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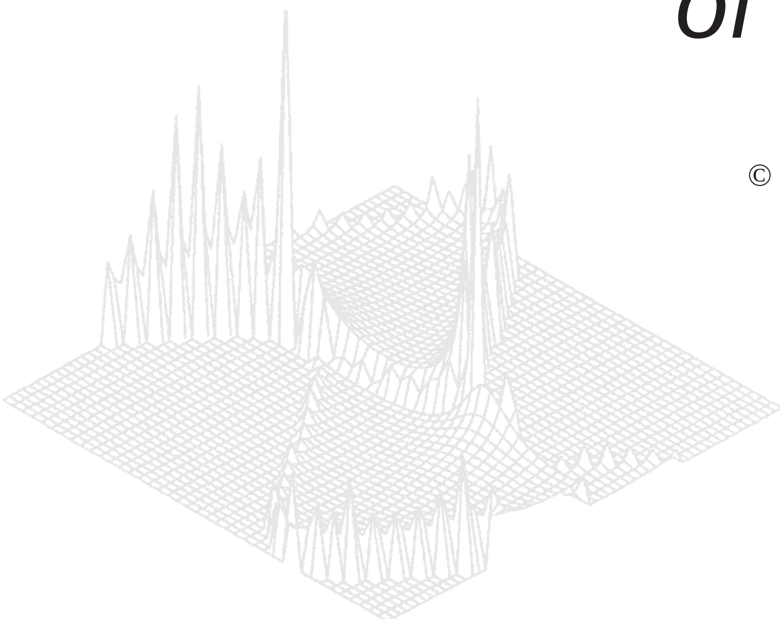

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Some Experimental Prospects involving Parabolic Quantum Wells*

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Abstract

We discuss two possible lines of experimental investigation based on parabolic quantum wells. In the first proposal, we note that the Generalised Kohn Theorem/Harmonic Potential Theorem forbids electron–electron damping of the Kohn mode in an electron layer gas under strictly parabolic confinement. This applies even for very strong driving. It is therefore interesting to attempt reduction of other sources of broadening in GaAlAs parabolic wells, so as to achieve a prominent narrow resonance in the far infrared. We concentrate here on phononic bandgap structures, which may be of interest for reduction of phonon effects in other systems as well. The second class of proposed experiment involves twinned parabolic wells in an attempt to observe van der Waals forces directly in GaAlAs systems. In a first approximation, the parabolic or Hooke's-law nature of the confinement allows one to use the well as a kind of spring balance to measure the weak van der Waals force. The influence of an applied magnetic field on these forces appears to be significant, and this system might provide the first measurement of such an effect.

1. Introduction

This paper is mainly concerned with oscillations of the electron gas in semiconductor nanostructures such as GaAlAs quantum wells. These oscillations can be regarded broadly as the plasmon modes of confined electron gases, though in many cases there are alternative interpretations in terms of modified single-particle transitions (see e.g. Das Sarma 1984; Schaich and Dobson 1994). In wide GaAlAs quantum wells these plasmon frequencies typically lie in the far infrared (FIR, TeraHerz) region.

We will first discuss a possible way to sharpen up the Kohn-mode or 'sloshing' resonance of the electron gas in a parabolic quantum well (Dobson 1992, 1993; Pinsukanjana *et al.* 1992). This approach relies on suppression of phonon damping. If successful this could provide a reliable sharp FIR absorption line in a portable solid-state device, with potential uses as a reference line. Before this can be achieved, however, substantial reduction in other sources of broadening, such as imperfect epitaxy, will probably need to be achieved.

We will also discuss the case of a double parabolic quantum well, containing two parallel but non-contacting (thick or thin) sheets of electron gas. The coupled

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plasmons of these two sheets mediate an attractive van der Waals force between the sheets. We explore the possibility of direct observation of this attraction by capacitive or tunneling measurements.

2. Generalised Kohn/Harmonic Potential Theorems: Unusual Properties of Parabolically Confined Electron Gases

In general plasmon motion of an electron gas, among various sources of line broadening, there are some of purely electronic origin. The first of these is Landau damping, in which the energy and momentum of a plasmon are transferred to an electron–hole pair. This process does not directly involve the electron–electron interaction and is therefore sometimes termed collisionless damping. The second electronic damping mechanism is multipair excitation which does require the electron–electron interaction (i.e. it involves electron–electron collisions): it can be broadly related to the viscosity of the electron gas (Vignale *et al.* 1997). This second mechanism is usually effective even where Landau damping is not, so that most quantum-well plasmon motions would have non-zero damping, even if influences such as defects and phonons were absent.

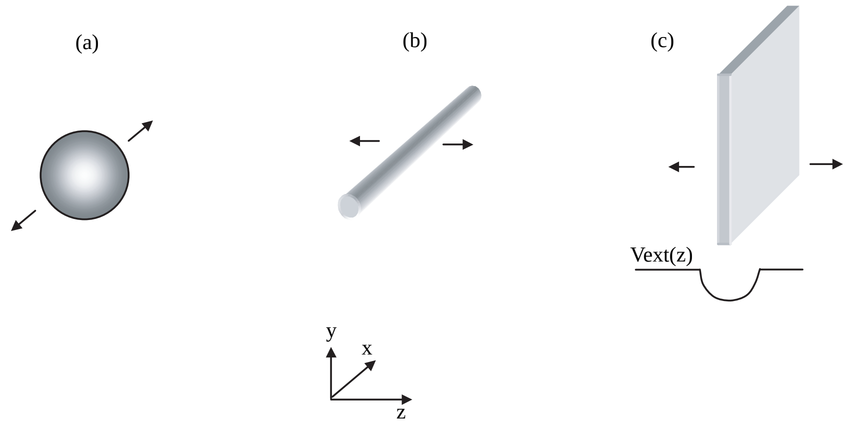

Fig. 1. Parabolic confinement of electrons in three, two or one dimensions, giving (*a*) a quantum dot, (*b*) a quantum wire or (*c*) a quantum well. The shaded areas represent the location of the ground-state electronic density, and the arrows represent rigid Kohn-mode motion allowed by the exact electron–electron dynamics. The graph in (*c*) represents the confining parabolic potential as a function of position z, for the case of a parabolic quantum well.

There is a remarkable exception to this observation, namely the case of electrons confined by an external potential $V^{\mathrm{ext}}(\vec{r}) = \frac{1}{2}\vec{r}.\mathbf{K}.\vec{r}$ which is a quadratic function of position $\vec{r}$. Particular cases are zero-dimensional quantum dots, 1D quantum wires, and most importantly parabolic quantum wells $V^{\mathrm{ext}}(\vec{r}) = \frac{1}{2}Kz^2 \equiv \frac{1}{2}m\omega_0^2 z^2$ which contain a thin (2D) or thicker (quasi-3D) electron gas layer. These three cases are illustrated in Fig. 1. It has been traditional to analyse such parabolic wells in terms of the number density $n_0 \equiv \rho_0/e$ of a fictitious uniform positive background which is regarded as the source of the parabolic potential. Then by Poisson's equation

$$K = n_0 e^2/\epsilon\,,$$

where ϵ is the macroscopic dielectric constant of the semiconductor. For the GaAlAs system, $\epsilon \approx 13\varepsilon_0$. A wide parabolic well, when filled with sufficient electrons, contains a wide slab of near-uniform electron gas (egas) with a 3D number density approximately equal to n_0. For this reason it is also commonplace to characterise the well curvature K by the dimensionless inter-electron spacing r_s^* of the corresponding egas:

$$n_0 = \left(\frac{4\pi r_s^{*3}}{3}\right)^{-1} a_B^{*-3} \,. \tag{1}$$

Here

$$a_B^* = \frac{\hbar^2 4\pi\epsilon}{m^* e^2} \tag{2}$$

is the effective Bohr radius for conduction electrons in the semiconductor, and m^* is the conduction electron effective mass. The degree of filling of a parabolic well is commonly described by two alternative quantities: the areal or surface electron density N_s, or an effective width of the electron gas

$$L = N_s / n_0 \,. \tag{3}$$

In the discussion below, for numerical work we will characterise parabolic wells by the quantities r_s^* and the dimensionless effective width $L^* = L/a_B^*$.

Consider now a parabolic quantum well containing a Coulomb-interacting gas in the absence of any coupling to defects or phonons, under excitation by a *spatially uniform* electric field $\vec{E}(t) = E_0 \hat{z} \cos(\omega t)$ directed along the semiconductor growth z direction. The Generalised Kohn Theorem of Brey *et al.* (1989) ensures that the linear response of the inhomogeneous electron gas displays an infinitely sharp resonance at $\omega = \omega_0$, and there is no resonance at any other frequency. Thus the electron–electron interaction is unable to introduce any damping to the Kohn mode of a parabolic well system, at the level of linear response. Furthermore, even for strong driving beyond the linear regime, i.e. for large values of E_0, the Harmonic Potential Theorem (Dobson 1994) ensures that the electron–electron interaction does not cause any damping or shift of the resonance. It also follows that there is no harmonic generation. (In reality, the parabolic potential cannot be grown to extend to $z = \pm\infty$, and more complex linear and nonlinear effects appear when the edges of the moving electron gas approach the edge of the region of parabolic potential.)

These conclusions for the perfectly quadratic case were originally proved via operator commutation rules for the linear response of a parabolic well system (Brey *et al.* 1989), and by explicit construction of the moving many-electron wavefunction, for nonlinear response of systems confined parabolically in 1D, 2D or 3D (Dobson 1994). A rather simple way to understand all of this physics, however, is to view the parabolic confining potential $V^{\text{ext}}(\vec{r}) = \frac{1}{2}\vec{r}.\mathbf{K}.\vec{r}$ from an accelerated reference frame whose origin $\vec{X}(t)$ is executing the same simple harmonic motion as experienced by a single electron driven by the uniform external field (Dobson 1995). Then $m d^2\vec{X}/dt^2 = -\mathbf{K}.\vec{X}(t) - e\vec{E}_0(t)$. In the accelerated reference frame, the external potential and the fictitious potential $-md^2\vec{X}/dt^2$ combine to create a new parabolic potential $\frac{1}{2}\vec{r'}.\mathbf{K}.\vec{r'}$+const., which is stationary in the moving (primed) frame. Thus for any stationary many-body state in the rest frame there is an identical one in the accelerated frame. Such a state is of course a moving state when observed from the rest frame.

From the present point of view, the remarkable thing is that this motion is completely undamped by electron–electron interactions, no matter how large the amplitude. In order to capitalise on this feature to obtain a narrow line, the other sources of broadening must be controlled. Some relevant considerations are discussed in the next section.

3. Controlling Sources of Broadening of the Kohn Mode

A number of physical phenomena will cause broadening of the Kohn mode. Some of these phenomena, and possible means of suppression where applicable, are discussed under the headings below.

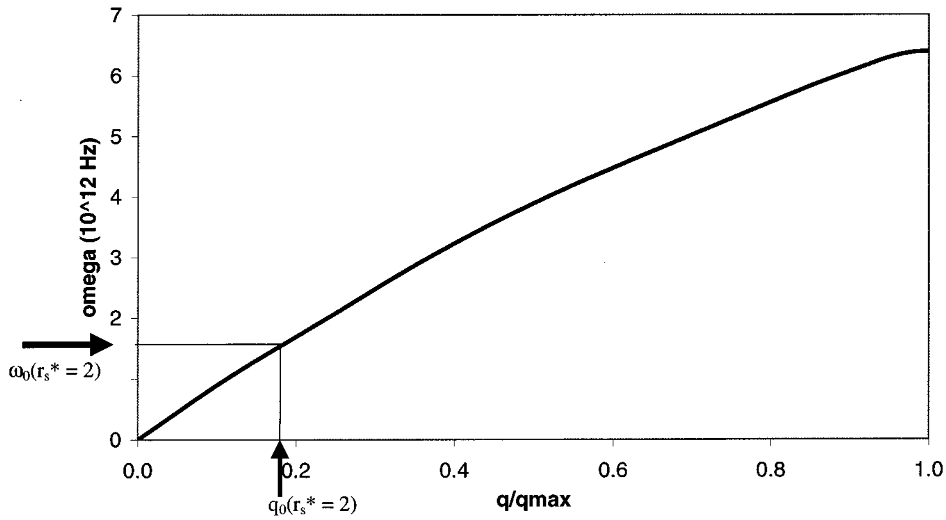

Fig. 2. Longitudinal acoustic phonon dispersion for GaAs, also showing the Kohn frequency for $Ga_{1-x}Al_xAs$ parabolic wells with $r_s^* = 2$. Phonon dispersion data were taken from Srivastava (1990): note that the LA dispersion curves in the [111], [110] and [100] directions all cover essentially the same frequency range.

(3a) Phonon Damping

In the Kohn-mode motion under discussion here, in the absence of defects and phonons, a (diffuse-edged) sheet of electronic charge executes rigid simple harmonic motion in the z-direction perpendicular to the sheet, at the bare harmonic oscillator frequency ω_0. This will couple to planar longitudinal acoustic (LA) phonons, leading to damping. This process occurs already in first order in the electron–phonon interaction, provided that ω_0 lies in the LA phonon band so that energy can be conserved. In fact typical values of ω_0 for $Ga_{1-x}Al_xAs$ parabolic wells do indeed lie in the LA phonon band (see Fig. 2). One way to prevent this source of damping, to first order at least, is to grow a layer structure (acoustic filter) on each side of the parabolic well as shown schematically in Fig. 3. This can create a phononic bandgap around frequency ω_0: see Fig. 4. For frequencies in the gap or filtering range, phonons will be reflected off the filters back into the region of the parabolic well, so that they are unable to cause broadening by carrying energy away. The layer thickness A of the required filter can be determined from the inverse of the wavenumber indicated as q_0 in Fig. 2, and would therefore be in the vicinity of a few semiconductor lattice spacings. A suitable composition for the filter, in order to create a sufficiently large gap in the phonon energy spectrum (see Fig. 4) is yet to be investigated.

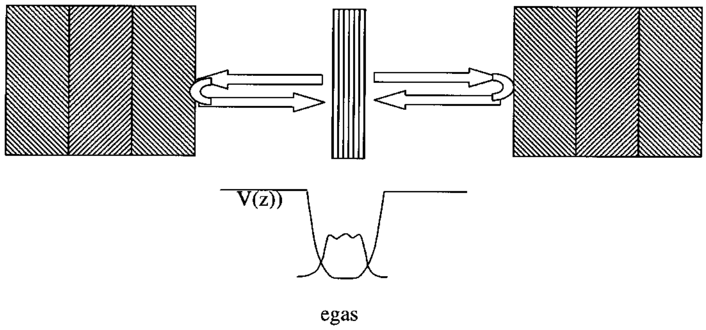

Fig. 3. Proposed acoustic filter setup (phononic bandgap structure) for suppression of first-order electron–phonon damping of the Kohn mode in a parabolic quantum well.

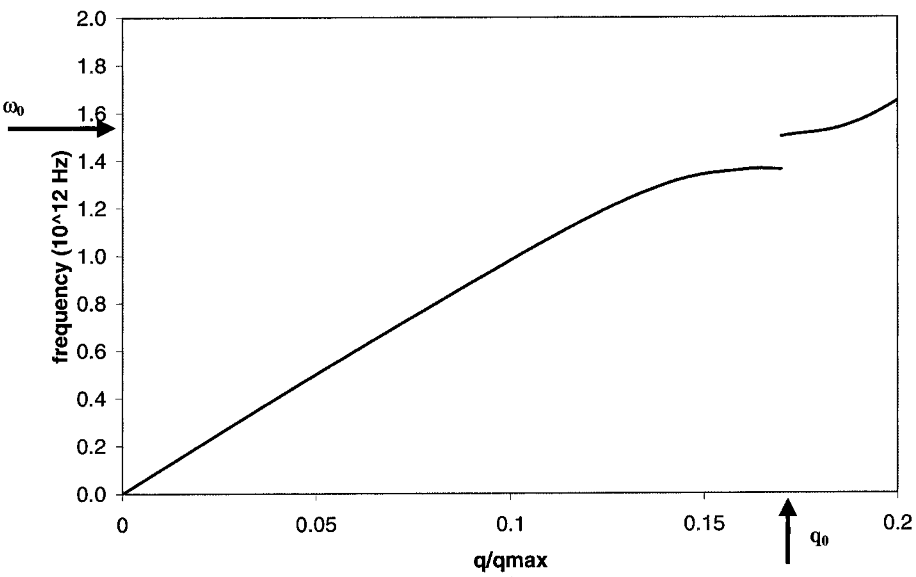

Fig. 4. Detail of the phonon spectrum in the presence of acoustic filters with a spatial period of $O(2\pi/q_0)$, showing the opening of a phonon gap at wavenumber q_0.

For other geometries where an electronically undamped Kohn motion also applies, such as the quantum dots and wires illustrated in Figs 1*a* and 1*b*, it may also be possible to grow phononic bandgap structures in order to avoid phonon damping. This will be more difficult than the essentially 1D quantum-well case discussed above, because one will need an absolute phonon gap, not just a gap for phonon motion in one space direction. Thus a fully 2D or 3D phononic bandgap superlattice structure will be required. Design and implementation of such structures in the analogous photonic case is already quite advanced (Yablonovitch 1993), but the phononic analogue is not so well developed.

Even with a phonon bandgap in place, electron–phonon interactions beyond first order are still effective and will cause damping of the Kohn mode at some level.

(3b) Imperfect Parabolic Growth

The generalised Kohn theorem relies on a perfectly parabolic effective external potential. The effects of departures from parabolicity of the grown potential have been investigated previously (Brey *et al.* 1990). Experimental workers to date (see e.g. Pinsukanjana *et al.* 1992) have sought to minimise these departures, achieving somewhere around 10%. Further improvement may be possible here.

(3c) Inhomogeneous $m^*(z)$ and $\varepsilon(z)$

In the parabolic quantum well case, the effective parabolic potential for electron confinement is grown in $Ga_{1-x}Al_xAs$ by varying the aluminium fraction x during layer-by-layer growth in the z-direction. The grown function $x(z) = Az^2$ determines the local bandgap $\varepsilon_g(z)$, within the envelope-function approximation (Bastard 1981). This in turn causes the conduction band-edge $V_{\rm cond}(z)$ to vary with z, thus supplying an effective external potential of form $V_{\rm ext}(z) = V_{\rm cond}(z) = \frac{1}{2}Kz^2$, which acts on the conduction electrons. Unfortunately this same spatial variation of the Al concentration also causes a spatially varying electron effective mass $m^*(z)$ and dielectric function $\epsilon(z)$. Even where the Al concentration has been grown exactly according to the desired parabolic space dependence, these effects can spoil the perfect adherence to the generalised Kohn theorem and the Harmonic Potential Theorem. It is possible that a deliberate departure from perfectly parabolic Al doping [i.e. $x(z) = Az^2$+correction] might achieve some cancellation of these effects. Certain effects of spatially varying m^* in an otherwise perfect parabolic well have been investigated previously by microscopic theory (Karrai *et al.* 1990; Stopa and Das Sarma 1993; Xu 1994, 1995). Both static effects, and also direct dynamic effects of inhomogeneous m^* on the Kohn mode have been studied (Le 1999), within a suitably generalised form of hydrodynamics (Dobson and Le 1999). Effects of inhomogeneous $\epsilon(z)$ were also included in Le (1999). [Note that a modified hydrodynamics was required because the usually accepted degenerate egas hydrodynamics does not exactly yield the Kohn mode for perfect parabolic systems, even when m^* and ϵ are taken as constant: see Dobson (1994) and Dempsey and Halperin (1992).] While the hydrodynamic treatment could perhaps be used to design a nonparabolic Al growth function $x(z)$, so as to restore the exact Kohn frequency despite the spatial variation of m and ϵ, this is not the issue here. In seeking a *sharp* resonance line, what we need is to reduce or remove any specifically electronic *damping* effects due to $m^*(z)$ or $\epsilon(z)$. The hydrodynamic approach cannot help here, since, in this formalism, damping is introduced on an ad hoc basis. Furthermore, a microscopic theory adequate to address this issue needs to go beyond the usual time-dependent Hartree and adiabatic LDA theories, because these only contain Landau (collision-less) damping, and fail to yield electron–electron damping. A recent dynamic LDA theory (Vignale and Kohn 1996, 1998; Dobson *et al.* 1997, Vignale *et al.* 1997) is capable of addressing this damping issue for parabolic wells with spatially varying m^* and ϵ, and this would indeed be an interesting calculation.

(3d) Imperfections, including Imperfect Epitaxy

Imperfections such as impurities, point defects of various types, interface roughness, layer thickness variations and other imperfections in epitaxial growth are already at a relatively low level in epitaxially grown parabolic well systems with spatially remote doping. Much experimental effort has already been invested to minimise these effects, so it will probably require substantial further effort to gain improvements here. Measures such as epitaxial growth under zero gravity might perhaps yield further significant improvement. The effects of these imperfections are possibly the main cause of the observed broadening, which amounts to about 10% of the line frequency in the parabolic-well experiments of Pinsukanjana *et al.* (1992). Clearly, to gain benefit from the phonon suppression mechanism proposed here, one must reduce the above effects below the level of phonon damping.

(3e) Failure of the Envelope-function Approximation

The true potential acting on electrons in a 'parabolic' quantum well is not parabolic, but is grainy on a scale of the underlying semiconductor crystal lattice period (or even on a coarser scale when modulation doping is adopted). The envelope function approximation (Bastard 1981), in which the external potential of the wells under consideration is taken to be simply parabolic, applies only to the outer envelope of the rapidly varying Bloch wavefunction. At some level this will limit the precision of the generalised Kohn and Harmonic Potential Theorems, which rely on perfect parabolicity. This should be investigated, but seems unlikely to be as important as the effects of imperfections and phonons described under other headings.

(3f) Summary of Linewidth Considerations

The discussion under the headings above has attempted to identify some likely causes of broadening of the Kohn mode of a parabolic quantum well. More detailed quantitative analysis is required to ascertain whether a significant narrowing can be achieved by the means suggested (phonon gap structures), or whether the remaining intractable items such as imperfect epitaxy and other defects are the dominant causes of the substantial linewidth observed in previous parabolic quantum well experiments. Nevertheless, it seems worth while to attempt further theoretical and experimental work, both from a technological point of view, and because of the fundamental interest inherent in phononic bandgap/acoustic filter physics.

We now leave the topic of phonon damping and turn to another aspect of parabolic-well physics.

4. Van der Waals Interaction between a Pair of Quantum-well Egases

It is now possible to grow a pair of parallel quantum wells in close proximity, separated by a distance D of order a few tens of nm or more. One can establish separate electrical contacts to the electron gases trapped in the wells (Gramila *et al.* 1991). To date this has been carried out mainly to investigate the Coulomb drag effect, in which a current in one confined electron-gas sheet induces a current or voltage in the other sheet.

Here we look instead at the van der Waals (vdW) attraction between the two electron gases. The mutual vdW energy $E_{\mathrm{vdW}}(D)$ of two egases separated by spacing D can be understood in a number of ways. It can be thought of as the Coulomb correlation energy of spontaneous charge density fluctuations on the two wells. In a purely hydrodynamic approach to electron dynamics, in which the physics of individual electron–hole pairs is neglected, all of the fluctuations can be understood in terms of the coupled plasmon modes of the two gases. The vdW energy can be regarded as that part of the total plasmon zero-point energy which depends on D. Whatever model we use for the vdW energy E_{vdW}, we can obtain from it an effective vdW attractive force $F_{\mathrm{vdW}} = -\partial E_{\mathrm{vdW}}/\partial D$. Here we propose to use a parabolic well, with its Hooke's law confining force, as a kind of 'spring balance' to measure this weak force.

The basic idea is to grow two or three quantum wells, at least one of which (well #2) is *parabolic*, and to measure the vdW force on the parabolic well by one of two fairly direct methods which we now describe—see Figs 5 and 6. In both these figures, we aim to measure the change in the vdW force on egas #2 due to the creation and destruction of the egas in well #1. This creation and destruction is achieved by changing the bias voltage V_1 between egas #1 and metal gate #1. As argued below, because well #2 is parabolic, this change in vdW force will cause a displacement of egas #2 in the z (growth, confinement) direction. In Fig. 5, this displacement is detected via its effects on the capacitance between egas #2 and metal gate #2. In Fig. 6, the displacement is detected instead by the change in tunneling current to a neighbouring steep-sided (non-parabolic) well #3.

The reason for choosing well #2 to be parabolic is as follows. For a parabolic well, and for no other type of well, a spatially uniform, externally imposed, static force, applied perpendicular to the plane of the confined electron gas, causes a *rigid* sideways displacement of the electron gas which is *strictly proportional to the applied force*. For the case of a static force, this follows because the linear potential $-Fz$ due to the uniform applied force adds to the grown parabolic potential $\frac{1}{2}Kz^2$ to form a new parabola $V(z) = \frac{1}{2}K(z - \Delta z)^2$+const., spatially shifted in the growth direction by an amount $\Delta z = F/K$ which is proportional to the force. Thus, if this displacement Δz can be measured, it provides a reliable measurement of any homogeneous external force applied to a parabolic-well egas. Furthermore, the Harmonic Potential Theorem (Dobson 1994) shows that when a force $F_0 \sin(\omega_{\mathrm{mod}} t)$ is applied, in the steadily oscillating state, the egas suffers a rigid displacement

$$\Delta z = F_0 \sin(\omega_{\mathrm{mod}} t)/(K - m\omega_{\mathrm{mod}}^2)\,.$$

The van der Waals attraction between two slabs of egas separated by distance D can be characterised by a force $F^{\mathrm{vdW}} = -\partial E^{\mathrm{vdW}}/\partial D$. The arguments in the

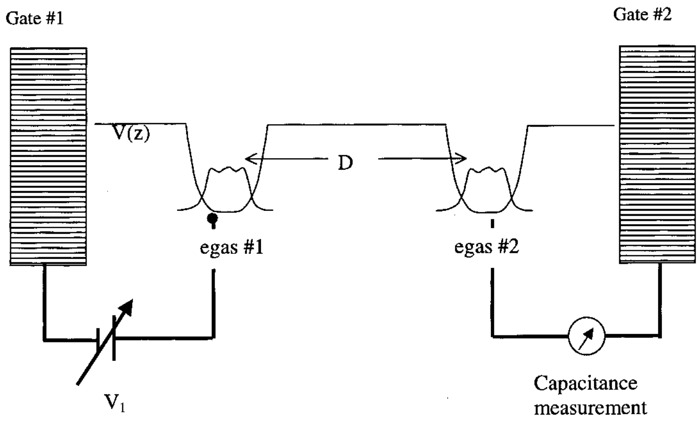

Fig. 5. Double quantum-well structure for observation of the van der Waals force in $Ga_{1-x}Al_xAs$. Quantum well #1 need not be parabolic, but well #2 must be parabolic. The electron gas in well #1 is created and destroyed by adjusting the bias voltage V_1 between egas #1 and gate #1. The switching of the vdW force due to the presence or absence of egas #1 causes movement of egas #2. This movement is detected by monitoring the capacitance between egas #2 and gate #2.

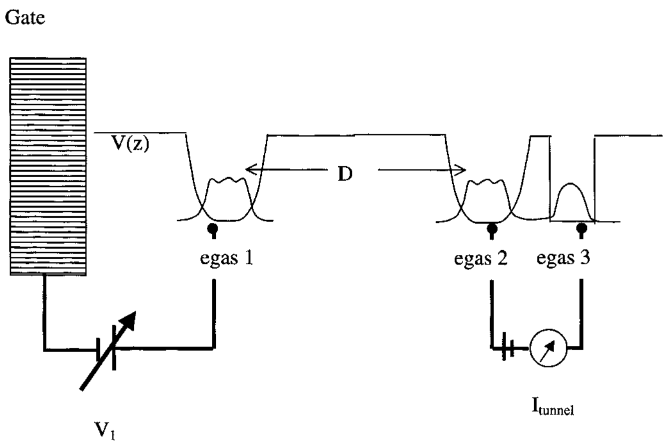

Fig. 6. As for Fig. 5, except that gate #2 is replaced by the steep-sided quantum well #3. The vdW-induced movement of egas #2 is now detected by the change in tunneling current between egas #2 and egas #3, rather than by capacitance measurements.

previous paragraph about an external force F do not apply rigorously to the vdW force F^{vdW} on egas #2 due to the presence of egas #1, because this is not a true external force but rather a form of internal exchange-correlation force. Nevertheless, it is clear that the egas #2 will lower its vdW energy by moving towards egas #1. We will assume for the present that the above analysis is still true to lowest nontrivial order in the interaction between the two egases, if one replaces the external force F by the van der Waals 'force' F^{vdW}. Thus the motion of egas #2 in its well can still be taken as a linear measure of the vdW interaction. A more complete theoretical treatment of this effect is, however, certainly of considerable interest (Kohn 1999).

The basic idea is to create and destroy egas #1 by changing the voltage V_1 between well #1 and a parallel metal gate #1, thus moving electrons between gate #1 and well #1. This in turn switches on and off the vdW force on egas #2 due to egas #1. This change in force on gas #2 is to be measured via the displacement of egas #2 within its confining parabolic well. Note that removal of some electrons from the thick high-density gas in gate #1 does *not* greatly change the vdW force on well #2 due to gate #1. This is because the vdW force from the thick egas in the gate is due basically to couplings involving the *surface* plasmons, not 2D plasmons, of the gate, and these plasmons are not strongly affected by partial charging of the gate. Furthermore, in transferring a sheet of charge from gate #1 to well #1, we have not changed the electrostatic force on gas #2, because the electrostatic force between two parallel sheets of charge is independent of their separation. Only fringing fields, i.e. a departure from truly planar geometry, will modify this conclusion. Such fringing-field forces can presumably be adequately quantified and subtracted out of the data, if they turn out to be significant in the vdW experiment.

The displacement of the edge of egas #2 in the growth or z-direction, when the vdW force due to egas #1 is switched on, can be measured in principle by monitoring the capacitance between egas #2 and metallic gate #2. This technique is the basis of the capacitance profiling technique used previously for experimental investigation of the progressive filling of quantum wells by transferring electrons from a gate via an applied potential (Pinsukanjana *et al.* 1992). This proposal is illustrated in Fig. 5.

Another, probably more sensitive, method of detecting the displacement of egas #2 is a tunneling method. In this approach (see Fig. 6) one grows a steep-sided non-parabolic well (well #3) close to the parabolic well #2, in such a way that wells #2 and #3 are both occupied because of remote doping. Wells #2 and #3 are arranged so that there is some tunneling between wells #2 and #3 (but none between wells #1 and #2). The vdW-mediated displacement of egas #2, occurring when egas #1 is created and destroyed, can be detected by changes in the tunneling current between wells #2 and #3. Note that egas #3 also feels vdW forces, but because of the steep-sided potential of well #3, these vdW forces do not cause significant movement of egas #3 in the z-direction.

As an initial step in quantitative analysis of these effects we study the vdW force assuming that both wells #1 and #2 are occupied only in the lowest subband state, so that the confined egases can be treated as two-dimensional. [Even in this case there are motions (Kohn modes in the case of the parabolic well #2) in which the egas oscillates in the z (confinement) direction. However, hydrodynamic analysis suggests that their contribution to the vdW force is negligible compared with that from the coupled 2D plasmon modes.] In the absence of metallic gates, the vdW interaction between non-overlapping 2D gases has been analysed in detail within the random phase approximation (Sernelius and Bjork 1998). The result for the mutual energy and force between two identical 2DEGs each with areal electron density N_s separated by a distance $D \gg \lambda_{\mathrm{Fermi}}^{\mathrm{2D}}$ is

$$E^{\mathrm{vdW}}/A = -0\cdot 00501_2 \hbar \sqrt{N_s e^2/(2\epsilon m^*)} D^{-5/2},$$

$$F^{\mathrm{vdW}}/A = -\partial(E^{\mathrm{vdW}}/A)/\partial D = -0\cdot 0\,1253_2 \hbar \sqrt{N_s e^2/(2\epsilon m^*)} D^{-7/2}.$$

where m^* is the band mass of a conduction electron in the semiconductor. This result can alternatively be obtained by a hydrodynamic treatment of the coupled 2D plasmons of the slabs, and also agrees with numerical results from microscopic theory of narrow egas slabs without the two-dimensional assumption (Dobson and Wang 1999).

The force per electron is then

$$F^{\mathrm{pe}} = (F^{\mathrm{vdW}}/A) N_s^{-1} = -0\cdot 0\,1253_2 \hbar \sqrt{N_s^{-1} e^2/(2\epsilon m^*)} D^{-7/2}.$$

Under the assumptions outlined in the last section, this leads to a displacement of the electron gas by a distance

$$\begin{aligned}\Delta z &= -0\cdot 0\,1253_2 \hbar \sqrt{N_s^{-1} e^2/(2\epsilon m^*)} D^{-7/2} K^{-1} \\ &= -0\cdot 0\,1253_2 \hbar \sqrt{N_s^{-1} e^2/(2\epsilon m^*)} D^{-7/2} \frac{\epsilon}{n_0 e^2}.\end{aligned}$$

This can be written in dimensionless units as

$$\Delta z^* = -2.\,1431 \times 10^{-2} r_s^{*9/2} L^{*-1/2} D^{*-7/2},$$

where starred lengths are measured in units of the effective Bohr radius a_B^* defined in equation (2). Clearly this displacement effect is maximised by choosing shallow parabolic wells (high r_s^*, weak 'spring constant') with only light filling (small L^*, narrow egases), as close as possible to one another without permitting significant tunneling (small D^*). For example $L^* = 1,\ r_s^* = 5,\ D^* = 5$ gives

$$\Delta z^* = -2.\,1431 \times 10^{-2} 5^{\frac{9}{2}} 1^{-1/2} 5^{-7/2} = -0.\,107.$$

That is, creation or destruction of one egas will cause a movement of the other egas by about $0\cdot 1 a_B^*$. Since $a_B^* \approx 10\,\mathrm{nm}$ in the GaAlAs system, we can expect a movement of about 1 nm. It is doubtful if this could be measured accurately by capacitive means as in Fig. 5, but it should be well within the ambit of a tunneling detection scheme as in Fig. 6.

As another variant, one can in principle observe the vdW interaction between a parabolic-well egas and a thick metal gate. For a parabolic well in the 2D limit analysed above, a simple hydrodynamic analysis to be presented elsewhere (Dobson 2000) shows that this force is about four times stronger than that between two similar 2D egases separated by the same distance D. The displacement of a parabolic-well egas due to switching off such a force should therefore be detectable either by tunneling current or capacitive measurements as described above. One cannot, of course, create and destroy the metal gate as proposed for egas #1 in the schemes outlined above. Instead, one can reduce the vdW force by applying a static magnetic field B aligned perpendicular to the egas plane. This restricts the electronic motions in the plane of the egas, and since these motions dominate the vdW interaction, there is a substantial effect of the B field on the vdW interaction. Estimates based on hydrodynamics (Dobson 2000) suggest that a field of 5 T will reduce the gate-induced vdW force on a GaAlAs parabolic-well egas with $r_s = 5$ to less than half of its $B = 0$ value. This effect is of interest in itself, and would be much harder to observe between two regular metals: the lower Fermi velocity of AlGaAs conduction electrons makes the effect more easily observable at moderate B fields.

A further variant of the above detection schemes for the small displacement of the parabolic-well egas could involve laser interferometric measurements. This would necessitate finding a laser frequency at which there is at least partial reflection of the beam off the egas, and transparency of the underlying semiconductor.

5. Summary

Two new classes of experiment have been considered, both involving a parabolic quantum well in the AlGaAs system.

In the first type of experiment, one attempts to obtain a sharper Kohn-mode resonance by suppressing the electron–phonon interaction using a phononic bandgap (acoustic filter) setup, and possibly also by growing a slightly non-parabolic well in order to compensate the effects of spatially inhomogeneous electron effective mass $m^*(z)$ and dielectric constant $\epsilon(z)$. More quantitative work is still required, to see whether other broadening influences (such as imperfect epitaxy, defects etc.) can be reduced sufficiently for the proposed measures to have a significant effect. Nevertheless, the issues involved are clearly of some interest.

In the second type of experiment, one uses the properties of parabolic confinement to create a type of force microscope. In this scheme the weak van der Waals force is measured via the movement of the parabolic-well electron gas in the confinement direction when the vdW force is turned on and off. This switching of the vdW force is achieved either by creating and destroying a second quantum-well egas, or by switching a magnetic field of order a few Tesla. The movement of the egas is detected either by capacitance measurements or by observing the tunneling current to a nearby non-parabolic quantum-well egas. The estimates provided here suggest that these effects should be observable. If feasible, this experiment should be of fundamental interest, as direct observation of magnetic field effects on van der Waals forces has not so far been attempted, to the authors' knowledge.

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