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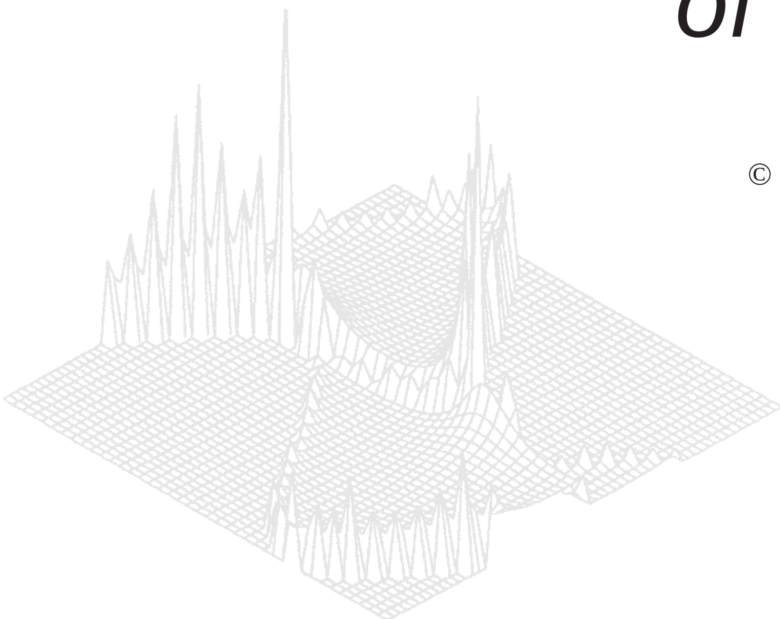

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Observation of Fractal Conductance Fluctuations over Three Orders of Magnitude*

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Abstract

Fractal magneto-transport properties of mesoscopic semiconductor billiards are highly topical. In these studies, the magnetic field range over which fractal behaviour can be observed is crucial. Previous observations have been limited to approximately one order of magnitude. We present fractal conductance fluctuations observed over three orders of magnitude and discuss the physical conditions required to extend this range.

1. Introduction

Measurements of device conductance as a function of magnetic field serve as a powerful probe of electron transport phenomena in semiconductor systems (Beenakker and van Houten 1991). This is particularly true for semiconductor billiards—cavities which are smaller than the electron mean free path. The use of electrostatic gates to define billiards within the two-dimensional electron gas (DEG) formed at the interface of AlGaAs/GaAs heterostructures has become well-established (Beenakker and van Houten 1991). In such a high mobility environment, ballistic electron trajectories are predominantly shaped by the confining walls of the billiard rather than scattering events induced by the host material. Magneto-conductance fluctuations (MCF) then act as 'magneto-fingerprints' of the classical electron trajectory statistics generated by the billiard's geometry (Jalabert *et al*. 1990; Marcus *et al*. 1992). Traditionally, billiards have been modelled using hard walls, where the billiard has a flat bottom and vertical walls of infinite height. More recent models, which incorporate the realistic softwall profiles defined by electrostatic gates, predict that the classical trajectory phase space will be 'mixed'—containing both regions of chaotic and stable behaviour (Ketzmerick 1996; Fromhold *et al*. 1998). At low temperatures,

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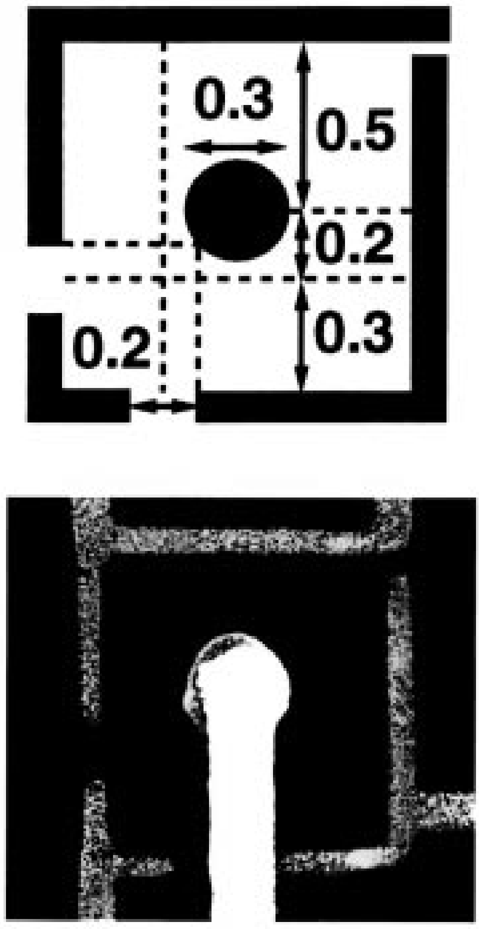

Fig. 1. Schematic diagram and corresponding scanning electron micrograph of the Sinai billiard. Dimensions are in microns.

the electrons maintain phase coherence whilst traversing the billiard and quantum interference effects become a probe of 'quantum chaos'—the quantum behaviour of a classical chaotic system. Following predictions that the quantum chaos of a soft-walled billiard would be detected as fractal behaviour in the MCF (Ketzmerick 1996), recent experiments have shown MCF to follow fractal scaling across one order of magnitude in the magnetic field (Micolich *et al.* 1998; and see the comment on Sachrajda *et al.* 1998 by Taylor *et al.* 1999). To investigate this novel phenomenon, we consider the Sinai billiard shown in Fig. 1. For a Sinai billiard described by hard walls, the circle at the centre of the square cavity acts as a 'Sinai diffuser,' generating exponentially divergent trajectories (Sinai 1970; Ott 1993). The role of a soft-walled Sinai diffuser, and in particular its effect on fractal MCF, is of considerable interest. We present experimental results where fractal MCF are observed over three orders of magnitude and discuss ways in which this range could be extended further.

2. Correlation Function F: Characterisation of Self-similarity

Fig. 2 shows the low temperature (50 mK) MCF generated by applying a magnetic field perpendicular to the plane of the billiard. The electronic mean free path ($l = 25$ μm) is significantly larger than the 1 μm cavity width, ensuring ballistic transport. A striking similarity exists between the MCF patterns observed at different magnifications. This scaling property is called exact self-similarity. In contrast, the scaling properties of previous observations of fractal MCF followed statistical self-similarity (Micolich *et al.* 1998; Taylor *et al.* 1999), where only the statistical properties of the trace were invariant under scaling. More generally, this occurrence of exact self-similarity in a physical system is rare: whereas many mathematically generated fractal patterns possess exact self-similarity, most physical fractals obey statistical self-similarity. To quantify this remarkable exact self-similarity, we have introduced a correlation function F which mathematically assesses the similarity of the MCF observed at different magnifications. The self-similarity occurs at the four magnifications shown in Fig. 2 and we label these levels as the ultra-coarse (uc), coarse (c), fine (f), and ultra-fine (uf) MCF levels. Consider the similarity between the coarse and fine levels as an example. Defining the c scale conductance amplitude as $\delta G_c(B) = G_c(B) - \langle G_c(B)\rangle$ (where $\langle\,\rangle$ represents an average performed over magnetic field) and the f scale conductance amplitude as $\delta G_f(B) = G_f(B) - \langle G_f(B)\rangle$, then conductance and magnetic field scaling factors λ_G and λ_B can be determined which give the required result $\delta G_c(B) \approx \lambda_G \delta G_f(\lambda_B B)$. The correlation function F quantifies the similarity:

$$F = 1 - \frac{\sqrt{\langle\{\delta G_c(B) - \lambda_G \delta G_f(\lambda_B B)\}^2\rangle}}{N}\,, \tag{1}$$

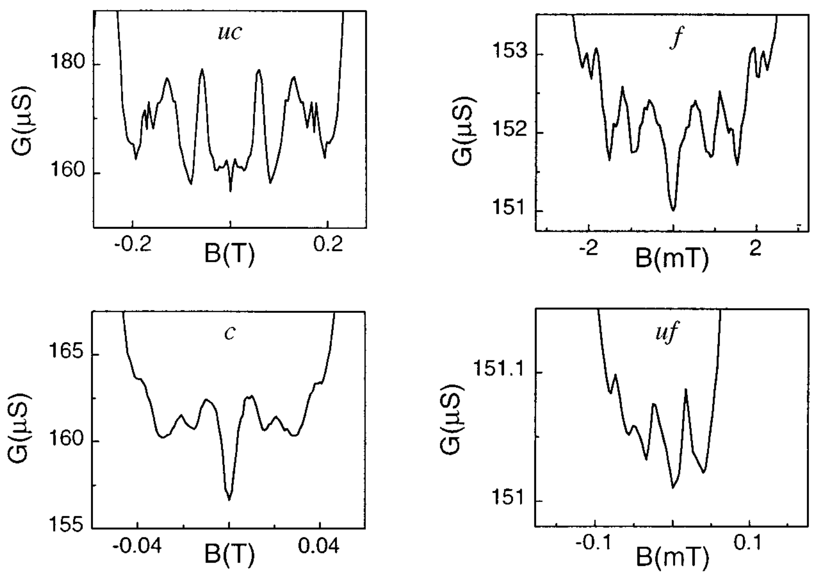

Fig. 2. Four levels of exact self-similarity observed in the magneto-conductance of the Sinai billiard: uc, c, f and uf. The temperature is $T = 50$ mK.

where N is a normalisation constant which sets $F = 1$ when $\delta G_c(B)$ and $\lambda_G \delta G_f(\lambda_B B)$ are identical traces and $F = 0$ if they are decorrelated. A comparison between the coarse and fine levels gives $F = 0{\cdot}97$, confirming the exact self-similarity of the MCF.

3. Fractal Dimension D_{F}: Characterisation of Fractal Behaviour

For the self-similar patterns to be fractal, the four levels must be described by a fractal dimension D_{F} which satisfies $1 < D_{\mathrm{F}} < 2$ (Mandelbrot 1998). Most definitions of D_{F} are based on a concept by Hausdorff, where the amplitude of the fractal structure observed at different magnifications has to scale as a power law and D_{F} is extracted from the exponent of this power law behaviour (Mandelbrot 1998). Extending this concept to MCF, we quantify the magnetic field scale of the nth level using ΔB_n, the full width at half maximum of the central trough feature. The amplitude ΔG_n of the fractal structure in the nth level is quantified by the height of this central trough feature. If fractal, ΔG_n should scale with ΔB_n according to the following expression:

$$\Delta G_n \propto (\Delta B_n^{\gamma}), \tag{2}$$

where $D_{\mathrm{F}} = 2 - \gamma$. This behaviour is confirmed in Fig. 3 for the four levels shown in Fig. 2. The four levels lie on a power law line (see later for the explanation of why the uf level lies slightly below this line) and the result $D_{\mathrm{F}} = 1{\cdot}5$ is extracted from the gradient. We note that the levels lie at equal intervals along

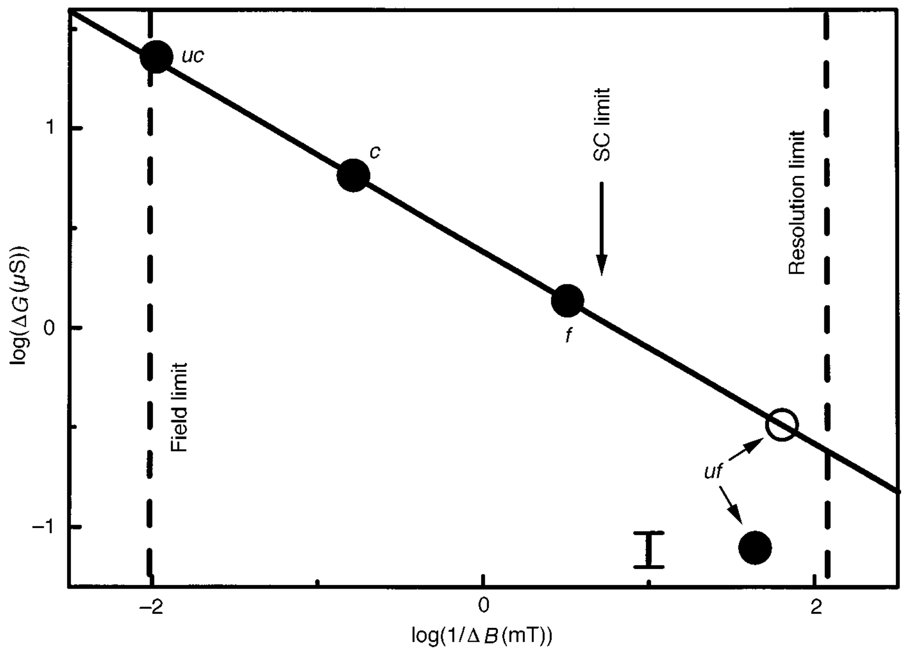

Fig. 3. A linear fit to $\log_{10}(\Delta G)$ versus $\log_{10}(1/\Delta B)$. The solid dots represent the measured values for the uc, c, f and uf levels. The open dot represents the anticipated zero-temperature coordinate for the uf level based on the λ_G and λ_B values.

this line, indicating a common magnetic field scaling factor $\Delta B_n/\Delta B_{n+1} = \lambda_B$ and a common scaling factor $\Delta G_n/\Delta G_{n+1} = \lambda_G$. For the data shown, $\lambda_G = 3{\cdot}7$ and $\lambda_B = 18{\cdot}6$.

The four MCF levels described by $D_{\rm F} = 1{\cdot}5$ are observed across $3{\cdot}7$ orders of magnitude in magnetic field. In contrast, previous observations of fractal MCF have been restricted to approximately 1 order of magnitude (Micolich *et al.* 1998; Taylor *et al.* 1999) and, more generally, a recent survey of fractal measurements in physical systems revealed that the average range over which fractals are observed is only $1{\cdot}3$ orders of magnitude (Avnir *et al.* 1998). Whereas fractals which are observed over limited ranges are, in theory, no less fractal than those observed over larger ranges (Mandelbrot 1998), the reliability of fitting to a power law behaviour is intrinsically linked to the range over which the fit can be applied. A larger range therefore lends confidence to the observation of fractal behaviour. In light of this, we now discuss the physical parameters which determine the range over which fractal behaviour is observed and also methods to extend this range.

4. Upper and Lower Cut-off Fields of Fractal Behaviour

The dashed vertical lines shown in Fig. 3 represent the observation limits. Self-similarity is currently restricted to points lying between these dashed lines—the experimentally observed levels of *uc*, *c*, *f* and *uf* MCF. We first consider the lower cut-off. The 'resolution limit' is determined by the minimum separation between data points taken in the experiment, which is $0{\cdot}01$ mT. Data lying to the right of this line, corresponding to smaller ΔB values, cannot be resolved in the present experiment. The close proximity of the *uf* structure to the resolution line restricts the comparison of similarity between the *f* and *uf* structure in Fig. 2. We note that the present resolution is already high: ΔB_u is two orders of magnitude smaller than the fields scales observed in typical billiard experiments and ΔG_u is less than $0{\cdot}01\%$ of the signal and approaches the experiment's noise floor. Further improvements to the resolution limits are expected to increase the resemblance between the *uf* and *f* structures.

Improvements to the resolution limit alone will not allow the observation of a fifth level at smaller ΔB. This is due to limitations imposed by the electrons' phase coherence length L_ϕ. The MCF originate from quantum interference of pairs of electron trajectories which form closed loops and ΔB is inversely proportional to the loops' enclosed areas (Beenakker and van Houten 1991; Jalabert *et al.* 1990; Marcus *et al.* 1992; Ketzmerick 1996). Thus points to the right of the diagram are generated by loops with larger areas A and hence longer trajectory lengths L. The fractal power law behaviour of Fig. 3 requires all the trajectories to remain phase coherent. Trajectories longer than the phase coherence length L_ϕ contribute less than expected to the quantum interference processes, resulting in smaller contributions to ΔG. This will cause deviations from the power law line. The condition $L \ll L_\phi$ must hold for the phase-breaking processes to have a minimal effect on ΔG. As a rough estimate, if we assume $\Delta B = h/2eA$ (Beenakker and van Houten 1991; Jalabert *et al.* 1990; Marcus *et al.* 1992; Ketzmerick 1996) and that the loop of area A is circular, then we obtain characteristic trajectory lengths L of $0{\cdot}6$, $1{\cdot}5$, 6 and 26 μm for the first (*uc*), second (*c*), third (*f*) and fourth (*uf*) levels. For the *uc*, *c* and *f* levels, the coordinates lie on the power law line, indicating that $L \ll L_\phi$. However, the *uf* coordinate lies below the power law line. The deviation is small (the adjacent bar represents $0{\cdot}01\%$ of the signal: less than $0{\cdot}05\ \Omega$) suggesting that L is of the order of L_ϕ for the *uf* level. Therefore the trajectories associated with higher levels (i.e. with L longer than that of the *uf* level) will not be phase coherent and self-similar MCF are not expected. There are two possible solutions to this problem. The first solution is to increase L_ϕ by defining billiards in higher quality host material. The second solution is to move the levels up the power law line. This can be achieved by reducing the billiard area $A_{\rm B}$, which in turn reduces the characteristic loop areas (hence reducing L and the damping of ΔG due to phase breaking) and increases the ΔB values of each level of MCF structure (hence allowing features to be better resolved).

It is not clear, however, how a decrease in $A_{\rm B}$ will affect the nature of the quantum interference processes. For the 1 μm billiard, the energy level spacing in the billiard $\Delta E = 2\pi\hbar^2/m^* A_{\rm B}$ is 9 μeV. The corresponding Heisenberg time $\tau_{\rm H} = \hbar/\Delta E$ can be converted into an approximate ΔB assuming $L = v_{\rm F}\tau_{\rm H}$ ($\approx$15 μm), $A = L^2/4\pi$ and $\Delta B = h/2eA$. This is indicated in Fig. 3 as the 'SC limit'. To the right of this value, the trajectory traversal times exceed $\tau_{\rm H}$ and the semiclassical picture of quantum interference processes do not necessarily hold (Clarke *et al.* 1995). For the 1 μm billiard, the *uc*, *c* and *f* levels satisfy the semiclassical condition but the *uf* data lie slightly beyond. If the size of the billiard is reduced, the SC limit will shift to the left and may start to affect the self-similarity of all the levels. Experiments are planned to investigate the effect of billiard size on self-similarity. This potential problem with scaling the billiard size also restricts improvements to the upper cut off. The 'field limit' represents the magnetic field $B_{\rm L} = \hbar k_{\rm F}/eW$ at which the radius of curvature of the electron trajectories becomes comparable to that of the Sinai billiard's width W. At higher magnetic fields the electron trajectories begin to form skipping orbits: data lying on different sides of this 'field limit' line will be in different transport regimes and cannot be used for comparisons of self-similarity. In Fig. 3, the *uc* level lies on the upper cut-off line and this causes the reduced similarity observed in Fig. 2 between the *uc* and *c* levels. One possible way to move the 'field limit' line to the left is to reduce the size of the billiard, but as discussed above, this might affect the self-similarity of the existing levels. An alternative solution is to lower the electron density $n_{\rm s}$ which reduces $k_{\rm F}$ through $k_{\rm F} = (2\pi n_{\rm s})^{1/2}$. In particular, billiards fabricated with a back-gate could fine tune $n_{\rm s}$ in order to move the field limit to the left.

Finally, in addition to increasing the magnetic field range over which the power law behaviour is observed, it is also desirable to increase the number of levels observed within a given magnetic field range. Our results indicate that an increase in the number of modes in the entrance and exit ports reduces λ_B and hence the spacing between the different levels. Experiments are planned to examine this effect further.

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