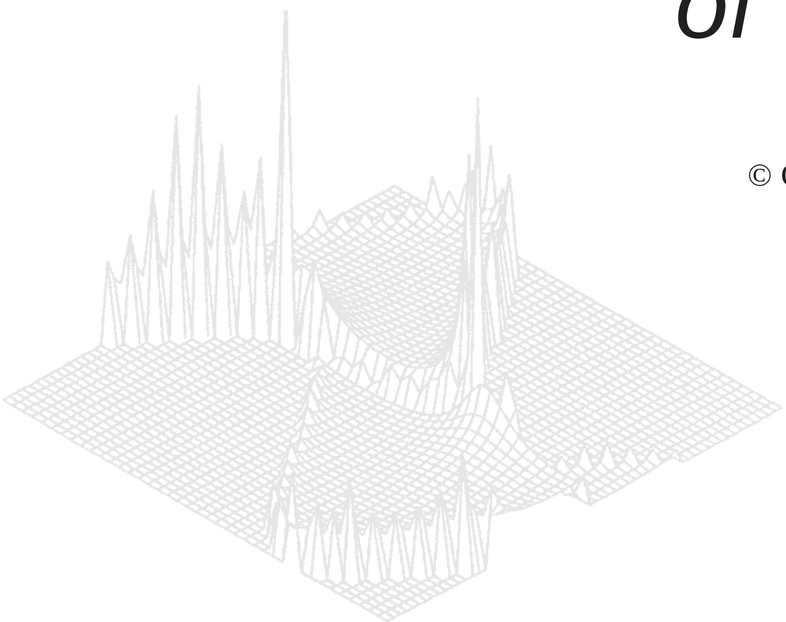
CSIRO PUBLISHING

Australian Journal of Physics

Volume 53, 2000



A journal for the publication of
original research in all branches of physics

www.publish.csiro.au/journals/ajp

All enquiries and manuscripts should be directed to
Australian Journal of Physics
CSIRO PUBLISHING
PO Box 1139 (150 Oxford St)
Collingwood
Vic. 3066
Australia
Telephone: 61 3 9662 7626
Facsimile: 61 3 9662 7611
Email: peter.robertson@publish.csiro.au

Published by **CSIRO** PUBLISHING
for CSIRO Australia and
the Australian Academy of Science

CSIRO AUSTRALIA

Academy of Science

Aust. J. Phys., 2000, **53**, 107–18

Theory of Frictional Drag in Coupled Quantum Wells: Beyond Weak Coupling*

Ben Yu-Kuang Hu

Mikroelektronik Centret, Danmarks Tekniske Universitet,
Lyngby, DK-2800 Denmark.
Permanent address: Department of Physics, 250 Buchtel Commons,
University of Akron, Akron, OH 44325–4001, USA.

Abstract

I give a brief review of the weak coupling theory of frictional drag of the coupled quantum well. I then present a theory of frictional drag based on the Kubo formalism that goes beyond weak coupling. Using the T-matrix approximation, I consider the Maki–Thompson contribution to the transconductivity and obtain a formal result for strong-coupling frictional drag in clean Fermi liquid systems. I discuss how the strong interlayer coupling could affect the temperature dependence of the drag transresistivity.

1. Introduction

The advent of fabrication and lithographic techniques in semiconductors has led to the development of an entire new branch of solid-state physics—the physics of electrons on the mesoscopic scale. Electrons which are confined in one or more spatial directions exhibit unique properties not found in bulk samples. Many of these properties can be traced to the restriction in phase-space and the increased influence of interactions which result from the reduced dimensionality.

The archetypical reduced-dimensionality electronic device is the two-dimensional electron gas (2DEG), which has been studied for several decades. The first two-dimensional electron gases with adjustable density were created at the interface between silicon and SiO_2 in MOSFETs (metal oxide semiconductor field effect transistors). With the development of molecular beam epitaxy, III–V materials such as GaAs were used to create 2DEGs. Molecular beam epitaxy (MBE) is currently the method of choice for producing the high quality devices which are used to study the properties of 2DEGs.

In addition to making single 2DEGs, it is possible using MBE to fabricate two 2DEGs very close to each other (i.e. within nanometres). Recently, experimentalists have succeeded in contacting each of the 2DEGs separately, which yields a host of opportunities to study 2DEGs in novel ways not possible with just single layer electron gases. For example, if the barrier between the 2DEG layers is thin enough, hopping from one layer to the other is possible. Then, tunneling spectroscopy can be done on this coupled system, which provides information on the spectral functions of the electron states (Murphy *et al.* 1995).

* Refereed paper based on a talk presented to the Workshop on Nanostructures and Quantum Confinements, held at the Australian National University, Canberra, in December 1998.

 10.1071/PH99050 0004-9506/00/010107$05.00

For thicker barriers the hopping is suppressed, but the separated layers still can affect each other if the Coulomb or other inter-layer interactions are significant. In particular, if electrons in one layer drift relative to the other, the inter-layer interactions will cause a net momentum transfer between the layers. This results in a friction-like force which opposes the relative motion of the electrons in the separated layers; i.e. the electrons on one layer appear to drag those in the other. The magnitude of this frictional drag force provides important information about the 2DEGs and the effective interlayer forces.

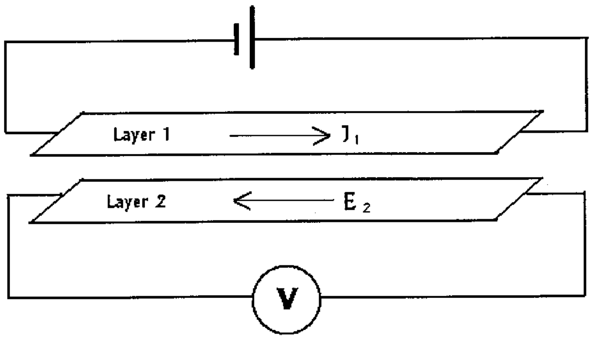


Fig. 1. Schematic diagram of a frictional drag experiment in a coupled 2DEG. A current $\mathbf{J}_1$ is passed through layer 1, and the interlayer interaction induces a transfer of momentum to the carriers in layer 2, dragging these carriers along. In the steady state, charges in layer 2 pile up on one side of the sample, producing an electric field $\mathbf{E}_2$ which negates the drag force. This electric field can be measured experimentally.

Experimental activity in this area is vigorous (Gramila *et al.* 1991; Sivan *et al.* 1992; Rubel *et al.* 1995, 1996*a*, 1996*b*, 1997; Hill *et al.* 1996, 1997; Patel *et al.* 1997; Lilly *et al.* 1998; Feng *et al.* 1998; Noh *et al.* 1998), and many observations are still not fully understood. A typical frictional drag experiment set-up is shown schematically in Fig. 1. In these experiments, current is driven through one layer, denoted in this paper by layer 1, and the voltage response caused by the interlayer coupling is measured in layer 2. Measurement of the current density in layer 1, $\mathbf{J}_1$, and the electric field response in layer 2, $\mathbf{E}_2$, in the linear-response regime yields the transresistivity tensor,

$$\overleftrightarrow{\rho}_{21} \cdot \mathbf{J}_1 = \mathbf{E}_2 \, . \tag{1}$$

In an isotropic system without a magnetic field, $\mathbf{E}_2$ and $\mathbf{J}_1$ are parallel by symmetry and the transresistivity is given by a scalar ρ_{21}. This holds true in many experimental systems such as double electron layers in GaAs quantum wells at $B = 0$. For simplicity, I assume in this paper that ρ_{21} is a scalar; the generalisation to non-isotropic systems is straightforward.

In many of the earlier experiments (e.g. Gramila *et al.* 1991; Sivan *et al.* 1992) the coupling between the electrons in the different layers was weak; that is, to describe these experiments, it was sufficient to employ theories which treat the interlayer coupling as a weak perturbation and to use a lowest non-vanishing order expansion in the perturbation parameter. Recently, however, one particular experiment (Lilly *et al.* 1998) reported results which are inconsistent with the weak coupling theory. The obvious conclusion from this is that the weak coupling theory is insufficient, and we need to consider the effects which go beyond weak coupling. The purpose of this paper is to describe the formalism of drag in the strong interlayer coupling regime, and to briefly describe possible experimental consequences of this theory.

The paper is organised as follows. In Section 2, I describe the Hamiltonian and the Kubo formalisms which are used to calculate the transresistivity. In Section 3 I briefly review the theory of drag in the weak-coupling regime. The formalism for the strong-coupling theory is described in Section 4. A discussion of the results and possible experimental consequences is given in Section 5.

2. The Hamiltonian and Kubo Formalisms

The model Hamiltonian I utilise for the system is

$$\begin{aligned} H = & \sum_{i=1,2} \sum_{\mathbf{k}\sigma} \epsilon_i(\mathbf{k})\, \hat{c}^\dagger_{i,\mathbf{k}\sigma}\, \hat{c}_{i,\mathbf{k}\sigma} \\ & + \sum_{\mathbf{q}} \hat{\rho}_1(\mathbf{q})\, \hat{\rho}_2(-\mathbf{q})\, U(\mathbf{q}) + \hat{H}_{\rm intra} + \hat{H}_{\rm imp}\,, \end{aligned} \tag{2}$$

where $\hat{c}_i^{(\dagger)}$ are the field operators of layer $i = 1,2$ (drive and drag layers respectively), $\mathbf{k}$ is the momentum, σ is the spin index, $\hat{\rho}_i(\mathbf{q}) = \sum_{\mathbf{k}\sigma} \hat{c}^\dagger_{i,\mathbf{k}\sigma}\, \hat{c}_{i,\mathbf{k}+\mathbf{q}\sigma}$ is the density operator and U is the Fourier transform of the interlayer interaction. The first term describes the two ideal noninteracting uncoupled 2DEGs, the second term describes the coupling between the electrons in different layers, $\hat{H}_{\rm intra}$ describes the intralayer interactions and $\hat{H}_{\rm imp}$ describes the effect of impurities and disorder in the system, which is normally modelled by uncorrelated static disorder potentials.

The most rigorous theoretical approach to calculating linear-response at non-zero temperatures is the Kubo formalism using Matsubara Green functions. In the case of drag in coupled 2DEGs, this formalism gives a formal expression for the trans *conductivity* $\sigma_{21} = J_2/E_1$ in terms of the current–current correlation function (Mahan 1990; Kamanev and Oreg 1995; Flensberg *et al.* 1995):

$$\sigma_{21}(\omega) = \frac{i}{\omega}\left[\lim_{i\Omega_n \to \omega + i0^+} \Pi_{xx}(i\Omega_n)\right], \tag{3}$$

$$\Pi_{xx}(i\Omega_n) = -\frac{1}{\mathcal{A}} \int_0^\beta d\tau \;\; \exp(i\Omega_n \tau)\, \langle \hat{j}_{2,x}(-i\tau)\hat{j}_{1,x}(0)\rangle\,, \tag{4}$$

where $\hat{j}_i(t) = e_i/m_i \sum_{\mathbf{k},\sigma} k_x\, \hat{c}^\dagger_{i,\mathbf{k}\sigma}(t)\, \hat{c}_{i,\mathbf{k}\sigma}(t)$ are the Heisenberg picture current operators in layer i, $\beta = T^{-1}$, Ω_n is a Matsubara boson frequency, and $\langle\cdots\rangle$ denotes thermal averaging. (Here e_i, m_i and $\mathcal{A}$ are the charge, effective mass and area of the ith layer. Note that there is no diamagnetic term because the current operators are in different layers. Also, $\hbar, k_B = 1$ in this paper.)

The transconductivity is related to the transresistivity by a simple matrix inversion,

$$\rho_{21} = -\frac{\sigma_{21}}{\sigma_{11}\sigma_{22} - \sigma_{21}\sigma_{12}} \approx -\frac{\sigma_{21}}{\sigma_{11}\sigma_{22}}\,. \tag{5}$$

The approximate equality comes from $|\sigma_{21}|, |\sigma_{12}| \ll \sigma_{11}, \sigma_{22}$ in most cases.

The current–current correlation in equation (4) can be evaluated by standard diagramatic techniques. When the interlayer coupling is weak, we can calculate σ_{21} by expanding the correlation function in powers of the interlayer coupling and evaluating the lowest non-vanishing order term, as I describe in the next section.

3. The Weak-coupling Regime

The vast majority of previous formal theoretical treatments of this system assume that the interaction between the carriers in the two layers is weak (Tso *et al.* 1992; Jauho and Smith 1993; Zheng and MacDonald 1993; Kamanev and Oreg 1995; Flensberg *et al.* 1995; Flensberg and Hu 1995; Bønsager *et al.* 1997), and hence the Born approximation is adequate in describing the physics. The weak-coupling regime holds for most of the earlier experiments done on drag. The transresistivities observed in these experiments appear to extrapolate quadratically towards zero as the temperature is lowered into the milli-Kelvin regime. This observation is consistent with the weak-coupling regime for Fermi liquids, as shown below.

In the weak-coupling regime, the static transconductivity to first order in the interlayer interaction is zero, and the lowest nonvanishing order is given in an isotropic system by the second-order term* (Flensberg *et al.* 1995*a*; Kamanev and Oreg 1995; Bønsager *et al.* 1997)

$$\sigma_{21} = \frac{e_1 e_2}{8T}\int \frac{d\mathbf{q}}{(2\pi)^2}\int_0^\infty \frac{d\omega}{2\pi}\,\frac{|U_{21}(\mathbf{q},\omega)|^2}{\sinh^2(\omega/2T)}\boldsymbol{\Delta}_1(\mathbf{q},\omega)\cdot\boldsymbol{\Delta}_2(\mathbf{q},\omega)\,, \tag{6}$$

where $U_{21}(\mathbf{q},\omega)$ is the Fourier transform of the effective interlayer interaction. The $\boldsymbol{\Delta}$ are given by the analytic continuation of the correlation function

$$\boldsymbol{\Delta}_i(\mathbf{q},\omega) = -\mathcal{A}^{-1}\lim_{\substack{i\omega_n\to\omega+i0^+\\ i\omega_n{}'\to\omega-i0^+}}\int_0^\beta d\tau_1\int_0^\beta d\tau_2\;\langle T_\tau\,\hat{\mathbf{j}}(0)\,\hat\rho(\mathbf{q},\tau_1)\,\hat\rho(-\mathbf{q},\tau_2)\rangle$$
$$\times\exp(i\omega_n\tau_1)\,\exp(i\omega_n{}'\tau_2)\rangle\,, \tag{7}$$

where T_τ is the imaginary-time ordering operator, and $\omega_n, \omega_n{}'$ are Matsubara boson frequencies.

It can be shown (Kamanev and Oreg 1995; Hu 1996) that $\boldsymbol{\Delta}(\mathbf{q},\omega)$ is related to the non-linear direct current density response. If a moving external potential $\phi(\mathbf{r},t) = \Phi_0 e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}$ is imposed on the system, then to second order in the potential the DC current density $\mathbf{J}_{\rm dc}$ in the $\mathbf{q}$-direction is given by

$$\frac{\mathbf{J}_{\rm dc}}{\Phi_0^2} = -\Delta(\mathbf{q},\omega)\,. \tag{8}$$

* Note that, to cut down the number of redundant variables, the definition of the $\boldsymbol{\Delta}_i$ in this paper differs slightly from the one given in Flensberg *et al.* (1995) and Bønsager *et al.* (1997).

It is obvious from symmetry that a static potential will not induce a DC current, and hence $\lim_{\omega\to 0} \mathbf{\Delta}(\mathbf{q},\omega) \to 0$. In many cases, the $\Delta(\mathbf{q},\omega)$ is given by the susceptibility of the system (Zheng and MacDonald 1993; Kamanev and Oreg 1995; Flensberg *et al.* 1995*a*), namely

$$\mathbf{\Delta}_i(\mathbf{q},\omega) \approx \frac{2\tau_{\mathrm{tr},i}\mathbf{q}}{m_i}\,\mathrm{Im}[\chi_i(\mathbf{q},\omega)]\,, \tag{9}$$

where τ_{tr} is the transport time and $\chi(\mathbf{q},\omega)$ is the bare polarisability of the 2DEG.

Let us examine equation (6) in the limit where $T \to 0$. Rescaling the ω integral by $x = \omega/2T$, we find

$$\sigma_{21} \sim \int_0^\infty dx\ \frac{1}{\sinh^2(x)}\ H(xT)\,; \tag{10a}$$

$$H(\omega') = \int d\mathbf{q}\ |U_{21}(\mathbf{q},2\omega')|^2\ \mathbf{\Delta}_1(\mathbf{q};2\omega')\cdot\mathbf{\Delta}_2(\mathbf{q};2\omega')\,. \tag{10b}$$

Let us assume that $H(\omega) \sim \omega^\alpha$ as $\omega \to 0$. Substituting this form into equation (10b) gives

$$\sigma_{21}(T\to 0) \sim \begin{cases} T^\alpha & \text{if } \alpha > 1\,; \\ \infty & \text{if } \alpha \le 1\,. \end{cases} \tag{11}$$

When $\alpha \le 1$, clearly perturbation theory breaks down and the above expressions are invalid. When $\alpha > 1$, perturbation theory is valid and equation (11) indicates that in the weak-coupling regime, as $T \to 0$ the transresistivity must go to zero faster than linearly in T. For example, in weakly Coulomb coupled two-dimensional Fermi liquid systems, $H(\omega) \sim \omega^2$, and hence $\rho_{21}(T\to 0) \sim T^2$, in agreement with the early frictional drag experiments (Gramila *et al.* 1991; Sivan *et al.* 1992).

It is clear from equation (11) that when the weak-coupling description holds, the transconductivity (and hence the transresisitivity) must go to zero at least linearly with temperature as the temperature goes to zero. There is, however, an experiment where the observed transresistivity seems to violates this weak-coupling condition.

4. The Strong-coupling Regime

In a recently reported experiment (Lilly *et al.* 1998), there is evidence that under certain conditions the transresistivity asymptotically approaches a non-zero value as $T \to 0$. This surprising result is still not completely understood. The experiment was done on very clean samples at very high magnetic fields. Drag was measured when the B-field gave a filling fraction in the lowest spin-split Landau level of $\nu = \frac{1}{2}$. At this filling fraction, the flux quanta tend to bind to the electrons, and since there are two flux quanta per electron, the composite particle plus magnetic flux behave like fermions with a renormalised mass (for a review of composite fermions see Jain 1997). The theory developed for electrons should in principle be valid in the case of composite fermions, with appropriately modified $\mathbf{\Delta}$ and U. This route has been taken to calculate $\rho_{21}(T)$ at $\nu = \frac{1}{2}$ (Kim *et al.* 1999; Sakhi 1997; Ussishkin and Stern 1997), and it was discovered that the diffusive nature of the effective $\Delta \propto \chi(q,\omega)$ at low q and ω for the composite fermions leads to the transresistivity which goes as $\rho_{21}(T \to 0) \propto T^{\frac{4}{3}}$. There is some evidence in Lilly *et al.* (1998) which points to this behaviour, and this temperature dependence is in accord with weak-coupling theory. The interesting part of the experimental data occurred at $T < 200\,\mathrm{mK}$. The transresistivity appears to saturate at a *non-zero* value as $T \to 0$. It is clear from the discussion in Section 3 that, if these observations are correct, a weak-coupling theory is insufficient.

(4a) Asmalazov–Larkin Contribution in Composite Fermion Drag

The first attempt to go beyond weak coupling was made by Ussishkin and Stern (1998). They specifically studied the composite fermion case. They considered the so-called Asmalazov–Larkin diagrams, which give the largest correction to the lowest-order result for the conditions of the experiments described in Lilly *et al.* (1998). Physically, these diagrams correspond to pairing fluctuations which occur above the predicted (but still not observed) critical temperature T_c at which a phase transition to a paired state occurs (Bonesteel *et al.* 1996). They concluded that the transresistivity would diverge as

$$\rho_{21}(T) \approx 0\cdot 03\rho_{xx}^2 \frac{e^2}{h}\frac{T_c}{T-T_c} + \rho_{21}^{(0)}\,, \tag{12}$$

where $\rho_{21}^{(0)}$ is the weak-coupling transresistivity.

The Azmalazov–Larkin diagrams were first considered in the context of superconducting fluctuations above the transition temperature. It was found that when the system was dirty, the Azmalazov–Larkin diagrams gave the dominant contribution (see e.g. Tinkham 1996, and references therein). As pair-breaking mechanisms are significant in the $\nu = \frac{1}{2}$ drag problem (Bonesteel *et al.* 1996), the system behaves as if it were ‘dirty’ and hence the Azmalazov–Larkin diagrams dominate.

(4b) Maki–Thompson Contribution

In clean superconducting systems, however, Maki (1968*a*, 1968*b*) and Thompson (1970) found that a different set of terms gave a bigger fluctuation contribution to the conductance when $T > T_c$. If correlations were to develop in clean coupled quantum wells at zero magnetic field, then the Maki–Thompson contributions would be dominant. One particular system of interest is a coupled electron–hole system. It has been postulated for some time that, at low enough temperatures, an exciton-condensate state develops (see e.g. Zhu *et al.* 1995, and references therein). The drag signature of this exciton-condensate below the transition temperature has been studied theoretically (Vignale and MacDonald 1996). However, the effect of pairing fluctuations on the behaviour of the transresistivity above the transition temperature has not yet been examined.

The physical interpretation of the Maki–Thompson contribution was described by Craven *et al.* (1973) as the ‘effect of ephemeral Cooper pairs in the conductivity of normal electrons’. In the case of drag transconductivity, the Feynman diagrams are shown in Fig. 2. The boxes in the middle of the figures denote the T-matrices, which describe the effect of the ephemeral pairing between the carriers in the different layers, before the pairs break up into quasiparticles. The key physics lies in the T-matrix, as it will describe the pairing which ultimately leads to a paired state condensate at low enough temperatures.

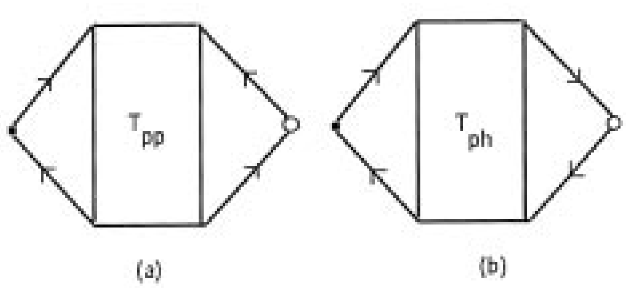


Fig. 2. Feynman diagrams corresponding to the Maki—Thompson contribution to the transconductivity: (a) particle–particle and (b) particle–hole scattering channels. The solid square and open circle indicate the current vertices in the first and second layers respectively.

In Fig. 2, T_{pp} and T_{ph} correspond to the particle–particle (pp) and particle–hole (ph) channels respectively. These T-matrices can be evaluated using various approximations, such as the Bethe–Salpeter equation. In this approximation scheme, if the interlayer interaction is assumed to be static, then T_{pp} and T_{ph} depend only on the sum of the energies of the vertices. In the following, I assume that the effective mass model is accurate at the Fermi surface, and hence the velocity is $\mathbf{v}_i = \mathbf{k}_i/m_i$. Furthermore, the interlayer interactions $U(q)$ are assumed to be static.

Following the standard diagramatic rules (Mahan 1990), the current–current correlation function given in equation (4) has the form

$$\Pi_{xx}(i\Omega_n) = -\frac{4}{\mathcal{A}^2}\sum_{\mathbf{k_1 k_2}} v_{1,x}(\mathbf{k}_1)\, v_{2,x}(\mathbf{k}_2)\; F(\mathbf{k}_1,\mathbf{k}_2;i\Omega_n), \tag{13}$$

$$\begin{aligned} F(\mathbf{k}_1,\mathbf{k}_2;i\Omega_n) = {} & \beta^{-1}\sum_{ik_{i,n}} G_1(\mathbf{k_i}, ik_{1,n}+i\Omega_n)G_1(\mathbf{k_i}, ik_{i,n}) \\ & \times \beta^{-1}\sum_{ik_{2,n}} G_2(\mathbf{k_2}, ik_{2,n}+i\Omega_n)G_2(\mathbf{k_2}, ik_{2,n}) \\ & \times \Big[\langle \mathbf{k}_{pp}|T_{pp}(\mathbf{P}_{pp}; ik_{1,n}+ik_{2,n}+i\Omega_n)|\mathbf{k}_{pp}\rangle \\ & + \langle \mathbf{k}_{ph}|T_{ph}(\mathbf{P}_{ph}; ik_{1,n}-ik_{2,n})|\mathbf{k}_{ph}\rangle\Big]\,. \end{aligned} \tag{14}$$

Here $\mathbf{P}$ and $\mathbf{k}$ are the centre-of-mass and relative coordinates defined by

$$\mathbf{P}_{pp} = \mathbf{k}_1 + \mathbf{k_2}\,, \tag{15a}$$

$$\mathbf{P}_{ph} = \mathbf{k}_1 - \mathbf{k_2}\,, \tag{15b}$$

$$\mathbf{k}_{pp} = x_2\mathbf{k}_1 - x_1\mathbf{k}_2\,, \tag{15c}$$

$$\mathbf{k}_{ph} = x_2\mathbf{k}_1 + x_1\mathbf{k}_2\,, \tag{15d}$$

and $x_i = m_i/(m_1+m_2)$ is the ratio of the mass of carrier i to the sum of the carrier masses. In obtaining equation (14), we have assumed for simplicity that the impurity scattering is δ-function-like and hence there are no vertex corrections in the current vertices (Mahan 1990). The generalisation to non-δ-function scatterers is not difficult.

(4c) Evaluation of F

In order to calculate the transconductivity, F has to be evaluated. It can be written as a sum of the particle–particle and particle–hole contributions,

$$F(\mathbf{k}_1,\mathbf{k}_2;i\Omega_n) = F_{pp}(\mathbf{k}_1,\mathbf{k}_2;i\Omega_n) + F_{ph}(\mathbf{k}_1,\mathbf{k}_2;i\Omega_n)\,, \tag{16}$$

where

$$F_{pp}(\mathbf{k}_1,\mathbf{k}_2;i\Omega_n) = \beta^{-1}\sum_{i\omega_n}\int_0^\beta d\tau\;\, S_1(\mathbf{k}_1,i\Omega_n,\tau)\,S_2(\mathbf{k}_2,i\Omega_n,\tau) \times \langle\mathbf{k}_{pp}|T_{pp}(\mathbf{P}_{pp};i\omega_n)|\mathbf{k}_{pp}\rangle\, e^{i(\Omega_n-\omega_n)\tau}\,, \tag{17a}$$

$$F_{ph}(\mathbf{k}_1,\mathbf{k}_2;i\Omega_n) = \beta^{-1}\sum_{i\omega_n}\int_0^\beta d\tau\;\, S_1(\mathbf{k}_1,i\Omega_n,\tau)\,S_2(\mathbf{k}_2,i\Omega_n,-\tau) \times \langle\mathbf{k}_{ph}|T_{ph}(\mathbf{P}_{ph};i\omega_n)|\mathbf{k}_{ph}\rangle\, e^{-i\omega_n\tau}\,. \tag{17b}$$

Here we have

$$S_i(\mathbf{k},i\Omega_n,\tau) \equiv \beta^{-1}\sum_{ik_n} G_i(\mathbf{k},ik_n+i\Omega_n)G_i(\mathbf{k},ik_n)\exp(ik_n\tau)\,, \tag{18}$$

where G_i is the Green function of layer i, k_n are Matsubara fermion frequencies and $-\beta \le \tau \le \beta$.

By Cauchy's theorem, the summation in equation (18) can be turned into an integral, yielding

$$S_j(i\Omega_n,\tau) = \mathrm{sgn}(\tau)\int_{-\infty}^{\infty}\frac{d\omega_j{}'}{2\pi}\; n_F(\mathrm{sgn}(\tau)\,\omega_j{}')\;\, A_j(\mathbf{k}_j,\omega_j{}') \times \left[G_j(\mathbf{k}_j,\omega_j{}'+i\Omega_n) + e^{-i\Omega_n\tau}G_j(\mathbf{k}_j,\omega_j{}'-i\Omega_n)\right]\;\exp(\omega_i{}'\tau)\,. \tag{19}$$

Using equation (19) in (17a) and (17b), one obtains a rather daunting expression for the analytically continued $F_{pp}(\mathbf{k}_1, \mathbf{k}_2; i\Omega_n \to \omega + i0^+)$ and $F_{ph}(\mathbf{k}_1, \mathbf{k}_2; i\Omega_n \to \omega + i0^+)$ in terms of double integrals over frequencies containing Fermi, Bose, Green and spectral functions, and interaction terms.

The result simplifies if one takes the static limit $\omega \to 0$ and assumes that the 2DEGs are clean. In the static limit, the expressions for F_{pp} and F_{ph} contain integrals over frequencies ω' involving terms $A_j(\mathbf{k}_j, \omega')G_i(\mathbf{k}_j, \omega' + \omega \pm i0^+)$. In clean samples these terms approach (Mahan 1990)

$$2 \lim_{\omega \to 0} A_j(\mathbf{k}_j, \omega')G_j(\mathbf{k}_j, \omega' + \omega \pm i0^+) \approx \mp i A_j(\mathbf{k}_j, \omega')A_j(\mathbf{k}_j, \omega')$$

$$\approx \mp 4i\pi\tau_j \delta(\xi_{j,\mathbf{k_j}} - \omega') , \tag{20}$$

where $\xi_{j,\mathbf{k}_j}$ is the energy of state $\mathbf{k}_j$ with respect to the chemical potential of layer j.

With these simplifying assumptions, after some algebra one obtains

$$\begin{aligned}\lim_{\omega \to 0} \mathrm{Im}[F(\mathbf{k}_1, \mathbf{k}_2; \omega + i0^+)] = \frac{\beta\omega}{2} &\int_{-\infty}^{\infty} \frac{d\omega_1{}'}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega_2{}'}{2\pi} \\ &\times A_1^2(\mathbf{k}_1, \omega_1{}')\, A_2^2(\mathbf{k}_2, \omega_2{}') \Big\{ n_F(\omega_1{}')\, n_F(\pm\omega_2{}') \\ &\times n_B(-\omega_1{}' - \omega_2{}') \\ &\times \mathrm{Im}[\langle \mathbf{k}_{pp} | T_{pp}(\mathbf{P}_{pp}; \omega_1{}' + \omega_2{}') | \mathbf{k}_{pp} \rangle] \\ &+ n_F(\omega_1{}')\, n_F(-\omega_2{}') \\ &\times n_B(-\omega_1{}' + \omega_2{}') \\ &\times \mathrm{Im}[\langle \mathbf{k}_{ph} | T_{ph}(\mathbf{P}_{ph}; \omega_1{}' - \omega_2{}') | \mathbf{k}_{ph} \rangle] \Big\} .\end{aligned} \tag{21}$$

The generalised optical theorem (Kadanoff and Baym 1962) can be used to rewrite $\mathrm{Im}[T_{pp}]$ and $\mathrm{Im}[T_{ph}]$ in terms of $\mathbf{q}$-integrals over $|T_{pp}|^2$ and $|T_{ph}|^2$ terms. Doing so gives an expression for F which, when substituted into equation (13), yields the formal expression for the transconductivity:

$$\begin{aligned}\overleftrightarrow{\sigma}_{21} = &-\frac{8\pi\tau_1\tau_2 e_1 e_2}{T} \int \frac{d\mathbf{k}_1}{(2\pi)^2} \int \frac{d\mathbf{k}_2}{(2\pi)^2} \int \frac{d\mathbf{q}}{(2\pi)^2} \\ &\times v_{1,x}(\mathbf{k}_1)\, v_{2,x}(\mathbf{k}_2) \Big\{ n_F(\xi_{\mathbf{k_1}})\, n_F(\xi_{\mathbf{k_2}})\, n_F(-\xi_{\mathbf{k_1+q}})\, n_F(-\xi_{\mathbf{k_2-q}}) \\ &\times |\langle \mathbf{k}_{pp} | T_{pp}(\mathbf{P}_{pp}; \xi_{\mathbf{k_1}} + \xi_{\mathbf{k_2}}) | \mathbf{k}_{pp} + \mathbf{q} \rangle|^2\end{aligned}$$

$$\times\, \delta\big(\xi_{\mathbf{k_1}} + \xi_{\mathbf{k_2}} - \xi_{\mathbf{k_1+q}} - \xi_{\mathbf{k_2-q}}\big)$$

$$- \, n_F(\xi_{\mathbf{k_1}})\, n_F(-\xi_{\mathbf{k_2}})\, n_F(-\xi_{\mathbf{k_1+q}})\, n_F(\xi_{\mathbf{k_2+q}})$$

$$\times \quad |\langle \mathbf{k}_{ph}| T_{ph}(\mathbf{P}_{ph}, \mathbf{k}_{ph}; \xi_{\mathbf{k_1}} - \xi_{\mathbf{k_2}}) |\mathbf{k}_{ph} + \mathbf{q}\rangle|^2$$

$$\times\, \delta\big(\xi_{\mathbf{k_1}} - \xi_{\mathbf{k_2}} - \{\xi_{\mathbf{k_1+q}} - \xi_{\mathbf{k_2+q}}\}\big)\Big\}\,. \qquad (22)$$

5. Discussion

The result (22) can be interpreted by comparing it to the expressions for the transconductivity derived from the semiclassical Boltzmann equation (Jauho and Smith 1993; Flensberg and Hu 1995). The integral over $\mathbf{q}$ corresponds to the sum of all possible momentum transfers between carriers in layer 1 and 2. The term containing $|T_{pp}|^2$ on the right-hand side of equation (22) can be understood as the contribution of momentum exchange from a particle in state $\mathbf{k}_1$ scattering off a particle in state $\mathbf{k}_2$, and the $|T_{ph}|^2$ term can be interpreted as the contribution of a particle in state $\mathbf{k}_1$ scattering off a hole in state $\mathbf{k}_2$. In the limit where the interlayer coupling is weak, the T-matrices approach the Born approximation result,

$$\langle \mathbf{k}_{pp}| T_{pp}(\mathbf{P}_{pp}; \omega + i0^+) |\mathbf{k}_{pp} + \mathbf{q}\rangle = \langle \mathbf{k}_{ph}| T_{ph}(\mathbf{P}_{ph}; \omega + i0^+) |\mathbf{k}_{ph} + \mathbf{q}\rangle = U(q)\,,$$

and the standard weak-coupling result is regained (Sivan *et al.* 1992; Tso *et al.* 1992; Jauho and Smith 1993; Zheng and MacDonald 1993; Kamanev and Oreg 1995; Flensberg *et al.* 1995; Flensberg and Hu 1995).

Nevertheless, the physical interpretation of the terms in equation (22) allows one to discuss qualitatively the temperature dependence of σ_{21}. As the temperature is decreased, either the T-matrices remain convergent or one of the T-matrices develops a singularity at some critical temperature T_c. I discuss both these cases below.

In the case of repulsive hard-sphere electrons, for example, as the temperature goes to zero the T-matrices do not exhibit any singularities. As mentioned previously, the T-matrices in equation (22) can be interpreted as the scattering amplitudes between carriers in the two different 2DEGs. In the limit where the temperature is zero, the phase space available for scattering goes to zero. Hence, if nothing dramatic occurs with the T-matrices, then the transconductivity must go to zero, as in the case of weak interlayer coupling. This conclusion is consistent with the recent findings of Yang and MacDonald (1999), who state that the transconductance at zero temperature is a topological invariant; in other words, unless there is a phase transition to a state with qualitatively different interlayer correlations, the transconductance at $T = 0$ vanishes. If there is to be something different at $T = 0$, something dramatic must occur.

The T-matrix does not always converge. The best known case of this is the conventional superconducting transition, where the magnitude of $T_{pp}(\mathbf{P}_{pp} = 0)$

diverges as $T \to T_c^+$. In the conventional superconductors, the divergence is caused by an effective attractive interaction mediated by phonons. A sufficiently strong phonon-mediated attractive interaction might occur in adjacent like-charged 2DEGs, as the phonon interaction in coupled two-dimensional quantum wells is fairly large (Bønsager *et al.* 1997). Additionally, in coupled 2DEGs, a possibility exists which is not possible in bulk superconductors. Electrons and holes with individually adjustable Fermi wavevectors can exist in the different layers. When the Fermi wavevectors in the two layers coincide, it is possible that the attractive Coulomb interaction between the electrons and holes could lead to an exciton or a superfluid transition. This possibility has been actively studied theoretically (Zhu *et al.* 1995) although no experimental signature of this transition has ever been observed.

From inspection of equation (22), when either T_{pp} or T_{ph} develops a precursor of a singularity as the temperature $T \to T_c^+$, the magnitude of σ_{21} increases. An increasing σ_{21} with decreasing T contrasts sharply with the standard weak-coupling result that $\sigma(T)$ always decreases as the temperature is lowered (simply because the amount of scattering phase space shrinks as the temperature goes down). An observation of an increase in σ_{21} with decreasing T could be an indication that the coupled 2DEG system is heading towards a transition to an excitonic or superfluid state. In fact, such an observation has already been reported for composite fermion drag (Lilly *et al.* 1998), but the experimental results are still controversial.

To conclude, in this paper I have described the formalism required to calculate the Maki–Thompson contribution in frictional drag of clean Fermi liquids. If the T-matrices remain finite for all temperatures (indicative of the absence of a phase transition), the transconductivity vanishes as the temperature goes to zero. When a T-matrix develops a singularity at a critical tempeture T_c, the transconductivity increases as $T \to T_c^+$, and hence the observation of an increasing transresistivity with decreasing temperature could be an indication that a transition to a new phase is imminent.

Acknowledgments

I thank K. Flensberg and A.-P. Jauho for useful discussions, and the organisers of the 1998 Workshop on Nanostructures at the Australian National University for their kind hospitality.

References

Bonesteel, N. E., McDonald, I. A., and Nayak, C. (1996). *Phys. Rev. Lett.* **77**, 3009.
Bønsager, M. C., Flensberg, K., Hu, B. Y.-K., and Jauho, A.-P. (1997). *Phys. Rev.* B **56**, 10,316.
Craven, R. A., Thomas, G. A., and Parks, R. D. (1973). *Phys. Rev.* B **7**, 157.
Feng, X. G., Zelakiewicz, S., Noh, H., Ragucci, T. J., Gramilla, T. J., Pfeiffer, L. N., and West, K. W. (1998). *Phys. Rev. Lett.* **81**, 3219.
Flensberg, K., and Hu, B. Y.-K. (1995). *Phys. Rev.* B **52**, 14,796.
Flensberg, K., Hu, B. Y.-K., Jauho, A.-P., and Kinaret, J. M. (1995). *Phys. Rev.* B **52**, 14,761.
Gramila, T. J., Eisenstein, J. P., MacDonald, A. H., Pfeiffer, L. N., and West, K. W. (1991). *Phys. Rev. Lett.* **66**, 1216.
Hill, N. P. R., Nicholls, J. T., Linfield, E. H., Pepper, M., Ritchie, D. A., Hamilton, A. R., and Jones, G. A. C. (1996). *J. Phys.* C **8**, L557.

Hill, N. P. R., Nicholls, J. T., Linfield, E. H., Pepper, M., Ritchie, D. A., Jones, G. A. C., Hu, B. Y.-K., and Flensberg, K. (1997). *Phys. Rev. Lett.* **78**, 2204.
Hu, B. Y.-K. (1996). Unpublished.
Jain, J. K. (1997). *In* 'Perspectives in Quantum Hall Effects' (Eds S. Das Sarma and A. Pinczuk), p. 265 (Wiley: New York).
Jauho, A.-P., and Smith, H. (1993). *Phys. Rev.* B **47**, 4420.
Kadanoff, L. P., and Baym, G. (1962). 'Quantum Statistical Mechanics' (Benjamin: New York; reprinted Addison–Wesley, Reading, Mass., 1989).
Kamanev, A., and Oreg, Y. (1995). *Phys. Rev.* B **52**, 7516.
Kim, Y. B., and Millis, A. J. (1999). *Physica* E **4**, 171.
Lilly, M. P., Eisenstein, J. P., Pfeiffer, L. N., and West, K. W. (1998). *Phys. Rev. Lett.* **80**, 1714.
Mahan, G. D., (1990). 'Many-Particle Physics', 2nd edn (Plenum: New York).
Maki, K. (1968*a*). *Prog. Theor. Phys. (Kyoto)* **39**, 897.
Maki, K. (1968*b*). *Prog. Theor. Phys. (Kyoto)* **40**, 193.
Murphy, S. Q., Eisenstein, J. P., Pfeiffer, L. N., and West, K. W. (1995). *Phys. Rev.* B **52**, 14,825.
Noh, H., Zelakiewicz, S., Feng, X. G., Gramila, T. J., Pfeiffer, L. N., and West, K. W. (1998). *Phys. Rev.* B **58**, 12,621.
Patel, N. K., Linfield, E. H., Brown, K. M., Pepper, M., Ritchie, D. A., and Jones, G. A. C. (1997). *Semicond. Sci. Technol.* **12**, 309.
Rubel, H., Linfield, E. H., Ritchie, D. A., Brown, K. M., Pepper, M., and Jones, G. A. C. (1995). *Semicond. Sci. Technol.* **10**, 1229.
Rubel, H., Fischer, A., Dietsche, W., von Klitzing, K., and Eberl, K. (1996*a*). Proc. 23rd Conf. on the Physics of Semiconductors (Eds M. Scheffler and R. Zimmermann), p. 2335 (World Scientific: Singapore).
Rubel, H., Linfield, E. H., Hill, N. P. R., Nicholls, J. T., Ritchie, D. A., Brown, K. M., Pepper, M., and Jones, G. A. C. (1996*b*). *Surf. Sci.* **361–2**, 134.
Rubel, H., Fischer, A., Dietsche, W., von Klitzing, K., and Eberl, K. (1997). *Phys. Rev. Lett.* **78**, 1763.
Sakhi, S. (1997). *Phys. Rev.* B **56**, 4098.
Sivan, U., Solomon, P. M., and Shtrikman, H. (1992). *Phys. Rev. Lett.* **68**, 1196.
Thompson, R. S. (1970). *Phys. Rev.* B **1**, 327.
Tinkham, M. (1996). 'Introduction to Superconductivity', 2nd edn (McGraw–Hill: New York).
Tso, H. C., Vasilopoulos, P., and Peeters, F. M. (1992). *Phys. Rev. Lett.* **68**, 2516.
Ussishkin, I., and Stern, A. (1997). *Phys. Rev.* B **56**, 4013.
Ussishkin, I., and Stern, A. (1998). *Phys. Rev. Lett.* **81**, 3932.
Vignale, G., and MacDonald, A. H. (1996). *Phys. Rev. Lett.* **76**, 2786.
Yang, K., and MacDonald, A. H. (1999). Report No. cond-mat/9904061.
Zheng, L., and MacDonald, A. H. (1993). *Phys. Rev.* B **48**, 8203.
Zhu, X., Littlewood, P. B., Hybertsen, M. S., and Rice, T. M. (1995). *Phys. Rev. Lett.* **74**, 1633.

Manuscript received 25 May, accepted 23 August 1999