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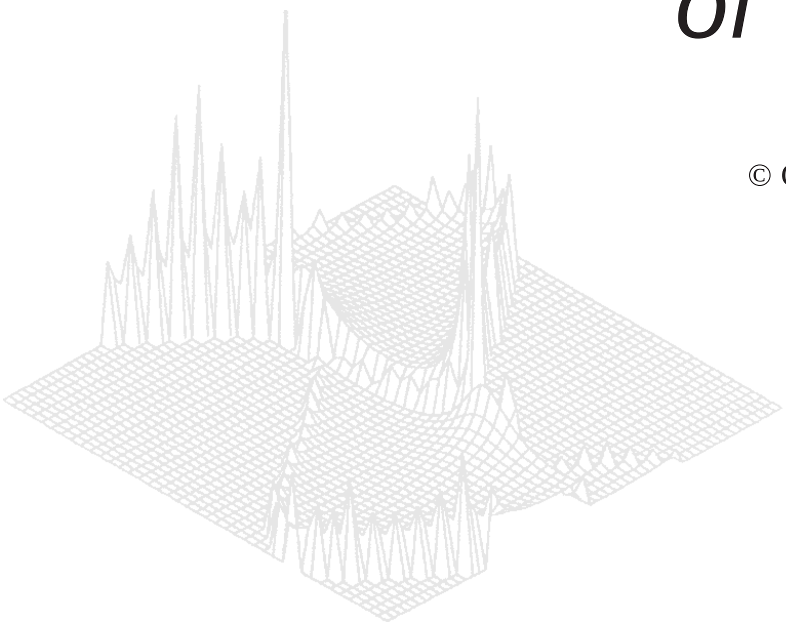

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Collapses and Revivals of the Coherence of a Mesoscopic Superposition of Motional States

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Abstract

A scheme is proposed to observe the collapses and revivals of a mesoscopic superposition of coherent states in the motion of a trapped ion. In the scheme the ion motion in the x-axis is first prepared in a Schrödinger cat state, and then coupled to the motion in the z-axis. With the vibrational excitation exchange between the two modes, the mesoscopic quantum coherence collapses and revives periodically, providing an example for complementarity.

1. Introduction

According to the superposition principle, superpositions of two or more different quantum states form new ones, exhibiting striking effects due to interference between the components in the superposition. The quantum superposition, however, does not appear at the classical level, otherwise the Schrödinger (1935) cat paradox arises. Recently, it has been shown that the environment is responsible for the nonexistence of macroscopic quantum superpositions (Zurek 1981, 1982, 1991). The coupling between the observed system and outside world reservoir results in the decoherence, turning the quantum superposition into a classical mixture. The coherence decay time scale is given by the energy dissipation time divided by a dimensionless number characterising the separation between the two parts. Thus, when the two superposed states are macroscopically separated, the coherence would decay too rapidly to be observed.

Recently, there has been much interest in the realisation of quantum superpositions at the mesoscopic level. Mesoscopic superpositions of two coherent states with different phases, referred to as phase Schrödinger cat-like states, have been observed in both the cavity QED (Brune *et al.* 1996) and the ion trap (Monroe *et al.* 1996). The experiment reported by Brune *et al.* has also explored the decoherence process of the Schrödinger cat states of a cavity field through subsequence two-atom correlation measurements (Davidovich *et al.* 1996).

Generally, the decoherence process is irreversible due to the large size of the reservoir. However, in a recent paper (Raimond *et al.* 1997), an experimental scheme has been proposed to show that the decoherence process may become a reversible process in a well controlled environment. In the scheme the cavity containing a superposition of two coherent fields with different phases is coupled to another cavity, which acts as a single-mode reservoir. Since the energy exchange

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between the two cavities is reversible, the coherence of the cat state exhibits collapses and revivals. This process can be interpreted in terms of complementarity. At the first stage of the coupling process, the phase information of the coherent fields leaks into the single-mode reservoir, resulting in the disappearance of the interference. When the reservoir is vacuum again, the 'which-path' information is lost, corresponding to the reappearance of the interference. In contrast to the case where the 'which-path' information is erased by manipulating the system in some way, as shown in quantum eraser experiments (Scully and Drühl 1982; Scully *et al.* 1991; Gerry 1996; Zheng and Guo 1997), the losses and revivals of the coherence during the two-cavity coupling are dynamical processes, occurring periodically. As pointed out by these authors, the main problem regarding the experiment is how one can realise the reversible coupling between two identical cavities.

In this paper we propose a scheme to observe the reversible decoherence of a mesoscopic superposition of motional states of a trapped ion. In the scheme the ion motion in one direction initially prepared in a cat state is coupled to the motion in another direction. Due to the transfer of vibrational excitations between the two vibrational modes, the mesoscopic superposition of motional states exhibits collapses and revivals. The motional Schrödinger cat states can be generated and probed through the laser-assisted Raman coupling between the internal and external degrees of freedom of the ion. Such a coupling can be achieved by resonant laser–ion interaction (Gerry 1997; Zheng and Guo 1998).

This paper is organised as follows. In Section 2 we propose a scheme to generate and observe a motional Schrödinger cat state in an ion trap. In Section 3 we suggest a scheme to realise the coupling between two motional modes, which results in the collapses and revivals of the coherence of the cat state. A summary is given in Section 4.

2. Generation and Observation of a Motional Schrödinger Cat State

Suppose a two-level ion is trapped in a three-dimensional anisotropic harmonic potential. The ion is first driven by two laser beams, tuned to the ion transition frequency, propagating along the x and y directions respectively. In the resolved sideband limit, where the trapping frequencies are much larger than other characteristic frequencies, the interaction Hamiltonian (in the interaction picture) for such a system is (Vogel and de Matos Filho 1995; de Matos Filho and Vogel 1996)

$$H_i = \tfrac{1}{2}[e^{-\eta_x^2/2}\Omega_x e^{-i\phi_x} f_x(a^+, a) + e^{-\eta_y^2/2}\Omega_y e^{-i\phi_y} f_y(b^+, b)]\, |e\rangle\langle g| + \text{H.c.}\,, \qquad (1)$$

where Ω_j and ϕ_j $(j = x, y)$ are the Rabi frequencies and phases for the laser beams in the x and y directions, $\eta_j = k_0/\sqrt{2M\nu_j}$ are the Lamb–Dicke parameters, with k_0 being the wave-vector, M the mass of the ion, ν_j the trap frequencies in the respective directions, and $|e\rangle$ and $|g\rangle$ are the internal excited and ground states for the two-level ion. The functions $f_x(a^+, a)$ and $f_y(b^+, b)$ are given by

$$f_x(a^+, a) = \sum_{k=0}^{\infty} \frac{(i\eta_x)^{2k}}{(k!)^2} a^{+k} a^k, \quad f_y(b^+, b) = \sum_{k=0}^{\infty} \frac{(i\eta_y)^{2k}}{(k!)^2} b^{+k} b^k\,. \qquad (2)$$

We assume that η_j are small enough so that we can retain terms only to the second order in η_j. Then we have

$$H_i = \tfrac{1}{2}[e^{-\eta_x^2/2}\Omega_x e^{-i\phi_x}(1-\eta_x^2 a^+ a) \\ + e^{-\eta_y^2/2}\Omega_y e^{-i\phi_y}(1-\eta_y^2 b^+ b)]\,|e\rangle\langle g| + \text{H.c.} \tag{3}$$

Choosing the intensities and phases of the laser beams so that $\Omega_x e^{-\eta_x^2/2} = \Omega_y e^{-\eta_y^2/2} = \Omega_0$ and $\phi_x = \pi$, $\phi_y = 0$, we obtain

$$H_i = \tfrac{1}{2}\Omega_0(\eta_x^2 a^+ a - \eta_y^2 b^+ b)(|e\rangle\langle g| + |e\rangle\langle g|)\,. \tag{4}$$

We assume the ion motion in the y direction is initially in the vacuum state. Then the effective Hamiltonian is

$$H_i^e = g a^+ a(|e\rangle\langle g| + |g\rangle\langle e|)\,, \tag{5}$$

where $g = \frac{1}{2}\Omega_0\eta_x^2$. This Hamiltonian describes the Raman coupling model (Knight 1986), and can be rewritten as

$$H_i^e = g a^+ a(|+\rangle\langle +| - |-\rangle\langle -|)\,, \tag{6}$$

where

$$|+\rangle = \sqrt{\tfrac{1}{2}}(|g\rangle + |e\rangle), \qquad |-\rangle = \sqrt{\tfrac{1}{2}}(|g\rangle - |e\rangle)\,. \tag{7}$$

We assume that the ion is initially in the ground state $|g\rangle$, which can be expressed in terms of $|+\rangle$ and $|-\rangle$ as

$$|g\rangle = \sqrt{\tfrac{1}{2}}(|+\rangle + |-\rangle)\,. \tag{8}$$

Suppose that the vibrational motion is initially in the coherent state $|\alpha\rangle$. Then after an interaction time τ we obtain

$$|\psi_1(\tau)\rangle = \sqrt{\tfrac{1}{2}}[|\alpha e^{-ig\tau}\rangle\,|+\rangle + |\alpha e^{ig\tau}\rangle\,|-\rangle]\,. \tag{9}$$

In this way the motional coherent state is phase-shifted by $-g\tau$ or $g\tau$ depending on whether the ion is in the state $|+\rangle$ or $|-\rangle$. We choose the interaction time τ appropriately so that $g\tau = \pi/2$. Thus we have

$$\begin{aligned} |\psi_1(\tau)\rangle &= \sqrt{\tfrac{1}{2}}[|-i\alpha\rangle\,|+\rangle + |i\alpha\rangle\,|-\rangle] \\ &= \tfrac{1}{2}[(|i\alpha\rangle + |-i\alpha\rangle)\,|g\rangle - (|i\alpha\rangle - |-i\alpha\rangle)\,|e\rangle]\,. \end{aligned} \tag{10}$$

We now perform a measurement of the internal state of the ion. The detection of the state $|g\rangle$ or $|e\rangle$ leaves the vibrational motion in the state

$$|\psi\rangle_m = \frac{1}{N_1}(|i\alpha\rangle + e^{i\varphi_1}\,|-i\alpha\rangle)\,, \tag{11}$$

where $N_1 = \sqrt{2[1+\cos\varphi_1 e^{-2|\alpha|^2}]}$, and $\varphi_1 = 0$ or π, corresponding to the detection of $|g\rangle$ or $|e\rangle$. For $|\alpha|^2 \gg 1$, the coherent state $|i\alpha\rangle$ is asymptotically orthogonal to $|-i\alpha\rangle$, and thus $N_1 = \sqrt{\frac{1}{2}}$.

In order to monitor the transition $|g\rangle \to |e\rangle$ we use the V-type electronic level scheme, where the two upper levels $|e\rangle$ and $|r\rangle$ couple to the common ground level $|g\rangle$. The transition $|g\rangle \to |e\rangle$ is electric-dipole forbidden, whereas the auxiliary transition $|g\rangle \to |r\rangle$ is allowed. After the above-mentioned Raman coupling another laser beam on resonance with the transition $|g\rangle \to |r\rangle$ is used to probe fluorescence. The presence of fluorescence corresponds to the detection of the state $|g\rangle$ and absence to $|e\rangle$. In order to avoid the modification of the motional state during the observation of fluorescence, a very small value of the Lamb–Dicke parameter $\eta_{g,r}$ for the transition $|g\rangle \to |r\rangle$ is required (de Matos Filho and Vogel 1996). If we detect the ion in the excited state, we then excite the ion only with the laser in the y direction tuned to the ion transition frequency. In this case the Hamiltonian for this system is

$$H_y = \Omega_y e^{-i\phi_y} |e\rangle\langle g| + \text{H.c.} \tag{12}$$

We choose the interaction time t carefully so that $\Omega_y t = \pi/2$. Then the state $|e\rangle$ flips back to $|g\rangle$. On the other hand, if we detect the ion in the ground state this step is unnecessary.

We now drive the ion, again with the two laser beams propagating along the$x$ and y directions. At this stage we assume there is no coupling between the motional modes. Then the initial state in this cycle is

$$|\psi_2(0)\rangle = \tfrac{1}{2}(|i\alpha\rangle + e^{i\varphi_1}|-i\alpha\rangle)(|+\rangle + |-\rangle)\,. \tag{13}$$

We also choose the ion–laser interaction time in such a way that the vibrational motion will be phase-shifted by -$\pi/2$ or $\pi/2$, conditional on the ion being in the state $|+\rangle$ or $|-\rangle$. This leads to

$$\begin{aligned} |\psi_2(\tau)\rangle &= \tfrac{1}{2}(|\alpha\rangle|+\rangle + |-\alpha\rangle|-\rangle + e^{i\varphi_1}|-\alpha\rangle|+\rangle + e^{i\varphi_1}|\alpha\rangle|-\rangle) \\ &= \tfrac{1}{2}\sqrt{\tfrac{1}{2}}[|g\rangle(1+e^{i\varphi_1})(|\alpha\rangle + |-\alpha\rangle) \\ &\quad + |e\rangle(1-e^{i\varphi_1})(|\alpha\rangle - |-\alpha\rangle)]\,. \end{aligned} \tag{14}$$

Now we again measure the internal state of the ion. Then the probability of finding the ion in the ground state $|g\rangle$ or excited state $|e\rangle$ is

$$P_g^{(2)} = \tfrac{1}{2}(1+\cos\varphi_1)\,, \qquad P_e^{(2)} = \tfrac{1}{2}(1-\cos\varphi_1)\,. \tag{15}$$

The correlation coefficient ξ is defined as $\xi = P_{g,g} - P_{g,e}$, where $P_{g,g}$ and $P_{g,e}$ are the conditional probabilities to detect the ion in states $|g\rangle$ and $|e\rangle$ in the second cycle, respectively, provided it is detected in $|g\rangle$ in the first cycle. From the expression (15), it can be easily seen that $\xi = 1$. Thus, there is a complete correlation between the outcomes of the two measurements. The correlation results from the presence of two paths via which the system can reach a certain state after the second Raman coupling. For example, the state $|g\rangle\,|\alpha\rangle$ can be obtained in two ways:

$$|g\rangle\,|-i\alpha\rangle \to |-\rangle\,|\alpha\rangle \to |g\rangle\,|\alpha\rangle\ , \tag{16}$$

$$|g\rangle\,|i\alpha\rangle \to |+\rangle\,|\alpha\rangle \to |g\rangle\,|\alpha\rangle\ . \tag{17}$$

These two paths are indistinguishable, which leads to the interference.

3. Collapses and Revivals of the Coherence of the Cat State

We now assume there is a coupling between the motional modes in the x and z directions during the intervals between the above-mentioned Raman couplings. Such a coupling can be achieved by the off-resonant excitations of the trapped ion (Wallentowitz and Vogel 1997; Agarwal and Banerji 1997; Steinbach *et al.* 1997). We drive the ion transition $|g\rangle \to |e\rangle$ with two off-resonant lasers of frequencies ω_L and $\omega_L + \Delta$ ($\Delta \ll \omega_L$), propagating along the x and z directions respectively. During the interaction, the ion stays in its electronic ground state. Suppose the relative detunings from the transition frequency ω_0 are small, i.e. $(|\omega_L - \omega_0|/\omega_0) \ll 1$ and $|\omega_L + \Delta - \omega_0|/\omega_0 \ll 1$. We choose the difference of the laser frequencies in such a way that $\Delta = \nu_x - \nu_z$. In the resolved sideband and Lamb–Dicke limits the interaction Hamiltonian is of the form (Wallentowitz and Vogel 1997)

$$H_i' = \lambda[e^{i(\phi_x' - \phi_z')} a c^+ + \mathrm{H.c.}]\,, \tag{18}$$

where c^+ and c are the creation and annihilation operators for the motion in the z direction, and ϕ_x' and ϕ_z' are the phases for the two laser beams. Here λ is given by

$$\lambda = \tfrac{1}{4}\eta_x'\eta_z' e^{-(\eta_x'^2 + \eta_z'^2)/2} \frac{\Omega_x'\Omega_z'}{\omega_0 - \omega_L}\,, \tag{19}$$

where η_j' and Ω_j' ($j = x, z$) are the respective Lamb–Dicke parameters and Rabi frequencies. We choose the phase difference appropriately so that $\phi_x' - \phi_z' = \pi/2$. Then we have

$$H_i' = i\lambda[a c^+ - a^+ c] \tag{20}$$

The Hamiltonian H_i' leads to the exchange of motional excitations between the x and z directions. If the motion in the x direction is initially prepared in the cat state $\sqrt{\frac{1}{2}}(|i\alpha\rangle + e^{i\varphi_1}\,|-i\alpha\rangle)$ and the motion in the z direction in the vacuum state $|0\rangle$, the two motional modes evolve into the entangled coherent state (Sanders 1992*a*, 1992*b*)

$$|\psi(t)\rangle = \sqrt{\tfrac{1}{2}}(|i\alpha(t)\rangle\,|i\beta(t)\rangle + e^{i\varphi_1}\,|-i\alpha(t)\rangle\,|-i\beta(t)\rangle)\,, \tag{21}$$

where

$$\alpha(t) = \alpha_0 \cos(\lambda t)\,, \quad \beta(t) = \alpha_0 \sin(\lambda t) \tag{22}$$

are the amplitudes of the coherent components in the x and z directions during the coupling. Entangled coherent states are also referred to as superpositions of two-mode coherent states (Chai 1992). Due to the correlation between the two modes, such states may exhibit various nonclassical features, such as two-mode squeezing and violation of the Cauchy–Schwartz inequality. After the second laser-assisted Raman coupling between the motion in the x direction and the internal state, the whole state-vector is

$$\begin{aligned} |\psi_2'(\tau)\rangle = \tfrac{1}{2}\sqrt{\tfrac{1}{2}}\{|g\rangle\,&[|\alpha(t)\rangle\,|i\beta(t)\rangle + |-\alpha(t)\rangle\,|i\beta(t)\rangle \\ &+e^{i\varphi_1}\,|-\alpha(t)\rangle\,|-i\beta(t)\rangle + e^{i\varphi_1}\,|\alpha(t)\rangle\,|-i\beta(t)\rangle] \\ &|e\rangle\,[|\alpha(t)\rangle\,|i\beta(t)\rangle - |-\alpha(t)\rangle\,|i\beta(t)\rangle \\ &+e^{i\varphi_1}\,|-\alpha(t)\rangle\,|-i\beta(t)\rangle - e^{i\varphi_1}\,|\alpha(t)\rangle\,|-i\beta(t)\rangle]\}\,. \end{aligned} \tag{23}$$

Thus the correlation coefficient is

$$\xi = \mathrm{Re}\{\langle i\beta(t)|\,-i\beta(t)\rangle\} = e^{-2n\sin^2(\lambda t)}\,. \tag{24}$$

In the case that $\lambda t \ll 1$, ξ can be approximated by

$$\xi \simeq e^{-2n\lambda^2 t^2}\,. \tag{25}$$

The coefficient ξ decays rapidly during the time $0 < t < \sqrt{1/2n\lambda^2}$. The disappearance of the interference is due to the leakage of the phase information of the motion in the x direction into the motion in the z direction. When the two coherent components in the z direction become asymptotically orthogonal the 'which-path' information is completely stored in these two components, resulting in the loss of coherence. At the time $t = \pi/2\lambda$, the motional excitations are completely transferred to the z direction, and thus the second Raman coupling has no effect on the state evolution of the system. At the time $t = \pi/\lambda$, the excitations in the x direction are completely taken back to the x direction. In this case the which-path information is lost and thus the interference reappears. In this way the coherence of the mesoscopic superposition in the x direction collapses and revives with the period of the vibrational excitation exchange between the x and z directions.

4. Summary

In conclusion, here we have made a proposal to show that the mesoscopic coherence of superpositions of two motional states of a trapped ion can exhibit collapses and revivals periodically by coupling the motion in one axis to that in another axis. Since the ion is relatively well isolated one can observe the collapses and revivals of superpositions of two coherent components with high stability in the ion motion. Another advantage of using a trapped ion is that the interaction time of the ion with laser fields can be more easily controlled than that of an atom with a cavity field.

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