular frequency ω, linearly polarised with linear polarisation vector $\hat{\epsilon}$. We also assume that the dipole approximation is valid. The Hamiltonian of the electron–atom system in the presence of a laser beam can be written as

$$H = H_\mathrm{f} + H_\mathrm{a} + V(\boldsymbol{r}_0,\, \boldsymbol{r}_\mathrm{a})\,, \tag{1}$$

where H_f and H_a are respectively the Hamiltonian of a free electron and atom in the presence of a laser field, $\boldsymbol{r}_0$ is the coordinate of the incident electron, $\boldsymbol{r}_\mathrm{a}$

that of the atomic electron, and $V(\boldsymbol{r}_0, \boldsymbol{r}_{\rm a})$ is the interaction between the incident electon and atom, defined as

$$V(\boldsymbol{r}_0, \boldsymbol{r}_{\rm a}) = \frac{-Z}{|\boldsymbol{r}_0|} + \sum_{j=1}^{Z} \frac{1}{|\boldsymbol{r}_{0j}|} \,. \tag{2}$$

In equation (2) Z is the charge of the target atom. Working in the Coulomb gauge we have the electric field $\boldsymbol{E}(t) = E_0\,\hat{\epsilon}\,\sin\omega t$, and the corresponding vector potential $\boldsymbol{A}(t) = \hat{\epsilon} A_0 \cos\omega t$, with $A_0 = cE_0/\omega$. In the presence of a laser field the incident electron of momentum $\boldsymbol{k}$ is represented as

$$\chi_{\boldsymbol{k}}(\boldsymbol{r}_0,\, t) = (2\pi)^{-3/2} \mathrm{e}^{i(\boldsymbol{k}\cdot\boldsymbol{r}_0 - \boldsymbol{k}\,\alpha_0 \sin\omega t - E_k t)} \,, \tag{3}$$

where $E_k = k^2$ and $\alpha_0 = E_0/\omega^2$ is a measure of the coupling between the field and projectile.

It is well known that the quantum levels of an atom are modified due to a laser field. These modified quantum levels are known as quasi-energy states (QES) and satisfy the equation

$$\mathrm{i}\,\frac{\partial \Phi}{\partial t} = H_{\rm a} \Phi \,, \tag{4}$$

where $H_{\rm a} = H_0(\boldsymbol{r}) + w(\boldsymbol{r},t)$, $H_0(\boldsymbol{r})$ being the Hamiltonian of the isolated sodium atom and $w(\boldsymbol{r},t)$ the interaction of the electromagnetic field with the sodium atom.

If we assume that the atom is interacting with radiation which is nearly resonant (i.e. $\epsilon_{n0} < \omega$, where ϵ_{n0} is n photon detuning) and that the intensity of radiation is not too high, then the solution of the Schrödinger equation (4) can be written as (Agre and Rapport 1982; Sharma and Mohan 1992; Prasad *et al.* 1996)

$$\Phi_n(\boldsymbol{r},\, t) = \mathrm{e}^{-\mathrm{i}(E_1 + \lambda_n t)} [\alpha_1^n\, U_1(\boldsymbol{r}) + a_2^n\, U_2(\boldsymbol{r})\, \mathrm{e}^{-\mathrm{i}\omega t} + a_3^n\, U_3(\boldsymbol{r})\, \mathrm{e}^{-2\mathrm{i}\omega t}] \,, \tag{5}$$

where $U(\boldsymbol{r})$ are the unperturbed (or bare) atomic states, E_1 is the ground state energy of the bare atom, the a_j are amplitudes corresponding to the bare atomic states, and λ_n are defined as quasi-energies. It is to be noted that we have written solution (5) only for three states, but it can be generalised for more (multiplets).

Substituting (5) into (4) and assuming that detuning is much smaller than the frequency of the laser beam, we get a quasi-energy matrix for electron impact excitation of the Na atom from an initial state i to final state f, in the presence of a laser field, given by

$$S_{\rm i\to f} = -\mathrm{i} \int_{-\infty}^{+\infty} \langle \chi_{k_{\rm f}}(\boldsymbol{r}_0,\, t)\, \Phi_{\rm f}(\boldsymbol{r},\, t) | V(\boldsymbol{r}_0,\, \boldsymbol{r}) | \chi_{k_{\rm i}}(\boldsymbol{r}_0,\, t) \Phi_{\rm i}(\boldsymbol{r},\, t) \rangle \,, \tag{6}$$

where V is the interaction potential defined by (2) and $\boldsymbol{k}_{\mathrm{i}}$ and $\boldsymbol{k}_{\mathrm{f}}$ are the initial and final momenta of the incident electron. We can write (6) as

$$S_{\mathrm{i}\to\mathrm{f}} = -\mathrm{i}\int_{-\infty}^{+\infty} \mathrm{d}t^{1}(2\pi)^{-3}\mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\alpha_0\sin\omega t}\,\mathrm{e}^{\mathrm{i}(E_{k_\mathrm{f}}-E_{k_\mathrm{i}})t}$$

$$\times\ \mathrm{e}^{\mathrm{i}(\lambda_\mathrm{f}-\lambda_\mathrm{i})t}[a_1^\mathrm{f}\,U_1(r)+a_2^\mathrm{f}\,U_2(r)\,\mathrm{e}^{-\mathrm{i}\omega t}+a_3^\mathrm{f}\,U_3^\mathrm{f}(r)\,\mathrm{e}^{-2\mathrm{i}\omega t}$$

$$\times\ |V(\boldsymbol{r}_0,\boldsymbol{r})|a_1^\mathrm{i}\,U_1(r)+a_2^\mathrm{i}\,U_2(r)\,\mathrm{e}^{-\mathrm{i}\omega t}+a_3^\mathrm{i}\,U_3(r)\,\mathrm{e}^{-2\mathrm{i}\omega t}]\,, \qquad (7)$$

where $\boldsymbol{q}=\boldsymbol{k}_\mathrm{i}-\boldsymbol{k}_\mathrm{f}$.

Using generating functions for Bessel functions we get

$$\mathrm{e}^{\mathrm{i}q\alpha_0}\sin\omega t = \sum_{L=-\infty}^{+\infty} J_\mathrm{L}(q\alpha_0)\mathrm{e}^{-\mathrm{i}L\omega t}\,, \qquad (8)$$

$$\Delta = E_{k_\mathrm{f}} - E_{k_\mathrm{i}} + \lambda_\mathrm{f} - \lambda_\mathrm{i}\,, \qquad \int_{-\infty}^{+\infty} \mathrm{e}^{\mathrm{i}\Delta L\omega t}\,\mathrm{d}t = 2\pi\,\delta(\Delta - L\omega)\,.$$

When specialised to the case of a single mode laser with occupation number $N \gg 1$, the contribution of the photon field reduces to a multiplicative factor common to each amplitude entering in (7) over all transition matrix elements. It is then easy to sum over the free wave projectile states, using in particular the known result

$$\sum_{j=1}^{Z}\langle x_{k_\mathrm{f}}|\frac{1}{r_{0j}} - \frac{1}{r_0}|x_{k_\mathrm{i}}\rangle = \sum_j (4\pi/q^2)(\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}_\mathrm{j}} - Z)\,.$$

Equation (7) becomes

$$S_{\mathrm{i}\to\mathrm{f}} = (-\mathrm{i})(2\pi)^{-2}\sum_{L=-\infty}^{+\infty}\delta[E_{k_\mathrm{f}} - E_{k_\mathrm{i}} + \lambda_\mathrm{f} - \lambda_\mathrm{i} - L\omega]$$

$$\times\ \{J_L(q\alpha_0)[a_1^\mathrm{f}\,a_1^\mathrm{i}\,T_{1,1}^{\mathrm{B1}}(q_n) + a_2^\mathrm{f}\,a_2^\mathrm{i}\,T_{2,2}^{\mathrm{B1}}(q) + a_3^\mathrm{f}\,a_3^\mathrm{i} T_{3,3}^{\mathrm{B1}}(q)]$$

$$+\ J_{L-1}(q\alpha_0)[a_1^\mathrm{f}\,a_2^\mathrm{i}\,T_{1,2}^{\mathrm{B1}}(q_\mathrm{i}) + a_2^\mathrm{f}\,a_3^\mathrm{i}\,T_{2,3}^{\mathrm{B1}}(q)]$$

$$+\ J_{L+1}(q\alpha_0)[a_2^\mathrm{f}\,a_1^\mathrm{i}\,T_{2,2}^{\mathrm{B1}}(q) + a_3^\mathrm{f}\,a_2^\mathrm{i}\,T_{3,2}^{\mathrm{B1}}(q)]$$

$$+\ J_{L+2}(q\alpha_0)[a_1^\mathrm{f}\,a_3^\mathrm{i}\,T_{1,3}^{\mathrm{B1}}(q)] + J_{L-2}(q\alpha_0)[a_3^\mathrm{f}\,a_1^\mathrm{i}\,T_{3,1}^{\mathrm{b1}}(q)]\}\,, \qquad (9)$$

where $J_n(q\alpha_0)$ is an ordinary Bessel function of order n and

$$
\begin{aligned}
T_{\mathrm{i,f}}^{\mathrm{B1}}(q) = &-(2\pi)^{-1}\langle\langle \mathrm{e}^{\mathrm{i}\boldsymbol{k}_\mathrm{f}\cdot\boldsymbol{r}_0}\,U_\mathrm{f}(r_0)|V|\mathrm{e}^{\mathrm{i}\boldsymbol{k}_\mathrm{f}\cdot\boldsymbol{r}_0}\,U_\mathrm{i}(r)\rangle\rangle \\
&= -(2\pi)^{-1}\int \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}_0}\langle U_\mathrm{i}|\frac{-Z}{r_0} + \sum_{j=1}^{Z}\frac{1}{r_{0j}}|U_\mathrm{f}\rangle \mathrm{d}r_0 \\
&= -(2\pi)^{-1}(4\pi/q^2)\langle U_\mathrm{i}(r)|\sum_{j=1}^{Z}(\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}_\mathrm{j}} - Z|U_\mathrm{f}(\boldsymbol{r})\rangle \\
&= -(2/q^2)\langle U_\mathrm{i}(r)|\sum_{j=1}^{Z}(\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}_\mathrm{j}} - Z)|U_\mathrm{f}(\boldsymbol{r})\rangle\,.
\end{aligned}
\tag{10}
$$

We can write (9) as

$$
S_{\mathrm{i}\to\mathrm{f}} = (2\pi)^{-1}\mathrm{i}\sum_{L=-\infty}^{+\infty}\delta(E_{k_\mathrm{f}} - E_{k_\mathrm{i}} + \lambda_\mathrm{f} - \lambda_\mathrm{i} - L\omega)f_{\mathrm{i}\to\mathrm{f}}^{\mathrm{B1}}\,, \tag{11}
$$

where

$$
\begin{aligned}
f_{\mathrm{i}\to\mathrm{f}}^{\mathrm{B1}} = &J_L(q\alpha_0)[a_1^\mathrm{f}\,a_1^\mathrm{i}\,T_{1,1}^{\mathrm{B1}}(q) + a_2^\mathrm{f}\,a_2^\mathrm{i}\,T_{2,2}^{\mathrm{B1}}(q) + a_3^\mathrm{f}\,a_3^\mathrm{i}\,T_{3,3}^{\mathrm{B1}}(q)] \\
&+ J_{L-1}(q\alpha_0)[a_1^\mathrm{f}\,a_2^\mathrm{i}\,T_{1,2}^{\mathrm{B1}}(q) + a_2^\mathrm{f}\,a_3^\mathrm{i}\,T_{2,3}^{\mathrm{B1}}(q)] \\
&+ J_{L+1}(q\alpha_0)[a_2^\mathrm{f}\,a_1^\mathrm{i}\,T_{2,1}^{\mathrm{B1}}(q) + a_3^\mathrm{f}\,a_2^\mathrm{i}\,T_{3,2}^{\mathrm{B1}}(q)] \\
&+ J_{L-2}(q\alpha_0)[a_1^\mathrm{f}\,a_3^\mathrm{i}\,T_{1,3}^{\mathrm{B1}}(q)] + J_{L+2}(q\alpha_0)[a_3^\mathrm{f}\,a_1^\mathrm{i}\,T_{3,1}^{\mathrm{B1}}(q)]\,,
\end{aligned}
\tag{12}
$$

with T elements defined by (10).

If the excited states of the optical or valence electron in an alkali atom are described as single electron states in a spherically symmetric potential, the matrix element (10) is defined as

$$
T_{\mathrm{i,f}}^{\mathrm{B1}} = -(2|q^2)\langle U_\mathrm{i}(\boldsymbol{r})|\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}} - 1|U_\mathrm{f}(\boldsymbol{r})\rangle\,,
$$

where $\boldsymbol{r} = (r,\theta,\phi)$ is the position vector of the atomic valence electron and $\boldsymbol{q} = \boldsymbol{k}_\mathrm{i} - \boldsymbol{k}_\mathrm{f}$ is the momentum transfer. Writing

$$
U_\mathrm{i}(r) = R_{nl}(r)\,Y_{lm}(\theta,\phi); \qquad U_\mathrm{f}(r) = R_{n'l'}(r)\,Y_{l'm'}(\theta,\phi)
$$

gives

$$
\begin{aligned}
T^{\mathrm{B1}}_{\mathrm{i,f}} = &- (2|q^2)\langle R_{nl}(r)\,Y_{lm}(\theta,\,\phi)|\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}} - 1|R_{n'l'}(r)\,Y_{l'm'}(\theta,\phi)\rangle \\
&= -(2|q^2)\langle R_{nl}(r)\,Y_{lm}(\theta,\,\phi)|\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}|R_{n'l'}(r)Y_{\,l'm'}(\theta,\,\phi)\rangle \\
&+ (2|q^2)\delta_{nn'}\,\delta_{ll'}\,\delta_{mm'} \\
&= -(2|q^2)\langle R_{nL}(r)\,Y_{lm}(\theta,\,\phi)|\sum_{\lambda=-\infty}^{+\infty}\sum_{\mu=-\lambda}^{+\lambda}\partial_\lambda(qr)4\pi. \\
&\times\; Y^*_{\lambda\mu}(\hat{q})\,Y_{\lambda\mu}|R_{n'l'}(r)\,Y_{l'm'}(\theta,\,\phi)\rangle + (2|q^2)\delta_{nn''}\delta_{ll'}\,\delta_{mm'} \\
&= (2|q^2)\delta_{nn'}\,\delta_{ll'}\,\delta_{mm'} - (2|q^2)\langle R_{nL}(r)\,Y_{lm}(\theta,\,\phi)|\sum_{\lambda=-\infty}^{+\infty}\sum_{\mu=-\lambda}^{+\lambda} \\
&\times\; \mathrm{i}^\lambda\partial_\lambda(qr)\,4\pi\,Y^*_{\lambda\mu}(\hat{q})\,Y_{\lambda\mu}(r)|R_{n'l'}(r)\,Y_{l'm'}(\theta,\,\phi)\rangle\,.
\end{aligned}
\tag{13}
$$

Here λ and μ are quantum numbers of the collision coupled angular momentum and $|l-l'| < \lambda < |l+l'|$, $\mu = -\lambda,\, -\lambda+1, \cdots, +\lambda$. Further, $\hat{q}$ is the unit vector in the direction of momentum transfer $\boldsymbol{q}$, and ∂_λ is a spherical Bessel function. The first Born differential cross section corresponding to various multiphoton processes is given by

$$
\frac{\mathrm{d}\sigma^{\mathrm{B1},L}_{\mathrm{i}\to\mathrm{f}}}{\mathrm{d}\Omega} = \frac{k_\mathrm{f}}{k_\mathrm{i}}|f^{\mathrm{Bl}}_{\mathrm{i}\to\mathrm{f}}|^2\,. \tag{14}
$$

Using (13) for $T^{\mathrm{Bl}}_{\mathrm{i}\to\mathrm{f}}$ along with (12) for $f^{\mathrm{Bl}}_{\mathrm{i}\to\mathrm{f}}$ we can get the differential cross section.

The total cross section is then evaluated as

$$
\sigma_{\mathrm{i}\to\mathrm{f}} = \int_{k_{\min}}^{k_{\max}}\left(\frac{\mathrm{d}\sigma_{\mathrm{i}\to\mathrm{f}}}{\mathrm{d}\Omega}\right)\mathrm{d}\boldsymbol{k}\,, \tag{15}
$$

with $k_{\min} = |\boldsymbol{k}_\mathrm{i} - \boldsymbol{k}_\mathrm{f}|$ (forward scattering $\theta = 0$) and $k_{\max} = |\boldsymbol{k}_\mathrm{i} + \boldsymbol{k}_\mathrm{f}|$ (backward scattering ($\theta = \pi$).

3. Results and Discussion

Here we have studied the laser assisted electron impact excitation of sodium atoms. The quasi-energy states of the atom are calculated as described earlier. Though we show results for only three transitions, in our calculations we have included the lowest five states of the sodium atom. The quasi-energy matrix has been diagonalised using standard diagonalisation routines to yield the quasi-energies and corresponding eigenvectors. In constructing the quasi-energy matrix we require dipole matrix elements between adjacent atomic states. We have calculated the dipole matrix elements numerically.

Table 1. Variation of the total cross section with laser frequency for the 3s–3p transition
Here $\hat{\epsilon}||\boldsymbol{q}$ and the laser intensity $I = 10^6$ W cm^{-2}. In all tables the values in parentheses give the power of ten by which the entry should be multiplied

Laser frequency (eV)	$\sigma_{3s\text{-}3p}$ (πa_0^2) $E_i = 50\ E_t$	$E_i = 100\ E_t$
0·2	0·3028(−9)	0·2686(−7)
0·04	0·2050(−10)	0·2095(−8)
0·6	0·3973(−11)	0·4916(−9)
0·8	0·12706(−11)	0·1711(−9)
1·0	0·1097(−11)	0·1483(−9)
1·2	0·2725(−11)	0·1396(−8)
1·4	0·3602(−10)	0·1206(−7)
1·6	0·4229(−10)	0·1734(−7)
1·8	0·3609(−9)	0·5439(−6)
2·0	0·2993(−9)	0·3673(0)
2·2	0·5156(+4)	0·8531(+4)
2·4	0·4867(+3)	0·8430(+3)
2·6	0·4788(+1)	0·8344(+1)
2·8	0·4616(+1)	0·7130(+1)
3·0	0·4447(0)	0·8010(0)
5·0	0·2218(0)	0·7999(9)

Table 2. Variation of the total cross section with laser frequency for the 3p–3d transition
Here $\hat{\epsilon}||\boldsymbol{q}$ and the laser intensity $I = 10^6$ W cm^{-2} and $E_i = 50\ E_t$

Laser frequency (eV)	$\sigma_{3p\text{-}3d}$ (πa_0^2)	Laser frequency (eV)	$\sigma_{3p\text{-}3d}$ (πa_0^2)
0·2	0·2667(−7)	1·8	0·2495(+4)
0·4	0·2095(−8)	2·0	0·8677(+2)
0·6	0·4816(−8)	2·2	0·7953(+1)
0·8	0·1711(−9)	2·4	0·7304(+1)
1·0	0·1483(−9)	2·6	0·6732(0)
1·2	0·1396(−8)	2·8	0·6203(0)
1·4	0·1207(−6)	3·0	0·5733(0)
1·6	0·1042(+5)	5·0	0·1503(0)

We have calculated the total cross section for the excitation of the sodium atom from the ground 3s states to 3p and 3d states and from the 3p to 3d state by electron impact in the presence of a lineraly polarised monochromatic laser field neglecting the exchange effect. In Table 1 we present the variation of the total cross section (in πa_0^2) with laser frequency (in eV) for the 3s–3p transition for two incident electron energies with the absorption of one photon (i.e. $L = +1$). Here E_t is the threshold energy. We have taken the laser intensity to be $I = 10^6$ W cm^{-2}, and $\hat{\epsilon}||\boldsymbol{q}$. As shown, the cross section increases quite sharply near $\omega = 1\cdot81$ eV, due to the fact that near $1\cdot81$ eV one photon resonance condition is satisfied. As is clear, the order of the cross section rises many times, a behaviour also seen by others (Yousef *et al.* 1988).

In Table 2 we present the variation of the total cross section with laser frequency (in eV) for the 3p–3d transition for the $\hat{\epsilon}||\boldsymbol{q}$ case. Here also the laser intensity is $I = 10^6$ W cm^{-2}. In this case the cross section shows a sharp enhancement near $\omega = 2 \cdot 0$ eV, again close to the 3p–3d transition frequency.

Table 3. Variation of the total cross section with laser frequency for the 3p–3d transition
Here $I = 10^6$ W cm^{-2} and $E_i = 50\ E_t$

Laser frequency (eV)	$\sigma_{3p\text{–}3d}\ (\pi a_0^2)$ $L = +1$	$L = -1$
0·2	0·1551(+3)	0·1471(+3)
0·4	0·1504(+3)	0·1403(+3)
0·6	0·1591(+3)	0·1374(+3)
0·8	0·1601(+3)	0·1233(+3)
1·0	0·1743(+3)	0·1583(+3)
1·2	0·1774(+3)	0·1596(+3)
1·4	0·1501(+3)	0·1616(+3)
1·6	0·1125(−2)	0·1070(−2)
1·8	0·0859(+3)	0·8040(−3)
1·81	0·2254(+5)	0·2137(+5)
2·0	0·14924(+5)	0·12705(+5)
2·2	0·52710(+4)	0·2010(+4)
2·4	0·69635(−2)	0·4099(−2)
2·6	0·2932(−2)	0·1080(−3)
2·8	0·1694(−2)	0·5829(−3)
3·0	0·1137(−2)	0·3805(−3)
3·2	0·8227303(−3)	0·2973(−1)
3·4	0·63433(−3)	0·3346(+1)
3·6	0·5754(−3)	0·567(−5)
3·61	0·3743(+1)	0·3349(+1)
5·0	0·38592(−4)	0·6617(−5)

In Table 3 we show a similar variation of the total cross section for the 3s–3d transition, but for the two cases $L = +1$ and $L = -1$. Here also the laser intensity is $I = 10^6$ W cm^{-2} and the energy of the incident is $E_i = 50\ E_t$. As shown, the one photon absorption cross section has larger values, as compared to the one photon emission case. Table 3 also shows that the 3s–3d cross section increases quite sharply near $\omega = 1 \cdot 8$ eV, which is because the two photon resonance condition is satisfied for this transition near 1·8 eV. Beyound 1·81 eV the cross section decreases and again it increases near 3·61 eV, which is due to the one photon resonance condition being satisfied, between the 3s and 3d states.

In Fig. 1*a* we show the variation of the total cross section for the 3s–3p transition with laser intensity. We have taken the $\hat{\epsilon}||\boldsymbol{q}$ case and the energy of the incident electron is $E_i = 50\ E_t$. As can be seen, the total cross section increases rapidly in the intensity range 10^6–10^{12} W cm^{-2}, and beyond this the cross section decreases. The decrease in the high intensity region is due to the mixing of all atomic states. Fig. 1*b* shows a similar variation for the 3p–3d transition.

Figs 2 and 3 show the variation of total cross section with incident electron energy for various laser parameters for different transitions. They show the usual variation of cross section with the incident electron energy. In Fig. 3 for comparision we show the variation of the zero field cross section (dashed line).

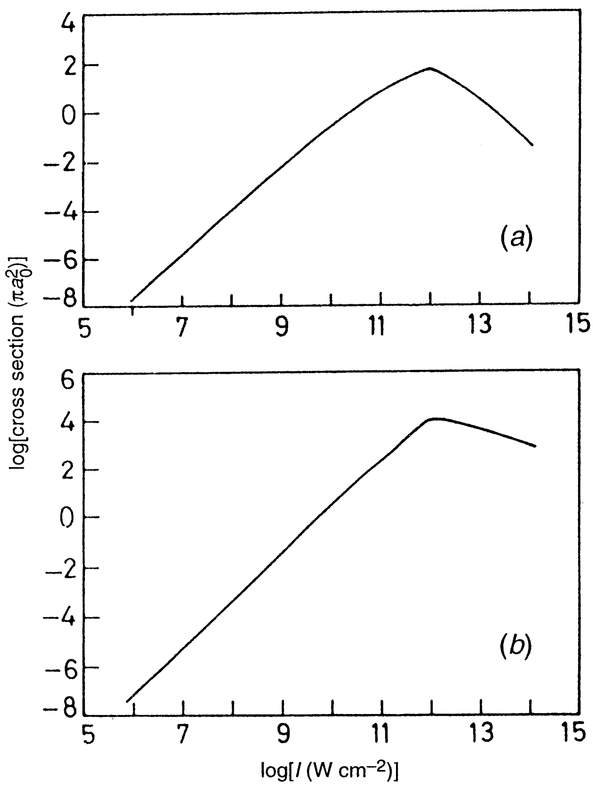

Fig. 1. Variation of the total cross section with laser intensity for (a) the 3s–3p transition and (b) the 3p–3d transition, for the case $\hat{e}||\boldsymbol{q}$. The laser frequency is $\omega = 0 \cdot 5$ eV and $E_{\mathrm{i}} = 50\ E_{\mathrm{t}}$.

As shown, the cross section for the near resonant case has a much larger value, thus illustrating the effect of the near resonance laser field on collisional excitation processes.

4. Conclusion

We have described a combination of the non-perturbative quasi-energy approach for the radiation–atom interaction and the Born approximation for the collision aided radiative excitation of a dressed atom. The advantage of the non-perturbative treatment is that the interaction can be taken to all orders. Our methods can be extended to any complex collision with an accurate interaction potential between the projectile and target, but the physics behind the laser assisted collision remains essentially the same.

At present no experimental or theoretical work is available to compare with the present study. However, with the advances in laser technology it is highly likely that such experiments on the electron–atom energy may be performed in the near future.

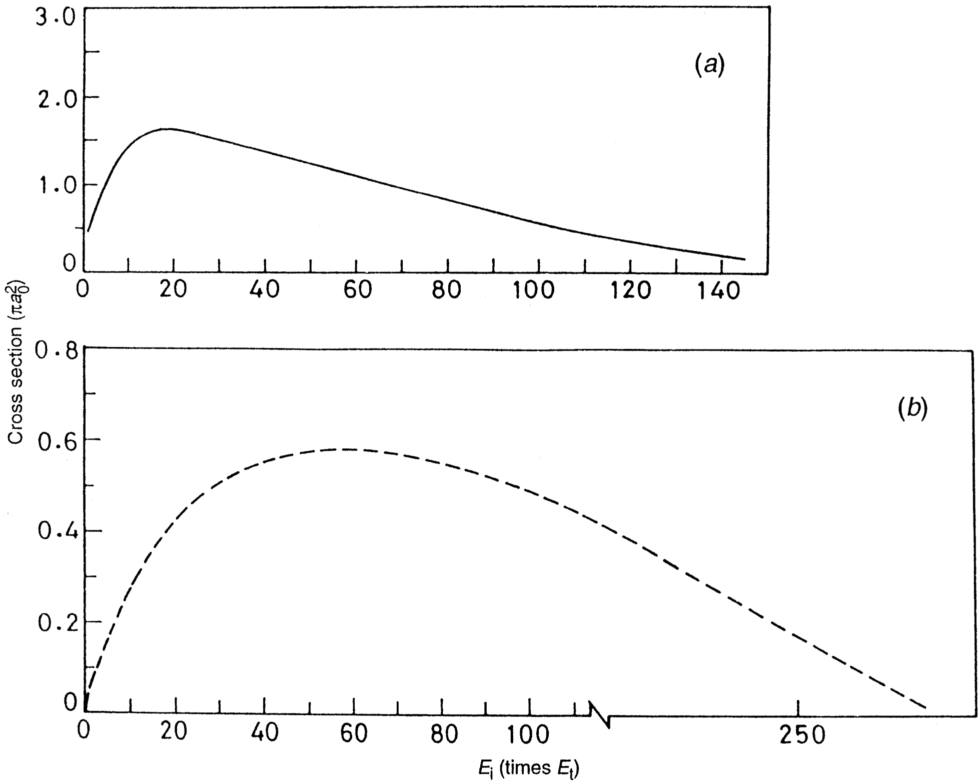

Fig. 2. Variation of the total cross section with incident electron energy for the 3p–3d transition with (*a*) $\omega = 0{\cdot}5$ eV and (*b*) $\omega = 3{\cdot}0$ eV and with laser intensity $I = 10^6$ W cm^{-2}.

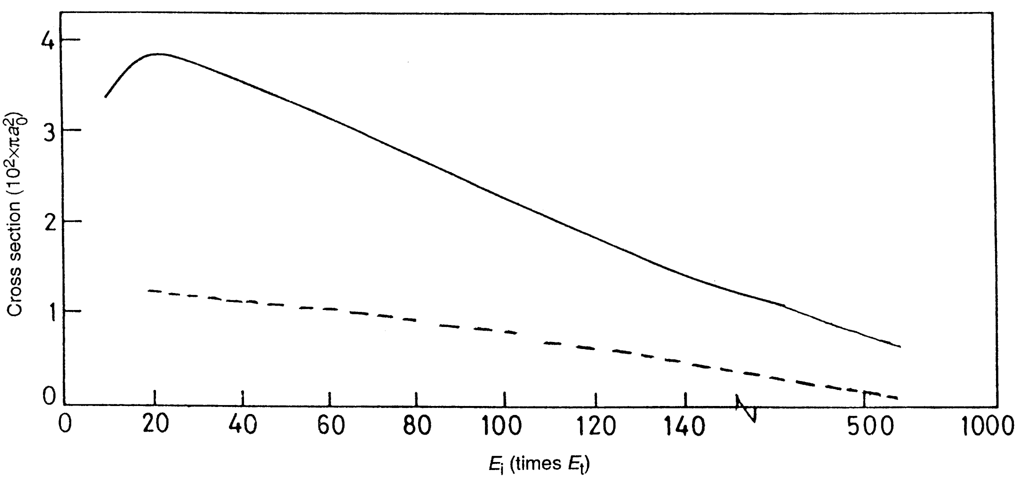

Fig. 3. Variation of the total cross section with incident electron energy for the 3s–3p transition for the laser assisted (solid line) and zero field (dashed line) cases. For the laser assisted cross section $\omega = 1{\cdot}81$ eV and $I = 10^6$ W cm^{-2}.

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